Elastic Property of an Aluminium Rod and its Crystal Grains

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Abstract

Among the elastic properties of aluminium rods composed of crystal grains of various sizes, Young's modulus has the same value independent of the crystal sizes. The value is almost the same that the crystal itself has. On the contrary the elastic limit represented by the elongation per unit length depends upon the crystal size. This seems to be due to the fact that the elastic limit at the inner part of the crystal is much affected by its boundary which is elastically stronger. Under such a consideration, the relation between the elastic limit and the grain number is expressed by a relatively simple formula.

The writer has already studied the relation between the elastic property and the crystal grain size for an aluminium rod[1]. According to that study, the crystal grain size affects the elastic limit but not the Young's modulus: that is, the Young's modulus is independent of the crystal grain size varied by cold-working or annealing and takes a special value proper to that crystal. In the following a further study of these points was made.

Fig. 1

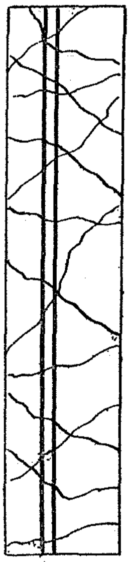
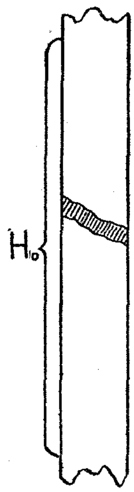


Fig. 2



I. Young's modulus

Consider an aluminium rod composed of many crystal grains as shown in Fig. 1. Let us consider in the rod a thin vertical column bounded by two thick lines as shown in this figure. Then such a column will cut many crystal boundaries. In Fig. 2, one of such boundaries is shown by the shaded part.

1. These Memoirs, A. **17**, 389 (1934).
These Memoirs, A. **20**, 27 (1937).
These Memoirs, A. **20**, 173 (1937).

If the specimen is elongated by applying a load, the original length H_0 will become H. Let the Young's moduli of the crystal itself and the crystal boundary be denoted by E_1 and E_2 respectively. By taking the vertical height of the boundary as uniform for the sake of simplicity, suppose its original height h_0 is elongated to h. Further let the original length of the crystal itself within the length H_0 be denoted by l_0 and the elongated length of l_0 be l.

Let the vertical load applied to the unit cross-sectional area of the rod be denoted by W, then in the case of a small elongation within the elastic limit the following relations hold for the crystal itself and for the boundary respectively.

$$\frac{l-l_0}{l_0}=\frac{1}{E_1}W \qquad \text{................................(1)},$$

$$\frac{h-h_0}{h_0}=\frac{1}{E_2}W \qquad \text{................................(2)}.$$

For the elongation as a whole, we have

$$\frac{(l+h)-(l_0+h_0)}{l_0+h_0}=\frac{(l-l_0)+(h-h_0)}{H_0}=\frac{\dfrac{l_0}{E_1}+\dfrac{h_0}{E_2}}{H_0}W.$$

Let the Young's modulus as a whole be denoted by E, then we get

$$\frac{\dfrac{l_0}{E_1}+\dfrac{h_0}{E_2}}{H_0}W=\frac{1}{E}W.$$

Therefore $$\frac{H_0}{E}=\frac{l_0}{E_1}+\frac{h_0}{E_2} \qquad \text{..............................(3)}.$$

When the thin vertical column cuts many crystal boundaries, we have

$$\frac{H_0}{E}=\frac{\Sigma l_0}{E_1}+\frac{\Sigma h_0}{E_2} \qquad \text{..........................(4)}.$$

When the actual test piece is supposed to be composed of many such thin vertical columns as we have here considered, we may consider that equation (4) holds for any column. Further we may consider that Σl_0 and Σh_0 in any column are equal respectively in an average. Therefore if we consider only one such column as above mentioned, we can infer the elastic property of the rod as a whole.

The writer has examined the Young's moduli of aluminium rods composed of crystals of various sizes extending over a very wide range. According to that experiment, Young's modulus has a constant value independent of the crystal grain size, and it is the same as that of the specimen composed of only a few crystals. Accordingly it can be stated, that even in the case of a rod composed of a number of fine crystal grains, the Young's modulus measured takes the value which is proper to the crystal itself.

If we assume that the Young's modulus of the crystal boundary is equal to that of the inner part of the crystal, the above-mentioned fact must hold as a matter of course. But this assumption is too dogmatic, and it seems to be more natural to consider that the crystal boundary and the inner part of the crystal have in general different Young's moduli. Even in this case, if Σh_0 is negligible in comparison with Σl_0, $E=E_1$ follows from equation (4). According to the writer's experiment, $E=E_1$ holds, as was stated before, even for the specimen composed of very fine crystal grains. Accordingly, even if the Young's modulus at the crystal boundary has a value different from that at the inner part of the crystal, this must be confined to a very thin atomic layer consisting of only a few atoms at the crystal boundary; so that Σh_0 can be neglected as compared with Σl_0.

II. Elastic limit

The elastic limit of an aluminium rod decreases according as the crystal grain becomes larger. This is entirely different from the case of Young's modulus. Now let the elastic limit of the crystal itself represented by the elongation per unit length be denoted by ε_1, and that of the crystal boundary by ε_2. Then it is reasonable to consider that ε_1 is smaller than ε_2. Accordingly, when the elongation of the inner part of the crystal reaches its elastic limit, the elongation of the specimen reaches its elastic limit as a whole, even though the elongation of the boundary is inferior to its elastic limit. By considering such a thin column as was considered before in the case of the Young's modulus, let us suppose that the elongation of the specimen reaches its elastic limit as a whole, then we have evidently

$$\frac{l-l_0}{l_0}=\varepsilon_1 \qquad \text{...(5).}$$

From equation (1), we have

$$W=\varepsilon_1 E_1.$$

From equation (2), we get

$$\frac{h-h_0}{h_0}=\frac{1}{E_2}W=\frac{E_1}{E_2}\varepsilon_1.$$

The elongation per unit length of the column as a whole at its elastic limit is

$$\frac{(l+h)-(l_0+h_0)}{H_0}=\frac{(l-l_0)+(h-h_0)}{H_0}=\frac{l_0\varepsilon_1+h_0\varepsilon_1\dfrac{E_1}{E_2}}{H_0},$$

When the thin column cuts many crystal boundaries, the elastic limit as a whole becomes

$$\frac{1}{H_0}\left\{\Sigma l_0\varepsilon_1+\Sigma h_0\varepsilon_1\frac{E_1}{E_2}\right\} \quad \text{.......................(6).}$$

If the part having a different elastic limit at the crystal boundary is restricted within a very thin atomic layer of the boundary as in the case of the Young's modulus, then the second term in (6) can be neglected in comparison with the first term, and formula (6) is reduced to

$$\frac{1}{H_0}\varepsilon_1\Sigma l_0=\varepsilon_1.$$

Accordingly the value of the elastic limit ought to be independent of the crystal size and must be the same as that of the crystal itself. But this is contradictory to the actual fact, and we must consider that the influence of the crystal boundary which has a higher elastic limit extends from the boundary to a considerable depth in the interior of the crystal, and accordingly that the elastic limit as a whole becomes larger than that of the crystal itself. This is entirely different from the case of the Young's modulus.

We can imagine, either from the presence of the Beilby's amorphous substance at the crystal boundary or simply from the irregular arrangement of the atoms at that point, that the elastic limit is larger at the crystal boundary than in the interior of the crystal itself. Next the fact that the influence of the boundary which has a higher elastic limit extends from the boundary to a considerable depth in the interior of the crystal seems to be easily explained by considering the slip interference of the crystals which is caused by the presence of its boundary.

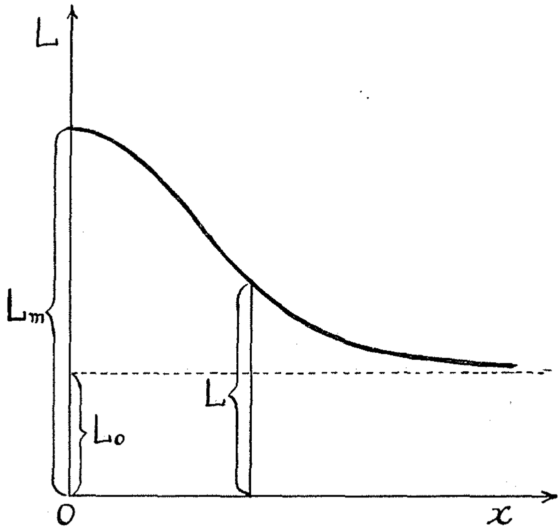
When a metal plate composed of a few large crystals is elongated plastically, the crystal adjacent to the boundary is elongated only slightly, and as we recede from the boundary its elongation becomes gradually larger, so that it is maximum at the middle part of the crystal. Hence it is not absurd to consider that the elastic limit in the crystal decreases inwards from the boundary not rapidly but gradually. Let the elastic limit at the crystal boundary represented by the elongation per unit length be denoted by L_m, and that of the single crystal by L_0. Next let the elastic limit at an inner part of the crystal whose distance from the boundary is x be denoted by L. Further let us assume that L is roughly represented by the following equation

$$L-L_0=(L_m-L_0)(1-e^{-\frac{a}{x}}) \qquad \text{.....................(7),}$$

where a is a certain constant. This relation is represented by a curve as shown in Fig. 3.

In elongating a specimen composed of many crystals, the middle parts of the crystals will be at first deformed by slipping. Thus if we take x as the distance from the boundary to the middle of the crystal, then x may be considered to be a measure of the linear size of the crystal. Accordingly $\frac{1}{x}$ is proportional to Z, the grain number per unit length of the crystal. Therefore equation (7) may be written as follows:

Fig. 3



$$L-L_0=(L_m-L_0)(1-e^{-CZ}) \qquad \text{.....................(8),}$$

where C is a certain constant.

If we neglect L_0 in comparison with L and L_m, we have

$$L=L_m(1-e^{-CZ}).$$

Fig. 4 (a)

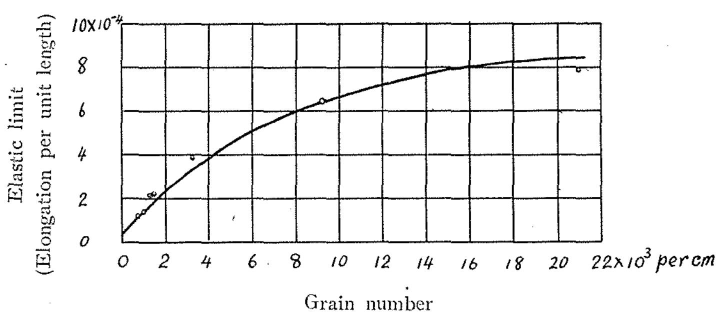
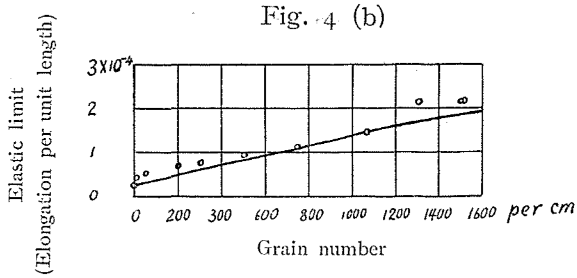


Fig. 4 (b)



This is the empirical formula obtained by the writer[1] previously. When L_0 is not neglected, the formula (8) is transformed as follows:

$$L = L_m - (L_m - L_0)e^{-cz} \qquad \text{(9)}.$$

By taking $L_m = 0.00089$ and $L_0 = 0.000025$ as the values of the elastic limit at the crystal boundary and in a single crystal of aluminium respectively, and by taking $C = 0.00014$, the relation given by this formula is represented by the curves in Figs. 4a and 4b. In these figures, small circles denote the observed values. As is seen in these figures, the agreement between the calculation and the observation is satisfactory; and the disagreement in the case of a smaller grain number observed in the former research[2] is almost removed by the new formula given by (9).

In conclusion, the writer wishes to express his sincere thanks to Prof. U. Yoshida for his kind guidance in the research.

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1. These Memoirs, A. **20**, 27 (1937).
2. These Memoirs, A. **20**, 173 (1937).