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One Possibility of Oscillation caused by Electrons in a Constant Magnetic Field

By

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Introduction

The author has previously[1] calculated the expressions of dielectric constant and conductivity in the space of a parallel plane condenser, through which electrons flow along a constant magnetic field, and verified the expression of the dielectric constant by comparing it with the data of the experiment in which the author has measured the variation of the dielectric constant with the variation of the magnetic field.

But the detailed discussion of the expression of conductivity has been omitted there.

The consideration of this expression makes the author anticipate the possibility of causing the negative resistance that is, one method of oscillation.

Current between Electrodes

Some of the results which the author has previously[2] made public are quoted here for the present necessity.

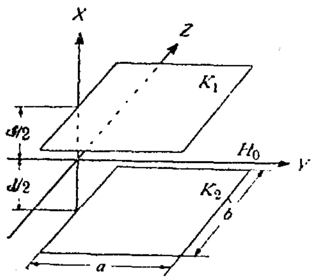
Electrons incident on yz-plane flow between parallel plane electrodes K_1 and K_2 along y-axis, that is, a constant magnetic field of intensity H_0 as in Fig. 1.

Then the current cirulating through K_1, K_2 and the external circuit connected is

1. I. Takahashi: '*Über die Dielektrizitätskonstante des Magneto-aktiven Ionisierten Gases*' (eingegangen am 20ten März, 1941), *These Memoirs*, A, **23**, No. 6, (1942), pp. 373~397.

2. I. Takahashi: *loc. cit.*

Fig. 1



$$J=\int_{t-T}^{t}\frac{nev_x}{d}dt_0-\frac{nes}{d}, \tag{1}$$

where

t: instant of time,

t_0: instant of incidence,

T: transit time through condenser space,

n: number of electrons incident per unit time,

v_x: velocity component of an electron perpendicular to electrodes,

d: distance between electrodes,

s: displacement of an electron at the exit of the condenser space,

e: charge of an electron,

m: mass of an electron,

c: light velocity,

though the last two do not explicitly appear here.

Thus

$$\frac{J}{ab}=E_0e^{j\omega t}(\sigma+j\omega\Delta\varepsilon), \tag{2}$$

where $E_0e^{j\omega t}$ is oscillating voltage between K_1 and K_2.

$$\sigma=\frac{ne^2T^2}{mabd}\frac{\omega}{\omega_0}\frac{1}{2}\left\{G(\omega_1T)-G(\omega_2T)\right\}, \tag{3}$$

$$\Delta\varepsilon=\frac{4\pi ne^2T^2}{mabd}\frac{1}{\omega_0}\frac{1}{2}\left\{F(\omega_1T)+F(\omega_2T)\right\}$$

with

$$\omega_0=\frac{eH_0}{cm}, \tag{4}$$

$$\left.\begin{aligned}\omega_1&=\omega_0-\omega,\\ \omega_2&=\omega_0+\omega,\end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned}G(X)&=\frac{1}{X^2}(1-\cos X),\\ F(X)&=\frac{1}{X}\left(1-\frac{\sin X}{X}\right).\end{aligned}\right\} \tag{6}$$

This expression of $\Delta\varepsilon$ has been previously verified by comparing it with the data of the experiment made by the author.

Character of σ

The graph of $G(X)$ with respect to X is plotted in Fig. 2.

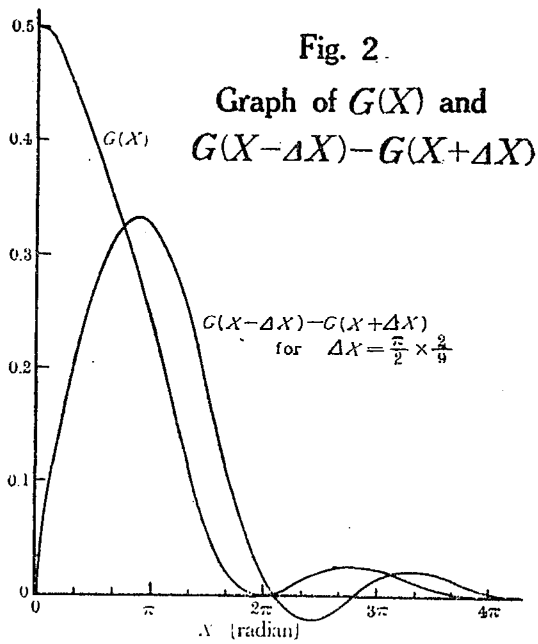


Fig. 2.
Graph of $G(X)$ and
$G(X-\Delta X)-G(X+\Delta X)$

If we put

$$\left.\begin{aligned}\omega_0 T &\equiv X,\\ \omega T &\equiv \Delta X,\end{aligned}\right\} \qquad (7)$$

then $G(\omega_1 T) \equiv G(X-\Delta X)$ and $G(\omega_2 T) \equiv G(X+\Delta X)$ are displaced to the right and to the left of $G(X)$ by ΔX respectively.

On the other hand, the mininum values of $G(X)$ occurs at $X=2n\pi$ with n: integers, and the maximum values of $G(X)$ at $X=(2n-1)\pi-\theta_n$ with n: integers, where $\theta_n \to 0$ with n becoming larger. Therefore we can see the possibility of $\sigma<0$ rises when the vicinity of the maximum of $G(X+\Delta X)$ overlaps that of the minimum of $G(X-\Delta X)$.

For example, in case

$$X+\Delta X=3\pi-\theta_1 \fallingdotseq 3\pi-\frac{\pi}{2}\times\frac{2}{9},$$

$$X-\Delta X=2\pi,$$

that is

$$\left.\begin{aligned}\Delta X=\omega T=\tfrac{1}{2}(\pi-\theta_1)\doteqdot\tfrac{1}{2}(\pi-\pi/9),\\ X=\omega_0 T=\tfrac{1}{2}(5\pi-\theta_1)\doteqdot\tfrac{1}{2}(5\pi-\pi/9),\end{aligned}\right\}\qquad(8)$$

$|\sigma|$ of $\sigma<0$ becomes the largest.

The graph of $G(X-\Delta X)-G(X+\Delta X)$ with respect to X is plotted in Fig. 2, where $\Delta X=\frac{1}{2}(\pi-\theta_1)$.

For $\omega=2\pi\times3\times10^9$, that is, wave length 10cm, the relation (8) become

$$\left.\begin{aligned}T=\frac{1}{4\pi\times3\times10^9}(\pi-\theta_1)\ \text{in sec.},\\ H_0=\frac{1}{2T}(5\pi-\theta_1)\frac{cm}{e}\ \text{in Gauss.}\end{aligned}\right\}$$

These are realizable figures.

Conclusion

The author proposes one method of producing oscillation which he has not yet experimentally verified in spite of his will.

The author hopes to be given kind instructions by the readers.