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On the Ultraviolet Radiation from the Solar Corona. II.

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§ 5. *Ionization of He I.* Denote by B_{1c} the absorption coefficient from the ground level 1 of *He I* to the continuous level c. We assume it to be equal to the value at the series limit with the effective range Δ_ν. Then we can write[4]

$$B_{1c} = \frac{4\pi^2 e^2}{mch\nu\Delta_\nu} f \ , \qquad (5,1)$$

in which $f = 1.55$ and $\Delta_\nu = 9.75 \times 10^{15}$.

Let A_{c1} and B_{c1} be the Einstein coefficients for the transition $c \rightarrow 1$. Then, we have, by the well-known consideration of thermodynamical equilibrium,

$$\frac{B_{1c}}{B_{c1}} = \frac{4(2\pi mkT)^{3/2}}{h^3} \ , \ A_{c1} = \sigma_\nu B_{c1}\Delta_\nu \ , \ \sigma_\nu = \frac{2h\nu^3}{c^2} \ , \qquad (5,2)$$

where T means the electron temperature of the chromosphere. We shall safely assume it to be equal to the effective temperature of photosphere.

Let N_e, N and N^+ be the particle density of electron, *He I* and *He II* respectively, and J_ν the mean intensity of the coronal ν-radiation. Then, the equation of ionization is given by

$$N_e N^+ q A_{c1} = N B_{1c} J_\nu \Delta_\nu \ , \qquad (5,3)$$

where we have introduced the factor q in order to include the capture to the excited levels. This becomes, by using (5,2),

$$\frac{N_e N^+}{N} = \frac{4(2\pi mkT)^{3/2}}{qh^3\sigma_\nu} J_\nu. \qquad (5,4)$$

As for J_ν, it changes with the depth, since the chromosphere is not transparent to the coronal radiation. We shall first calculate the optical depth of the chromosphere. The optical depth at the height z is, by definition,

$$\tau = \int_z^\infty \frac{h\nu}{4\pi} B_{1c} N(z)\, dz\,. \tag{5,5}$$

Following A. Unsöld[5], assume the density distribution of the lower chromosphere to be of the form:

$$N(z) = N(0)\,(0.982e^{-\alpha z} + 0.018e^{-\alpha' z})\,, \tag{5,6}$$

where

$$a = 5.9 \times 10^{-8}\text{cm}^{-1}\;,\; a' = 2.6 \times 10^{-8}\text{cm}^{-1}\,.$$

We shall first consider $N(z)$ to be the particle density of the total gas, or practically that of hydrogen. It is not known exactly. In the following, we shall treat three cases: $\log N(0) = 16$, 15 and 14. *He* abundance is also not known accurately. Using Payne's estimation[6] from the stellar data, we take for the logarithm of the relative abundance of *He* to *H* to be -2.7. Thus, we shall consider for H_e the following three cases: $\log N(0) = 13.3$, 12.3 and 11.3.

As will be seen later, *He* ionization is weak throughout the chromosphere, so that we can consider (5,6) as the distribution of H_e *I*. The optical depth , (5,5), is then expressed by this and (5,1) as follows:

$$\tau = \frac{\pi e^2 f}{mc\Delta_\nu} N(0) \left(\frac{0.982}{a} e^{-\alpha z} + \frac{0.018}{a'} e^{-\alpha' z} \right). \tag{5,7}$$

With this formula we find for the optical thickness $\tau_1 = 1500$, 150, 15, corresponding to $\log N(0) = 13.3$, 12.3 and 11.3 respectively.

Coronal radiation will be absorbed or scattered in the chromosphere. In place of the exact treatment of the problem, we shall consider two extreme cases: namely, the case of pure absorption and of pure scattering. As is easily seen by the rough estimation, the probability of de-excitation from the excited level of *He I* by the electron collision is larger than that of ionization and capture. Therefore, the actual condition in the chromosphere may be near to the case of pure absorption.

In the case of pure absorption, we have at once

$$J_\nu = J_\nu^* e^{-\tau}\,, \tag{5,8}$$

while in the case of pure scattering, it can easily be shown that

$$J_\nu = J_\nu^* \,(5 - 2e^{-\tau})\,, \tag{5,9}$$

where we have denoted by J_ν^* the mean intensity of incident coronal light.

§ *6*. The ionization formula (5,3) combined with (5,8) becomes

$$\frac{N_e N^+}{N} = 3.1 \times 10^5 e^{-\tau}\,. \tag{6,1}$$

In the numerical calculation we have taken $q = 2$ and $T_e = 10^6$. Assume that the distribution of the free electron follows the law (5,6), with Cillié and Menzel's estimation[3] $N_e(0) = 4 \times 10^{11}\,\text{cm}^{-3}$ at the base. Then, for $N_e \sim 10^9\,\text{cm}^{-3}$ and $e^{-\tau} \sim 1$ near $z = 1000\,\text{km}$ we have $N^+/N = 3 \times 10^{-4}$.

Population N' of the excited levels are connected with the ionized level by the usual Saha formula:

$$\frac{N_e N^+}{N'} = K(T)\,. \tag{6,2}$$

Observed *He* spectra are emitted by the scattering of the photospheric radiation. They are thus proportional to N' and hence to NJ_ν, according to (6,2). Three light curves in Fig. 1 represent the quantity

$$E(z) = \frac{N(z)}{N(0)} e^{-\tau}\,, \tag{6,3}$$

for $\log N(0) = 13.3$, 12.3 and 11.3 respectively. At the great height, $E(z)$ decreases with $N(z)$, while at the deeper layers, it again decreases, since the coronal radiation is weakened rapidly by the absorption. $E(z)$ thus takes a maximum at certain height.

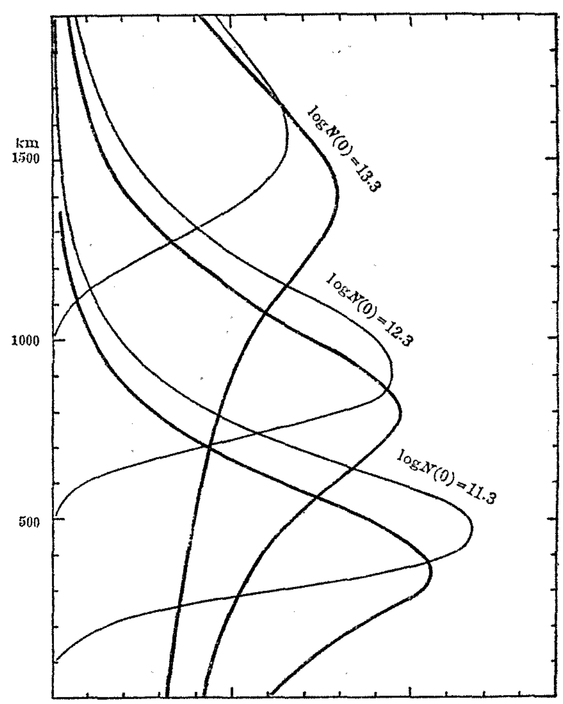

Fig. 1 light curves: $E(z)$, scale unit 2×10^{-4}, 10^{-3} and 10^{-2}
heavy curves: Integrated quantity, scale unit 10 km, 100 km and 1000 km, each for $\log N(o) =$ 13.3, 12.3 and 11.3 respectively.

Integrating $E(z)$ to the line of sight, we obtain the quantity directly comparable to the intensity of the flash. The heavy curves in Fig. 1 represent the result of integration. The height of the maximum is shifted scarcely, but there remains a considerable order of intensity even at the base $z = 0$.

A. Pannekoek and M. Minnaert[7] have observed that the *He* intensity increases with height and first above $\sim$ 1000 km it begins to decrease. Appearance of the maximum is also observed by E. J. Perepelkin and O. A. Melnikov[8] in D_3-line. Theoretically, the maximum place is a function of the chromospheric density. It occurs near 1000 km for $\log N(0) = 12.3$.

So far, we have neglected the absorption due to the neutral hydrogen. It must be taken into account for the lower chromosphere. The absorption decreases rapidly with increasing height, since the hydrogen begins to ionize. Consideration of this effect will not change our conclusion.

In the case of pure scattering, coronal radiation takes a constant intensity throughout the chromosphere. Consequently, there appears no maximum. When the hydrogen absorption is effective, coronal radiation will be weakened in the lower chromosphere and again we can expect a maximum.

§ 7. *Concentration of the excited atom.* We shall make some estimation about the absolute density of the excited *He* atoms, which can be compared with those deduced from the flash observation. At the height $\sim$ 1000 km and for $\log N(0) = 12.3$, we have, as before, $\log N^+/N = -3.5$. Next, by (6,2) with $\chi = 4.74 eV$ for the ionization potential from the excited level, we have $\log N^+/N' = 6.5$. Therefore, $\log N'/N = -10$ and with $\log N = 10$, we have $N' \sim 1\ \mathrm{cm}^{-3}$. These figures are in accord with observation in their order of magnitudes.

§ 8. *Ionization of He II.* Ionization formula for *He II* can be written as

$$N_e N^{++} q^+ A_{c1} = N^+ B_{1c} J_\nu \Delta_\nu\,, \tag{8,1}$$

where J_ν^+ is the mean intensity at the limit of *He II* ionization $\chi = 54.14\ eV$ and other notations are parallel to (5,3).

The probability of absorption per sec per atom is given by

$$B_{1c} J_\nu \Delta_\nu = \int_{\nu_0}^{\infty} B_{1c}(\nu) J_\nu d\nu\ ,\ \ B_{1c}(\nu) = \frac{2^8 \pi^5}{3\sqrt{3}} \frac{m Z^4 \varepsilon^{10}}{c h^7 \nu^4} \cdot (\,Z = 2\,) \tag{8,2}$$

Coronal intensity J_ν is given by (4,1). Introducing it into (8,2), we get

$$B_{1c} J_\nu \Delta_\nu = \frac{2^9 \pi^4 m Z^4 \varepsilon^{10}}{3\sqrt{3}\, c h^7} \frac{C}{T_\varepsilon^{1/2}} \left(\frac{h}{kT_\varepsilon}\right)^3 \int_{h\nu_0/KT_\varepsilon}^{\infty} \frac{e^{-x}}{x^4} dx \tag{8,3}$$

For the capture probability, we have

$$A_{c1} = \frac{2^9\pi^5}{(6\pi)^{3/2}} \frac{\varepsilon^{10}}{m^2c^3h^3} \left(\frac{m}{k}\right)^{3/2} Z^4 e^{\frac{h\nu}{kT}} \frac{1}{T^{3/2}} E_i\left(\frac{h\nu}{kT}\right). \qquad (8,4)$$

Using these expressions and taking $q^+ = 2$, $T_\varepsilon = 10^6$, $T = 5600$, we get for (8,1),

$$\frac{N_e N^{++}}{N^+} = 2.1 \times 10^5 . \qquad (8,5)$$

Owing to its small concentration, *He II* is transparent to the coronal radiation near 54.14 eV, but it will be weakened in the lower chromosphere by the absorption due to *He I* and *He II*.

λ 4686 of *He II* is emitted by the capture. Its intensity may be proportional to $N_e N^{++}$, which, in turn, is proportional to N^+ by (8,5). According to (6,1), N^+ is proportional to N/N_e at great height. Thus, a slow decrease of λ 4686 with increasing height can be expected, in accord with observation.

In § 4, it has been shown that the contact temperature raises to $T = 16500$ at $\chi = 54.14\,eV$. If we make the formal application of Saha's formula, we must take $\Phi(54.14\,eV) = \chi\, 5040/T = 16.5$. This is in agreement with the estimation $\Phi = 15.67$ derived by A. Unsöld from other directions.[9]

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