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# On the Possibility of Barkhausen-Kurz Oscillation. II

By

**Isao Takahashi**

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## 1. Introduction

While in the previous paper[1] with the same title the author treated the case when no electrons are caught by the anode in Barkhausen-Kurz tube, in this paper, the interelectrode current of Barkhausen-Kurz tube is calculated in the case when the electrons of larger amplitude are captured by the anode and the occurrence of negative resistance is demonstrated in some numerical examples.

## 2. Currents between Electrodes, when the Electrons of larger Amplitude are captured by the Anode.

We consider this case for ( a ) and ( b ) with the anode d. c. voltage $V_a = 0$ in Fig. 1. The amplitude $d_m$ of an electron from $G$ to $A$ can be obtained by means of $u_2$ in Part I :

$$\begin{aligned} d_m - d_2 &= \frac{\beta_2}{\omega^2} e^{j\omega(t_0+T_1)} ( e^{j\omega T_2} - 1 - j\omega T_2 ) \\ &\quad + \frac{T_2}{T_1} \frac{\beta_1}{\omega^2} e^{j\omega t_0} (e^{j\omega T_1} - 1 - j\omega T_1 e^{j\omega T_1}) \\ &= \frac{\beta_2}{\omega^2} \gamma e^{j(\omega t_0 - \varphi)}, \end{aligned} \tag{1}$$

where $\beta_1 = \dfrac{ev_1}{md_1}$ for (a) and (b), and

$$\beta_2 = \begin{cases} \dfrac{ev_1}{md_2} & \text{for (a)} \\ \dfrac{-ev_2}{md_2} & \text{for (b)} \end{cases}$$

and $T_1$ is the transit time from $K$ to $G$, while $T_2$ is the transit time

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1. I. Takahashi: On the possibility of Bankhausen-Kurz oscillation, I. This Memoirs, A, Vol. **25**, No 1, Article 3, 1947.

from $G$ to the turning point. Thus the electrons leaving $K$ at $t_0$ which makes the real part of (1) positive, that is, which satisfies

$$\left(2N-\frac{1}{2}\right)\pi<\omega t_0+\varphi<\left(2N+\frac{1}{2}\right)\pi,$$

are destined to be caught by the anode, and the electrons starting at $t_0$ which makes the real part of (1) negative, that is, which satisfies

$$\left(2N+\frac{1}{2}\right)\pi<\omega t_0+\varphi<\left(2N+\frac{3}{2}\right)\pi,$$

are to turn back in front of the anode.

Now, we know that the function $f(\omega t_0+\varphi)$ such that

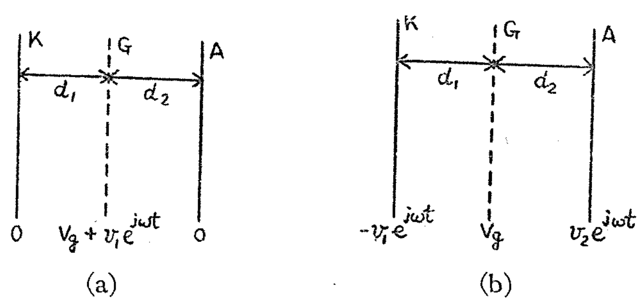
$$f(\omega t_0+\varphi)=0 \quad \text{for} \left(2N-\frac{1}{2}\right)\pi<\omega t_0+\varphi<\left(2N+\frac{1}{2}\right)\pi,$$

and

$$f(\omega t_0+\varphi)=1 \quad \text{for} \left(2N+\frac{1}{2}\right)\pi<\omega t_0+\varphi<\left(2N+\frac{3}{2}\right)\pi,$$

is expressed as:

$$f(\omega t_0+\varphi)=\frac{1}{2}+\frac{2}{\pi}\left(-\frac{\cos(\omega t_0+\varphi)}{1}+\frac{\cos 3(\omega t_0+\varphi)}{3}\cdots\cdots\right)$$
$$=\frac{1}{2}+F(t_0) \qquad (2)$$




$A$: anode $V_g$: positive grid voltage
$G$: grid $ve^{j\omega t}$: oscillating voltage
$K$: cathode $d$: distance between electrodes

Fig. 1. Voltage distribution

We distinguish two classes of electrons. The first class consists of electrons which are not caught by the anode, thus being emitted by $K$ in $n\{1/2+F(t_0)\}$ per unit time, and the second class consists of electrons which are caught by the anode, thus being emitted in $n\{1/2-F(t_0)\}$

per unit time. We proceed as in Part I. The current $J'_A$ flowing into $G$ from outside on account of electron motions between $G$ and $A$ is

$$J'_A = -\frac{ne}{d_2}\int_{t-T_1-2T_2-\delta T_1''-\delta T_2''}^{t-T_1-\delta T_1'}\left(\frac{1}{2}+F(t_0)\right)u_2\,dt_0 - \frac{ne}{d_2}\int_{t-T_1-T_2-\delta T_1'-\delta T_4'}^{t-T_1-\delta T_1'}\left(\frac{1}{2}-F(t_0)\right)u_2\,dt_0\,,$$

where $\delta T_4$ is included in the instant $t = t_0+T_1+T_2+\delta T_1(t_0)+\delta T_4(t_0)$ at which the second class electron reaches $A$ and $\delta T'_4$ is equal to $\delta T_4(t-T_1-T_2)$. Then, the d.c. part of $J'_A$ is

$$J'_1 = -\frac{ne}{d_2}\left\{\int_{t-T_1-T_2}^{t-T_1}\frac{1}{2}u'_2dt_0 + \int_{t-T_1-2T_2}^{t-T_1}\frac{1}{2}u'_2dt_0\right\} = -\frac{nea_2}{4d_2}T_2^2$$

and the a.c. part of frequency $\omega$ of $J'_A$ is

$$i'_1 = -\frac{ne}{d_2}\left\{\int_{t-T_1-2T_2-\delta T_1''-\delta T_2''}^{t-T_1-2T_2}\frac{1}{2}u'_2dt_0 + \int_{t-T_1-2T_2}^{t-T_1}\frac{1}{2}u''_2dt_0 - \int_{t-T_1-2T_2}^{t-T_1}\frac{2}{\pi}\cos(\omega t_0+\varphi)\,u'_2\,dt_0\right.$$
$$-\int_{t-T_1-\delta T_1'}^{t-T_1}\frac{1}{2}u'_2dt_0 + \int_{t-T_1-T_2-\delta T_1'-\delta T_4'}^{t-T_1-T_2}\frac{1}{2}u'_2dt_0 + \int_{t-T_1-T_2}^{t-T_1}\frac{1}{2}u'_2\,dt_0$$
$$\left.+\int_{t-T_1-T_2}^{t-T_1}\frac{2}{\pi}\cos(\omega t_0+\varphi)\,u'_2dt_0 - \int_{t-T_1-\delta T_1'}^{t-T_1}\frac{1}{2}u'_2dt_0\right\}.$$

Thus

$$i'_1 = -\frac{ne}{d_2\omega^2}\frac{2a_2}{\pi}e^{j\omega t}\left\{e^{j\varphi}\left(e^{j\omega T_1}+1+j\omega T_2 e^{j\omega T_1}\right)\right\}$$
$$+\frac{ne}{d_2\omega^2}\beta_1 e^{j\omega t}\left\{\left(\frac{2a_1+a_2}{2a_1}\right)\left(1-e^{j\omega T_1}-j\omega T_1 e^{-j\omega T_1}\right)e^{-j2\omega T_2}\right.$$
$$-j\omega T_2\left(1-e^{-j\omega T_1}\right)e^{-j2\omega T_2}$$
$$+\left(1-e^{-j\omega T_1}-j\omega T_1 e^{-j\omega T_1}\right)$$
$$-\frac{1}{2}\left(2-e^{-j\omega T_1}-e^{-j2\omega T_2}\right)$$
$$\times\left[\left(\frac{a_1+a_2}{a_1}\right)\frac{\left(1-e^{-j\omega T_1}-j\omega T_1 e^{-j\omega T_1}\right)}{j\omega T_1}-\left(1-e^{j\omega T_1}\right)\right]$$
$$+\frac{1}{2}\frac{\beta_2}{\beta_1}\left(1-e^{-j2\omega T_2}-j2\omega T_2 e^{-j2\omega T_2}\right)$$
$$\left.+\frac{1}{2}\frac{\beta_2}{\beta_1}\left(2-e^{-j\omega T_1}-e^{j2\omega T_2}-j3\omega T_2\right)\right\}, \qquad (3)$$

where $a_1 = \frac{eV_g}{md_1}$, $x_2 = \frac{eV_g}{md_2}$.

The current $J_K'$ flowing into $\dot{G}$ from outside on account of electron motions between $K$ and $G$ is

$$J_K' = \frac{ne}{d_1}\int_{t-T_0-\delta T_0'}^{t-T_1-2T_2-\delta T_1''-\delta T_2''}\left(\frac{1}{2}+F(t_0)\right)u_3\,dt_0 .$$

Then, $J_2'$, the d. c. part and $i_2'$, a. c. part of frequency $\omega$ are respectively:

$$J_2' = \frac{ne}{d_1}\left\{\int_{t-T_0}^{t-T_1-2T_2}\frac{1}{2}u_3'dt_0 + \int_{t-T_1}^{t}u_1'dt_0\right\} = \frac{nex_1}{4d_1}T_1^2 ,$$

$$i_2' = \frac{ne}{d_1}\left\{\int_{t-T_0-\delta T_0'}^{t-T_0}\frac{1}{2}u_3'dt_0 + \int_{t-T_0}^{t-T_1-2T_2}\frac{1}{2}u_3''dt_0 - \int_{t-T_0}^{t-T_1-2T_2}\frac{2}{\pi}\cos(\omega t_0+\varphi)\,u_3'dt_0\right.$$

$$\left. -\int_{t-T_1-2T_2-\delta T_1''-\delta T_2''}^{t-T_1-2T_2}\frac{1}{2}u_3'dt_0 + \int_{t-T_1-\delta T_1'}^{t-T_1}u_1'dt_0 + \int_{t-T_1}^{t}u_1''dt_0\right\}.$$

Thus, we have

$$\begin{aligned} i_2' = &-\frac{ne}{d_1\omega^2}\frac{2d_1}{\pi}e^{j\omega t}\left\{e^{j\varphi}(1-e^{j\omega t}+j\omega T_1e^{j\omega T_1})\right\} \\ &+\frac{ne}{d_1\omega^2}\beta_1e^{j\omega t}\left\{\left(\frac{2a_1+a_2}{2a_2}\right)(1-e^{-j\omega T_1}-j\omega T_1e^{-j\omega T_1})\,e^{-j2\omega T_2}\right. \\ &\quad -j\omega T_2(1-e^{-j\omega T_1})\,e^{-j2\omega T_2} \\ &\quad +(1-e^{-j\omega T_1}-j\omega T_1e^{-j\omega T_1}) \\ &\quad +\frac{1}{2}\left[3(1-e^{-j\omega T_1}-j\omega T_1)\right. \\ &\qquad -(1-e^{-j\omega T_1})^2e^{-j2\omega T_2} \\ &\quad \left.-\frac{a_1+a_2}{x_2}\frac{2(1-e^{-j\omega T_1})(1-e^{-j\omega T_1}-j\omega T_1)\,e^{-j2\omega T_2}}{j\omega T_1}\right] \\ &\quad +\frac{1}{2}\frac{\beta_2}{\beta_1}(1-e^{-j2\omega T_2}-j2\omega T_2e^{-j2\omega T_2}) \\ &\quad +\frac{1}{2}\frac{\beta_2}{\beta_1}(1-e^{-j\omega T_1})\times\left[(1-e^{-j2\omega T_2})\right. \\ &\qquad \left.\left.-\frac{a_1+a_2}{a_2}\frac{(1-e^{-j2\omega T_2}-j2\omega T_2e^{-j2\omega T_2})}{j\omega T_2}\right]\right\} \end{aligned}$$

Now, the total d. c. current becomes

$$J'_1 + J'_2 = \frac{ne}{4}\left(\frac{a_1}{d_1}T_1^2 - \frac{a_2}{d_2}T_2^2\right) = 0\,, \tag{4}$$

because of $a_1T_1 = a_2T_2$.

## 3. Some Numerical Examples

$i'_1$ and $i'_2$ consist respectively of two components, one of which contains the factor $\alpha$ and the other the factor $\beta$.

We attach the suffixes $a$ and $b$ to $i'_1$ and $i'_2$ for the cases (a) and (b) in Fig. 1 respectively. For (b) $-i'_{1b}$ is chosen instead of $i'_{1b}$, on account of the sign of the oscillating voltage between $G$ and $A$.

Hence

$$i'_{1a} = \frac{ne}{d_2\omega^2}\frac{2a_2}{\pi}e^{j\omega t}\,(A_1 + jB_1) + \frac{ne}{d_2\omega^2}\beta_1 e^{j\omega t}\,(R_1 + jI_1),$$

$$i'_{2a} = \frac{ne}{d_1\omega^2}\frac{2a_1}{\pi}e^{j\omega t}\,(A_2 + jB_2) + \frac{ne}{d_1\omega^2}\beta_1 e^{j\omega t}\,(R_2 + jI_2),$$

$$-i'_{1b} = \frac{ne}{d_2\omega^2}\frac{2a_2}{\pi}e^{j\omega t}\,(A_3 + jB_3) + \frac{ne}{d_2\omega^2}\beta_1 e^{j\omega t}\,(R_3 + jI_3),$$

$$i'_{2b} = \frac{ne}{d_1\omega^2}\frac{2a_1}{\pi}e^{j\omega t}\,(A_4 + jB_4) + \frac{ne}{d_1\omega^2}\beta_1 e^{j\omega t}\,(R_4 + jI_4),$$

and we obtain the following table:

| $\omega T_1$ | $\omega T_2$ | $A_1$ | $R_1$ | $A_2$ | $R_2$ | $A_3$ | $R_3$ | $A_4$ | $R_4$ |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{\pi}{2}$ | $\frac{\pi}{2}$ | +0.57 | +0.66 | +0.57 | +0.52 | −0.30 | +4.43 | −1.11 | +2.03 |
| $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | −2.22 | +3.16 | +0.49 | −0.32 | −1.80 | +2.65 | −0.43 | +0.00 |
| $\frac{\pi}{4}$ | $\frac{3\pi}{4}$ | −0.97 | +9.72 | +0.30 | −0.13 | −2.36 | +1.43 | −0.12 | −0.26 |

As the first terms in $i'_1$ and $i'_2$ are larger in magnitude than the second terms, corresponding to the ratio of $a$ to $\beta$, so we expect the oscillation, when both $A$'s are negative, that is, for $-i'_{1b}$ and $i'_{2b}$ in these examples. Here are presented only the possibility of oscillation but not the limit of oscillation.