On Minor Variations of Latitude at Greenwich

by S. Kawasaki

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Contents

I. Introduction

1. Observations of latitude at Greenwich. After an experience of ten years as an observer of the International Latitude Station at Mizusawa, the author of this paper was ordered by the government to study abroad. The great interest which he had in the observations made with the Cookson floating zenith telescope at the Royal Observatory, Greenwich, caused him to visit there first. This was in the summer

of the year 1932. He was permitted to stay at that observatory studying subjects concerning latitude variation. The greater part of this paper is the result of investigations made or commenced at that time. At this point the author wishes to express his sincere thanks to Sir Frank Dyson, the then Astronomer Royal, Dr. J. Jackson, Mr. W. M. H. Greaves and others on the staff of the Royal Observatory. Through their generosity he was able to make use of valuable observed results and to have access to other necessary data. Particular acknowledgment should be made of the kindness of Mr. H. H. Furner and Mr. H. W. Acton, who were always helpful to him and afforded him every convenience during his stay there.

Formerly, the observations of latitude at Greenwich were made with the Reflex Zenith Tube. Those observations were discontinued because of the insufficient accuracy of the results obtained and the difficulty in the choice of suitable stars. Observations with the floating zenith telescope were then commenced in the year 1911. This telescope was designed by the late Bryan Cookson who used it for two years at Cambridge in connection with Prof. Küstner's method for determining the accurate value of the constant of aberration. He prepared a preliminary discussion based on his materials but died before reaching final conclusions. Further discussions were continued, mainly by Prof. Stratton, and published in the Memoirs of the Royal Astronomical Society, Vol. 60 (1911) part II. Even though the results were not very satisfactory, they suggested that this new telescope was suitable for the determination of latitude variations. Observations made with it at the Royal Observatory verified this supposition. The observations for the years 1911–1927 were set forth by the Astronomer Royal in two volumes,[1] from which the whole material discussed in the present paper has been taken.

To make the following discussions clear, short explanations concerning the floating telescope, the observing hut and the observing programme follow herewith:

2. The Telescope. By courtesy of the Astronomer Royal, the author of this paper was allowed to make observations with the Cookson floating zenith telescope, that he might compare it with the ordinary zenith telescope. The Cookson floating zenith telescope is very simple

1. Observations Made with the Cookson Floating Zenith Telescope in the years 1911–1918. London, 1921. Observations Made with the Cookson Floating Zenith Telescope in the years 1919–1927. London, 1928.

in its construction and use. It is superior in at least two respects to the ordinary zenith telescope, now being used at all International Latitude Stations. It avoids the use of levels, and it adopts the photographic method. As is well known, the small micrometer and the levels are the most essential parts of the ordinary zenith telescope. But they are at the same time its most defective parts. Recently the author investigated the readings of the Talcott levels of latitude observations taken during ten years 1902.0 to 1912.0 at Mizusawa, and found that the bubbles of the levels move constantly even during the bisection of a single star.[1] Both diurnal and annual variations exist in their motions. The position of the telescope and the declination of the latitude star are also closely correlated with these respective motions of the bubbles. It is not yet known how the final results of the latitude observations are affected by these motions, but some anomalies must certainly result from such unsteadiness of the levels. On the other hand, any one who has had experience in observing latitude with the ordinary zenith telescope, or in deducing the results obtained with it, knows how difficult it is to get the true angular value of one revolution of the small micrometer attached to the telescope. Much more difficult it is to find accurately the inequality of the screw. And yet on the angular value and the inequality depend the final results of latitude observations. Any slight error in the micrometer gives rise to a systematic variation in the observed values of latitude. In the floating zenith telescope, however, spilit levels are replaced by a mercury surface, and a photographic plate is attached in place of a small micrometer. Accordingly, the results of observation, taken with this telescope, are quite free from those instrumental errors. Moreover, the use of levels and the small micrometer being thus avoided, the method of latitude observation is much simplified.

The object glass is a triplet of 6.5 inches aperture and 65.4 inches focal length. The tube is of brass and has trunnions resting on V's which are carried on an iron annulus floating on mercury. Full details of the construction have been given by the designer, Mr. Bryan Cookson,[2] and short descriptions are to be found in any of the previously mentioned publications.

3. The Observing hut. The present observing hut is that which was once used at Cambridge by Mr. Cookson, who himself had designed

1. Read before the 12th General Meeting of the Japanese Association for the Advancement of Science. Oct. 18th, 1935. 2. M. N., **61**, 315, 1901.

it, paying special attention to the matter of ventilation. It is built of wood and has two doors but no windows. The wall is double, and the narrow opening at the top of the roof used for making observations has external and internal shutters. Just under this opening, there is a sliding screen, by the pulling of which the photographic plate is exposed. The hut itself is thus carefully designed, but the conditions of the surroundings are rather unfavourable. At the immediate north side of the hut is a sharply inclined hill sloping downwards, and nearby to the south, are large buildings. Fearing that the latitude observations are largely influenced by meteorological conditions, observations at this site were discontinued at the end of 1935. The telescope was then rehoused at a different site under better conditions.[1]

Table I

Distribution of the Groups in Right Ascension

Group	Approx. Limits of R.A.	Group	Approx. Limits of R.A.
	h h		h h
I	0 — 1	IX	14½ —16
II	2½— 3½	X	16 —17
III	3½— 4½	XI	17 —18
IV	7½— 8½	XII	18 —19
V	8½— 9½	XIII	19 —20
VI	11 —12	XIV	20 —21
VII	12 —13	XV	21 —22
VIII	13½—14½	XVI	23 —24

4. Observing programme. The observing programme for the International Latitude Service comprises twelve groups of stars, each group extending over about two hours of right ascension, at any epoch two consecutive groups being observed in cyclic order. But the programme for Greenwich consists of sixteen groups, whose approximate limits of right ascension are represented in Table I. Each of the above groups contains four or five pairs of latitude stars. All stars belonging to the same group are photographed on the same plate. The observations during one year are divided into ten periods. During the summer periods, May 30–June 21 and June 22–July 9, three groups are observed in one night. During the other periods, four groups are observed.

Unlike the observations made at the International Latitude Stations, which are intended to determine the variations of latitude itself, these observations made with the floating telescope at Greenwich were begun primarily to determine the value of the constant of aberration. Accordingly, the groups are observed as near 6^h and 18^h as possible,

1. Transactions of the International Astronomical Union, **5**, 122, 1935. M. N., **96**, 303, 1936.

while at the International Latitude Stations, the observations are carried on near midnight throughout the year.

It should also be remembered that the International Latitude Stations are distributed on the same parallel at 39° 8′ N., and that the latitude of the Royal Observatory is 51°29′ N..

II. Kimura Term

1. Derivation of the Kimura term. When the existence of the z-term, or the Kimura term, as it is generally called, was first ascertained by Prof. Kimura, this term was thought to have the same value at all the International Latitude Stations at any given time, and to vary with the time in a period of one year. But this conception has been changed by further investigations. To-day it is generally believed that this term is of rather local nature, each station having its own Kimura term. Thus, there is a Kimura term in the latitude variation observations at Greenwich which is peculiar to that observatory. It is derived by comparing the results of observations taken there with the corresponding values of the x-component of the motion of the terrestrial pole which are determined by the International Latitude Service. In Vols. II, III, IV and V of the Transactions of the International Astronomical

Table II

Greenwich Kimura Term Derived by Prof. Kimura (unit 0.″001)

year	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1919.0-1927.0	+ 6	+13	+ 4	−12	−26	− 9	+15	+17	0	−12
1923.0-1931.0	− 4	+ 8	+12	− 6	−26	−14	0	− 1	− 6	−15
1931.0-1934.0	−20	−23	−20	−33	−50	−37	+ 7	+ 7	−10	−17

Union, are given the numerical values of this term by Prof. Kimura. They are quoted in Table II. In the publications containing the observations of latitude taken at Greenwich in the years 1911–1927 are also given values for the Kimura term for those years. But all of such numerical values, hitherto obtained, were based on the provisional results of the International Latitude Service.

The definitive discussions of the observations at the International Latitude Stations for the years 1912.0–1931.0 were recently published in two volumes by Profs. Mahnkopf[1] (Vol. VI) and Kimura[2] (Vol. VII).

1. Ergebnisse des Internationalen Breitendienstes von 1912.0 bis 1922.7, Potsdam, 1932.
2. Results of International Latitude Service from 1922.7 to 1931.0, Mizusawa, 1935.

The author has utilized them to derive the Greenwich Kimura term anew. In doing this, however, attention had to be paid to the position of the mean pole of the earth. The mean latitudes of the three stations, Mizusawa, Carloforte and Ukiah, which were adopted respectively by the above two investigators, are changed as shown in the table.

	Mahnkopf	Kimura
	° ′ ″	″
Mizusawa	39 8 3.602	3.397
Carloforte	8.940	8.855
Ukiah	12.119	12.031

Hence to reduce Prof. Kimura's results to Prof. Mahnkopf's co-ordinate, $+0.''048$ was to be added to the x's given in Vol. VII. This correction having been applied, the numerical values of x of the International Latitude Service were subtracted from the corresponding "smoothed values" of latitude at Greenwich. The residuals thus obtained should be regarded as the Kimura term in the latitude observations at Greenwich. They are given in Tables III*a* and III*b*.

In the following, the nature of this term is discussed.

2. Yearly mean value of the Kimura term. In the last lines of Tables III*a* and III*b* are shown the mean values of the Kimura term taken for each year. They were calculated by giving half weight to the values for the epochs 0.00 and 1.00. It is at once noticed that these mean values change year by year in cycles of about seven years, the maximum occurring in 1913, 1919 and 1926, and the minimum occurring in 1916 and 1923. It is very curious to note that in the years 1913, 1919 and 1926, the ranges of the variations, or the amounts of the variations of both x and the observed value of latitude at Greenwich reached their minimum points, and in 1916 and 1923, they were nearly in their maxima. To express the amount of a variation numerically, there may be some other suitable methods, but in this paper the absolute value of the increase or decrease of the observed latitude per unit time was calculated for each tenth of a year, and their yearly mean values were taken for this purpose. In this case one tenth of a year was adopted as a unit time. The mean amount of the variation of x for each year was also determined by the same method. They are given in Table IV under the headings $\frac{\Delta\varphi}{\Delta T}$ and $\frac{\Delta x}{\Delta T}$ respectively. The second column of this table contains the yearly mean values of the Kimura term after subtracting the algebraic mean of those values given in the last lines of Tables III*a* and III*b*. The above-mentioned correlation is well represented in this table. This

Table III*a*

Greenwich "Smoothed Value" minus International *x* (1912–1918)
(unit 0.″001)

year	1912	1913	1914	1915	1916	1917	1918	mean
.00	-66	-51	-80	-10	-58	-118	-24	-58
.05	-45	-28	-66	-31	-70	-149	-25	-59
.10	-32	-4	-60	-54	-77	-161	-8	-57
.15	-34	+20	-71	-78	-79	-147	-14	-58
.20	-54	+37	-85	-101	-76	-119	-38	-62
.25	-81	+22	-102	-112	-75	-95	-68	-73
.30	-101	+1	-111	-115	-71	-97	-95	-84
.35	-112	-22	-99	-113	-70	-101	-110	-90
.40	-112	-38	-79	-101	-57	-98	-93	-83
.45	-110	-49	-62	-85	-44	-68	-66	-69
.50	-103	-53	-52	-55	-35	-30	-48	-54
.55	-92	-46	-43	-11	-31	-1	-47	-39
.60	-79	-30	-38	+38	-46	+11	-58	-29
.65	-72	-21	-29	+35	-58	+11	-51	-26
.70	-66	-20	-17	-18	-61	+8	-42	-31
.75	-69	-26	-7	-52	-61	+7	-29	-34
.80	-72	-39	+2	-59	-64	+6	-19	-35
.85	-78	-65	+11	-63	-61	+3	-15	-38
.90	-75	-85	+11	-62	-62	-4	-4	-40
.95	-66	-93	+6	-52	-89	-12	+3	-43
1.00	-51	-80	-10	-58	-118	-24	+12	-47
mean	-76	-30	-47	-56	-64	-55	-42	

Table III*b*

Greenwich "Smoothed Value" minus International *x* (1919–1927)
(unit 0.″001)

year	1919	1920	1921	1922	1923	1924	1925	1926	1927	mean
.00	-1	-39	-61	-125	-96	-130	-75	-80	+10	-66
.05	+5	-52	-48	-89	-99	-122	-60	-74	-2	-60
.10	0	-68	-37	-68	-106	-118	-45	-65	-17	-58
.15	-33	-78	-38	-68	-108	-116	-39	-38	-28	-61
.20	-72	-84	-56	-80	-117	-119	-48	-17	-39	-70
.25	-142	-89	-77	-97	-143	-141	-75	-16	-51	-92
.30	-179	-94	-94	-114	-177	-153	-88	-7	-60	-107
.35	-179	-95	-101	-124	-205	-158	-100	-5	-64	-115
.40	-152	-91	-103	-124	-214	-158	-112	-6	-68	-114
.45	-111	-74	-95	-117	-202	-158	-112	-19	-61	-105
.50	-60	-50	-96	-100	-174	-149	-101	-30	-51	-90
.55	-11	-24	-86	-65	-122	-154	-87	-36	-40	-69
.60	+20	-16	-77	-24	-89	-155	-75	-39	-32	-54
.65	+47	-6	-66	+7	-84	-141	-68	-43	-23	-42
.70	+59	-10	-58	+16	-88	-129	-71	-44	-17	-38
.75	+54	-3	-54	-59	-90	-118	-78	-38	-15	-45
.80	+34	-27	-81	-107	-104	-105	-93	-30	-19	-59
.85	+16	-59	-124	-123	-131	-92	-92	-15	-28	-72
.90	-2	-79	-155	-132	-151	-84	-94	-2	-39	-82
.95	-20	-75	-150	-111	-139	-76	-87	+9	-45	-77
1.00	-39	-61	-125	-96	-130	-75	-80	+10	-51	-72
mean	-37	-56	-84	-84	-133	-127	-80	-28	-36	

correlation will be discussed later on, but there is another which should be mentioned here. That is the systematic difference between the numerical values given in the third and fourth columns of this table. The amount of the variation of the observed latitude is always larger than that of x. This means that the observed latitude increases and decreases more rapidly than the corresponding x, thus making the range of latitude variation always larger. But the reason why such a systematic difference exists between them is not yet well known.

Table IV

Variation of Yearly Mean Value of the Kimura Term (unit 0.″001)

year	Kimura term	$\frac{\Delta\varphi}{\Delta T}$	$\frac{\Delta x}{\Delta T}$
1912	−11	73	63
1913	+35	47	29
1914	+18	61	64
1915	+ 9	108	86
1916	+ 1	91	82
1917	+10	74	65
1918	+23	58	37
1919	+28	45	26
1920	+ 9	56	51
1921	−19	65	65
1922	−19	87	73
1923	−68	74	60
1924	−62	38	38
1925	−15	28	26
1926	+37	13	24
1927	+29	24	12

3. Annual and semi-annual variation.

From the mean values given in the last columns of Tables III*a* and III*b*, their algebraic mean values were subtracted and reasonable corrections for the secular variations were also applied to them. In this way Table V was constructed so that it exhibited most clearly the annual variations of the Greenwich Kimura term for the two periods 1912–1918 and 1919–1927.

It is mentioned in the original report for the first period 1911–1918, that the values of the Greenwich Kimura term for that period when plotted against the fraction of the year gave an almost perfect simple cosine curve. Dr. Jones[1] also wrote a foot-note in his paper, that for this period the secondary maximum and minimum did not appear. Thus it is generally believed that for the first period, 1911–1918, only the primary maximum and minimum were noticeable, while for the period 1919–1927 the existence of the double maximum and double minimum was quite remarkable. The same conclusion is reached by inspecting Table V. But non-appearance by no means indicates non-existence. The Kimura term is essentially the sum of many factors which are independent of the motions of the terrestrial axis, and hence it is composed of many terms of various origins. Some

1. M. N., **96**, 131, 1935.

Table V

Annual Variation
(unit 0."001)

epoch	1911 to 1918	1919 to 1927
.00	0	+ 5
.05	− 1	+11
.10	+ 1	+14
.15	− 1	+11
.20	− 6	+ 2
.25	−17	−20
.30	−29	−34
.35	−35	−41
.40	−28	−41
.45	−16	−32
.50	− 1	−16
.55	+14	+ 5
.60	+23	+20
.65	+25	+33
.70	+20	+37
.75	+16	+31
.80	+15	+17
.85	+11	+ 4
.90	+ 8	− 6
.95	+ 5	− 1

Table VI

Variation in Residuals
(unit 0."001)

epoch	1912 to 1918	1919 to 1927
.00	− 4	− 5
.05	+ 2	+ 8
.10	+10	+19
.15	+13	+23
.20	+12	+20
.25	+ 3	+ 2
.30	− 9	−10
.35	−17	−17
.40	−13	−20
.45	− 6	−16
.50	+ 3	− 6
.55	+11	+ 8
.60	+14	+15
.65	+11	+21
.70	+ 2	+19
.75	− 4	+ 9
.80	− 5	− 7
.85	− 7	−20
.90	− 7	−27
.95	− 5	−17

of those terms may remain always constant and others may change their amplitudes and phases year by year. In addition to this, to some part of the Greenwich Kimura term thus obtained, the observations taken at the International Latitude Stations must be responsible. Accordingly it is very probable that the secondary maximum and minimum are occasionally masked and occasionally amplified by other terms. Thus if the harmonic analysis is applied to the numerical values given in the second column of Table V, the annual terms are taken out of the form

$$+0.''0035\cos T - 0.''0203\sin T,$$

where T represents the year and is measured from the beginning of it. By subtracting the variations represented by these terms from the original variation, residuals which are given in the second column of Table VI are obtained. It is easily noticed in this column that these residuals have double maxima and double minima; and what is remarkable is the fact that the phase of this variation is nearly the same as that of the Kimura term for the period 1919.0–1927.0, which is given in Table II.

The secondary maximum and minimum appear clearly in the last column of Table V, and the harmonic analysis gives annual terms,

$$+0.''0098\cos T - 0.''0219\sin T,$$

together with the remainders which are shown in the third column of Table VI. Thus it is proved that a term of semi-annual nature always exists in the Greenwich Kimura term, and that the phase of this term does not vary with the year even though the phase of the annual variation may vary year after year.

4. Annual variation due to $\varkappa$. To discuss more fully the Greenwich Kimura term, one should remember how the "smoothed values" of latitude variation were determined.

At Greenwich, three or four groups of latitude stars were observed during one night, each group being photographed on one plate. After measuring this plate, the residuals were computed and a numerical value, which is called "corrected residual" was derived for that plate. The mean of these corrected residuals taken during the same period gave the "mean residual." Then an equation of condition was constructed of the form :—

$$z_p + t_q + a_{pq}\varkappa = \text{mean residual},$$

in which z_p is the correction to be applied to the mean adopted zenith distance of the pth group, t_q the latitude variation at the epoch q, and a_{pq} the aberration factor corresponding to p and q. For the sake of convenience, $\varkappa$ was so defined that $20.47\varkappa$ is the correction to the adopted value of the aberration constant, viz.: $20.''470$. By weighing the equations according to the number of observations, normal equations for the z's, the t's and $\varkappa$ were formed. These normal equations were solved by quite an elaborate method and the solutions for the t's were obtained in the term of $\varkappa$, i. e.,

$$t_q = \tau_q + k_q\varkappa.$$

These τ's were next plotted on squared paper and by considering the relative weights, a smooth curve was drawn to represent them as closely as possible. This smooth curve was dealt with as representing the definitive results of latitude variation, and the above-mentioned "smoothed values" were the readings of this curve. Thus, in drawing a smooth curve, or in determining the "smoothed values" of latitude variation, the term $k_q\varkappa$ was omitted, because $\varkappa$ was thought to be too small. It was considered that the application of this correctional term would not generally have affected the values of latitudes by more than 0.″01. The numerical value for $\varkappa$ is as follows:—

$$20.47\ \varkappa = -0.''028, \quad \text{for the period 1911–1918,}$$

and

$$= -0.''023 \quad \text{for the period 1919–1927.}$$

Hence so long as the latitudes alone are concerned, the negligence of this term, $k_q\varkappa$, is permissible. But in case the Kimura term at Greenwich is discussed, this term should never be neglected, because it gives rise to small but appreciable variations in latitudes of an annual nature.

Dr. Jones[1] has recently pointed out the fact that the sign of the term of $\varkappa$ in the equations of condition was erroneously given in the original publications. Moreover, he reached a conclusion that it is impossible to derive the constant of aberration even by the programme of the zenith telescope observations at Greenwich where the determination of the value of this constant was primarily planned for.

Even if the sign of the term for $\varkappa$ was erroneously given, and even if the correction to the adopted constant of aberration thus obtained must be regarded, as he said, as spurious, it is nevertheless reasonable to regard $\varkappa$ as a constant factor related to a systematic error existing in the residuals: an error which is the same in nature as the closing error in the international latitude observations. So, considering the $\varkappa$ as such a constant factor, let us discuss how the "smoothed values" would have been changed if the term $k_q\varkappa$ had been adopted.

To consider it, Figs. 1*a* and 1*b* were constructed first. In both figures small circles represent the numerical values of the term, $k_q\varkappa$, for each value of q; and the black dots represent their mean values at respective epochs of a year, −0.″00136 and −0.″00112 being respectively used as the numerical values of $\varkappa$ for these two periods 1911–1918 and 1919–1927. Curves were drawn connecting these black dots. It will be at once apparent that the curve in Fig. 1*a* has no secondary maximum and minimum, while the curve in Fig. 1*b* has double maximum and double minimum. This fact reminds us of the variations in the Kimura term which were given in Table V. For the years 1911–1918, the secondary maximum and minimum did not appear in the Kimura term, whereas for the years 1919–1927, there occurred a marked double maximum and double minimum. It will also be noticed that the phases of these two curves are just reverse

1. M. N., **96**, 122–131, 1935.

to that of the Kimura term. The range of the variations represented by them is about 0.″015, and in plotting the black dots, values were taken in the same sense as the smoothed values of latitude variation. Hence if the term, $k_0 \varkappa$, were properly adopted in tracing the smooth curve, the range of the Kimura term must have been certainly diminished by this amount.

After all, it is concluded that in the "smoothed values" of latitude variation, there is an annual variation having a range of about 0.″015, which is caused by the negligence of the terms of $\varkappa$, and that the range of the Kimura term is accordingly much increased by the same cause.

Table VII

Annual Variation in the Term $k\varkappa$

(unit 0.″001)

1911–1918		1919–1927	
epoch	$k\varkappa$	epoch	$k\varkappa$
.054	+ 2	.027	+ 7
.179	+ 2	.163	+ 3
.279	+ 7	.279	+10
.358	+ 9	.366	+ 7
.438	+ 8	.441	+ 1
.495	+ 2	.505	− 3
.565	− 5	.575	− 6
.669	− 3	.670	− 4
.759	+ 1	.781	+ 1
.923	+ 2	.910	+ 2

In Table VII are given the numerical values which were used in plotting the black dots of the figures. These values are the corrections to be applied to both of the "smoothed values" and the Kimura term.

5. Variation diminished by drawing a curve. A smooth curve is very often drawn to represent as closely as possible some plotted

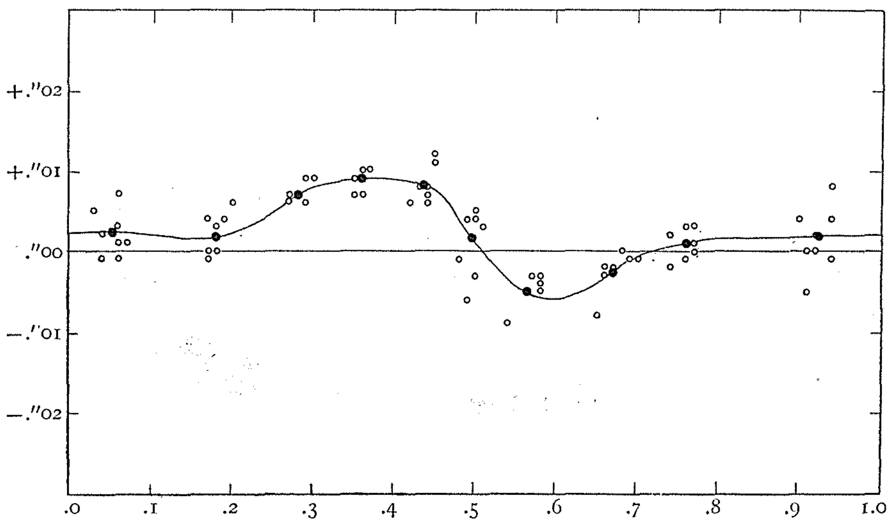


Fig. 1*a*

Annual variation in $k\varkappa$ (1911–1918).

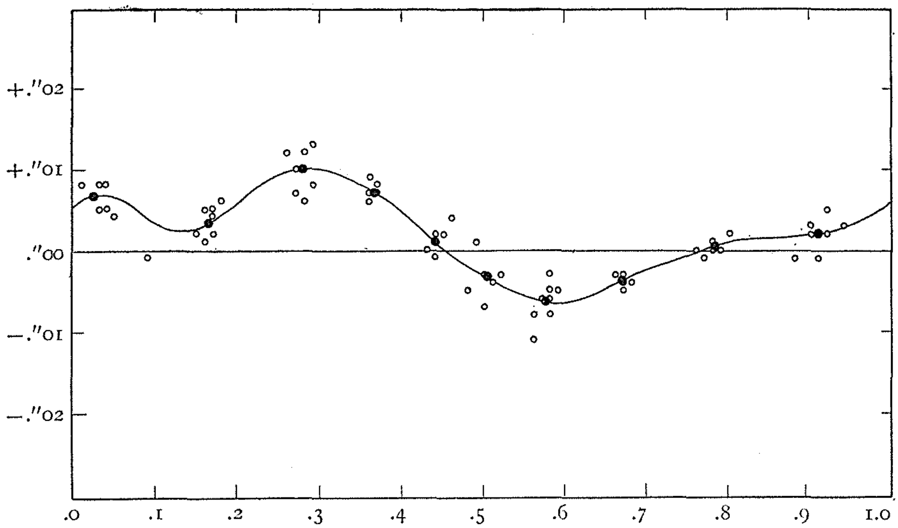


Fig. 1*b*

Annual variation in *kx* (1919–1927).

values. But practically this drawing is very difficult. Though the primary variation is usually represented quite well, the secondary fluctuation has a tendency to escape attention. In the following, the smooth curves representing the Greenwich latitude variations for the two periods, 1911–1918 and 1919–1927, are examined in order to make the nature of the Kimura term clearer.

Table VIII*a*

Deviation of τ_η from the Smooth Curve (1911–1918).
(unit 0.″001)

epoch	.06	.18	.28	.36	.44	.50	.57	.67	.76	.92
1911								+20	−51	− 4
1912	−98	+112	−47	− 9	+73	+ 81	− 16	+19	−51	+30
1913	−37	+ 13	−29	−11	+ 3	−160	− 34	+ 3	− 4	− 5
1914	−11	+ 53	− 3	−13	+ 2	− 8	+136	+ 4	−16	+ 1
1915	−13	+ 37	+31	−59	+13	− 40	+ 34	+25	−58	+37
1916	−11	− 1	+10	−22	+ 9	+ 46	− 7	−39	− 1	+16
1917	−23	+ 19	+ 6	−12	− 4	+ 53	+ 5	+ 4	0	+ 5
1918	− 3	− 5	+29	−11	+49	− 6	− 8	−15	+29	−13
1919	+11									
mean	−19	+ 28	+ 4	−21	+19	+ 5	+ 8	0	−16	+ 7

Table VIII*b*

Deviation of τ_q from the Smooth Curve (1919–1927).
(unit 0.″001)

epoch	.03	.16	.26	.36	.43	.50	.56	.67	.78	.91
1919		−38	+27	−29	+14	−19	+ 2	+20	−21	0
1920	+28	− 6	+48	−43	+ 4	−31	+12	0	+18	− 1
1921	−13	+15	−25	−51	+47	− 1	−20	+ 6	+ 3	−43
1922	−17	+32	+71	−37	−60	+ 4	− 8	0	+ 3	−13
1923	+14	− 1	+ 3	−16	+18	−18	− 1	+ 3	0	0
1924	− 3	+ 2	−32	+22	+ 2	+ 1	− 1	+30	−11	−46
1925	+39	− 1	−25	+30	− 5	+ 6	+17	−29	+21	+ 7
1926	− 2	+26	− 9	−93	0	+ 1	−17	+40	+24	−55
1927	+22	+ 2	+ 1	− 9	+ 4	−16	− 1	+ 3	+ 2	−77
1928	+56									
mean	+17	+ 7	+ 5	−23	+ 2	− 7	− 2	+ 8	+ 4	−23

As mentioned before, the solutions of the normal equations for the t's gave the results which were written in the form:

$$t_q = \tau_q + k_q \chi.$$

These τ's were then plotted on squared paper and a curve was drawn to represent them as closely as possible. The "smoothed values" of latitude variation at Greenwich were obtained in this way. To see if any systematic variation was neglected in drawing this curve, these τ's were compared with corresponding values which were read anew from the smooth curve. The differences, or the deviations of these τ's from the curve were taken in this way and arranged in Tables VIII*a* and VIII*b* with respect to the epoch to which they referred.

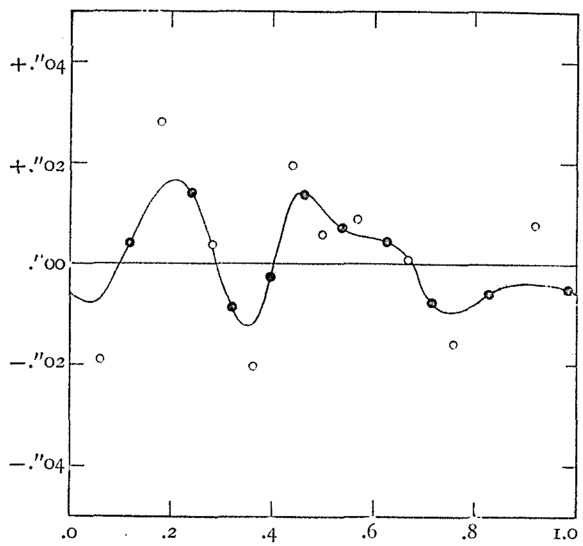


Fig. 2*a*

Deviation of τ from the Smooth Curve (1911–1918)

Their weighted mean values are also shown in the last lines of these tables.

It is easily noticed that the negative sign predominates in the second, fifth and tenth columns of Table VIII*a* and that the positive sign predominates in the sixth column of the same table. A similar fact is also noticed in Table VIII*b*. As such domination of a sign could not happen by chance, it is reasonable to suspect that at some epochs the smooth curve should have been drawn so that it might represent larger values, and at other epochs smaller values.

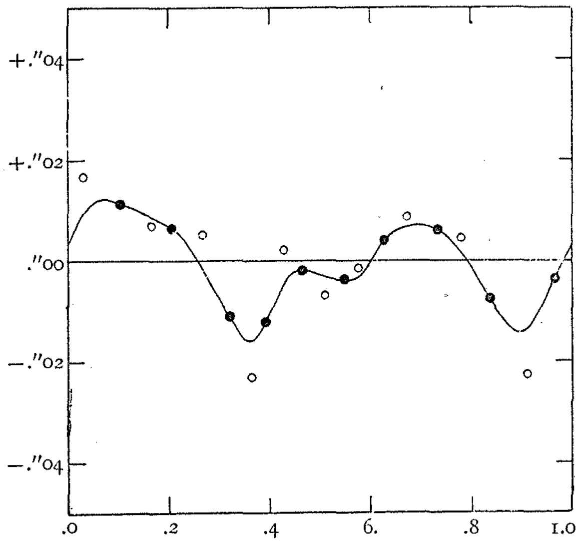


Fig. 2*b*

Deviation of τ from the Smooth Curve (1919–1927)

The mean values in each column may seem to represent no clear systematic variation. But by considering relative weights, and taking the mean value of the results given in each two adjacent columns, the mean values as shown in Table IX, reveal the existence of clear variations in both series of observations for the years 1911–1918 and 1919–1927. Both of these two variations have double maximum and double minimum with a range of about 0.″02. Not only do the variations in Table IX resemble each other in their nature, but their phases are nearly equal to those of the semi-annual variations represented in Table VI. These resemblances can not be regarded as a meaningless coincidence, and hence the variations here obtained should be looked upon as real. The numerical values given in Table IX were taken in the same sense as the smoothed values of latitude variation. Therefore, the conclusion is reached that the amplitude of a variation of semi-annual nature, which existed in the observations of latitude at Greenwich, was diminished by the drawing of a smooth curve. n other words, it would be justifiable to consider that the "smoothed values" of latitude variation, which are used as corrections to be applicable to the declination observations at Greenwich, have a systematic error of semi-annual nature. Accordingly some corrections for this systematic error are to be applied also to the Kimura term which was derived from those "smoothed values."

Table IX

Latitude Diminished by Drawing a Curve

(unit 0."001)

1911–1918		1919–1927	
epoch	dim'shed amount	epoch	dim'shed amount
.12	+ 4	.10	+11
.24	+14	.20	+ 6
.32	− 9	.32	−11
.40	− 3	.39	−12
.46	+13	.46	− 2
.54	+ 7	.54	− 4
.63	+ 4	.63	+ 4
.72	− 8	.73	+ 5
.83	− 7	.84	− 8
.99	− 6	.97	− 4

Small circles in Figs. *2a* and *2b* represent the mean values which are given in the last lines of Tables VIII*a* and VIII*b*. Black dots represent the weighted mean values given in Table IX. Curves were drawn connecting the black dots. But in doing so, attention was also paid to the accuracy of the positions of small circles. These curves may not represent the real variations. But regarding them for the while as representing the first approximate values, necessary corrections can be read off from them. If these correctional values were added to the Kimura term, the secondary maximum and minimum would clearly appear even in the results for the years 1911–1918.

6. Chandler period in the Kimura term. As is well known, the movement of the terrestrial pole is composed mainly of two motions: one having an annual period and the other a period of about fourteen months. The author has long thought that so long as the former of the above two periods exists in the Kimura term, the latter might also be found in it. But up to the present, he had no opportunity to confirm this supposition. Since the Kimura term began to be included in the discussions of the observations of latitude variation, numerous papers have been written concerning the annual period. But attention has rarely been paid to other periods which also may be contained in it. It may seem rather ridiculous to make any attempt in this chapter to detect the fourteen month period, or the Chandler period, in the Greenwich Kimura term. Yet this attempt is not utterly meaningless. Because, though very many terms of the form $f(\odot, \alpha)$ have been brought out recently to represent the numerical values of the Kimura term, those terms will remain inadequate for that

purpose in case any fairly large variation of non-annual nature should exist in it.

As to the origin of the Kimura term, it has been generally concluded that it is caused mainly by the seasonal variations of local meteorological conditions, although little research has been done to determine the meteorological correlation with the Kimura term. The author supports this theory and has contributed some papers on the subject, but he would not neglect other natural phenomena that may account for some smaller part of the Kimura term.

Soon after the variations of latitude had become one of the most attractive subjects in astronomy and geodesy, Prof. Milne discussed the correlation between the frequencies of earthquakes and the variations of latitude. He was followed by many investigators who compared not only the seismic frequencies but also the time intervals of the volcanic eruption with the movement of the axis of the earth. If such geophysical phenomena are really related to the motions of the terrestrial axis, both annual and Chandler periods must exist in the activity of the internal matter of the earth. Accordingly, it is reasonable to expect the same period in the local change of the gravity the origin of which is the variations of the internal conditions of the earth, and which gives rise to the local and abnormal variations of latitude. This reasoning may not be perfect, but it suggests the possibility of the existence of the Chandler period in the Kimura term.

As already mentioned, the yearly mean value of the Greenwich Kimura term is closely correlated with the amount of variation of the international x. This apparent correlation constitutes the first reason why the author suspected the existence of a variation with the Chandler period in the Greenwich Kimura term. It was ascertained that the variation of the yearly mean values of the Greenwich Kimura term has a period of seven years. This seven year periodicity can be regarded as being caused to some extent by the combination of the two periods of twelve months and fourteen months. Thus, with great promise that the Chandler period would be found because of these considerations, research was commenced with the first series of observations, i. e., the observations for the years 1911–1918. The mean values of the Kimura term for each twentieth of a year, given in the second column of Table V, were first subtracted respectively from corresponding individual values of Table III*a*. The annual variation of the Kimura term being eliminated in this way, the residuals were arranged again as shown in Table X, where the maxima occurring at

1912.10, 1913.20, 1914.45, 1915.60, 1916.75 and 1917.85 are specially marked with asterisks. Another maximum occurs just after the beginning of the year 1919. Of course there are many other maxima in these residuals, but these asterisked maxima suggest that there is contained in the observed data of this table a periodical variation which made six complete cycles in these seven years. For this second reason the author believes the possibility of the existence of a period of one sixth of seven years, or a fourteen month period, in the Greenwich Kimura term.

The residuals given in Table X were then plotted on squared paper and were connected with a curve from which the values for each tenth of the Chandler period were read off.

In the paper "New Study of the Polar Motion Based on the Luni-Solar Actions[1]," Prof. Kimura discussed the periodicity of both

Table X

Chandler Period in Kimura Term

(unit 0.″001)

year	1912	1913	1914	1915	1916	1917	1918
.00	−13	+ 2	−27	+43	− 5	− 65	+29
.05	+ 9	+26	−12	+23	−16	− 95	+29
.10	*+20	+48	− 8	− 2	−25	−109	+44
.15	+20	+74	−17	−24	−25	− 93	+40
.20	+ 5	*+96	−26	−42	−17	− 60	+21
.25	−11	+92	−32	−42	− 5	− 25	+ 2
.30	−19	+83	−29	−33	+11	− 15	−13
.35	−24	+66	−11	−25	+18	− 13	−22
.40	−31	+43	+ 2	−20	+24	− 17	−12
.45	−41	+20	*+ 7	−16	+25	+ 1	+ 3
.50	−49	+ 1	+ 2	− 1	+19	+ 24	+ 6
.55	−53	− 7	− 4	+28	+ 8	+ 38	− 8
.60	−49	0	− 8	*+68	−16	+ 41	−28
.65	−44	+ 7	− 1	+63	−30	+ 39	−23
.70	−33	+13	+16	+15	−28	+ 41	− 9
.75	−32	+11	+30	−15	*−24	+ 44	+ 8
.80	−34	− 1	+40	−21	−26	+ 44	+19
.85	−36	−23	+53	−21	−19	*+ 45	+27
.90	−30	−40	+56	−17	−17	+ 41	+41
.95	−18	−45	+54	− 4	−41	+ 36	+51
1.00	+ 2	−27	+43	− 5	−65	+ 29	+65

1. Proc. Imp. Acad., **2**, 470, 1926.

Table XI*a*

Residuals arranged against Each Tenth of Chandler Period (1912–1918)
(unit 0.″001)

.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
−13	+21	− 7	−24	−45	−51	−33	−35	−21	+29
+87	+85	+39	− 5	+ 6	+ 9	−33	−30	− 9	−30
−12	+ 6	− 7	+15	+44	+55	+23	−31	−36	−20
+ 9	+71	−16	−19	− 3	−26	−12	+17	+25	− 2
−29	−25	−30	−91	−85	−17	−17	+28	+40	+44
+45	+32	+44	+14	−21	+ 4	−17	−13	+20	+46
+15	+32	+ 4	−18	−17	− 4	−15	−11	+ 3	+11

Table XI*b*

Residuals arranged against Each Tenth of Chandler Period (1920–1926)
(unit 0.″001)

.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
									+ 5
−10	+13	+36	+ 57	+48	+53	+21	+25	+29	+11
+12	+ 4	− 8	+ 10	−33	−54	0	− 8	−13	− 7
+21	+69	−24	− 28	−17	−41	−58	−98	−75	−21
−29	−37	−46	− 47	−47	−49	−45	−73	−80	−35
+19	+10	+28	+ 13	+ 8	− 1	− 6	−15	− 3	+ 6
+ 2	+51	+99	+105	+49	+13	+40	+99	+89	,,
+ 2	+18	+14	+ 18	+ 1	−13	− 8	−12	− 9	− 7

the x- and the y-components of the motions of the terrestrial axis, and tried to separate all periods into two, namely, the Chandler group and the annual group. Among the Chandler group, a variation whose annual angular velocity is 306° has the largest amplitude. But he concluded that its period, 429.7 days, is still uncertain, having an estimated error of about 0.5 day, and that 428.5 days, which had been originally determined by Chandler, will not be very far from the true period. So this 428.5 day period (=306.°86 per year) was adopted for the present research. 1912.0 was taken as the original epoch, and the values for each tenth of the Chandler period, which had been read from the above mentioned curve, were arranged in Table XI*a*. The mean values given in the last line of this table represent an unexpectedly large range of a systematic variation. This is the third reason for concluding that the Greenwich Kimura term contains a clear fourteen month variation besides the annual one.

As to the second series of observations, i. e., the observations for the years 1919–1927, investigations were made by the same process. But in this case, the author utilized the observations for 1920.0–1927.0 only. Those made in 1919 and 1927 were not used, because the adoption of observations made over a period of exactly seven years facilitated the complete separation of the twelve month period from the fourteen month period. Moreover, the fact that the number of plates taken in the first half of the year 1919 is comparatively small, is another reason for rejecting the observations for that year. The details of the observations for the year 1927 are not given in the report, and so it was thought best not to use them.

The seasonal variation of the Kimura term was newly determined for those seven years and was eliminated from the Kimura term given in Table III*b*. It was impossible, however, to detect the Chandler period from the remaining variations as nicely as we did in the former case, because, during these seven years, the Kimura term did not vary so uniformly. In some years there occurred double maximum and double minimum, and in others not. Moreover, even though a correction, $+0.''048$, was adopted in order to transform Prof. Kimura's co-ordinate into Prof. Mahnkopf's, some discrepancies very likely still remain between the respective results obtained by these two investigators. The Chandler variation could nevertheless be detected as shown in the last line of Table XI*b*. In forming this table, 1912.0 was taken again as the original epoch, so 1920.094 corresponded to 0.9 and 1927.014 corresponded to 0.8 of the Chandler period.

To these Chandler variations of both periods, (1912–1918 and 1920–1926) were applied reasonable corrections for the secular variation, and then their respective algebraic mean values were subtracted. The results are contained in the second and third columns of Table XII. Though the ranges are nearly the same, some phase difference exists between these two variations. But if the individual values of Table XI*a* and XI*b* are examined, this phase difference can not be regarded as real. The mean numerical values of these two variations are given in the last column of the same table. They are also represented by small circles in Fig. 3, together with a cosine curve:

$$+0.''0168\cos(\theta-37.^\circ3),$$

where θ means the Chandler period, 428.5 days, and is measured from the beginning of the year 1912. This curve probably represents very nearly the real Chandler variation that existed in the Greenwich Kimura term during the whole period 1912 to 1927.

By analysing the Greenwich Kimura term in this way, we were able to ascertain the existence of a new variation whose period is fourteen months and which has an unexpectedly large range of about 0.″03. Of course this new variation has no mathematical connection with the motions of the terrestrial axis. From the physical point of view, the Chandler period is one which belongs only to the movement of the poles of the earth. If there were any physical basis for the existence of the same period in the Kimura term, which is of quite a local nature, it would be a very interesting fact for geodetists and geophysicists. In the above discussion, annual terms were rejected to make the phase and range of the Chandler variation clear, but even though they had not been rejected, we could have obtained results similar to those given in Tables XI and XII. And so the author can not believe that this Chandler variation is due to some defect in the process of deriving it; but he is still not sure whether this term represents a real physical phenomenon, or is caused by a very trifling matter. As the present results are not enough to prove a physical basis, further discussion should be postponed until more research of this kind is done

Table XII

Chandler Variation in the Greenwich Kimura Term
(unit 0.″001)

fraction of period	1912 to 1918	1920 to 1926	mean
.0	+20	+ 4	+12
.1	+36	+19	+28
.2	+ 7	+14	+11
.3	−16	+18	+ 1
.4	−17	+ 1	− 8
.5	− 5	−15	−10
.6	−17	−10	−13
.7	−14	−14	−14
.8	− 1	−12	− 7
.9	+ 6	− 5	0

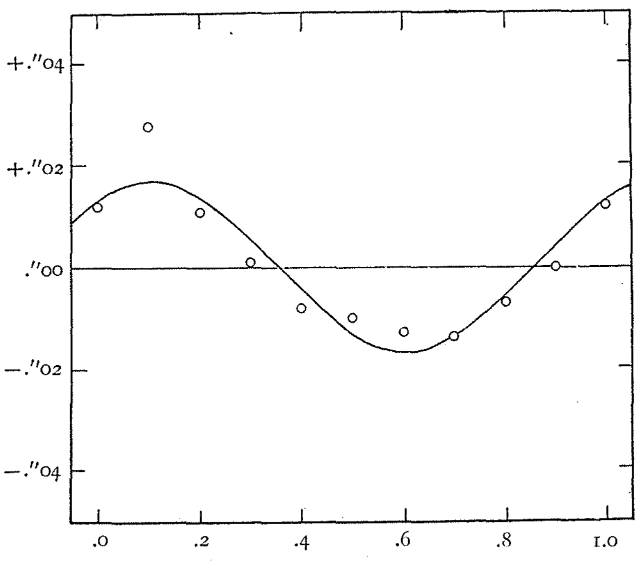


Fig. 3

Chandler variation in the Kimura term

on the observations taken at other observatories. It should, however, be remembered that this variation was derived by nearly the same process as that adopted in the determination of the annual variation in the Kimura term. Hence, so long as some attempts are made to explain the annual term, full attentions should also be paid to this new variation. Again, if some trifling matter should be regarded as the cause of this Chandler period in the Kimura term, then at least some part of the annual variation should also be attributed to the same trifling origin. These variations in the Kimura term were determined by comparing the " smoothed values " of latitude variation at Greenwich with corresponding values for the international x. Accordingly, the results of the International Latitude Service may be thought to be responsible for this existence of the Chandler period in the Greenwich Kimura term. But it is hardly possible that the results given by the two authoritative experts, Profs. Mahnkopf and Kimura, would be so improper. In any case, this new variation should not be neglected in discussions of the nature of the Kimura term.

III. Diurnal Variation of Latitude

1. Diurnal variatiou of the form sin $(\odot - \alpha)$. While the present author was making a study on the diurnal variation of latitude at Greenwich, Dr. Jones[1] published his valuable paper, making many suggestions pertinent to this investigation. Even though the author recognises that both his methods and conclusions bear some resemblance to those of Dr. Jones', he believes that he still has some things worth publishing.

To derive the group corrections, the latitude variations and the correction to the adopted value of the constant of aberration from the observations, the equation of condition was constructed of the form:

$$z_p + t_q + a_{pq}\varkappa = \text{mean residual.}$$

But as Dr. Jones said, it is impossible to derive an accurate value of the constant of aberration from the latitude variation observations, because the effect of errors of a seasonal or diurnal nature is too large. This means that the term $a_{pq}\varkappa$ should not be used in forming the equation of condition. So this term being rejected, the equation of condition for each group and period becomes

$$z_p + t_q = \text{mean residual.}$$

From many equations of condition of the above form, normal

1. M. N., **96**, 122-131, 1935.

equations are constructed for the z's and t's. But in practice, it is not necessary to solve these normal equations anew. These solutions are obtained from those which are given in the original publications, by making

$$z=0.$$

To ascertain the existence of the diurnal variation of latitude at Greenwich, it is necessary to subtract the group correction and latitude variation from each of the mean residuals. In this paper, however, the daily value of the "corrected residual" which is given in Table I of the original publications was used instead of the mean residual. Thus $\Delta v_{pq'}$ was obtained by making

$$\Delta v_{pq'}=(\text{corrected residual})_{pq'}-z_p-t_{q'}.$$

The (corrected residual)$_{pq'}$ is that which was computed from the result of observations of the pth group made on the date q'; and $t_{q'}$ is the variation of latitude for that date and can be read from the smooth curve, whose nature was discussed in the foregoing chapter. Of course $t_{q'}$, thus determined is of only a first approximate value. But it is enough for the present purpose.

Table XIII*a*

Mean Value of Δv (1911–1918)
(unit 0.″001)

group	epoch	Δv	group	epoch	Δv
I	7 9	−24 −11	IX	1 2 3	−36 + 2 +33
II	0 8	+42 +21	X	2 3 4	+ 5 + 3 − 2
III	0 8	+17 + 8	XI	3 4 5	+16 −21 + 7
IV	1 9	−28 −12	XII	4 5 6	−26 − 6 − 4
V	1 9	+16 −54	XIII	5 6 7	− 6 0 + 7
VI	0 2	+19 −11	XIV	6 7 8	−30 +14 +23
VII	0 2	+ 6 + 6	XV	6 8	− 6 +12
VIII	1 3	−53 +20	XVI	7 9	−29 +46

Table XIII*b*

Mean Value of Δv (1919–1927)
(unit 0.″001)

group	epoch	Δv	group	epoch	Δv
I	7 9	−44 +30	IX	1 2 3	−30 +25 +31
II	0 8	− 1 −22	X	2 3 4	−59 +38 + 3
III	0 8	−18 − 8	XI	3 4 5	+ 5 + 5 +17
IV	1 9	+ 7 −22	XII	4 5 6	−11 + 9 +10
V	1 9	− 9 +11	XIII	5 6 7	− 6 + 6 − 9
VI	0 2	−28 + 3	XIV	6 7 8	−24 − 5 − 3
VII	0 2	−11 0	XV	6 8	− 9 + 3
VIII	1 3	− 9 +29	XVI	7 9	+ 4 +13

There are as many Δv's as the number of photographic plates. They were separated according to the group number and the period of observation, and then the mean values for these separated Δv's were taken, giving the results shown in Table XIII*a* and XIII*b*. As mentioned in the introduction, the observations made over one year are divided into ten periods. For the sake of convenience, the mean dates of these ten periods were respectively represented by ten figures from 0 to 9, and were written in the second column of these tables under the heading of "epoch."

Close examinations of these two tables show that there still remained certain errors in the z's and the t's. Let these errors be represented by Δz_p and Δt_Q, where Q means the above-mentioned epoch, taking numerical values from 0 to 9. Let again Δv_{pQ} represent each value contained in Table XIII*a*. Then we can make

$$\Delta v_{pQ} = \Delta z_p + \Delta t_Q.$$

From the 38 equations of this form, normal equations are constructed for 16 Δz's and 10 Δt's. Their solution gives the results which are arranged in the second column of Table XIV. The third column of the same table contains similar results deduced from the data given in Table XIII*b*. By subtracting these results from the values contained in Tables XIII*a* and XIII*b*, all errors due to inadequate group corrections and improper latitude variations were eliminated. Hence in case the group corrections and latitude variations were all that were contained in the mean residuals (or corrected residuals), or, in other words, if the equation of condition of the form:

$$\text{mean residual} = z_p + t_q$$

were correct, the remainders, δv's, which were obtained by these subtractions, must take random values, without any systematic relationship existing between them. But practically these δv's vary systematically as shown in Table XV. This table was constructed by combining each two of the corresponding remainders which were respectively derived from Tables XIII*a* and XIII*b*. Bracketed small figures mean the time of observation represented in hours by Greenwich civil time. It is noticed in this table first, that the remainder δv decreases gradually as the time of observation advances, and second, that this variation in remainders seems to remain nearly constant throughout the whole course of a year. Thus it became clear that in the mean residual, there must be contained at least one more systematic variation besides group correction and latitude variation. Accordingly it came to be necessary to add another term to the above equation of condition.

Table XIV

Solutions for Δz and Δt

	1911 to 1918	1919 to 1927
	$''$	$''$
Δz_1	−0.0091	+0.0062
Δz_2	+0.0183	−0.0048
Δz_3	−0.0001	−0.0073
Δz_4	−0.0113	−0.0003
Δz_5	−0.0046	−0.0019
Δz_6	−0.0115	−0.0067
Δz_7	−0.0057	−0.0035
Δz_8	−0.0192	−0.0008
Δz_9	−0.0026	−0.0003
Δz_{10}	−0.0036	−0.0055
Δz_{11}	−0.0031	−0.0009
Δz_{12}	−0.0004	+0.0030
Δz_{13}	+0.0144	+0.0040
Δz_{14}	+0.0157	+0.0016
Δz_{15}	+0.0085	+0.0060
Δz_{16}	+0.0142	+0.0112
Δt_0	+0.0208	−0.0066
Δt_1	−0.0147	−0.0056
Δt_2	+0.0062	+0.0024
Δt_3	+0.0257	+0.0283
Δt_4	−0.0123	+0.0012
Δt_5	−0.0055	+0.0048
Δt_6	−0.0190	−0.0057
Δt_7	−0.0153	−0.0163
Δt_8	+0.0051	−0.0063
Δt_9	−0.0023	+0.0073

In the international latitude observations, there are variations of a diurnal nature, which Prof. Kimura used to represent by terms of the form $f(\odot, \alpha)$. Since this variation in the Greenwich observations

Table XV

Variation in δv

(unit 0.″001)

	Jan.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Dec.
z_2	+7 (19.6)									
z_3	−4 (20.6)									
z_4		+6 (21.4)								
z_5		+18 (22.4)								
z_6	−2 (3.8)		+2 (22.4)							
z_7	−5 (4.6)		+4 (23.2)							
z_8		−11 (3.3)		+8 (22.8)						
z_9		−21 (4.3)	+11 (1.9)	+7 (23.7)						
z_{10}			−27 (3.1)	−2 (1.0)	+11 (23.2)					
z_{11}				−15 (2.1)	−1 (0.2)	+15 (22.8)				
z_{12}					−15 (1.2)	+1 (23.8)	+14 (22.1)			
z_{13}						−15 (0.9)	+7 (23.3)	+6 (20.8)		
z_{14}							−24 (0.3)	+11 (21.8)	+2 (19.5)	
z_{15}							−2 (1.0)		+1 (20.5)	
z_{16}								−10 (0.9)		+15 (19.1)
z_1								−17 (1.8)		+9 (19.8)
z_2									−7 (1.8)	
z_3									+4 (2.9)	
z_4										−14 (3.4)
z_5										−21 (4.4)

is the same in nature as those found in the international latitude observations, it will also be possible to represent the variation in Table XV by one of those terms.

Results arranged in Table XV suggest that such terms of the form $f(\odot, \alpha)$ take positive values so long as the observations are carried on before midnight and that the sign is changed after midnight, $f(\odot, \alpha)$ taking negative values in the morning. But it is very clear that it can not be a linear function of $\odot$ and α. Let the remainders be grouped into four according to the time of observation, and take their mean values. Then the results become as follows:

civil time	$20.^h3$	$23.^h0$	$1.^h1$	$3.^h5$
mean δv	+0.″006	+0.″009	−0.″008	−0.″012

These results also suggest the same fact. After all, sin $(\odot - \alpha)$, which is one of the most common terms used by Prof. Kimura, is found to be quite suitable for representing this variation. Hence, the numerical values given in Tables XIII*a* and XIII*b* were discussed again by constructing the conditional equation whose form is

$$\Delta v_{pQ} = \Delta z_p + \Delta t_Q + s\ \sin(\odot_{pQ} - \alpha_p).$$

The numerical value of sin $(\odot - \alpha)$ at the mean time of observation is found easily in the Nautical Almanac. By forming again the normal equations for Δz's and Δt's, their solutions were obtained in the term of s, i. e.,

$$\Delta z_p = \Delta \zeta_p + \sigma_p s,$$

and $$\Delta t_Q = \Delta \tau_Q + \sigma_Q s.$$

The numerical values of $\Delta\zeta$'s and $\Delta\tau$'s are the same as those of Δz's and Δt's which are given in Table XIV. The σ's, the coefficients of s, take values contained in the second and fourth columns of Table XVI. The normal equation for s was also formed and on substitution of the values for the Δz's and Δt's in it, we obtained

$$s = -0.''01764 \quad \text{for the years 1911–1918,}$$

and $$= -0.''01852 \quad \text{for the years 1919–1927.}$$

Substituting back these results in the solutions for Δz's and Δt's, their final numerical values were determined as given in the third and fifth columns of Table XVI. They are the most probable amounts of errors in the adopted group corrections and latitude variations.

Fig. 4 shows both calculated and observed values of the diurnal variation of latitude at Greenwich. The sine curve represents the calculated variation, −0.″018 being adopted as the numerical value for *s*. This is the mean of the above-mentioned values for the two series of observations. The numerical values given in Table XV were corrected for the term σs, and those corrected results are represented by small circles in this figure. Black dots are the mean values of four separated groups of the small circles, 22^h, 0^h and 2^h being taken as the criteria of this separation. It is noticed that on the whole all observed values are in accordance with the calculated curve. There may still exist some other diurnal variations of different form in the latitudes observed at Greenwich, but we can conclude that the variation represented by the curve $-0.''018 \sin(\odot - \alpha)$, is the most remarkable one.

Table XVI

Numerical value of σ, Corrected Δz and Δt

	1911–1918		1919–1927	
	σ	corrected value	σ	corrected value
		"		"
Δz_1	+0.6027	−0.0197	+0.6279	−0.0055
Δz_2	+0.2124	+0.0146	+0.1344	−0.0073
Δz_3	+0.0964	−0.0018	−0.0353	−0.0066
Δz_4	−0.1825	−0.0081	−0.1708	+0.0029
Δz_5	−0.3297	+0.0012	−0.3283	+0.0042
Δz_6	−0.5206	−0.0023	−0.7746	+0.0076
Δz_7	−0.7340	+0.0073	−0.9432	+0.0140
Δz_8	−0.6201	−0.0083	−0.6951	+0.0120
Δz_9	−0.8678	+0.0127	−0.9692	+0.0176
Δz_{10}	−0.6773	+0.0084	−0.4820	+0.0035
Δz_{11}	−0.3598	+0.0033	−0.1753	+0.0023
Δz_{12}	+0.1490	−0.0030	+0.3677	−0.0038
Δz_{13}	+0.6941	+0.0031	+0.8139	−0.0110
Δz_{14}	+1.0047	−0.0020	+0.9969	−0.0169
Δz_{15}	+0.8884	−0.0072	+0.9137	−0.0109
Δz_{16}	+0.6440	+0.0029	+0.7187	−0.0021
Δt_0	+0.2765	+0.0159	+0.5408	−0.0166
Δt_1	+0.4135	−0.0220	+0.5404	−0.0156
Δt_2	+0.6004	−0.0044	+0.8817	−0.0139
Δt_3	+0.5841	+0.0154	+0.6437	+0.0164
Δt_4	+0.3514	−0.0185	+0.1278	−0.0012
Δt_5	−0.1212	−0.0034	−0.2458	+0.0094
Δt_6	−0.5379	−0.0095	−0.5678	+0.0048
Δt_7	−0.5140	−0.0062	−0.5369	−0.0064
Δt_8	−0.3735	+0.0117	−0.2958	−0.0008
Δt_9	−0.0798	−0.0009	−0.0286	+0.0078

Dr. Jones introduced by somewhat different method a diurnal variation represented by $-0.''016 \sin(\odot - \alpha)$ very similar to the present result.

Fig. 4 shows that the results of latitude observations are always larger in the evening than in the morning. This may seem to be just opposite to the daily variation of latitude, observed at the International Latitude Stations, where the observed latitude is generally smaller in the evening than in the morning. But in this chapter, "mean residuals" or "corrected residuals" were utilized for our discussion, and it must be remembered that these residuals were obtained by subtracting the tabular values from the observed values of the zenith distance N. of star pairs. So their sense is just opposite to the

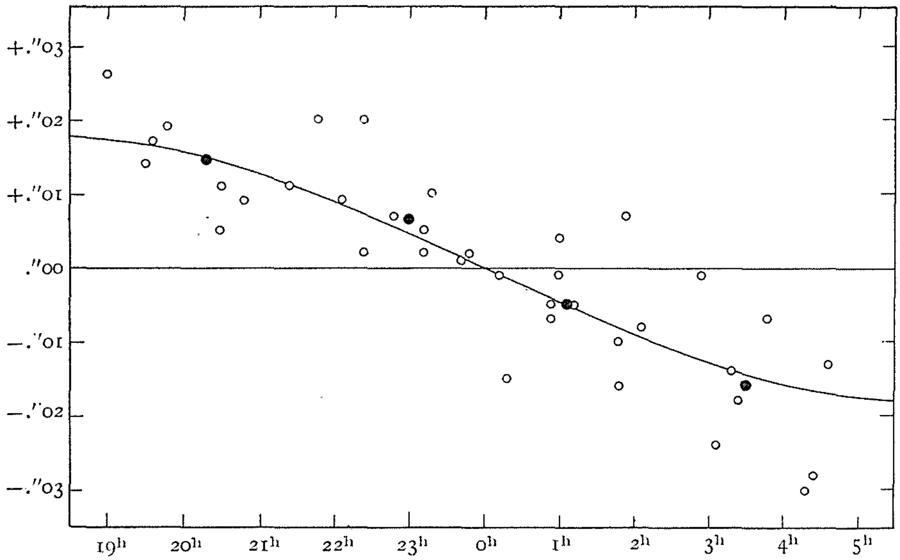


Fig. 4

Diurnal variation of latitude

"smoothed values" of latitude variation, and accordingly the diurnal variation at Greenwich occurs in the same way as that of the international results.

2. Correctional term σs. The latitude variations at Greenwich were deduced by constructing conditional equations of the form :

$$z_p + t_q + a_{pq}\varkappa = \text{mean residual.}$$

Solutions for z's and t's thus obtained were given in term of $\varkappa$. Their forms were

$$z_p = \zeta_p + k_p\varkappa,$$

and $$t_q = \tau_q + k_q\varkappa.$$

As the term $\varkappa$ was omitted in determining the "smoothed values" of latitude variation, no alternation is needed in those "smoothed values" even if the numerical value for $\varkappa$ were found to be spurious.

Now, it was ascertained in the foregoing paragraph that there was a remarkable diurnal variation in the observations of latitude made at Greenwich, and that this variation could be fairly well represented by the term $\sin(\odot - \alpha)$. So, to discuss the latitude observations, the conditional equations should be written in the form

$$z_p + t_q + s \sin(\odot_{pq} - \alpha_p) = \text{mean residual,}$$

where a term $s \sin(\odot_{pq} - \alpha_p)$ is introduced instead of $a_{pq}\varkappa$. $\odot_{pq}$ is the mean longitude of the sun at the mean date when the pth group is observed during the qth period, and α_p is the mean right ascension of the pth group.

In this case, the solutions for the z's and t's are given in term of s, or

$$z_p = \zeta_p + \sigma_p s,$$

and $$t_q = \tau_q + \sigma_q s.$$

In them, ζ_p and τ_q are essentially the same as those of the old solutions, while the correctional terms, the σs's, remain unknown. To determine the values that will be taken by the σs's, normal equations for the z's, t's and s were solved anew. To obtain these solutions for the first period, 1911–1918, the author employed all observations taken before the date 1919.20, though the observations taken after 1918.57 were not used in the original publication.

Tables XVII*a* and XVII*b*, which contain the coefficients of s in the normal equations, are given here instead of the whole equations. Normal equations were solved by the usual method and the coefficients of s in those solutions for z's and t's are given in Tables XVIII*a* and XVIII*b*.

Normal equations for s gave

$$s = -0.''01947 \quad \text{for } 1911\text{–}1918,$$

and $$= -0.01718 \quad \text{for } 1919\text{–}1927.$$

Substituting these values in the solutions for z's and t's, the final values for them may be determined if they are necessary.

3. Annual variation produced by the term $\sin(\odot - \alpha)$. The numerical values of the correctional terms in the solutions for the z's and t's are calculated by multiplying the coefficients given in Tables XVIII*a* and XVIII*b* by s. There are ten periods of observation in a year, and so the results obtained in this way were divided into ten parts according to those periods, and their mean values were taken. In the first and third columns of Table XIX are given the mean epochs for each of ten periods and in the second and fourth columns are the mean values for σs. It will be noticed in this table that the mean value of σs varies systematically over a period of one year. This variation is also shown in Fig. 5, where small circles represent the values for the years 1911–1918 which are contained in the second column of Table XIX, and black dots represent the values for the years 1919–1927.

The annual variation in the term kx has already been represented in Figs. 1*a* and 1*b*. Comparing these figures, one will find that the curve drawn connecting the small circles in Fig. 5 resembles the curve

Table XVII*a*

Coefficients of *s* in the the Normal Equations

(1911–1918)

z_1	−19.358	t_8	+0.478	t_{31}	+0.117	t_{54}	− 0.304
z_2	− 7.316	t_9	+0.037	t_{32}	−1.513	t_{55}	− 0.398
z_3	− 2.898	t_{10}	+0.349	t_{33}	−0.562	t_{56}	+ 1.978
z_4	+ 0.324	t_{11}	−3.543	t_{34}	+2.902	t_{57}	+ 0.470
z_5	+ 5.704	t_{12}	−0.106	t_{35}	+3.886	t_{58}	+ 0.046
z_6	+ 2.330	t_{13}	−3.876	t_{36}	+3.452	t_{59}	− 0.961
z_7	+15.742	t_{14}	−1.997	t_{37}	+3.587	t_{60}	+ 0.115
z_8	+ 6.733	t_{15}	−1.190	t_{38}	+0.579	t_{61}	− 5.070
z_9	+20.052	t_{16}	−0.142	t_{39}	−0.836	t_{62}	− 6.162
z_{10}	+21.899	t_{17}	+1.589	t_{40}	−1.288	t_{63}	+ 0.671
z_{11}	+ 6.598	t_{18}	−1.103	t_{41}	−4.878	t_{64}	− 8.831
z_{12}	− 6.031	t_{19}	+0.312	t_{42}	−3.453	t_{65}	− 1.166
z_{13}	−28.786	t_{20}	−1.876	t_{43}	+3.834	s_{66}	+ 1.547
z_{14}	−61.167	t_{21}	−8.115	t_{44}	+0.303	t_{67}	+ 0.921
z_{15}	−32.827	t_{22}	−7.604	t_{45}	−1.412	t_{68}	+ 0.088
z_{16}	−19.090	t_{23}	+0.346	t_{46}	+2.002	t_{69}	− 2.050
t_1	− 3.487	t_{24}	+2.492	t_{47}	+0.703	t_{70}	− 3.123
t_2	− 5.333	t_{25}	+5.472	t_{48}	+1.516	t_{71}	− 6.705
t_3	− 9.644	t_{26}	+0.379	t_{49}	+0.065	t_{72}	− 8.516
t_4	− 1.546	t_{27}	+2.666	t_{50}	−5.540	t_{73}	− 4.675
t_5	+ 1.033	t_{28}	−2.074	t_{51}	−4.079	t_{74}	+ 0.691
t_6	+ 2.408	t_{29}	−1.092	t_{52}	−6.880	t_{75}	− 1.579
t_7	+ 0.580	t_{30}	−1.535	t_{53}	−2.461	$\varkappa$	+406.26

Table XVII*b*

Coefficients of *s* in the Normal Equations

(1919–1927)

z_1	−37.410	t_{12}	+ 2.099	t_{36}	− 5.205	t_{60}	− 0.955
z_2	−25.511	t_{13}	+ 1.104	t_{37}	− 9.581	t_{61}	− 5.714
z_3	−10.975	t_{14}	+ 0.406	t_{38}	− 9.337	t_{62}	+ 1.984
z_4	−17.132	t_{15}	− 0.658	t_{39}	− 7.938	t_{63}	− 0.674
z_5	− 4.366	t_{16}	+ 0.383	t_{40}	−13.120	t_{64}	+ 1.569
z_6	+ 0.569	t_{17}	− 2.545	t_{41}	− 5.055	t_{65}	+ 0.019
z_7	+14.126	t_{18}	−12.993	t_{42}	− 0.777	t_{66}	− 5.759
z_8	+ 7.930	t_{19}	− 0.201	t_{43}	− 0.642	t_{67}	− 7.640
z_9	+28.462	t_{20}	− 0.032	t_{44}	+ 0.098	t_{68}	− 3.439
z_{10}	+ 6.477	t_{21}	+ 1.227	t_{45}	− 1.689	t_{69}	−14.979
z_{11}	+ 3.502	t_{22}	− 0.880	t_{46}	− 1.680	t_{70}	−11.823
z_{12}	−13.663	t_{23}	+ 0.227	t_{47}	− 6.764	t_{71}	− 7.173
z_{13}	−46.713	t_{24}	− 0.184	t_{48}	−10.333	t_{72}	+ 1.986
z_{14}	−80.401	t_{25}	− 2.534	t_{49}	− 8.024	t_{73}	− 1.011
z_{15}	−45.503	t_{26}	− 3.531	t_{50}	− 5.830	t_{74}	− 0.208
z_{16}	−48.366	t_{27}	− 5.961	t_{51}	+ 1.105	t_{75}	− 1.583
t_4	− 0.671	t_{28}	− 5.464	t_{52}	− 2.218	t_{76}	− 3.729
t_5	+ 0.119	t_{29}	− 7.039	t_{53}	+ 0.068	t_{77}	− 8.960
t_6	− 3.195	t_{30}	− 0.569	t_{54}	− 0.800	t_{78}	− 2.153
t_7	− 8.225	t_{31}	− 6.325	t_{55}	− 1.378	t_{79}	− 7.776
t_8	− 6.323	t_{32}	− 3.995	t_{56}	− 1.917	t_{80}	−10.192
t_9	+ 1.202	t_{33}	+ 0.855	t_{57}	− 4.781	t_{81}	+ 0.602
t_{10}	− 1.339	t_{34}	+ 0.954	t_{58}	− 6.984	$\varkappa$	+527.28
t_{11}	+ 3.487	t_{35}	− 0.671	t_{59}	−17.312		

Table XVIII*a*

Coefficients of s in the Solutions for t's and z's
(1911–1918)

z_1	+.547	t_8	+.254	t_{31}	−.685	t_{54}	+.265
z_2	+.193	t_9	−.154	t_{32}	−.459	t_{55}	+.446
z_3	+.073	t_{10}	−.670	t_{33}	−.139	t_{56}	+.525
z_4	−.185	t_{11}	−.374	t_{34}	+.094	t_{57}	+.587
z_5	−.279	t_{12}	−.438	t_{35}	+.305	t_{58}	+.306
z_6	−.469	t_{13}	+.061	t_{36}	+.505	t_{59}	+.083
z_7	−.668	t_{14}	+.327	t_{37}	+.469	t_{60}	−.657
z_8	−.583	t_{15}	+.582	t_{38}	+.312	t_{61}	−.454
z_9	−.814	t_{16}	+.639	t_{39}	−.011	t_{62}	−.325
z_{10}	−.647	t_{17}	+.465	t_{40}	−.520	t_{63}	−.141
z_{11}	−.354	t_{18}	+.525	t_{41}	−.414	t_{64}	+.482
z_{12}	+.131	t_{19}	−.422	t_{42}	−.372	t_{65}	+.474
z_{13}	+.673	t_{20}	−.395	t_{43}	−.374	t_{66}	+.541
z_{14}	+.956	t_{21}	−.453	t_{44}	+.209	t_{67}	+.592
z_{15}	+.840	t_{22}	−.235	t_{45}	+.505	t_{68}	+.346
z_{16}	+.586	t_{23}	−.156	t_{46}	+.544	t_{69}	+.021
t_1	−.376	t_{24}	+.069	t_{47}	+.580	t_{70}	−.393
t_2	−.271	t_{25}	+.241	t_{48}	+.243	t_{71}	−.488
t_3	+.260	t_{26}	+.625	t_{49}	−.304	t_{72}	−.300
t_4	+.328	t_{27}	+.539	t_{50}	−.417	t_{73}	+.017
t_5	+.303	t_{28}	+.458	t_{51}	−.447	t_{74}	+.211
t_6	+.499	t_{29}	−.080	t_{52}	−.307	t_{75}	+.451
t_7	+.556	t_{30}	−.422	t_{53}	−.053		

Table XVIII*b*

Coefficients of s in the Solutions for t's and z's
(1919–1927)

z_1	+.551	t_{12}	+.573	t_{36}	−.433	t_{60}	+.405
z_2	+.098	t_{13}	+.553	t_{37}	−.393	t_{61}	+.527
z_3	−.070	t_{14}	+.046	t_{38}	−.219	t_{62}	+.679
z_4	−.134	t_{15}	−.279	t_{39}	+.118	t_{63}	+.599
z_5	−.296	t_{16}	−.786	t_{40}	+.617	t_{64}	+.014
z_6	−.729	t_{17}	−.508	t_{41}	+.573	t_{65}	−.437
z_7	−.930	t_{18}	−.149	t_{42}	+.820	t_{66}	−.356
z_8	−.604	t_{19}	−.238	t_{43}	+.588	t_{67}	−.433
z_9	−.860	t_{20}	+.401	t_{44}	+.090	t_{68}	−.252
z_{10}	−.429	t_{21}	+.379	t_{45}	−.253	t_{69}	−.001
z_{11}	−.155	t_{22}	+.927	t_{46}	−.573	t_{70}	+.578
z_{12}	+.338	t_{23}	+.551	t_{47}	−.540	t_{71}	+.660
z_{13}	+.748	t_{24}	+.165	t_{48}	−.197	t_{72}	+.663
z_{14}	+.942	t_{25}	−.208	t_{49}	−.043	t_{73}	+.684
z_{15}	+.865	t_{26}	−.430	t_{50}	+.633	t_{74}	+.179
z_{16}	+.667	t_{27}	−.482	t_{51}	+.359	t_{75}	−.227
t_4	+.136	t_{28}	−.283	t_{52}	+.952	t_{76}	−.469
t_5	−.350	t_{29}	+.016	t_{53}	+.566	t_{77}	−.409
t_6	−.497	t_{30}	+.410	t_{54}	+.226	t_{78}	−.319
t_7	−.430	t_{31}	+.591	t_{55}	−.231	t_{79}	+.039
t_8	−.283	t_{32}	+.986	t_{56}	−.568	t_{80}	+.657
t_9	−.228	t_{33}	+.479	t_{57}	−.475	t_{81}	+.159
t_{10}	+.473	t_{34}	+.039	t_{58}	−.233		
t_{11}	+.343	t_{35}	+.010	t_{59}	+.174		

in Fig. 1*a*, and that the curve connecting the black dots in Fig. 5 represents a variation which is quite similar to that given in Fig. 1*b*. Hence, on the nature of the curves in Fig. 5, discussion is made in the same way as was done concerning the curves in Figs. 1*a* and 1*b*. Fig. 5 shows that for the years 1919–1927 the annual variation produced by the diurnal variation has a secondary minimum in the first quarter of a year, while for the years 1911–1918 there is no such minimum. Such difference in the features of variations is supposed to be due to the difference in the observed groups of latitude stars, viz., the existence of the secondary minimum and the secondary maximum in this variation probably depends upon the number of observations of the evening groups compared with that of the morning groups during the period in question.

The circles and dots in Fig. 5 were plotted purposefully in the same way as the smoothed values of latitude variation, and the variation represented by these curves are evidently just the reverse in phase of the Kimura term. It should be remembered that this new correctional term σs was of course not used in drawing the curve from which the "smoothed values" of the latitude variation were taken. Hence, in the "smoothed values," there must remain an annual term due to this avoidance of the correctional term. Accordingly, the Kimura term determined in Chapter II must contain an annual term caused by the diurnal variation of latitude, and so if this correctional term is properly added to τ's, the range of the Kimura term will be decreased by an amount of about 0.″02.

Table XIX

Annual variation produced by sin $(\odot-\alpha)$

(unit 0.″001)

1911–1918		1919–1927	
epoch	σs	epoch	σs
.054	+ 5	.035	+ 9
.179	+ 8	.163	+ 8
.279	+11	.277	+14
.358	+11	.365	+10
.438	+ 7	.440	+ 2
.495	− 2	.502	− 4
.565	−10	.574	− 9
.669	− 9	.670	− 8
.759	− 7	.780	− 4
.923	− 1	.908	− 1

Dr. Jones has deduced a term of the same nature. But the range of his term was 0.″015, whereas the range shown in Table XIX is more than 0.″02.

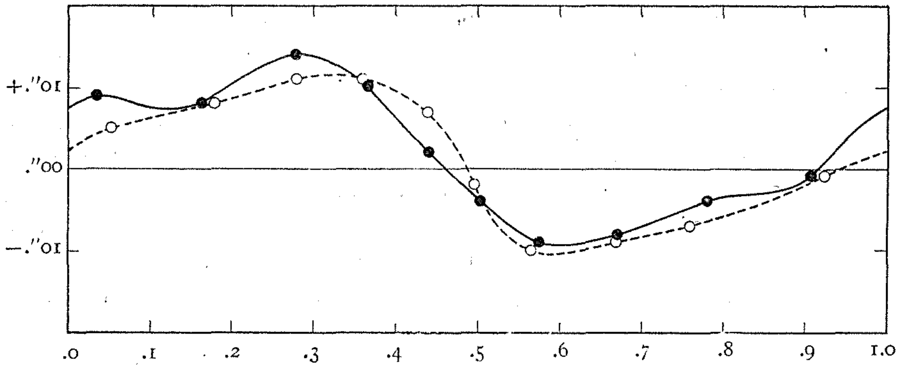


Fig. 5

Annual variation caused by sin (⊙−α)

IV. Lunar Semi-Diurnal Variation

1. Variation in the results of the International Latitude Service.

Though the longitude determinations or the meridian circle observations were sometimes utilized for finding the tidal variations in the direction of the vertical referred to the earth's axis, the latitude observations furnish the most suitable data for this purpose. Hence with the appearance of the publications containing the results of the International Latitude Service, harmonic analysis was applied to those data for finding the Lunar semi-diurnal tide. In "Rapports Généraux Établis à l'Occasion de la Sixième Assemblée Générale, Édimbourg, 16–26 Septembre 1936," Dr. Lambert gave a table in which the results of various determinations were collected. That table is reproduced here instead of making an historical sketch of this branch of study. "These results have been put in the form

$$a\cos(2t-\alpha)$$

where t is the hour-angle, mean or true, of the moon and where α is the difference (in the sense of a lag when α is positive) in phase between the tidal variation of latitude and the tide-producing forces." This explanation is quoted from the same report. Dr. Stetson has published many papers[1] in which he discussed the semi-diurnal and diurnal variations in the latitude observations correlated with the moon's hour-angle and declination. His data were taken from the same sources as those which were used by the authors mentioned in table XX. But the amplitudes of the variations detected by him were much larger.

1. e.g. Trans. Amer. Geophys. Union (1932), 85. Nature, **131**, 437, 1933.

Table XX

Various Determinations of the Tidal Effect on the Variation of Latitude
(International Stations on 39th Parallel)

Station	a	α	Period Covered	Author
Carloforte	0".00878	− 4°	1900–08 incl.	Shida & Matsuyama
	.0054	− 2	1900.0–1911.9	Przybyllok (1)
	.0065	− 3.1	1900.0–1912.0	Przybyllok (2)
	.0118	+ 26	1912.0–1922.7	Nisimura
	.0082	− 3	1922.7–1931.0	Kawasaki
Mizusawa	.0077	+ 27.7	1900.0–1912.0	Przybyllok (2)
	.0032	− 100	1912.0–1922.7	Nisimura
	.0114	− 11	1922.7–1931.0	Kawasaki
Ukiah	.0024	+ 7.5	1900.0–1912.0	Przybyllok (2)
	.0105	+ 11	1912.0–1922.7	Nisimura
	.0123	− 21	1922.7–1931.0	Kawasaki
Gaithersburg	.0074	+ 20.5	1900.0–1912.0	Przybyllok (2)
Theory— All Stations	.0077 $\times(1+k-l)$	0	——	——

Thus, nearly all available data have already been fairly well discussed. But all of those data were limited to the results of the International Latitude Service. Hence, in the following, the same subject is discussed and an analysis is applied to the observations taken at Greenwich.

2. Variation in the latitude observations at Greenwich. All known systematic terms were first subtracted from the "corrected residuals" for each plate. $\zeta_p+\sigma_p s$ and $\tau_{q'}+\sigma_{q'} s$ were respectively adopted as the group correction and latitude variation. But, $\tau_{q'}$ and $\sigma_{q'} s$ were treated separately. The daily values of the former were read off directly from the smooth curve given in the original publications, and the daily values of the latter were taken from a curve which was newly drawn by connecting the values of $\sigma_q s$, which had been plotted against each value of q. The numerical values of $s \sin(\odot_{pq'}-\alpha_p)$ were also calculated for each individual residual. After making subtractions of all these values, the remainder, Δv, which is represented by the formula:

$$\Delta v_{pq'}=(\text{corrected residual})_{pq'}-z_p-t_{q'}-s\sin(\odot_{pq'}-\alpha_p)$$

was obtained. Then all Δv's were arranged with respect to the hour-angle of the moon, and their mean values were taken as shown in Table XXI. The harmonic analysis was applied to the results given in this table, the result being

$0.''0108\cos(2t-9.^{\circ}3)$.

In this investigation, the hour-angle of the real moon was used. So the amplitude here obtained may not be the same as that which would have resulted from the adoption of the hour-angle of the mean moon, but the difference will be very small.

The theoretical mean amplitude of the Lunar Semi-diurnal tide for Greenwich is practically the same as that given in Table XX, because the mean latitude of Greenwich is $51^{\circ} 28' 38''$. Hence, it is ascertained that the amplitude deduced from the observations is quite close to the theoretical one, and that the observed time lag is very small.

The results of the present discussion are shown graphically in Fig. 6.

Table XXI

Lunar Semi-Diurnal Tide
(unit $0.''0001$)

hour-angle	variation
$0^h - 12^h$	+102
1 - 13	+195
2 - 14	+100
3 - 15	− 34
4 - 16	− 80
5 - 17	− 48
6 - 18	−141
7 - 19	− 62
8 - 20	− 68
9 - 21	− 1
10 - 22	+ 66
11 - 23	− 28

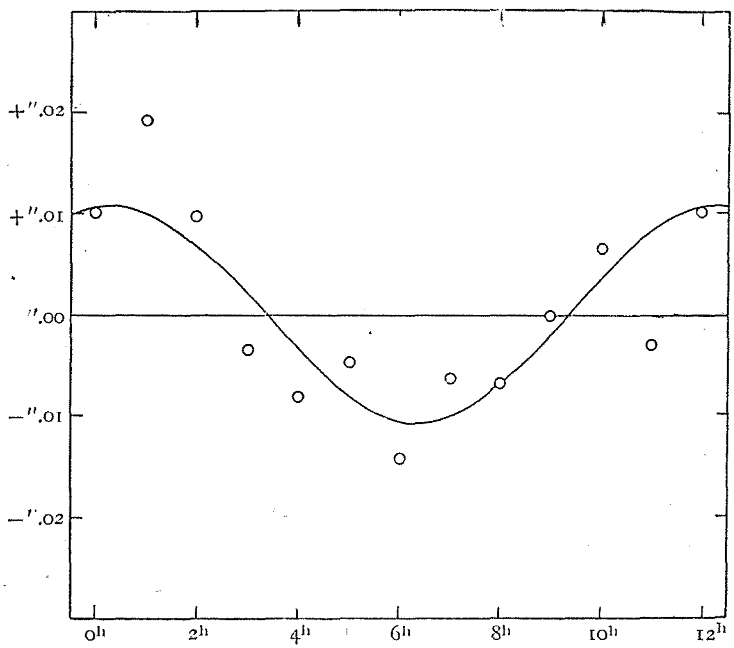


Fig. 6

Lunar semi-diurnal variation

V Wind Correlations with Latitude Variation

1. Direction of the wind and plate residual. The wind correlation with the latitude observations at Greenwich was for the first time noticed by the astronomers of that observatory. In the report of the latitude observations made during the first seven years, they concluded that the correlation between the magnitudes of the plate residuals and the directions of the wind could be represented by the simple harmonic

$$+0''.05\cos(\theta-45^\circ), \qquad (1)$$

where θ is the direction of the wind measured from N. through E., S. and W. This correlation was derived by simply plotting the residuals of each plate on squared paper against the direction of the wind. Nothing was mentioned, however, about such a correlation in the second report of latitude observations made in subsequent years. To make the results of the meridian observations as accurate as possible, research of this kind must thoroughly be done. So the present author attempted to reinvestigate the same problem utilizing the latitude observations taken in the years 1919–1927.

Instead of plotting the plate residuals against the direction of the wind, the remainder $\Delta v_{pq'}$ was first obtained for each plate by making the following subtraction :—

$$\Delta v_{pq'} = (\text{corrected residual})_{pq'} - (z_p + t_{q'} + a_{pq'}\varkappa), \qquad (2)$$

where $(\text{corrected residual})_{pq'}$ means the residual which is given in columns under the heading "corrected residual" of Table I of the original publication. z_p and $\varkappa$ were also taken from the original publication and $t_{q'}$ was read off from the smooth curve representing the observed results of latitude variation. $a_{pq'}$ is not published but the author was kindly allowed to find it in the original data. It has already been mentioned that $\varkappa$ should not be regarded as the correction to the assumed aberration constant; but as discussed in the foregoing chapters, there can be no serious error even if $a\varkappa$ is adopted as a term representing a systematic variation which exists in the latitude observations.

At Greenwich the wind is observed by means of a self-registering apparatus which is placed at the top of a 40 foot tower, located west of the zenith telescope hut at a distance of about 60 feet. By reading the records anew, the author found the direction of the wind at the time of the observations of latitude. The mean velocity of the wind at that time was calculated as well. Then all Δv's were classified in eight groups according to the direction of the wind, and the mean

Table XXII

Correlation between the Direction of the Wind and Latitude Variation
(unit 0.″001)

direction of wind	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
Δv_{mean}	+83	+47	+36	+16	−23	−40	−13	+27
probable error	± 9	±11	± 5	± 6	± 7	± 3	± 3	±10

values for each group were obtained. These results are given in Table XXII, together with their probable errors, which were found by calculating the deviation of each Δv from their mean value. If the mean values should be represented by an expression of the form :—

$$A+B\cos(\theta-C),$$

where θ has the same meaning as that of (1), the numerical values of these three constants can be found by giving suitable weights and forming normal equations. The solution is

$$A=+0.''015\pm0.''003,$$
$$B=+0.''052\pm0.''004,$$
$$C=+\ 34^\circ\ \pm\ 4^\circ. \qquad (3)$$

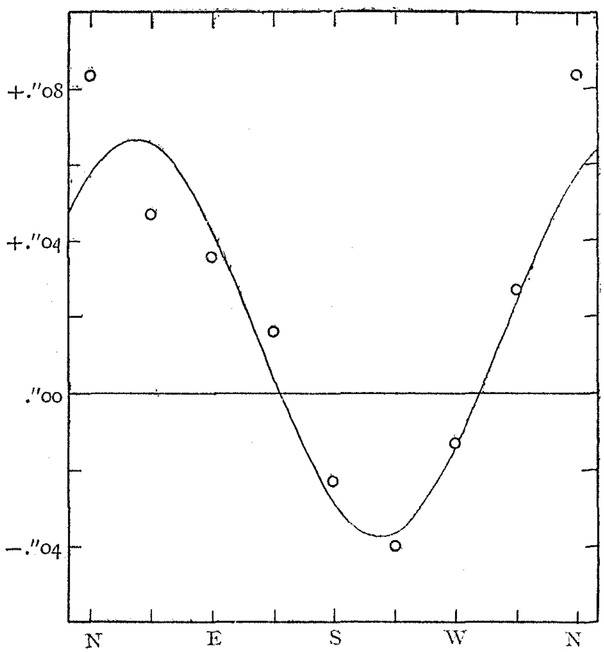


Fig. 7

Direction of the wind and latitude variation

In Fig. 7, a curve is drawn employing these values and the small circles in it represent the results given in Table XXII.

The positive value of A means that at Greenwich S., S.W. and W. winds predominate throughout the year. Comparison of the values of B and C with those of the expression (1) shows that they are in good agreement respectively; and these statistical investigations suffice to show beyond doubt that the latitude observations at Greenwich have a close connection with the direction of the wind.

2. Frequency of the wind and the annual term. Generally the mean direction of the wind makes a systematic change during the course of a year. Therefore, if the observed latitude is affected by the change of the direction of the wind, it must naturally follow that a systematic term, which is independent of the real variation of the terrestrial pole, becomes included in the observed variations of latitude. In the following, the Kimura term at Greenwich is considered from this point of view.

It is easily seen that the correlation between the direction of the wind and the corresponding wind factor in the observed latitude is represented by an expression of the form :—

$$U_i = G\cos\theta_i + H\sin\theta_i, \qquad (4)$$

where θ_i represents as before the direction of the wind measured from north to eastward, i being changed from 0 to 7, so that in the case of the N.E. wind $i=1$, accordingly $\theta_1=45^\circ$, and so on. Then the variation of latitude due to the variation in the direction of the wind during any tenth of a year is obtained by the formula :—

$$\boldsymbol{U}_j = \frac{\sum_{i=0}^{7} U_i N_{i,j}}{\sum_{i=0}^{7} N_{i,j}}, \qquad j=0, 1, \ldots\ldots, 9, \qquad (5)$$

where $N_{i,j}$ is the number of latitude observations made when the wind was blowing from the direction corresponding to i during the interval of the jth tenth of a year.

From (4) and (5) we get

$$\boldsymbol{U}_j = GQ_j + HR_j, \qquad j=0, 1, \ldots\ldots, 9, \qquad (5')$$

in which

$$Q_j = \frac{\sum_{i=0}^{7} N_{i,j}\cos\theta_i}{\sum_{i=0}^{7} N_{i,j}},$$

$$R_j = \frac{\sum_{i=0}^{7} N_{i,j}\sin\theta_i}{\sum_{i=0}^{7} N_{i,j}}.$$

Since G and H are both constant, (5′) means that $\boldsymbol{U}$ varies according to the variations of Q_j and R_j. Moreover, Q_j and R_j are the functions of $N_{i,j}$, or of frequencies of each direction of the wind.

Therefore it follows that so long as the expression (4) exists, the variation of latitude due to the variation of the direction of the wind is simply derived by counting the frequencies of the wind for each direction. Practically, to find the accurate values of G and H is rather difficult, and accordingly the determination of the range of the variation of $\boldsymbol{U}$ is also difficult. But as we can find the numerical values of the Q_j's and R_j's fairly accurately, the period and the phase of $\boldsymbol{U}$ are comparatively accurately determined, even when absolute values of G and H are lacking and only relative values are known.

Returning to the observations at Greenwich, the numerical values of $N_{i,j}$ during the years 1919.6–1927.1 are shown in Table XXIII, where the figures in the first line represent the direction of the wind and those in the first column, the tenth of a year. From this table the numerical values of Q_j and R_j are derived. Their results are tabulated in Table XXIV. It is easily noticed in this table that both Q_j and R_j vary exhibiting two annual maxima and minima. Therefore, even if the Kimura term at Greenwich were not yet known, it

Table XXIII

Numerical Value of N

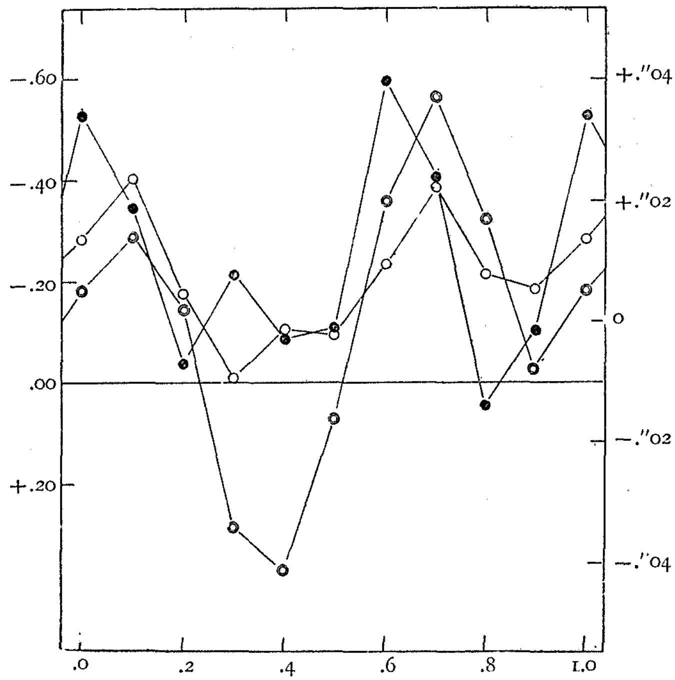
j \ i	0	1	2	3	4	5	6	7
0	7	1	3	8	18	48	37	15½
1	6	6	6	12	12	58	16½	4
2	14½	11	27½	10½	15	44	10½	9
3	12½	17	16½	5½	5	27	37	3½
4	11	29½	21½	16	6½	54	24	8½
5	17	13½	27	19	11½	34	38	9
6	5	5	3	4	10	53	45	13½
7	9	7½	4	12	30½	71	33	9
8	18½	12	27½	24½	19½	44½	8	9
9	13½	13	13½	12½	15½	36½	14	6

Table XXIV

Numerical Values of Q and R

j	0	1	2	3	4	5	6	7	8	9
Q	—.283	—.402	—.175	—.008	—.106	—.095	—.233	—.389	—.214	—.186
R	—.528	—.345	—.037	—.211	—.085	—.109	—.597	—.408	+.046	—.101

could be anticipated that the wind effect must introduce in the latitude observations a variation of a semi-annual type. But the Greenwich Kimura term for the years 1919–1927 is already given in Table V, and it really has features similar to those anticipated. Comparison of this Kimura term with Q_j and R_j is made in Fig. 8, where the scale for Q_j and R_j is marked at the left-hand side and the Kimura term



○ represents the values of *Q*.
● " " *R*.
◎ " " Kimura term.

Fig. 8

is measured by that on the right-hand side. It is very clear that all of these three quantities fluctuate similarly; the period and the phase being common to all, and the maximum occurring near 0.0 or 0.1 and 0.6 or 0.7 of a year. Thus it is concluded that at least some part of the Kimura term, though of course not the whole of it, is caused by the seasonal variation of the mean direction of the wind.

Not only in the case of the latitude, but in any other meridian observation made at Greenwich, a correlation between the direction of the wind and the astronomical data may exist. If so, it will probably be able to represent that correlation by a simple harmonic resembling (4). Then a supposition can be made again that in that meridian observation, there must exist a term which has the same nature as the Kimura term, though its range will be very small.

3. Numerical representation of the effect of the wind. Numerical values of G and H are considered in the following :—

There would be no great mistake even if these values were derived directly from B and C, whose values are given in (3). And yet there remains a question about the nature of B and C. The greatest trouble is this. Both B and C were determined by analysing $\Delta v_{pq'}$. $\Delta v_{pq'}$ is a remainder obtained by subtracting $t_{q'}$ from the corrected residual, while $t_{q'}$ is the observed variation of latitude containing a variation due to the effect of the wind. This means that from the data, from which the effect of the wind on the latitude observations is to be derived, some part of the effect is already rejected. To answer this question, the following method was adopted :—

Let Δv_{pq} be the remainder corresponding to the epoch j and the wind direction i, then we can say

$$\Delta v_{pq} \equiv \Delta v_{i,j},$$

and

$$\Delta v_{i,j} = \Delta v + U_i - \boldsymbol{U}_j,$$

where Δv is a certain constant common to all $\Delta v_{i,j}$. Accordingly Δv_{mean} of Table XXII is written down in the form

$$\frac{\sum_{j=0}^{9} \Delta v_{i,j} N_{i,j}}{\sum_{j=0}^{9} N_{i,j}}, \qquad i = 0,\ 1,\ \ldots\ldots,\ 7,$$

which becomes

$$\Delta v + \beta_i G + \gamma_i H,$$

where

$$\beta_i = +\frac{\sum_{j=0}^{9} (\cos\theta_i - Q_j) N_{i,j}}{\sum_{j=0}^{9} N_{i,j}},$$

$$\gamma_i = +\frac{\sum_{j=0}^{9} (\sin\theta_i - R_j) N_{i,j}}{\sum_{j=0}^{9} N_{i,j}}.$$

The numerical values of Q_j, R_j and $N_{i,j}$ being known, those of β_i and γ_i are also known, and thus eight equations of condition are obtained for eight values of Δv_{mean} in Table XXII. By giving suitable weights to them, normal equations are formed and the solution of those equations gives

$$\begin{aligned} \Delta v &= -0.''0006 \pm 0.''0024 \\ G &= +0.0455 \pm 0.0046 \\ H &= +0.0324 \pm 0.0034. \end{aligned} \qquad (6)$$

Δv is negligibly small as expected, and from these values of G and H, we can get the expression

$$U = +\underset{(\pm 0.''004)}{0.''056}\cos(\theta - \underset{(\pm 4^\circ)}{36^\circ}). \qquad (6')$$

Though the difference are not great, these values will be more reasonable than those of B and C.

4. Strength of the wind and latitude variation. The present data are not enough to give any definitive idea as to the correlation between the strength of the wind and the latitudes observed at Greenwich. Yet Table XXV suggests that the stronger wind probably gives the greater effect on the observed latitudes. In computing the results given in this table, all observations corresponding to the same direction of the wind were separated into two groups according to the strength of the wind. Different criteria were chosen for different directions of the wind, so that both of the two groups in each direction might contain nearly the same number of observations. These

Table XXV

Wind Strength and Observed Latitude

(unit 0.''001)

direction of wind		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
criterion		11	10	7	7	9	10	12	12
Δv_{mean}	strong wind	+110	+62	+44	+37	−16	−33	−21	+25
	weak wind	+ 62	+36	+31	− 1	−29	−47	− 7	+28
probable error		± 13	±14	± 7	± 8	±10	± 4	± 4	±14

criteria are given in the second line of the table: for example, in the case of N. wind, the latitude observation corresponding to the wind strength of, or greater than, 11 miles per hour is put into the group "strong wind." Δv_{mean}'s of this table were computed in the same way as those given in Table XXII. Probable errors were obtained by simply multiplying those given in Table XXII by $\sqrt{2}$, because the weight of the results in this table is always nearly half of that of Δv_{mean}'s given in Table XXII.

5. Wind factor in the conditional equation. It was ascertained that the direction of the wind had a close correlation with the latitude observations, and this correlation was expressed by the simple harmonic

$$U=+0.''056\cos(\theta-36^\circ). \qquad (6')$$

In the previous paragraphs, (6′) was regarded as representing the effect of the wind on the observed latitude. But, strictly speaking, U is not the variation in the final results of observed latitude. It is only the wind factor in the corrected residual from which the group correction, latitude variation and a constant $\varkappa$ are to be derived. Accordingly, to examine fully to what amount the final value of latitude at Greenwich is affected by the wind, some more calculations should be made.

We are given

$$(\text{mean residual})_{pq}\equiv v_{pq}=z_p+t_q+a_{pq}\varkappa. \qquad (7)$$

If the total effect of the wind gave to v_{pq} a variation of an amount dv_{pq}, then the corresponding variation in z_p, t_q and $\varkappa$ will be given by the expression:

$$dv_{pq}=dz_p+dt_q+a_{pq}d\varkappa. \qquad (8)$$

On the other hand, by modifying the expression (5′), we get

$$dv_{pq}=Q_{pq}G+R_{pq}H, \qquad (9)$$

where

$$Q_{pq}=\frac{\sum_{i=0}^{7}N_{i,pq}\cos\theta_i}{\sum_{i=0}^{7}N_{i,pq}}, \qquad R_{pq}=\frac{\sum_{i=0}^{7}N_{i,pq}\sin\theta_i}{\sum_{i=0}^{7}N_{i,pq}}.$$

The numerical values for G and H are given by (6), and those for Q_{pq} and R_{pq} can be easily calculated as the direction of the wind at the time of the zenith telescope observation is already known. Hence, by comparing (8) and (9), the necessary conditional equation for the three unknown quantities, dz_p, dt_q and $d\varkappa$ is obtained.

6. Wind factor in the normal equation. Suitable weight being given to each of the equations of condition of the form (8), a system of normal equations for the dt's, dz's and $d\varkappa$ is formed. Then, by subtracting dv_{pq} from v_{pq}, we get a remainder v'_{pq} which is a factor quite independent of the effect of the wind. These factors furnish us with data for forming another system of normal equations for t''s, z''s and $\varkappa'$, whose meaning are given by expressions:

$$t'_q=t_q-dt_q,$$
$$z'_p=z_p-dz_p$$

and $$\varkappa'=\varkappa-d\varkappa.$$

Table XXVI

Constant Terms in Normal Equations

	c	dc		c	dc
	″	″		″	″
t_4	+0.327	−0.141	t_{52}	+2.446	+0.225
t_5	+0.028	+0.078	t_{53}	+3.839	−0.771
t_6	−0.096	−0.353	t_{54}	+1.461	−0.256
t_7	−0.245	−0.700	t_{55}	+2.757	−0.397
t_8	+2.019	−0.477	t_{56}	+2.230	−0.441
t_9	+1.580	−0.313	t_{57}	+0.651	−0.749
t_{10}	+0.670	−0.341	t_{58}	+0.415	−0.088
t_{11}	+0.641	−0.437	t_{59}	+2.171	−0.654
t_{12}	−0.973	−0.473	t_{60}	+0.146	−0.982
t_{13}	−1.683	−0.763	t_{61}	+1.208	−0.568
t_{14}	−0.870	−0.018	t_{62}	+1.254	−0.251
t_{15}	−0.670	−0.250	t_{63}	+1.133	−0.746
t_{16}	−2.697	−0.633	t_{64}	+1.469	+0.491
t_{17}	−1.158	−0.119	t_{65}	+0.263	−0.105
t_{18}	−1.535	+0.488	t_{66}	+0.820	−0.615
t_{19}	+1.907	−0.294	t_{67}	+1.169	−0.588
t_{20}	+1.343	−0.110	t_{68}	+0.465	−0.432
t_{21}	+2.603	−0.716	t_{69}	+2.495	−0.447
t_{22}	+0.283	+0.101	t_{70}	+3.187	−1.055
t_{23}	−0.750	+0.836	t_{71}	+0.830	−0.618
t_{24}	−3 509	+0.625	t_{72}	+0.517	−0.244
t_{25}	−4.214	+0.436	t_{73}	+0.491	+0.276
t_{26}	−1.251	−0.665	t_{74}	+0.904	−0.643
t_{27}	−2.687	−0.522	t_{75}	+0.370	+0.262
t_{28}	−1.553	−0.253	t_{76}	+0.386	+0.213
t_{29}	+2.356	+0.046	t_{77}	−0.443	−0.685
t_{30}	+4.052	−0.434	t_{78}	−0.135	−0.572
t_{31}	+3.159	−0.474	t_{79}	+0.730	−0.373
t_{32}	−0.437	+0.301	t_{80}	−0.165	−1.152
t_{33}	+2.659	−0.467	t_{81}	−0.238	−0.138
t_{34}	+0.819	+0.069	z_1	+6.001	−2.004
t_{35}	−0.228	−0.220	z_2	+8.601	−2.237
t_{36}	−3.538	−0.697	z_3	+6.020	−2.065
t_{37}	−6.317	−0.420	z_4	+10.476	−1.281
t_{38}	−2.795	−0.193	z_5	+4.646	−1.632
t_{39}	+1.565	+0.104	z_6	+0.888	−0.327
t_{40}	+4.111	−0.290	z_7	+3.466	−0.765
t_{41}	+3.777	−0.353	z_8	+3.412	−0.913
t_{42}	+3.361	−0.094	z_9	+8.332	−1.889
t_{43}	+4.852	−0.600	z_{10}	+2.996	−0.843
t_{44}	+1.463	−0.101	z_{11}	+0.970	−0.673
t_{45}	+2.504	−0.228	z_{12}	+3.643	−1.058
t_{46}	−0.188	−0.502	z_{13}	−5.605	−2.939
t_{47}	−3.032	−1.155	z_{14}	−8.478	−2.878
t_{48}	−1.864	−1.014	z_{15}	−4.567	−1.393
t_{49}	+1.129	−0.922	z_{16}	+3.175	−1.992
t_{50}	+1.488	−0.144			
t_{51}	+4.744	+0.016	$\varkappa$	−14.41	+81.20

Of course nothing of the effect of the wind is contained in t''s, z''s and x'.

Thus, there are three different systems of normal equation, that is, (i) normal equations for t's, z's and x, which are given on pages 16 to 18 of the original publication, (ii) normal equations for dt's, dz's and dx, and (iii) those for t''s, z''s and x'.

Close relations naturally exist among the corresponding equations of these three systems. For example, as the equation for t_4, we are given

$$19t_4+7z_{10}+6z_{11}+6z_{12}-5.5x=+''.186 \qquad (=c),$$

for dt_4, we have

$$19dt_4+7dz_{10}+6dz_{11}+6dz_{12}-5.5dx=-''.141 \qquad (=dc),$$

and for t_4',

$$19t_4'+7z_{10}'+6z_{11}'+6z_{12}'-5.5x'=+.''327 \qquad (=c').$$

In this way the constant term, c, is analysed into two; one, dc, being entirely attributable to the effect of the wind, and the other, c', having nothing to do with the direction of the wind.

In Table XXVI are given the numerical values for c' and dc. In some cases, it will be found that $c'+dc$ is not exactly equal to c, which is given in the original publication. This is because minor corrections were applied to some of the data in forming these normal equations.

7. Wind factor in group correction. Solutions for the z''s and dz's were obtained by the usual procedure and are given in Table XXVII. Thus for example

$$z_1'=+.''0640+8.66x',$$

and

$$dz_1=-.''0020+8.66dx.$$

The former is quite free from the effect of the wind and the latter is an amount produced by this effect.

dz's are very small, but it may be worth noting that, with the exception of dz_5, which is somewhat abnormal, they seem to bear some connection with the mean right as-

Table XXVII

Group Corrections

	z'	dz	coef. of x and dx
	$''$	$''$	
z_1	+.0640	−.0020	+ 8.66
z_2	+.0699	+.0016	+ 2.12
z_3	+.0471	+.0037	− 0.57
z_4	+.0152	+.0038	− 2.69
z_5	−.0431	−.0002	− 5.19
z_6	−.1028	+.0057	−11.55
z_7	−.0496	+.0032	−14.33
z_8	−.0510	+.0004	− 9.24
z_9	−.0068	−.0069	−13.02
z_{10}	−.0237	−.0048	− 6.33
z_{11}	−.0312	−.0029	− 2.40
z_{12}	+.0412	−.0020	+ 5.21
z_{13}	+.0165	−.0013	+11.37
z_{14}	+.0104	+.0013	+14.29
z_{15}	+.0088	+.0012	+13.27
z_{16}	+.0351	−.0009	+10.40

cension of the star group and that in them there probably exists a variation which takes two maxima between 0^h and 24^h.

8. Wind factor in latitude variation. By substituting these values of the z''s in the normal equations for the t''s, solutions for latitude variations are obtained in term of x'. Solutions for the dt's were also derived in the same way. They are collected in Table XXVIII. We

Table XXVIII

Latitude Variation

	t'	dt	coef. of x' and dx		t'	dt	coef. of x' and dx
	"	"			"	"	
t_4	+.023	−.004	+ 1.7	t_{43}	+.277	−.027	+ 6.9
t_5	.000	+.028	− 4.9	t_{44}	+.160	−.007	+ 0.9
t_6	−.027	−.022	− 5.5	t_{45}	+.107	−.009	− 3.1
t_7	−.035	−.028	− 2.8	t_{46}	−.036	−.041	− 6.7
t_8	+.047	−.020	− 0.0	t_{47}	−.122	−.035	− 4.5
t_9	+.088	−.021	− 1.1	t_{48}	−.099	−.041	+ 1.1
t_{10}	+.067	−.041	+ 4.9	t_{49}	+.016	−.036	+ 1.8
t_{11}	+.038	−.012	+ 1.0	t_{50}	+.093	−.015	+ 7.3
t_{12}	−.043	−.038	+ 5.1	t_{51}	+.141	.000	+ 1.5
t_{13}	−.020	−.018	+ 6.0	t_{52}	+.210	+.011	+11.0
t_{14}	−.128	+.001	− 0.1	t_{53}	+.192	−.029	+ 6.6
t_{15}	−.106	−.034	− 3.4	t_{54}	+.175	−.025	+ 3.2
t_{16}	−.163	−.034	−10.1	t_{55}	+.150	−.021	− 2.9
t_{17}	−.141	−.011	− 4.1	t_{56}	+.118	−.027	− 6.8
t_{18}	−.077	+.013	+ 1.8	t_{57}	+.009	−.044	− 3.4
t_{19}	+.087	−.017	− 0.8	t_{58}	−.015	−.005	+ 0.5
t_{20}	+.175	−.017	+ 3.4	t_{59}	+.041	−.024	+ 4.3
t_{21}	+.137	−.031	+ 1.5	t_{60}	+.007	−.049	+ 4.1
t_{22}	+.095	+.008	+10.4	t_{61}	+.059	−.019	+ 3.8
t_{23}	−.004	+.038	+ 6.4	t_{62}	+.121	−.014	+ 6.5
t_{24}	−.176	+.036	+ 1.7	t_{63}	+.091	−.037	+ 7.1
t_{25}	−.170	+.019	− 2.3	t_{64}	+.055	+.020	− 0.6
t_{26}	−.106	−.044	− 4.3	t_{65}	+.069	−.028	− 5.9
t_{27}	−.147	−.023	− 3.6	t_{66}	+.016	−.029	− 3.1
t_{28}	−.079	−.009	− 0.1	t_{67}	+.019	−.023	− 2.8
t_{29}	+.096	+.003	+ 2.0	t_{68}	−.012	−.024	+ 0.2
t_{30}	+.203	−.025	+ 4.1	t_{69}	+.027	−.010	+ 2.1
t_{31}	+.153	−.020	+ 4.7	t_{70}	+.083	−.042	+ 7.0
t_{32}	+.044	+.014	+11.7	t_{71}	+.054	−.030	+ 5.7
t_{33}	+.128	−.014	+ 5.4	t_{72}	+.068	−.010	+ 6.7
t_{34}	+.053	+.007	− 0.2	t_{73}	+.101	+.036	+ 8.2
t_{35}	−.038	−.042	+ 0.8	t_{74}	+.067	−.035	+ 2.2
t_{36}	−.190	−.033	− 4.3	t_{75}	+.006	+.013	− 2.8
t_{37}	−.275	−.016	− 2.3	t_{76}	−.004	+.011	− 5.1
t_{38}	−.117	−.008	+ 0.7	t_{77}	−.041	−.028	− 2.5
t_{39}	+.067	+.007	+ 3.1	t_{78}	−.044	−.031	− 0.6
t_{40}	+.120	−.014	+ 7.5	t_{79}	+.009	−.021	+ 2.9
t_{41}	+.235	−.022	+ 4.6	t_{80}	−.037	−.054	+ 7.4
t_{42}	+.252	−.008	+ 8.8	t_{81}	−.034	−.032	− 0.9

see that, if there were no wind, the real variation of latitude at the epoch $q=4$, for example, is

$$t'_4=+.''023+1.7z',$$

while the amount of variation, which is caused only by the effect of the wind, is

$$dt_4=-.''004+1.7dz.$$

Of course the latter one is independent of the motion of the pole of the earth.

The latitude variations given in the original publication from Greenwich are the sums of these two sorts of variation.

dz is a very small quantity, and so in making statistical investigations on the values of the dt's, their second terms can be neglected.

If the yearly mean values of the dt's are taken after giving suitable weight to each of them, then the results given in Table XXIX are obtained. In this table, 1919 begins in reality with the epoch 1919.450; and 1926 contains the extra epochs 1927.044 and 1927.092. Hence, mean dt corresponding to the year 1919 is the mean of only six values, while the same corresponding to the year 1926 is the result from twelve values. Accordingly, these two are influenced by a certain seasonal variation.

The numerical values which are given in the last column of this table were found by subtracting the algebraic mean value $+0.''016$ from each of those in the foregoing column. In them, it is noticed that if the mean latitude at Greenwich is calculated from the results of zenith telescope observation, there is contained in it an irregular variation, which is attributable only to the effect of the wind, the range of which is more than $0.''02$.

The dt's were then separated into ten groups according to their corresponding

Table XXIX

Yearly Mean Value of dt

year	mean dt	
	"	"
1919	+.018	+.002
1920	+.015	—.001
1921	.000	—.016
1922	+.012	—.004
1923	+.025	+.009
1924	+.016	.000
1925	+.018	+.002
1926	+.021	+.005
mean	+.016	

Table XXX

Seasonal Variation of dt

epoch	mean dt	
	"	"
.034	+.034	+.019
.163	+.017	+.002
.277	+.005	—.010
.365	+.011	—.004
.440	—.004	—.019
.502	+.002	—.013
.574	+.026	+.011
.670	+.027	+.012
.780	+.014	—.001
.908	+.016	+.001
mean	+.015	

periods of observation, and mean values for each groups were taken. In Table XXX, these results are arranged together with the mean epochs represented with fractions of a year. Deviations of these ten values from their simple mean value $+0.''015$ are also given in the last column. These deviations show an annual variation of latitude which does not depend on the motions of the terrestrial axis. Hence this must be a component of the Kimura term at Greenwich, and what should be remembered is that this variation of latitude arises only from the effect of the wind. The range of this variation is about half that of the Kimura term given in the last column of Table V. But the maxima and minima of the present variation coincide very closely with those of the Kimura term. Hence it must be concluded that some part of the Greenwich Kimura term is caused by the effect of the wind.

VI. Conclusion

We have discussed in this paper the minor variations of latitude observed at Greenwich with the Cookson floating zenith telescope in the years 1911–1927. In the first place, the Kimura term was derived by comparing the "smoothed values" of latitude variation at Greenwich with the x-components of the motions of the terrestrial axis which had been computed by Profs. Mahnkopf and Kimura. The newly derived Kimura term was found to have a large range of variation. It is about twice that determined by Prof. Kimura from the observations taken in the years 1919–1927. Generally the Kimura term for the first period, 1911–1918, has been thought to have no secondary maximum and minimum, while the same for the second period, 1919–1927, has two remarkable maxima and minima in a year. The same conclusion was reached in this paper. But, by analysing the Kimura term for the first period, we were able to point out the existence of a semi-annual variation in it. So it is concluded that the semi-annual variation, which, according to the present author's opinion, is correlated with the seasonal variation of the direction of the wind, always exists in the Greenwich Kimura term. Though the phase of the annual variation of the Kimura term varies year by year, the phase of the semi-annual variation seems to remain nearly constant in any period of observation.

Seven years periodicity was noticed in the Greenwich Kimura term. This periodicity is closely correlated with the yearly range of the variation in x. This fact suggested the existence of the Chandler period in the Greenwich Kimura term, and at last the Chandler variation, which had an unexpectedly large range of about 0.″03, was found existing there. Physical interpretation of this variation is postponed until farther investigation of this subject can be made on the observations taken at other observatories. But, as there is little doubt about its existence, full attention should be paid to it, in case any attempts are made to explain the cause of the Kimura term.

The "smoothed values" of latitude variation, which are given in the original publications, are regarded as the definitive values for variation of latitude at Greenwich. They were read off from a smooth curve drawn so that it might represent the observed values as closely as possible. In this paper, we discussed the nature of this curve with respect to two points: viz., (i) the negligence of a correctional term, and (ii) the difficulty in drawing such a curve properly.

Solutions, obtained by solving normal equations for observed values of latitude variation, contained a correctional term of the form $k_q\varkappa$. But this term was neglected in drawing the smooth curve. $\varkappa$ takes indeed a very small value, and so the latitude itself is not much affected by this negligence. When the Kimura term is discussed, however, this negligence can not be overlooked, because the term $k\varkappa$ varies with a period of one year. The range of this variation is about 0.″015, and its phase is just opposite to that of the Kimura term. Hence it was concluded that the negligence of this correctional term gave rise to an annual term in the smoothed values of latitude variation, and that this annual term contributed to the Kimura term in the Greenwich observations.

On the other hand, it was feared that some part of the secondary variation in the observed latitude could not be well represented by this smooth curve; and investigations on this point showed that, as the consequence of drawing this curve, the range of the variation was very likely made smaller. If the smooth curve were properly drawn, the secondary maximum and secondary minimum would have clearly appeared even in the Kimura term for the first period.

It was found that a term of the form $s\sin(\odot - a)$ represented fairly well the diurnal variation of latitude at Greenwich. The numerical value of s was calculated. Probably 0.″018 is the most reasonable value for it. This diurnal variation causes an annual variation which should be regarded as one composing some part of the Kimura term. Its range is about 0.″02. Other features of this annual variation are quite the same as those of the variation due to the correctional term kz.

Hitherto, several investigators derived the variation of the plumb-line referred to the terrestrial axis from the latitude variation observations. But the data of those discussions were always limited to the results of the International Latitude Service. So the author utilized the observations taken at Greenwich for the same purpose, and deduced the lunar semi-diurnal tide from them. The result is $0.''0108\cos(2t-9^\circ.3)$. This is very near to what might be theoretically expected.

The existence of the correlation between the direction of the wind and the mean residuals of the plate taken in the years 1911–1918 has been ascertained already by Greenwich astronomers. In this paper a similar correlation was found to exist in the observations for the years 1919–1927. The strength of the wind seems to be also correlated with the magnitude of the residuals. So far as discussed in this paper, it is suggested that a stronger wind produces a larger effect on the latitude observations. As a result of the seasonal variation of the direction of the wind, there arises an annual term in the observed latitudes, and it was ascertained that this term contributes to the Kimura term at Greenwich.

It is vey probable that, at any other observatory, not to speak of the case of the zenith telescope observations, variations in the meteorological conditions influence precise astronomical observations of all kinds. Hence it is very important to observe carefully all meteorological phenomena, wherever astronomical observation, especially meridian observation, is carried on.

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International Latitude Station,
Mizusawa, Iwate-ken.