The Beppu Hot Springs and the Tide, with the Effect of the Atmospheric Pressure*

By **Takaharu Nomitu****and **Kinzo Seno**

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Abstract

There are many reports concerning the tidal effect on ordinary coast-groundwater or wells, but there are few about the tidal effect on hot springs, especially about its comprehensive quantitative study. In 1934 we made consecutive observations of the rate of discharge, spring temperature, electric conductivity, etc. of 21 hot springs in the city of Beppu for 1 month, and comparing them with records of the tide and harmonically analysing them, we determined the effect of each component of tide. Moreover, eliminating the tidal effect only by taking the mean value of every 25 hours, we succeeded in showing clearly the effect of the atmospheric pressure.

The variation in the rate of discharge is in proportion to that in the tide, amounting to in maximum as much as 94% of the mean rate of flow and diminishing greatly with the distance from the coast. In the city of Beppu, however, there is no hot spring which is not more or less influenced by the tide.

The spring temperature also has a positive correlation with tide while the electric conductivity and the mineral content show negative correlations with tide, all being the secondary effects caused by the change in discharge-rate.

The influence of the barometric fluctuation upon the hot springs is also clearly seen but in the opposite direction to the tidal effect. Thus it is concluded that the tide acts on the rate of discharge making the hot water head change according to the sea water pressure, while the atmospheric pressure acts on it just over the spring mouth, not through the sea surface.

That the tide causes variations in the rate of flow or in the temperature of a hot spring on the sea front has been well known from various traditions. As to our Beppu hot springs by the sea, such a tidal influence also has been early known among the people. The late Dr. Sida instructed certain men to investigate this subject, but it has not been studied analytically, and the results have not yet been made public. Therefore taking up this problem again and with abundant data for its analytical study, we have tried to learn the characteristics of the tidal influence. For this purpose we chose 21 welling mouths convenient for us to observe night and day, and measured the spring

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** Nomitsu has recently changed the spelling of his name to "Nomitu."

discharge, temperature, and electric conductivity every 2 hours, and especially we measured the discharge and temperature every 10∼30 minutes before and after the ebb- or flood-tide for 2 days∼1 month. The present paper is a report based upon the data thus obtained.

I. The position of the hot springs observed, and the method of observing and its results

As to hot springs to be used for the investigation of the tidal influence, first of all, it is necessary that their welling mouths should be as near to the coast as possible, but this alone is not enough. To study how the influence of tide varies according to the distance from the coast, or whether or not the phase takes a definite time, as many springs as possible which are considerably inland from the coast must also be observed. From this theoretical point of view and convenience for measurement or circumstances connected with the owners, we selected the 21 springs shown in Fig. 1. As the Beppu hot springs

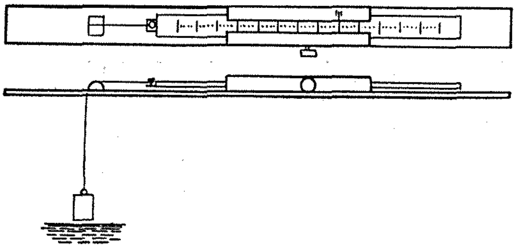
Fig. 1

Position of the springs whose tidal effects were investigated.

have the construction reported in 'Geophysics',[1] the rate of flow can be measured directly from the discharge running into the bath in a natural state. Measuring the sectional area of the bath and after drawing hot water out of it to an adequate amount, we measured the velocity with which the water level in the bath was raised by the hot water newly running in. Though the sectional area was almost equal at all heights, various errors could be eliminated by fixing measuring spaces. To fix perpendicular space accurately, we used the apparatus shown in Fig. 2, which we found simpler and more accurate than such implements as a common rule, a rule inserted in an open glass tube, or an electric contact method. A weight with a plane base is hung with a piece of string, at the other end of which a movable scale is tied. Raising or lowering the base of the weight by moving the scale, we measure the time when the water level in the bath touches the base of the weight at the two positions determined on. By the shade of the weight on the water, we can see the water level gradually approaching the base and accurately judge the moment when the water level touches the base of the weight by the ripple caused on the water surface.

Fig. 2

Simple apparatus used for measuring discharge.



The spring temperature, though it would be desirable to measure it at the welling mouth, was measured at the 'Moto-Bako' (subsidiary box) because of various difficulties. This measurement was not so inappropriate except to produce a small effect as if the pipes had to be buried a little deeper than actual. As to the thermometer used, that of 1/10°C scale was used, and especially for 1 month's observation a Beckmann thermometer of 1/100°C used for minute observation.

It is very convenient, if the change of chemical ingredients of a hot spring water can be made clear by chemical analysis. The hot springs we examined, however, belong to the type of simple spring, in which the changes are as small as the ordinary analytical error. In such a case, its variation can be inspected more minutely by electric conductivity

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[1] General View of Hot-Springs in Beppu City. (in Japanese). Tikyûbuturi (Geophysics), 1 (1937), 20-27: 267-284.

which is parallel with the amount of the ingredients. Since the electric conductivity is affected by the variation in the temperature on the spot, the hot spring water was carried back to the Laboratory and measured in a thermostat. It was ascertained that a minute difference in the temperature of the thermostat itself, if below 0°.02, did not markedly influence the measurement.

The rate of discharge, spring temperature, and electric conductivity thus observed have all been given in 'Geophysics'.[1] Here, a summary of them is tabulated in Table 1. Some of their variations are shown in Fig. 3, 4, 5, 6, and 7 for concrete understanding. Both the spring

Table 1 Summary of the tidal effect on the Beppu springs.

Spring No.	Depth	Pipe		Distance from coast	Flow rate		Temperature		Electric conductivity	Atmosph. factor		Tidal factor		$k\overline{\frac{\partial T}{\partial r}}$
		nature	dia.		mean	change	mean	change		p	c	h	$c\lambda$	
	m		cm	m	L/M	L/M	°C	°C	mho $\times 10^{-3}$	L/M cm	min^{-1}	L/M cm	min^{-1}	cal/cm² min
No. 211 (S)	45	bamboo	4.5	30	10	8.5	51	2.9	1.25			0.0490	3.08	0.446
„ (N)	66	„	4.5	30	16	5	58	2.7	1.50			0.0343	2.16	
No. 235	73	„	3.0	84	14	8.5	64	0.9	1.69			0.0502	4.97	0.174
No. 309	72	„	4.2	180	24	11	66	0.5	1.62			0.0407	2.94	0.215
No. 1197	78	Iron	5.1	0	35	19.74	66	0.5	1.82			0.109	5.65	0.206
No. 801	122	„	2.8	690	**1.3**	0.65	38	2.2	—	0.086	10.5	0.00336	0.55	0.038
No. 671	40	bamboo	4.2	870	11	1	52.3	0.3	1.13	0.112	6.1	0.00430	0.32	
No. 491	55	„	5.0	390	7.6	1.1	53.7	0.4	1.17	0.023	0.9	0.00358	0.18	
No. 427	91	„	4.9	−18	14	10	65.5	3.0	2.17			0.0372	1.97	0.308
No. 551	102	„	3.8	420	12	3	53.7	0.5	1.71	0.300	20.0	0.0060	0.53	
No. 365	54	„	4.9	240	5	1.3	49	2.2	1.29			0.0148	0.78	0.165
No. 199	98	„	6.1	24	8	15	60	9.33	1.62			0.101	3.46	0.087
No. 594	69	„	3.8	780	5.2	0.71	55.5	0.5	1.07	0.187	12.5	0.00235	0.21	0.122
No. 4	104	„	3.3	42	17	7.0	55.7	0.7	1.49			0.0367	4.29	0.260
No. 1165	62	„	5.8	18	13	18.44	53	4.32	4.42			0.0127	4.81	0 208
No. 35	73	„	6.0	318	8.2	0.8	46.3	0.22	1.18	0.178	4.8	0.00409	0.14	
No. 1299	182	„	3 9	42	**57**	6	43.3	0.12	1.20			0·0334	2.79	0.340
No. 316	45	„	4.5	96	12	6	61.2	0.8	—			0.0256	1.61	0.285
No. 315	58	„	4.5	120	8	8	58.3	2.7	—			0.0342	2.15	0.208
No. 319	58	„	4.5	30	8	5	62.7	0.9	—			0.0278	1.75	0.139
No. 220	119	„	4.0	24	21.2	14.4	62.1	1.18	1.66	0.085	5.1	0.0573	4.56	0.201
											mean 8.6			mean 0.213

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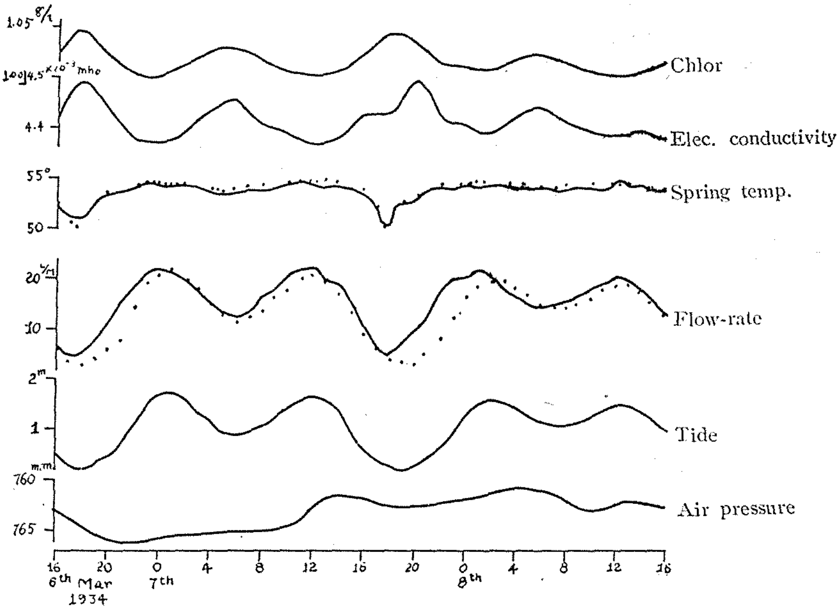
[1] Table of Observations on the Tidal Effect upon the Beppu Hot-Springs. (in Japanese). Tikyûbuturi (Geophysics), **2** (1938), 41–96.

Fig. 3

Springs near the coast.

(Dots are calculated values)

Sp. No. 1165



Sp. No. 4

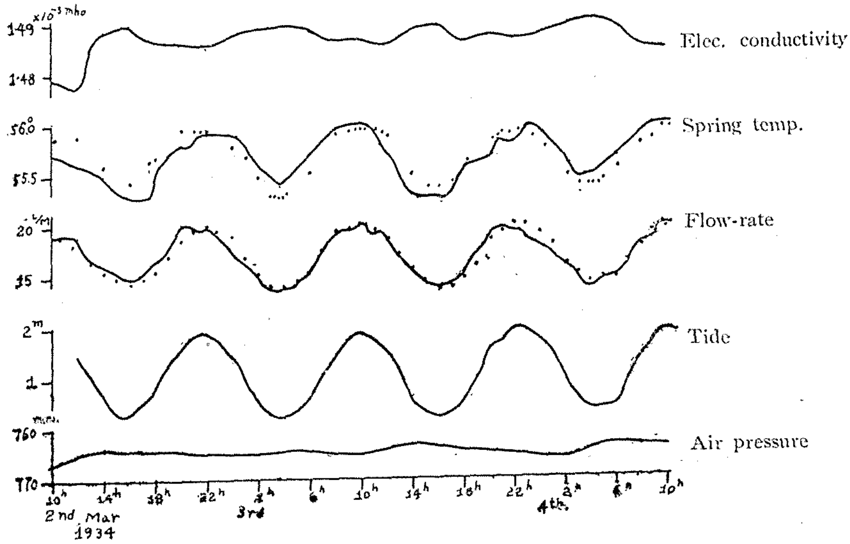
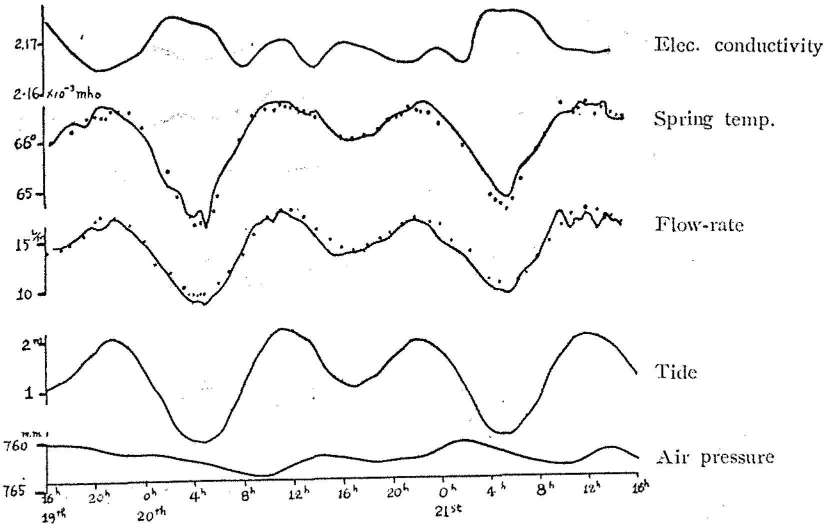


Fig. 4

Springs near the coast.

(Dots are calculated values)

Sp. No. 427



Sp. No. 1197

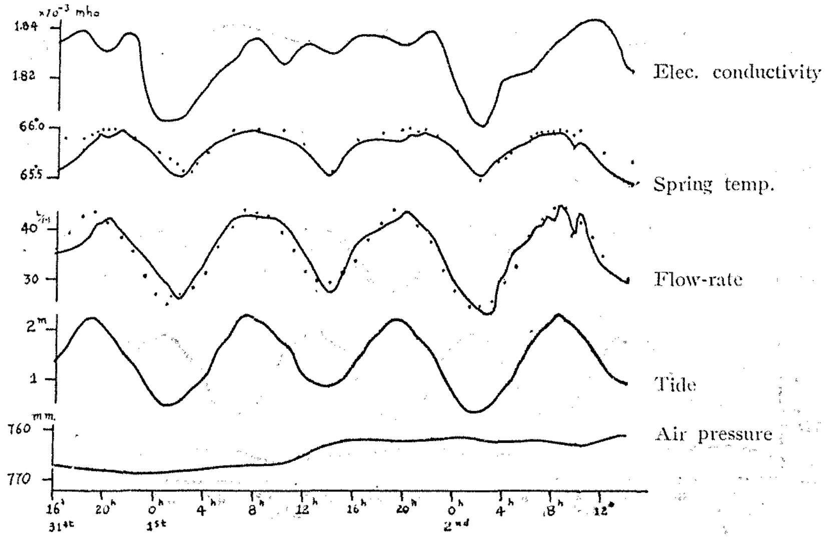
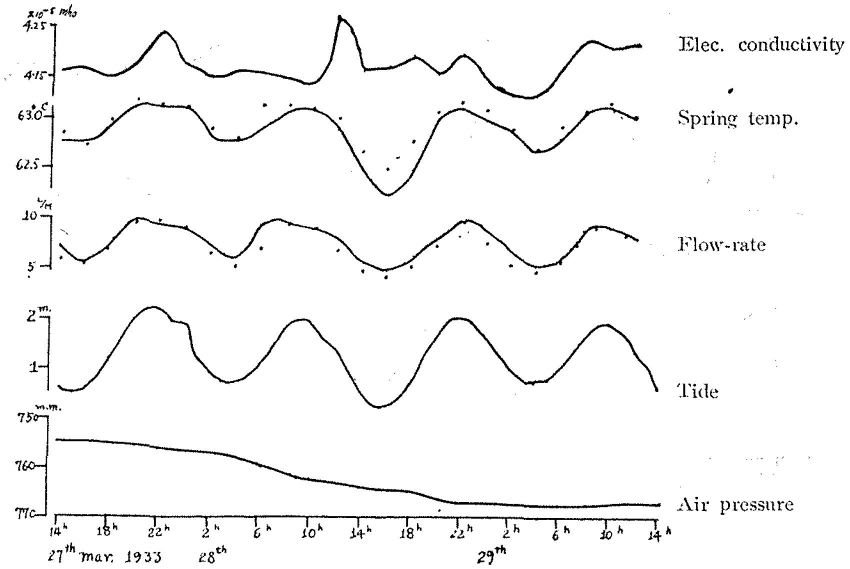


Fig. 5

Spring near the coast.

(Dots are calculated values)

Sp. No. 319



temperature and electric conductivity have been since consecutively recorded by the recording gauges set near the coast.

Concerning other important data, as the record of tide, that of the Ôita Harbour (1/20 diminution) was adopted from the Ôita Meteorological Station, for at that time there was no tide gauge set on the Beppu coast. Afterwards our Laboratory also set a tide gauge on the coast of Beppu, and so harmonically analysing the tides at Ôita and Beppu by the simultaneous tidal records on the two coasts, we obtained Table 2, which will serve for comparing the two.

As meteorological elements, the results[1] observed at the Beppu Laboratory of the Kyoto Imperial University (74 m above the sea level, 1.75 km from the coast) was adopted.

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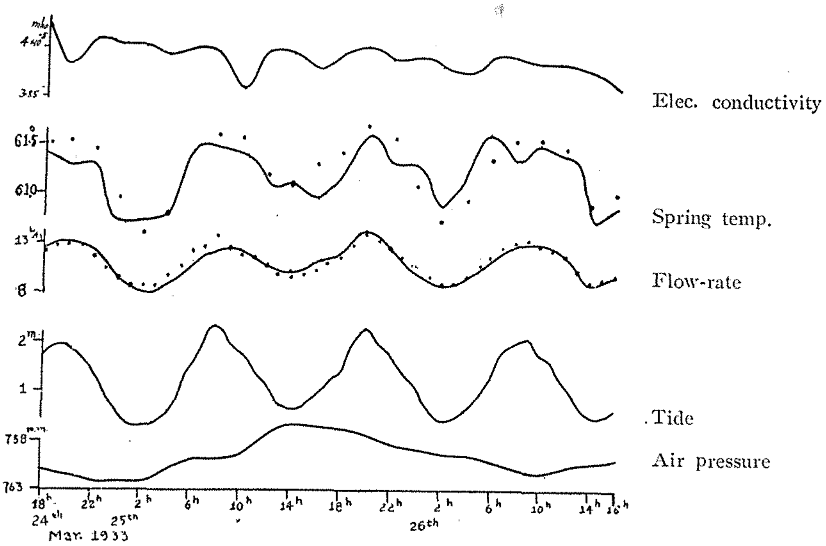
[1] Reports of Meteorological Observations at Beppu. (in Japanese). Ṭikyûbuturi (Geophysics), **1** (1937), 104-163.

Fig. 6

Springs not so near the coast.
(Dots are calculated values)

Sp. No. 316



Sp. No. 365

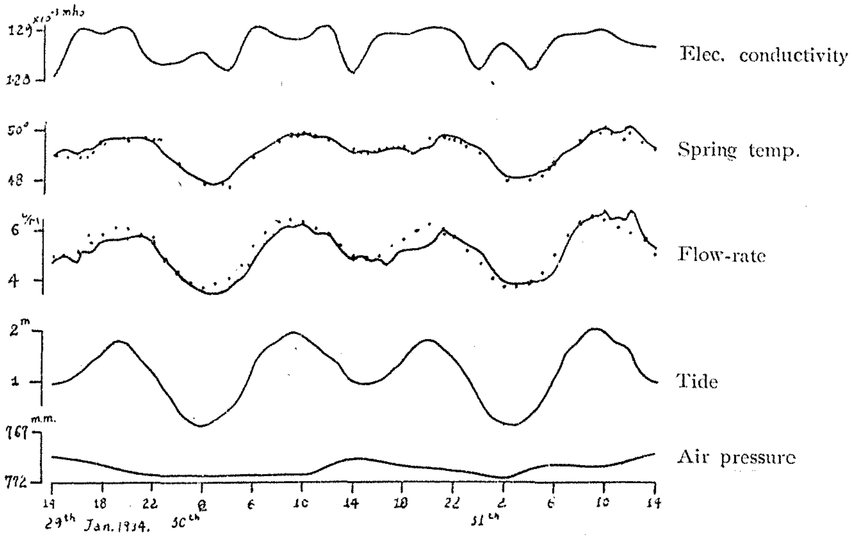
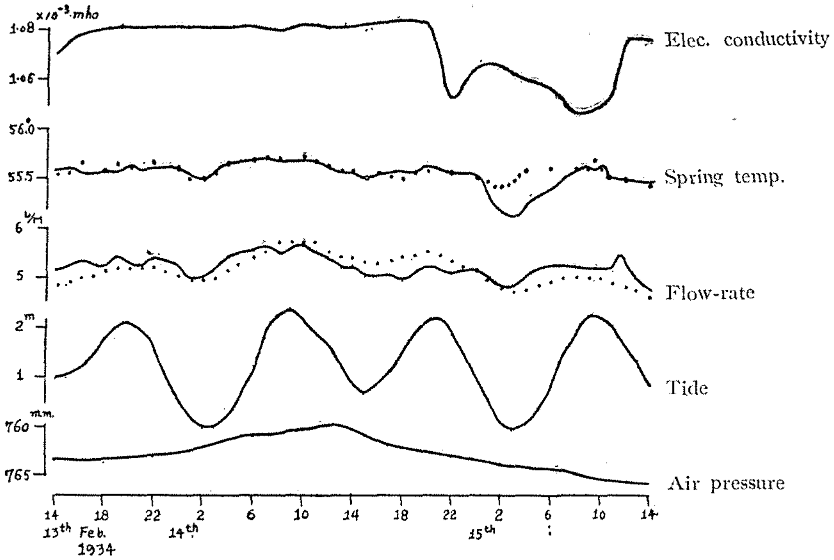


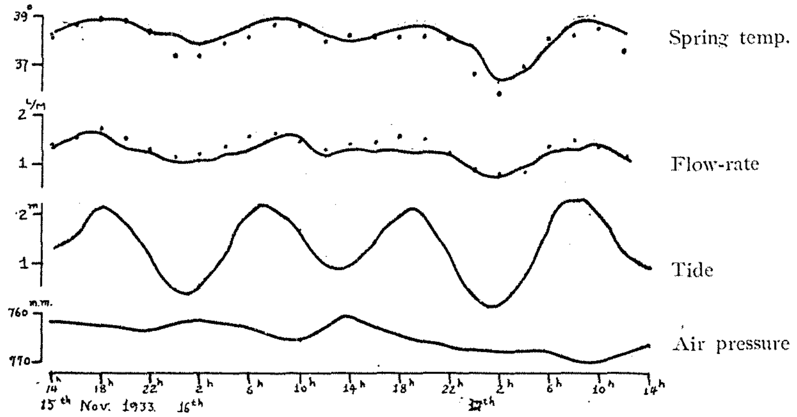
Fig. 7

Springs distant from the coast.
(Dots are calculated values)

Sp. No. 594



Sp. No. 801



II. The variation in the spring discharge due to the tide and the atmospheric pressure

The consecutive observation of the spring discharge showed the following facts.

1) *The variation in the rate of discharge occurs in proportion to that in the tide level*, and there holds the following linear relation positively between these two as first approximation.

$$\partial q / \partial H = h \quad \text{(constant)} \tag{1}$$

where q is the rate of discharge, H the tide level, and h is a constant peculiar to the welling mouth, which we call the *tidal coefficient*. This relation may almost be inferred even from Figs. 3∼6. Drawing the q-H curves makes it unmistakable, as the dots arrange themselves almost linearly as shown in Fig. 8.

The value of h for each hot spring observed is given in Table 1. For the calculation of h, taking Δq and ΔH as the difference between the maximum and the minimum of q and H respectively, we obtained its mean value from $\dfrac{\Delta q}{\Delta H}$. The dots marked on the curves of the variations in the spring discharge in Figs. 3∼6 indicate the rate of flow calculated from the tide level with the above obtained value of h. They almost agree with the values observed, which justify equation (1).

One month's consecutive observation was made at spring No. 220 to learn how each of the component tides shows an individual effect. Harmonically analysing the spring discharge and tide observed at the

Fig. 8

Examples of q-H relation.
Sp. No. 4

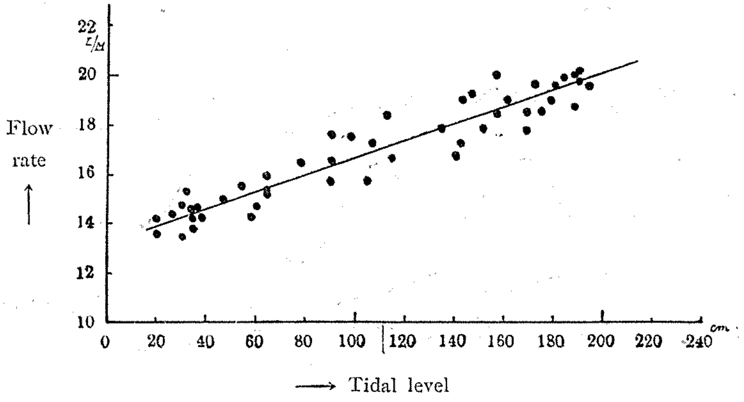
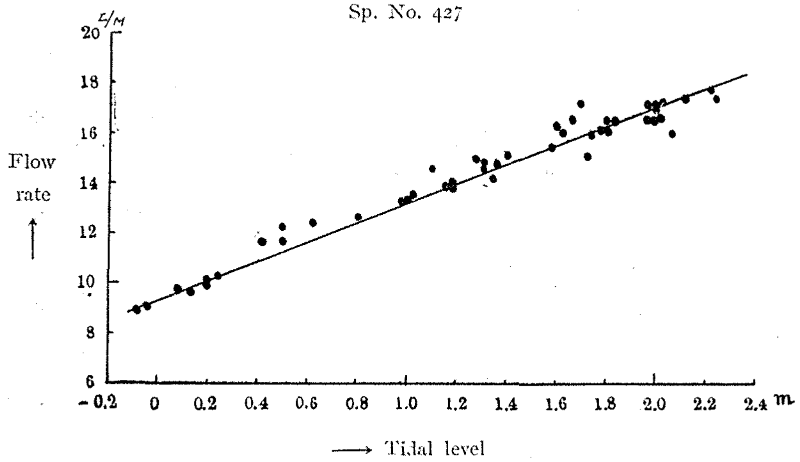
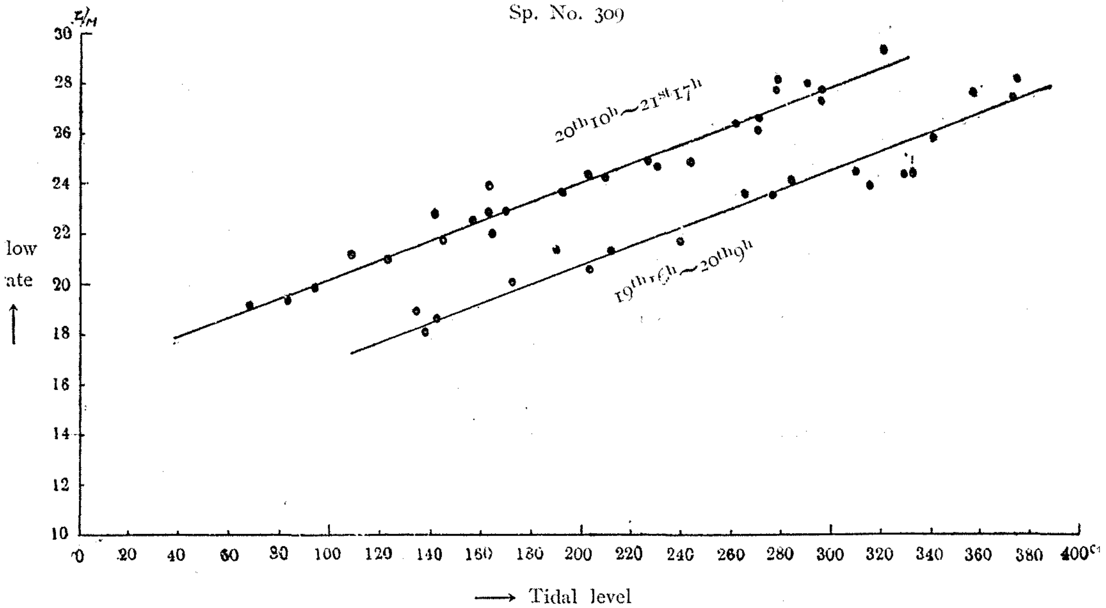


Fig. 8 Continued

Sp. No. 427



Sp. No. 309



Ôita Harbour during the same period, we obtained Table 2. Though varying a little in regard to each component, the value of h may be said to be rather equal for all components. Taking the ratio of the discharge-rate to the component tides obtained afterwards from the tidal record at the Beppu coast by our Laboratory, we clearly recognize that the value of h is more nearly identical than with Ôita data.

Table 2

Resumé of Harmonic analysis of tide, discharge and temperature.

(i) 1934, May 17∼June 16.

Comp. tide		Tide at Ôita	Flow rate	Spring temp.	$h=\frac{\text{Flow-rate}}{\text{tide at Ôita}}$	Phase diff. of flow & Ôita tide	Phase diff. of temp. & flow
M_2	k_m	252°	250°	270°		−2°	+20°
	H_m	cm 56.6	L/M 3.116	o 0.2179	L/M/m 5.40		
O	k_o	196°	193°	209°		−3°	+16°
	H_o	cm 18.3	L/M 1.045	o 0.1000	L/M/m 5.70		
S_2	k_S	287°	285°	292°		−2°	+7°
	H_S	cm 22.0	L/M 1.241	o 0.1004	L/M/m 5.64		
K_2	k''	287°	285°	292°		−2°	+7°
	H''	cm 6.0	L/M 0.338	o 0.0274	L/M/m 5.64		
K_1	k'	203°	201°	212°		−2°	+11°
	H'	cm 23.1	L/M 1.283	o 0.1090	L/M/m 5.56		
P	k_p	203°	201°	212°		−2°	+11°
	H_p	cm 7.7	L/M 0.428	o 0.0363	L/M/m 5.56		

(ii) 1937, March 1∼31 (iii) Obtained from (i) & (ii)

Comp. tide		Tide		$\frac{\text{Ôita tide}}{\text{Beppu tide}}$	Phase diff. of tides at Ôita & Beppu	$h=\frac{\text{Flow rate}}{\text{Beppu tide}}$	Phase diff. of flow & Beppu tide
		at Ôita	at Beppu				
M_2	k_m	250°	251°		+1°		−3°
	H_m	cm 58.6	cm 61.2	0.959		L/M/m 5.18	
O	k_o	187°	190°		+3°		−6°
	H_o	cm 17.3	cm 19.0	0.912		L/M/m 5.20	
S_2	k_S	280°	280°		0°		−2°
	H_S	cm 24.9	cm 26.6	0.935		L/M/m 5.28	
K_2	k''	280°	280°		0°		−2°
	H''	cm 6.8	cm 7.3	0.935		L/M/m 5.28	
K_1	k'	203°	207°		4°		−6°
	H'	cm 28.3	cm 30.7	0.924		L/M/m 5.14	
P	k_p	203°	207°		4°		−6°
	H_p	cm 9.4	cm 10.2	0.924		L/M/m 5.14	

Ogura's analysis of tide at Ôita (Suiro Yôhô, Vol. 12, No. 11)

M_2		S_2		K_2		K_1		O		P	
H_m	k_m	H_S	k_S	H''	k''	H'	k'	H_o	k_o	H_p	k_p
m		m		m		m		m		m	
0.59	242°	0.24	270°	0.07	270°	0.26	208°	0.19	183°	0.09	208°

It has been found, as will be reported in another paper,[1] that the rate of discharge in the Beppu hot springs varies in proportion to the difference of the water pressure. Let P_0 represent the still water pressure of the hot spring; P_l, the pressure caused by the water pillar as far as the upper end of the welling mouth; and a, the sectional area of the welling mouth, then the rate of discharge q is as follows;

$$q=\frac{c}{g\rho}a(P_0-P_l) \qquad (2)$$

where g is the gravity acceleration, ρ, the specific gravity of the hot spring water, and c, a constant particular to each welling mouth.

In order that the rate of discharge may be proportional to the tide level, P_0 is presumed from equation (2) to be in proportion to the tide level. When λ is a constant,

$$P_0=\lambda\cdot g\rho H. \qquad (3)$$

As we may regard $\rho \fallingdotseq 1$ in the accuracy of the present problem, from (1), (2), and (3) we obtain

$$c\lambda=h/a \qquad (4)$$

where c is a constant concerning the underground stratum or the character of conduit pipes. The constant λ, as seen from (3), is a diminution coefficient of the tide level acting as the sea water pressure, but $c\lambda$ cannot be obtained individually, and so they were calculated as if they were $c\lambda=h/a$. The values obtained are tabulated in Table 1.

2) *In Beppu there is, on the whole, scarcely any phase difference between the variations of the tide level and the rate of discharge.* Scarcely any phase difference was recognized in many hot springs, so far as they were observed every 30 minutes. Of course, it was sometimes evident at some springs, but no distinct relation held between the distance from the coast and the phase difference. While such a phase advance of the spring discharge as No. 1165 (Fig. 3) was seen for about 1 hour, the discharge of spring No. 1254–1 distinctly showed the lag of about 30 minutes. At No. 1197 (Fig. 4) and No. 365 (Fig.

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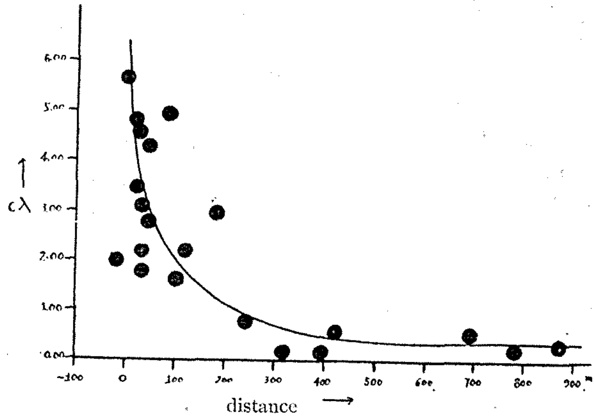
[1] The Correlation between the Rate of Discharge and the Pressure Head in the Beppu Hot Springs. These Memoirs, next No.

6) a lag of about 1 hour was seen, a little advance being seen at No. 427 (Fig. 4). In the case of 1 month's consecutive observation of No. 220, it was measured every 10 minutes before and after the ebb- or flood-tide for minuteness, but the phase difference was not clearly observed as appears in Fig. 10. Seeing the results (Table 2) got by harmonic analysis of this one month's observation, we can acknowledge an advance of several degrees of the phase at each component of tide. But such a small difference was within the error of harmonic analysis for the observation made only for 1 month, and it is evident from comparing it with Dr. Ogura's harmonical constant of the Ôita Harbour appended to Table 2. In short, from the above mentioned facts it is found that in Beppu there is, on the whole, no phase difference between the rate of discharge and the tide.

3) *The tidal influence decreases according to the distance from the coast.* Taking $c\lambda$ of the 21 hot springs in Table 1 as the ordinate and their distance from the coast as the abscissa, we have Fig. 9, from which it is apparent that the value of $c\lambda$ varies according to the distance from the coast having no relation to the depth of the hot spring or the rate of discharge.

Fig. 9

Relation between $c\lambda$ and the distance from the coast.



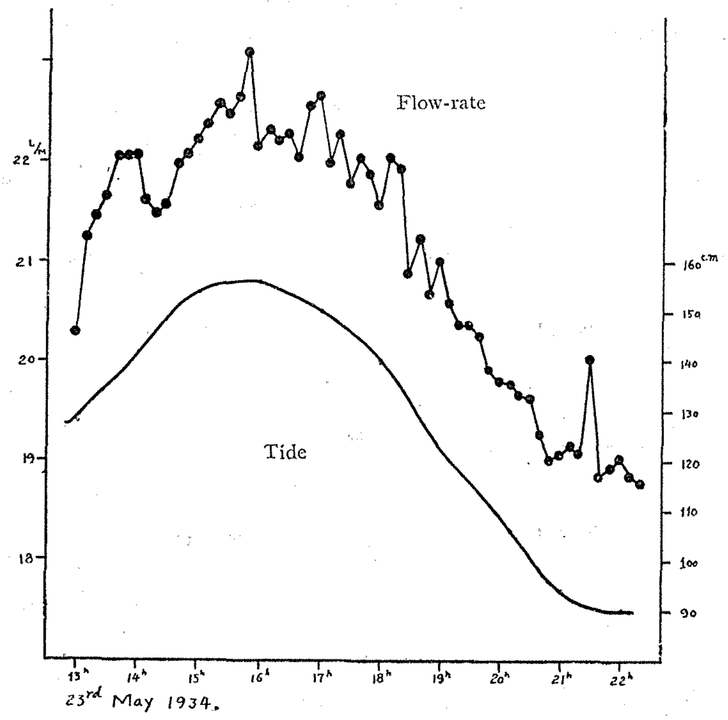
Having relation also to the character of stratum, etc., $c\lambda$ is not a function of the distance from the coast only, but it is known from the figure that it is, on the whole, influenced chiefly by the distance from the coast. This fact proves that the tidal influence on the spring discharge decreases according to the distance from the coast.

Now it is natural that the smaller the tidal influence becomes, the more irregular and indistinct it becomes. This is partly caused by the fact that the degree of the influence reduces to the point of the observation error, but partly by other causes, such as the fact that the variation in the spring discharge caused by the atmospheric pressure or various circumstances in the strata disturbs the tidal influence.

For instance, in the every 10 minute observations at Sp. No. 220, small fluctuation appeared (supposed as the effect of sea-seiche), as shown in Fig. 10.

Fig. 10

An example of observation every 10 minutes at Sp. No. 220.

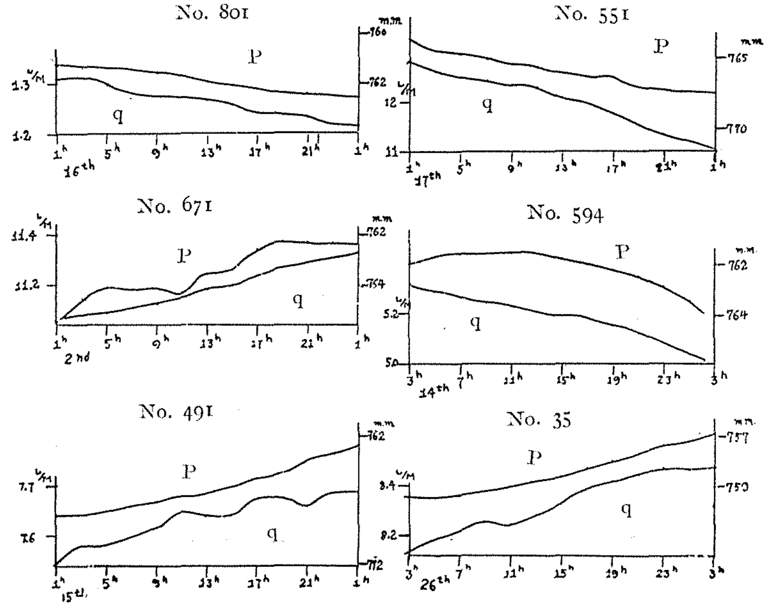


4) *The influence of the atmospheric pressure.* According as the influence of the tide is reduced by leaving the coast, the influence of the variation in the atmospheric pressure appears on the spring discharge. For example, from the result of the observation at No. 801 and No. 594 (Fig. 7), the 4 ebb- and flood-tides for 2 days were almost equally high and regular, but the spring discharge seemed to be influenced by the atmospheric pressure as well as by the tide; the rate of discharge was little when the atmospheric pressure was high and was large when it was low.

To realize this relation more clearly, it is necessary to isolate the influence of the atmospheric pressure, eliminating the effect of the tide from the whole variation in the spring discharge. This purpose is answered by taking the mean value of every 25 hours as the discharge value at its middle time. Thus the influence of tide at the place which is far from the coast and little influenced by the tide can be eliminated. However, in the case where the atmospheric pressure varies with a period of 1 day, even its influence is also eliminated by the procedure, and so avoiding such a day, we must choose a day where the atmospheric pressure either rises or falls steadily. With this idea, we observed No. 671, No. 491, No. 551, and No. 35 besides No. 801 and No. 594 already mentioned, and obtained Fig. 11.

Fig. 11

Effect of atmospheric pressure P upon the flow-rate q (Springs distant from the coast)



These hot springs are situated far (320~870 m) from the coast, and the relation between the atmospheric pressure and the spring discharge is seen quite clearly. The relation is approximately of the negative linear:

$$\partial q/\partial P=-p \quad \text{(constant)}, \qquad (5)$$

where P is the atmospheric pressure, q, the rate of discharge, and p is a constant particular to each welling mouth, which we will call the *barometric coefficient* hereafter. The barometric coefficient of the 6 springs shown in Fig. 11, calculated from (5), are given in Table 1.

The rate of discharge influenced by both the tide and the atmospheric pressure can be given from (1) and (5) as follows:

$$q = hH - pP + C \qquad (6)$$

where C is the discharge rate which seems to be almost definite during the rainless season and corresponds to the base rate of discharge in the case where H and P are zero. The dots marked on the curves of the spring discharge at No. 594 and No. 801 denote the discharge-rate obtained from equation (6) with the tide level and the atmospheric pressure observed. It may be said that the value observed corresponds fairly closely to that calculated. Thus, the fact that the effects of sea water pressure and atmospheric pressure have the linear relation to the spring discharge suggests that the Darcy Law which is applicable to the common ground water also applies to the under-ground of the Beppu hot springs, and that the under-ground is in the state of the so-called *hot water layer*.

It must be noted here that it seems as if the variation in the discharge-rate of the springs near (not over 20~30 m) the coast could be explained by the effect of tide alone without taking the influence of atmospheric pressure into account. Fig. 3, 4, and 5 all seem so, and especially in the case of No. 319 (Fig. 5) the influence of atmospheric pressure cannot be directly recognized in spite of its constant rise. But this is not because there is truly no influence of atmospheric pressure. The meteorological tide, which varies according to the atmospheric pressure acting on the sea surface, is characterised by its impossibility to influence the spring discharge. But when it is regarded as having the effect as well as the astronomical tide and the whole tide level and spring discharge are related, any effect of the atmospheric pressure merely seems to be no more there. To make this point clearer, it will be better to study it by making a daily mean by long, consecutive observation and eliminating the conspicuous daily astronomical tides. Our 1 month's consecutive observation at spring No. 220 was planned for studying this point as well as for the purpose of making harmonic analysis. This hot spring is only 24 m from the coast and it is difficult to recognize directly the influence of the atmospheric pressure, because the influence of the tide twice a day is so large. However, drawing Table 3 and Fig. 12 from a daily mean value, we can distinctly recognize that the spring discharge and temperature vary according to the atmospheric pressure. Here it must be noted that the mean daily sea water level also varies parallel with the rate of

Table 3

Daily mean of the observed data for Sp. No. 220.

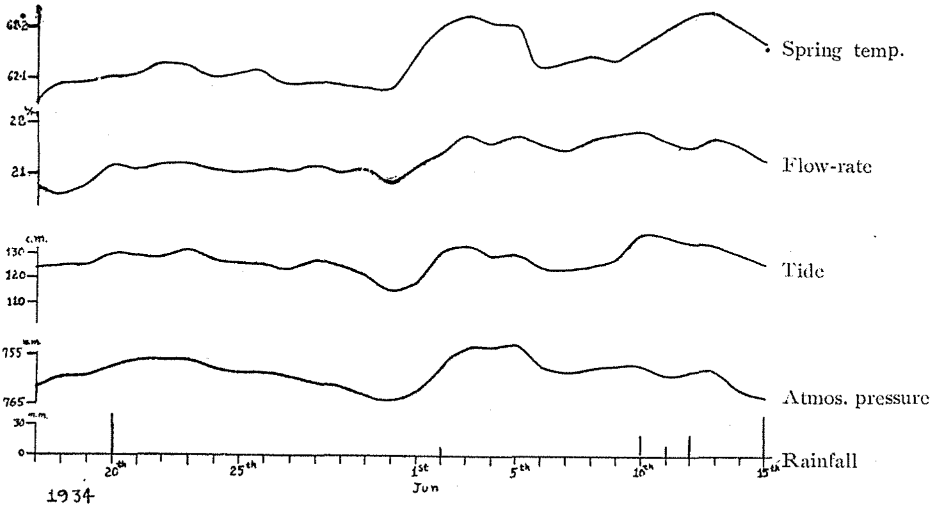
May 17~June 15, 1934.

Date	Flow rate	Spring temp.	Electric conduc.	Tide	Air pressure	Air temp.	Humidity
May	L/M	$60^{\circ}.005+$	$\times 10^{-3}$mho	cm	mm	°C	%
17	20.735	2.0507	1.6565	124	760.69	19.0	75
18	20.601	2.0908	1.6540	126	59.28	19.0	81
19	20.792	2.0954	1.6532	125	59.33	19.7	67
20	21.149	2.1053	1.6497	131	59.40	18.3	83
21	21.093	2.1106	1.6494	130	56.09	19.1	89
22	21.184	2.1285	1.6496	129	55.73	21.1	62
23	21.207	2.1286	1.6521	132	55.62	19.4	68
24	20.852	2.0938	1.6501	128	57.62	18.5	64
25	21.016	2.1139	1.6474	127	58.46	18.2	65
26	21.113	2.1136	1.6509	126	58.67	19.7	62
27	21.095	2.0880	1.6513	124	59.23	19.9	69
28	21.220	2.0973	1.6505	128	60.40	19.2	78
29	21.022	2.0918	1.6482	126	62.09	21.3	65
30	21 098	2.0837	1.6490	122	63 42	20.5	69
31	20 866	2.0814	1.6521	136	64.44	20.9	77
June							
1	21.122	2.1503	1.6480	138	62.39	21.4	77
2	21.400	2.2035	1.6475	131	57.23	20.7	81
3	21.841	2.2156	1.6476	133	53.63	22.7	70
4	21.646	2.1816	1.6471	129	54.43	21.8	56
5	21.813	2.1777	1.6469	130	53.76	20.7	64
6	21.560	2.1216	1.6452	125	57.47	21.1	38
7	21.514	2.1360	1.6457	124	58.44	20.9	49
8	21.692	2.1504	1.6453	124	58.26	21.1	71
9	21.796	2.1374	1.6461	128	57.87	20.9	69
10	21.916	2.1784	1.6460	138	56.99	19.1	90
11	21.690	2.2014	1.6475	137	58.87	22.4	76
12	21.590	2.2363	1.6463	135	58.56	19.9	96
13	21.767	2.2389	1.6440	134	58.64	19.9	81
14	21.549	2.2184	1.6456	131	62.00	22.1	57
15	21.321	2.1790	1.6450	127	63 32	18.9	81

discharge. Some readers may suspect that the very variation in the spring discharge is caused by that in the sea water level, not by that in the atmospheric pressure. But we regard it as caused by the atmospheric pressure, and would call attention to the fact that even the

Fig. 12

Continuous observations of Sp. No. 220 during 1 month (daily mean).



variation in the sea water level itself is nothing but the secondary meteorological tide caused by the atmospheric pressure. As has been shown by one of the present authors (Nomitu) in his 'The Causes of the Annual Variation of the Mean Sea Level along the Japanese Coast',[1] this is explained by the fact that the variation in the atmospheric pressure acting on the sea surface causes the negative variation of the sea surface balanced with it and consequently it exerts no influence as submarine pressure. The variation in the sea surface shown in Fig. 12 supplements the variation in the atmospheric pressure on the sea, and exerts no influence on the submarine pressure since the two cancel each other. Accordingly we cannot look for effect on the spring discharge from that source. Thus in spite of the fact that the atmospheric pressure acts over both the spring mouth and the sea, only the atmospheric pressure immediately over the spring mouth is effective and it influences the hot springs farther inland as well as near the coast.

Thus it is not true in the case of No. 319 that there seems to be no influence of the atmospheric pressure. The influence of the atmospheric pressure is verified by the fact that the whole tide observed (including the meteorological tide) is linearly related to the spring discharge.

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[1] T. Nomitsu and M. Okamoto, These Memoirs, **10** (1926), 125,

If, as we have said, we may regard the atmospheric pressure as acting only over the spring mouth, the barometric effect will not decrease according to the position (the distance from the coast) of the spring, differing from the nature of the tidal effect. Therefore, to calculate the coefficient of the atmospheric pressure from (5) in the same way by which $c\lambda$, the coefficient of the effect of tide, has been obtained from (1) as (4), we may take λ to be 1, that is,

$$c = p/a \qquad (7)$$

by which we calculated the coefficient of atmospheric pressure for each hot spring, as tabulated in Table 1. The above mentioned supposition is proved by the result that the coefficients agree well with each other, being independent of the distance from the coast.

5) *Mechanism of the effect of the tide and the atmospheric pressure.* How does the rate of discharge of the Beppu hot springs vary by the influence of the tide and the atmospheric pressure? The atmospheric pressure acts just over the welling mouth so to check the flowing power of the hot spring. Such a checking action, varying according to the atmospheric pressure, directly influences the rate of discharge in much the same way that we turn the hydrant on or off. The tide acts on the flowing power of the Beppu hot springs in the sea as submarine pressure and consequently makes the hot spring pressure change at the bottom of the spring pipe on land, so that it influences the rate of discharge just as the amount of the service-water used at the lower part of a pipe influences the amount at the hydrant at the upper part.

It is probable that the ebb- or flood-tide or the variation in the atmospheric pressure, acting as the load on ground, squeezes out the spring water through the variation in velocity caused by the inclination of ground or by the compression of the hot water-bearing strata. But according to Nisimura's observation with clinographs in Beppu, the inclination of ground caused by the tide is only about $0''.05$, being negligible as compared with the natural slope of hot water head. Judging from the state of the springs and various other points, there is also no action of squeezing by compression, or at most very little. For examples, (i) the actions of the tide and the atmospheric pressure, being equally loads on the ground, are correlated with the spring discharge in opposite sign, and (ii) the barometric coefficient has no relation to the distance from the coast while the tidal coefficient diminishes rapidly according to the coastal distance. Moreover as reported in another paper,[1] (iii) although the area that is loaded with the tide and the area with the atmospheric pressure are entirely different from each other (one over the sea and the other over the land), the mean value of the barometric coefficient is not only of the same order as the tidal coefficient near the coast, but also equal to the coefficient of the effect of change in height of the flowing level of the spring, showing that the action of the barometric pressure is equivalent to that of the water column applied on the spring pipe. The above three facts sufficiently indicate that the tide acts at the bottom of the hot spring pipe through sea and the atmospheric pressure acts only over the spring mouth but not through sea, both actions being of hydraulic nature but not caused by squeezing.

6) *The comparison of the coefficients of the tide and the atmospheric pressure.* From Table 1 we know that the coefficient of the atmospheric pressure is always larger than that of the tide in the same hot spring. The action of the atmospheric pressure has no relation to the distance from the coast, but the influence of the tide markedly varies according to it.

These facts can be easily realized from the interpretation of 5). The atmospheric pressure, as it acts directly over the spring mouth, can do without its decrease in power and has no relation to the distance from the coast. On the contrary, the tide, as it acts on the bottom of the spring pipe through the stratum as submarine pressure, is related to its location and its coefficient being somewhat decreased even near the coast and increasingly according to the distance from the coast is smaller than that of the atmospheric pressure. From the state of the under-ground of the Beppu hot springs, the conditions of the water system of the hot springs and the fact that there are actually some submarine hot springs having connection with the bottom of the sea, we infer that the action of the tide on the Beppu hot springs comes not from the shore itself but from the offing.

III. The influence of the tide and the atmospheric pressure on the spring temperature

From Fig. 3~7, it is evident that the variation in the *hot spring temperature* as well as the rate of discharge *is positively correlated with the variation in the tide level.* However, it is known from obser-

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[1] loc. cit. p. 315.

vation at No. 427, No. 1165, etc. (Fig. 3 & 4) that it is not a simple linear correlation.

1) *It is not owing to the direct flowing in of the sea water that the hot spring temperature varies according to the tide.* For, since the tide affects the temperature of the hot springs which are considerably far from the coast, it is difficult to assume that the sea water goes through the stratum covering such a large range at the ebb- or flood-tide. Moreover, the amount of salt and the electric conductivity, which will be considered in the next section, are generally smaller when the hot spring temperature increases. Besides, Fig. 12, the result of observation at No. 220, shows that the hot spring temperature is parallel to the spring discharge and the tide level. In this case, as already mentioned, the variation in the daily mean of tide level shows no influence on the submarine pressure and so we cannot believe the sea water goes through the stratum. Thus it is clear that the variation in the hot spring temperature is not caused by the direct flowing in of the sea water. By what is it caused, then?

2) *The variation in the spring temperature following the tide is the secondary influence caused by the variation in the rate of discharge and this phenomenon is caused by the difference of cooling of the hot spring water as it passes through the conduit pipe.* When the tide level is high and the rate of discharge increases, it takes a short time for the hot spring water to well out to the earth surface and consequently it can keep almost the original temperature. On the contrary, when the tide level is low and the discharge decreases, it takes a long time to well out to the surface and the temperature of the spring water falls remarkably, being cooled on the way. We proved this phenomenon, measuring the distribution of the hot spring temperature through the conduit pipe at ebb- and flood-tide, as we have reported in another paper[1] with firm conviction.

Now, when the hot spring water wells from the hot water layer of high temperature parallel to the earth surface, passing through the stratum of low temperature by a small perpendicular conduit pipe, the variation in the earth temperature T is represented by the following differential equation, using the cylindrical co-ordinates, r and z:

$$s\rho\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \qquad (8)$$

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[1] K. Seno: Distribution of Certain Geophysical Elements of the Hot Spring District of Beppu; Jap. Jour. Ast. Geophys., **17**, No. 2 (1940).

where k is the heat conductivity through the stratum, s the specific heat, ρ the density, t the time, and the vertical co-ordinate z is measured from the earth surface downward.

Conceive the following conditions, and assume $s\rho \doteqdot 1$:

$T=f(r)$ at $z=0$ (at the upper end),

$T=$constant at $z=l$ (at the lower end),

$q\frac{\partial T}{\partial z}=-2\pi r_1 k\frac{\partial T}{\partial r}$ at $r=r_1$ (on the outer walls of a conduit pipe),

$\partial T/\partial r=0$ at $r=\infty$.

Equation (8) will not be completely solved, but, for simplicity, using the mean value of $k\partial T/\partial r$ from the lower end to the upper one of a conduit pipe and representing the rate of discharge by q, we have

$$-q(T_b-T_u)=2\pi r_1 lk\overline{\frac{\partial T}{\partial r}} \tag{9}$$

where, $T_b=$spring temperature at the lower end, $T_u=$that at the upper end. Accordingly this becomes:

$$\varDelta T_u=\frac{1}{\varDelta q}\cdot 2\pi r_1 lk\overline{\frac{\partial T}{\partial r}}. \tag{10}$$

Using equation (10), $k\partial T/\partial r$ can be obtained from the variations in the rate of discharge and spring temperature.

The absolute value of $k\partial T/\partial r$ for each spring is appended to Table 1. Conversely using this value, we calculated the amount of cooling through the conduit pipe from the variation in the rate of discharge observed, and obtained the theoretical variation in the hot spring temperature, which is marked with dots on the variation curve of the observed temperature. No calculation, however, was made for springs where the variations of temperature and rate of discharge are very small, because the above mentioned way of calculation is inadequate.

To know the distribution of $k\partial T/\partial r$, the spring mouths are given classified according to $k\partial T/\partial r \gtrless 0.213$ in Fig. 1. From the figure, the value is small at the Nagarekawa, Hamawaki, and the neighbourhood of the Takegawara hot spring. This indicates that the neighbouring earth has a comparatively high temperature, if the conductivity k of the stratum makes little difference throughout the area. It is indeed interesting to notice that these places correspond to the zone which has been called the natural welling zone.

From the value of $k\overline{\frac{\partial T}{\partial r}}$ we can learn the original temperature at the lower end of the spring pipe from that at the upper end of it, and this calculation will be dealt with in a later paper.

3) *The influence of the atmospheric pressure upon the spring temperature.* The influence of the atmospheric pressure is recognized and the spring temperature varies negatively as the barometric pressure, as seen in Fig. 12. This effect is also a secondary one due to the change in flow-rate, just as the tidal effect upon the spring temperature.

4) *The influence of the air temperature upon the spring temperature.* The influence of the air temperature is also recognized in any hot spring where the hot spring water, after welling from the earth, is led by a long horizontal conduit pipe, especially in the case where the vertical pipe is not deeply buried or the flow-rate is small. No. 491 and No. 551 are good examples.

5) *The phase difference between the hot spring temperature and the rate of discharge is not recognized in most cases.* If cooling of the spring water passing through the conduit pipe truly causes the variation in the hot spring temperature, the phase of the spring temperature will be a little retarded. From the harmonic analysis of the result observed at No. 220, a lag of 30 minutes~1.5 hours is seen. But since it takes about 7 minutes for the spring water to flow from the lower to the upper end this is insufficient to explain the reason of the lag. In fact, there may be a little cooling before the hot spring water flows into the conduit pipe. Strictly speaking, it may be because equation (9) does not hold exactly. Moreover, the harmonic analysis is insufficient since it is based upon only 1 month's observation, so that we cannot discuss in authoritatively. It would be safe to say that generally there is not a striking phase difference.

IV. The variation in electric conductivity due to the tide

When the solute in hot spring water is extremely little, no regular variation is found with regard to either its chemical analysis or electric conductivity, for it is little influenced by the tide. But concerning the hot spring where a large amount of the solute is included, we can observe a fairly regular variation. For example, this phenomenon is shown in No. 1197, No. 309, No. 427, No. 551, No. 4, and No. 1165 given in Fig. 3 & 4. Only in the case of No. 1197, the variation in the electric conductivity is positively correlated with that in the tide, but all the others are negatively correlated with it. It is also similar as to the amount of chlor at No. 1165.

Generally, as has been mentioned in the preceding section, the sea water does not directly flow into the hot spring. If it were caused by the flowing in of the sea water, it would be impossible to explain why with the rise of the tide level the hot spring temperature becomes higher, while Cl and the electric conductivity decreases. Therefore, this phenomenon is also presumed to be caused by the secondary influence of the rate of discharge. The distribution graph concerning the amount of the solute in the Beppu hot spring is fairly complex. One of the causes producing a solute seems to lie in the stratum beneath the spring mouth. The hot spring water, dissolving the chemical substances, brings them to the spring mouth. When the rate of discharge increases and accordingly the velocity of the hot spring water grows large, the amount of the solute does not increase in proportion to the flow-rate and consequently the concentration diminishes. Only at No. 1197, the electric conductivity increases as well as the rate of discharge. As this spring lies on the natural welling vein and there is a submarine spring hard by it, the sea water may flow in directly.

V. Conclusion

The chief points are summarised.

1) The variation in the rate of discharge is in proportion to that in the tide. The largest difference reaches 19.74 litre per minute, some being as much as 94% of the mean rate of discharge.

2) The tidal influence on the rate of discharge is large near the coast, but suddenly diminishes with distance from the coast. In the city of Beppu, however, there is no hot spring which is not more or less influenced by the tide.

3) The rate of discharge oppositely varies according to the increase or decrease in the atmospheric pressure.

4) The tide acts on the rate of discharge making the hot water head change according to the sea water pressure; the atmospheric pressure acts on it just over the spring mouth, not through the sea surface.

5) There is no phase difference between the variation in the rate of discharge and that in the tide.

6) The hot spring temperature varies parallel with the tide, its difference sometimes reaching 9.3°C.

7) The hot spring temperature varies negatively as the atmospheric pressure.

8) The variation in the hot spring temperature is not caused by the direct influence of the tide or atmospheric pressure, but by the

secondary effect of the variation in the rate of discharge which causes difference in cooling through the conduit pipe.

9) The amount of cooling through the conduit pipe is small in the neighbourhood of the belt where the hot spring water naturally wells. This may be ascribed to the fact that the stratum near that place is at comparatively high temperatures.

10) The electric conductivity and the amount of *Cl* vary negatively as the tide. This is also not caused by the direct influence of the tide, but it seems to be a secondary variation caused by the change in discharge-rate.

In short, the tide acts directly on the rate of discharge as the submarine pressure, and the variations in the hot spring temperature and in the dissolved substances seem to be caused by the variation in the rate of discharge.

As to the characteristics or the causes of the long period variations (e. g. the annual variation), further research will be made.

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