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On the Possibility of Barkhausen-Kurz Oscillation. I

By

Isao Takahashi

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Introduction

The frequency of Barkhausen-Kurz oscillation is nearly equal to the inverse of the period of the pendulum motion of an electron or to its integral multiples. This is an experimental fact and it can easily be imagined that the frequencies are such ones, when oscillation occurs. But this relation of frequency can never afford any explanation to the occurrence of oscillation, because if the electrons which can contribute to the oscillation are respectively in different phases of motion, their effects are nothing in all. On the other hand, the occurrence of oscillation as an experimental fact must be attributed to the existence of the negative resistance, in an electron tube.

The author has calculated the currents between electrodes in an electron tube for two cases. The first case is one when no electrons are caught by the anode by means of sufficient negative anode voltage and the second is one when the electrons of larger amplitude are caught by the anode. By use of these expressions, the author demonstrates that the negative resistance can occur between electrodes at the expected frequencies. In this paper, only the first case is treated and the second case is to appear in part II.

Currents between Electrodes, when no Electrons are caught by the Anode

We assume the parallel plane electrodes as (a), (b) and (c) in Fig. 1, and neglect the effect of the space charges. In the following, where v_1 and v_2 are assumed to be small against V_g, we calculate, neglecting the terms of the order of $(v/V_g)^2$ and the higher terms.

The notations to be used are summarized:

T_1: transit time from K to G when $v=0$,

T_2: transit time from G to turning point when $v=0$.

$$T_0=2(T_1+T_2),$$

$$\alpha_1=\frac{eV_g}{md_1},\quad \alpha_2=\frac{e(V_g+V_a)}{md_2},$$

$$\beta_1=\frac{ev_1}{md_1},$$

$$\beta_2=\begin{cases}\dfrac{ev_1}{md_2} & \text{for (a),}\\[2mm] \dfrac{e(v_1+v_2)}{md_2} & \text{for (b),}\\[2mm] \dfrac{-ev_2}{md_2} & \text{for (c),}\end{cases}$$

m, $-e$ being the mass and the charge of an electron.

n: the number of electrons emitted per unit time by K to contribute to the oscillation.

The relation

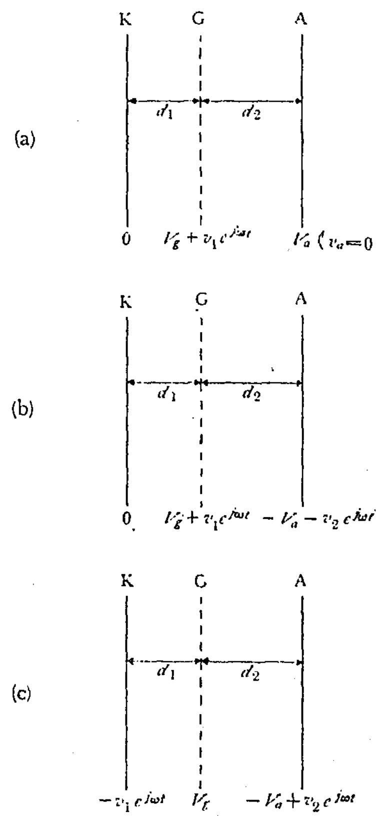
$$\alpha_1T_1=\alpha_2T_2 \tag{1}$$

exists.

From the equation of motion, we obtain the following velocity expressions for an electron leaving K at the instant $t=t_0$:

Voltage distribution according to the circuit

Fig. 1



A: anode
G: grid
K: cathode
V_g: positive grid voltage
$-V_a$: negative anode voltage
$ve^{j\omega t}$: oscillating voltage
d: distance between electrodes

from K to G,

$$u_1=u_1'+u_1'',\qquad u_1'=\alpha_1(t-t_0),\qquad u_1''=\frac{\beta_1}{j\omega}(e^{j\omega t}-e^{j\omega t_0})\,; \tag{2}$$

from G to turning point and thence to G again:

$$u_2=u_2'+u_2'',$$
$$u_2'=a_2(t_0-t)+(a_1+a_2)T_1, \tag{3}$$
$$u_2''=(a_1+a_2)\delta T_1+\frac{\beta_1}{j\omega}e^{j\omega t_0}(e^{j\omega T_1}-1),$$
$$\delta T_1(t_0)=\frac{\beta_1}{a_1T_1\omega^2}e^{j\omega t_0}(e^{j\omega T_1}-1-j\omega T_1);$$

and from G to K,

$$u_3=u_3'+u_3'',\quad u_3'=a_1(t-t_0)-a_1T_0,$$
$$u_3''=-(a_1+a_2)\delta T_2+\frac{\beta_1}{j\omega}(e^{j\omega t}-e^{j\omega(t_0+T+2T_2)}) \tag{4}$$
$$+\frac{\beta_1}{j\omega}e^{j\omega t_0}(e^{j\omega T_1}-1)-\frac{\beta_2}{j\omega}e^{j\omega(t_0+T_1)}(e^{j2\omega T_2}-1),$$
$$\delta T_2(t_0)=\frac{2a_1}{a_2}\delta T_1(t_0)+\frac{2\beta_1}{a_2j\omega}e^{j\omega t_0}(e^{j\omega T_1}-1)$$
$$+\frac{\beta_2}{a_2T_2\omega^2}e^{j\omega(t_0+T_1)}(e^{j2\omega T_2}-1-j2\omega T_2).$$

$\delta T_1(t_0)$ and $\delta T_2(t_0)$ are such that the electron passes through G at $t=t_0+T_1+\delta T_1(t_0)$ on the way to A and at $t=t_0+T_1+2T_2+\delta T_1(t_0)+\delta T_2(t_0)$ on the way back, when δT_1 and δT_2 mean their real parts here.

If the instant at which the electron comes back to K is $t_0+T_0+\delta T_0(t_0)$, $\delta T_0(t_0)$ is not needed to the calculation of currents, as can be indicated later and so we do not seek its functional form. Now, the current J_A flowing into G from outside on account of electron motions between G and A is

$$J_A=\frac{-ne}{d_2}\int_{t_0''}^{t_0'}u_2dt_0,$$

where t_0' is the t_0 of the electron which passes through G at t on the way to A and t_0'' the t_0 of the electron which passes through G at t on the way back. Therefore,

$$t_0'=t-T_1-\delta T_1(t_0')\doteqdot t-T_1-\delta T_1',$$
$$t_0''=t-T_1-2T_2-\delta T_1(t_0'')-\delta T_2(t_0'')$$
$$\doteqdot t-T_1-2T_2-\delta T_1''-\delta T_2'',$$

with

$$\delta T_1' \equiv \delta T_1(t-T_1), \qquad \delta T_1'' \equiv \delta T_1(t-T_1-2T_2),$$
$$\delta T_2'' \equiv \delta T_2(t-T_1-2T_2). \tag{5}$$

Then

$$J_A = \frac{-ne}{d_2}\left\{\int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2} u_2 dt_0 + \int_{t-T_1-2T_2}^{t-T_1} u_2 dt_0 - \int_{t-T_1-\delta T_1'}^{t-T_1} u_2 dt_0\right\}.$$

Let the d.c. part of J_A be J_1, and the oscillating part of frequency ω be i_1, then

$$J_1 = \frac{-ne}{d_2}\int_{t-T_1-2T_2}^{t-T_1} u_2' dt_0 = 0\,,$$

$$i_1 = \frac{-ne}{d_2}\left\{\int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2} u_2' dt_0 - \int_{t-T_1-\delta T_1'}^{t-T_1} u_2' dt_0 + \int_{t-T_1-2T_2}^{t-T_1} u_2'' dt_0\right\},$$

$$\therefore\ i_1 = \frac{ne}{d_2\omega^2}\beta_1 e^{j\omega t}\left\{\frac{\beta_2}{\beta_1}(2-2e^{-j2\omega T_2}-j2\omega T_2-j2\omega T_2 e^{-j2\omega T_2})\right.$$
$$+(1-e^{-j\omega T_1}-j\omega T_1 e^{-j\omega T_1})\left[1+\left(\frac{2\alpha_1+\alpha_2}{\alpha_2}\right)e^{-j2\omega T_2}-\left(\frac{\alpha_1+\alpha_2}{\alpha_2}\right)\right.$$
$$\left.\left.\times\frac{1-e^{-j2\omega T_2}}{j\omega T_1}\right]+(1-e^{-j\omega T_1})(1-e^{-j2\omega T_2}-j2\omega T_2 e^{-j2\omega T_2})\right\}. \tag{6}$$

In the next place, the current J_K flowing into G from outside on account of electron motions between G and K is

$$J_K = \frac{ne}{d_1}\left\{\int_{t-T_1-\delta T_1'}^{t} u_1 dt_0 + \int_{t_0'''}^{t-T_1-2T_2-(\delta T_1''+\delta T_2'')} u_3 dt_0\right\},$$

where t_0''' is the t_0 of the electron which come back to K at t, i.e.

$$t_0''' = t-T_0-\delta T_0(t_0''') \doteqdot t-T_0-\delta T_0''', \quad [\delta T_0''' \equiv \delta T_0(t-T_0)]\,.$$

$$J_K = \frac{ne}{d_1}\left\{\int_{t-T_1-\delta T_1'}^{t-T_1} u_1 dt_0 + \int_{t-T_1}^{t} u_1 dt_0 + \int_{t-T_0-\delta T_0'''}^{t-T_0} u_3 dt_0 + \int_{t-T_0}^{t-T_1-2T_2} u_3 dt_0 - \int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2} u_3 dt_0\right\},$$

and the d.c. part J_2 of J_K is

$$J_2 = \frac{ne}{d_1}\left\{\int_{t-T_1}^{t} u_1' dt_0 + \int_{t-T_0}^{t-T_1-2T_2} u_3' dt_0\right\} = 0\,;$$

and the oscillating part i_2 of frequency ω of J_K is

$$i_2=\frac{ne}{d_1}\left\{\int_{t-T_1-\delta T_1'}^{t-T_1}u_1'dt_0+\int_{t-T_0-\delta T_0'''}^{t-T_0}u_3'dt_0-\int_{t-T_1-2T_2-(\delta T_1''+\delta T_2'')}^{t-T_1-2T_2}u_3'dt_0+\int_{t-T_1}^{t}u_1''dt_0+\int_{t-T_0}^{t-T_1-2T_2}u_3''dt_0\right\},$$

$$\therefore\quad i_2=\frac{ne}{d_1\omega^2}\beta_1 e^{j\omega t}\left\{\frac{\beta_2}{\beta_1}\left[(1-e^{-j2\omega T_2}-j2\omega T_2e^{-j2\omega T_2})-\left(\frac{\alpha_1+\alpha_2}{\alpha_2}\right)\frac{T_1}{T_2}\right.\right.$$
$$\times\frac{1}{j\omega T_1}(1-e^{-j\omega T_1})(1-e^{-j2\omega T_2}-j2\omega T_2e^{-j2\omega T_2})+(1-e^{-j\omega T_1})$$
$$\left.\times(1-e^{-j2\omega T_2})\right]+(1-e^{-j\omega T_1}-j\omega T_1e^{-j\omega T_1})\left[1+\left(\frac{2\alpha_1+\alpha_2}{\alpha_2}\right)e^{-j2\omega T_2}\right]$$
$$+(1-e^{-j\omega T_1})\left[-j2\omega T_2e^{-j2\omega T_2}-\left(\frac{\alpha_1+\alpha_2}{\alpha_2}\right)\frac{1}{j\omega T_1}2(1-j\omega T_1\right.$$
$$\left.-e^{-j\omega T_1})e^{-j2\omega T_2}+(1-e^{-j2\omega T_2}+e^{-j2\omega T_2}\cdot e^{-j\omega T_1})\right]$$
$$\left.+(1-e^{-j\omega T_1}-2j\omega T_1)\right\}. \tag{7}$$

Here it is noticed that in the calculation of i_2,

$$\int_{t-T_0-\delta T_0'''}^{t-T_0}u_3'dt_0=0$$

holds in the first order, independent of the form of $\delta T_0(t_0)$.

Interelectrode impedances and some numerical examples.

In order that oscillation may occur in the system of an electron tube and an external circuit, it is needed that the negative resistance between electrodes in an electron tube occurs.

For the case when no electrons are caught by the anode, we define interelectrode impedance z_1 between G and A and z_2 between G and K by the following expressions, the suffix a, b, c specifying the circuit related respectively:

$$\frac{1}{z_{1a}}=\frac{i_{1a}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1d_2\omega^2}F_1(X,\ Y,\ \varepsilon_a),$$
$$\frac{1}{z_{2a}}=\frac{i_{2a}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1^2\omega^2}F_2(X,\ Y,\ \varepsilon_a),$$
$$\frac{1}{z_{1b}}=\frac{i_{1b}}{(v_1+v_2)e^{j\omega t}}=\frac{ne^2}{md_1d_2\omega^2}\left(\frac{1}{1+x}\right)F_1(X,\ Y,\ \varepsilon_b),$$
$$\frac{1}{z_{2b}}=\frac{i_{2b}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1^2\omega^2}F_2(X,\ Y,\ \varepsilon_b),$$

$$\frac{1}{z_{1e}}=\frac{-i_{1e}}{v_2e^{j\omega t}}=\frac{ne^2}{md_1d_2\omega^2}\left(\frac{1}{-\varkappa}\right)F_1(X,\ Y,\ \varepsilon_e),$$

$$\frac{1}{z_{2e}}=\frac{i_{2e}}{v_1e^{j\omega t}}=\frac{ne^2}{md_1^2\omega^2}F_2(X,\ Y,\ \varepsilon_e),$$

where $$X=\omega T_1,\quad Y=\omega T_2,\quad \varkappa=v_2/v_1,$$

$$\varepsilon_a=\frac{\beta_2}{\beta_1}=\frac{d_1}{d_2},\qquad \varepsilon_b=\frac{\beta_2}{\beta_1}=\frac{d_1}{d_2}\frac{v_1+v_2}{v_1}=(1+\varkappa)\frac{d_1}{d_2},$$

$$\varepsilon_c=\frac{\beta_2}{\beta_1}=-\frac{d_1}{d_2}\frac{v_2}{v_1}=-\varkappa\frac{d_1}{d_2},$$

$$\begin{aligned}F_1(X,\ Y,\ \varepsilon)\equiv{}&\varepsilon(2-2e^{-j2Y}-j2Y-j2Ye^{-j2Y})+(1-e^{-jX}-jXe^{-jX})\\&\times\left[1+\frac{X+2Y}{X}e^{-j2Y}+j\frac{X+Y}{XY}(1-e^{-j2Y})\right]\\&+(1-e^{-jX})(1-e^{-j2Y}-j2Ye^{-j2Y}),\end{aligned}\tag{8}$$

$$\begin{aligned}F_2(X,\ Y,\ \varepsilon)\equiv{}&\varepsilon\Big[(1-e^{-j2Y}-j2Ye^{-j2Y})+j\frac{X+Y}{XY}(1-e^{-jX})\\&\times(1-e^{-j2Y}-j2Ye^{-j2Y})+(1-e^{-jX})(1-e^{-j2Y})\Big]+(1-e^{-jX}\\&-jXe^{-jX})(1+\frac{X+2Y}{X}e^{-j2Y})+(1-e^{-jX})\Big[-j2Ye^{-j2Y}+j\frac{X+Y}{X^2}\\&\times 2(1-jX-e^{-jX})e^{-j2Y}+(1-e^{-j2Y}+e^{-jX}e^{-j2Y})\Big]+(1-e^{-jX}-j2X).\end{aligned}\tag{9}$$

Now the oscillations can be most expected, when the negative resistances occur in both gaps G, A and G, K. On the other hand, from $\alpha_1T_1=\alpha_2T_2$, it follows that

$$\frac{T_1}{T_2}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}=\begin{cases}\varepsilon_a\\ \varepsilon_b/(1+\varkappa)\\ -\varepsilon_c/\varkappa\end{cases}\tag{10}$$

must be fulfilled. We present these possibilities in some numerical examples following.

i) $X=\pi/2,\quad Y=\pi/2,\quad T_1/T_2=1.$

The condition that the negative resistances occur both between G and A, and between G and K is

$$0.3<\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<1 \text{ for (a)},\quad \frac{0.3}{1+\varkappa}<\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{1}{1+\varkappa} \text{ for (b)}.$$

These are possible for a tube with $d_2>d_1$, making $T_1=T_2$.

ii) $X=\frac{1}{4}\pi,\quad Y=\frac{3}{4}\pi,\quad T_1/T_2=1/3.$

The condition is

$$-\frac{0.2}{\varkappa}<\frac{1}{3}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{0.37}{\varkappa}\qquad \text{for (c).}$$

This is possible for a tube with $d_2>3d_1$, making $\varkappa<1.11$, and $T_1/T_2=1/3$.

iii) $X=\frac{1}{2}\pi,\quad Y=\frac{3}{2}\pi,\quad T_1/T_2=1/3.$

The condition is

$$-0.05<\frac{1}{3}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<1.67\qquad \text{for (a),}$$

$$-\frac{0.05}{1+\varkappa}<\frac{1}{3}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{1.67}{1+\varkappa}\quad \text{for (b).}$$

These are possible for a tube with $d_2>3d_1$, making $T_1/T_2=1/3$.

iv) $X=\frac{1}{2}\pi,\quad Y=\frac{5}{2}\pi,\quad T_1/T_2=1/5.$

The condition is

$$-0.17<\frac{1}{5}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<2.6\qquad \text{for (a),}$$

$$\frac{-0.17}{1+\varkappa}<\frac{1}{5}\frac{V_g}{V_g+V_a}=\frac{d_1}{d_2}<\frac{2.6}{1+\varkappa}\quad \text{for (b).}$$

These are possible for a tube with $d_2>5d_1$, making $T_1/T_2=1/5$.

$X+Y=\pi$ means that the oscillating frequency is Barkhausen-Kurz frequency and $X+Y=n\pi$ with n (integers) means the oscillation at its dwarf waves. It has been shown that the oscillation can be expected at B.K. frequency and its dwarf wave frequencies. But the author never means to present the limit of the oscillation.