Memoirs of the College of Science, University of Kyoto, Series A,
Vol. XXV, Nos. 3 & 4, Article 21, 1949.

On the Electronic Admittance in Reflex Klystron. I

By

Isao Takahashi and **Kenji Mitani**

(Received August 24, 1949)

Introduction

The possibility of oscillation of the klystron, including the case of the reflex klystron, has been discussed by many authors, [3, 4, 7 *] but in their calculations, the effect of the transit time of electron in the resonator gap and the effect of the oscillatory voltage on the electron motion in the drift space have been neglected. We have calculated the interelectrode admittance of the reflex klystron, taking these effects into consideration. The method of calculation is similar to that which the senior author (I. T.) has adopted in the case of B. K. oscillation.[2]

In the case of reflex klystron too as in the case of B. K. oscillation, we could demonstrate the production of the negative conductance.

The Velocity of Electron in Each Space

The sectional view of the reflex klystron in principle is shown in Fig. 1 (a).

F : filament
R : cavity resonator
G_1, G_2 : grids forming the gap of cavity resonator
d : the distance between G_1 and G_2
A : repeller
D : the distance between G_2 and A

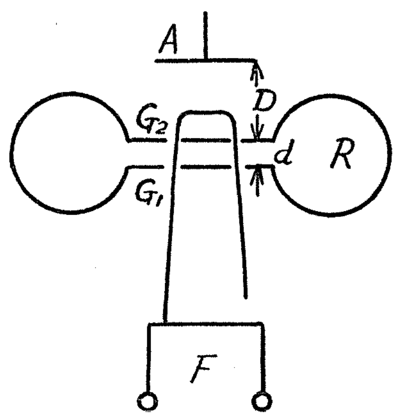

Fig. 1. (a)

* These numbers indicate the numbers of papers listed at the end of this paper.

Fig. 1 (b) shows the potential distribution in this klystron; V_0 is the accelerating voltage; $v_g e^{j\omega t}$ is the oscillatory one which G_2 has against G_1, and $-V_a$ is the voltage of A against G_1, consequently the repelling voltage acting between G_2 and A is $-(V_a + v_g e^{j\omega t})$.

In the following, we consider the induced current flowing into G_2 by way of the resonator from G_1.

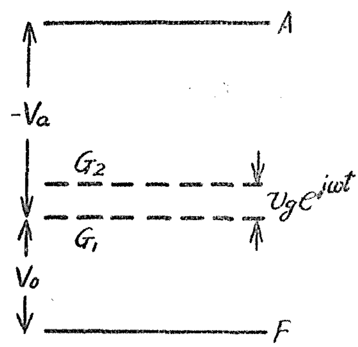

Fig. 1. (b)

First we calculate the velocity of electron in each space. The velocity u_0 which the electron has when it reaches G_1 starting from the filament, is

$$u_0 = \sqrt{\frac{2eV_0}{m}}, \qquad (1)$$

where m is the mass of an electron and $-e$ its charge.

(a) From G_1 to G_2.

The equation of motion is

$$m\frac{du_1}{dt} = e\frac{v_g}{d}e^{j\omega t}.$$

From this equation, we get the velocity u_1 in the resonator gap:

$$u_1(t, t_0) = u_0 + \frac{\alpha_1}{j\omega}(e^{j\omega t} - e^{j\omega t_0}), \qquad (2)$$

where

$$\alpha_1 \equiv \frac{e}{m}\frac{v_g}{d}, \qquad (3)$$

and t_0 is the instant at which the electron passes through G_1.

The instant t_1 at which the electron reaches G_2 can be put:

$$t_1 = t_0 + T_1 + \delta T_1, \qquad (4)$$

where T_1 is the transit time from G_1 to G_2 when $v_g = 0$, namely

$$T_1 = \frac{d}{u_0}. \qquad (5)$$

And the deviation δT_1 from T_1 is calculated in the first approximation as:

$$\delta T_1(t_0) = \frac{\alpha_1}{\omega^2 u_0} e^{j\omega t_0}(e^{j\omega T_1} - 1 - j\omega T_1) . \qquad (6)$$

Thus, the velocity which the electron has when it reaches G_2, becomes, in the first approximation,

$$u_1(t_1) = u_1(t_1, t_0) = u_0 + \frac{\alpha_1}{j\omega} e^{j\omega t_0}(e^{j\omega T_1} - 1) .$$

(b) From G_2 to A and again to G_2.

The equation of motion is

$$m\frac{du_2}{dt} = -\frac{e}{D}V_a(1 + Re^{j\omega t}), \text{ where } R = \frac{v_g}{V_a}. \qquad (7)$$

From this equation, the velocity u_2 in this space is

$$u_2(t, t_0) = u_1(t_1) - \alpha_2(t - t_1) - \frac{\alpha_2 R}{j\omega}(e^{j\omega t} - e^{j\omega (t_0 + T_1)}), \qquad (8)$$

where

$$\alpha_2 \equiv \frac{e}{m}\frac{V_a}{D}. \qquad (9)$$

The instant, when the electron comes back to G_2, can be put

$$t_2 = t_1 + T_2 + \delta T_2 = t_0 + (T_1 + \delta T_1) + (T_2 + \delta T_2), \qquad (10)$$

where T_2 is the transit time from G_2 to G_2 before A, when $v_g = 0$, that is

$$T_2 = \frac{2u_0}{\alpha_2}. \qquad (11)$$

δT_2 is the deviation from T_2 caused by v_g, and is calculated from (8) as:

$$\delta T_2(t_0) = \frac{2\alpha_1}{j\omega\alpha_2} e^{j\omega t_0}(e^{j\omega T_1} - 1) + \frac{2R}{\omega^2 T_2} e^{j\omega (t_0 + T_1)}(e^{j\omega T_2} - 1 - j\omega T_2) . (12)$$

Accordingly, the velocity which the electron has when it is back to G_2, is

$$u_2(t_2) = u_2(t_2\,, t_0)$$
$$= -\, u_0 \frac{\alpha_1}{j\omega} e^{j\omega t_0} (e^{j\omega T_1} - 1)$$
$$+ R \frac{\alpha_2}{\omega^2 T_2} e^{j\omega (t_0 + T_1)} \left\{(-\,2 + j\omega T_2)\, e^{j\omega T_2} + 2 + j\omega T_2\right\} \,. \quad (13)$$

(c) From G_2 to G_1.

By solving the equation of motion, $m\frac{du_3}{dt} = e\frac{v_g}{d} e^{j\omega t}$, the velocity of the electron moving from G_2 to G_1 is found to be equal to

$$u_3(t, t_0) = u_2(t_2) + \frac{\alpha_1}{j\omega}(e^{j\omega t} - e^{j\omega (t_0 + T_1 + T_2)})\,. \quad (14)$$

If we put the time t_3 when the electron comes back to G_1, as follows:

$$t_3 = t_2 + T_1 + \delta T_3\,, \quad (15)$$

we get, by integrating (14), the following form for δT_3:

$$\delta T_3(t_0) = -\, e^{j\omega (T_1 + T_2)}\, \delta T_1(t_0) + \frac{\alpha_1}{\omega^2 u_0}(j\omega T_1) e^{j\omega t_0} (e^{j\omega T_1} - 1)$$
$$+ R \frac{\alpha_2}{\omega^2 u_0} \frac{T_1}{T_2} e^{j\omega (t_0 + T_1)} \left\{(-\,2 + j\omega T_2) e^{j\omega T_2} + 2 + j\omega T_2\right\} \,. \quad (16)$$

Induced Current in the Resonator Gap

We calculate the induced current flowing into G_2 by way of the resonator from G_1, caused by electron motion in like manner as in the previous papers.[1, 2]

The induced current J_1 caused by the repelled electrons is

$$J_1 = \frac{ne}{d} \int_{t_0''}^{t_0'} u_3\, dt_0\,, \quad (17)$$

where n is the number of electrons emitted per unit time by F to contribute to the oscillation. t_0' is the t_0 of the electrons which pass through G_2 at t on their way back from before A and t_0'' the t_0 of the electrons which come back to G_1 at t on their way back too.

We have

$$t_0' = t - T_1 - T_2 - \delta T_1(t_0') - \delta T_2(t_0') \fallingdotseq t - T_1 - T_2 - \delta T_1' - \delta T_2',$$
$$t_0'' = t - 2T_1 - T_2 - \delta T_1(t_0'') - \delta T_2(t_0'') - \delta T_3(t_0'')$$
$$\fallingdotseq t - 2T_1 - T_2 - \delta T_1'' - \delta T_2'' - \delta T_3''\,,$$

with

$$\delta T_1' \equiv \delta T_1(t - T_1 - T_2)\,, \qquad \delta T_2' \equiv \delta T_2(t - T_1 - T_2)\,,$$
$$\delta T_1'' \equiv \delta T_1(t - 2T_1 - T_2)\,, \qquad \delta T_2'' \equiv \delta T_2(t - 2T_1 - T_2)\,,$$
$$\delta T_3'' \equiv \delta T_3(t - 2T_1 - T_2)\,.$$

Then we get, to the first approximation,

$$\begin{aligned} J_1 = &- ne + \frac{ne\alpha_1}{d\omega^2} e^{j\omega t}\Big[2 - j\omega T_1 - (2 + j\omega T_1)\, e^{-j\omega T_1} \\ &+ (2 - j\omega T_2)\, e^{-j\omega T_2} + (2 + 2j\omega T_1 - j\omega T_2)\, e^{-j\omega(T_1 + T_2)} \\ &+ (-4 - 2j\omega T_1 + 2j\omega T_2)\, e^{-j\omega(T_1 + T_2)}\Big] \\ &+ \frac{ne\alpha_2 R}{d\omega^2} e^{j\omega t}\Big[1 - e^{-j\omega T_2} - j\omega T_2 e^{-j\omega T_2} - e^{-j\omega T_1} \\ &+ e^{-j\omega(T_1 + T_2)} + j\omega T_2 e^{-j\omega(T_1 + T_2)} \\ &- \frac{T_1}{T_2}\Big\{(-2 + j\omega T_2)\, e^{-j\omega T_1} + (2 + j\omega T_2)\, e^{-j\omega(T_1 + T_2)}\Big\} \\ &+ \frac{1}{j\omega T_2}\Big\{(-2 + j\omega T_2) + (2 + j\omega T_2)\, e^{-j\omega T_2}\Big\}\,(1 - e^{-j\omega T_1})\Big]\,. \end{aligned} \tag{18}$$

Similarly, the induced current J_2 caused by the electrons moving from G_1 to G_2 is

$$J_2 = \frac{ne}{d}\int_{t_0'''}^{t} u_1 dt_0\,, \tag{19}$$

where t_0''' is the t_0 of the electrons which pass through G_2 at t on their way to A. Therefore

$$t_0''' = t - T_1 - \delta T_1(t_0''') \doteqdot t - T_1 - \delta T_1'''\,,$$

with $\delta T_1''' \equiv \delta T_1(t - T_1)$. Thus we get, to the first approximation,

$$J_2 = ne + \frac{ne\alpha_1}{d\omega^2} e^{j\omega t}\Big[2 - j\omega T_1 - (2 + j\omega T_1)\, e^{-j\omega T_1}\Big]\,. \tag{20}$$

Thus, by (18) and (20), the total induced current J flowing into G_2 by way of the resonator from G_1 is given by

$$J = J_1 + J_2\,. \tag{21}$$

The Electronic Admittance in the Gap

In order that the oscillation may occur in this krystron, it is necessary that the negative conductance is produced in the resonator gap.

We can define the electronic admittance in the resonator gap by the following expression:

$$Y = \frac{J}{v_g e^{j\omega t}}, \tag{22}$$

whose real part is the conductance.

For simplicity, we take the case, where $\alpha_2 R = \dfrac{ev_g}{mD}$ can be neglected against $\alpha_1 = \dfrac{ev_g}{md}$ because of $d \ll D$. In this case, we get for Y

$$\begin{aligned} Y_1 = \frac{ne^2}{md^2\omega}\Big[4 - 2j\omega T_1 - (4 + 2j\omega T_1)\, e^{-j\omega T_1} \\ + (2 - j\omega T_2)\, e^{-j\omega T_2} + (2 + 2j\omega T_1 - j\omega T_2) e^{-j\omega(2T_1+T_2)} \\ + (-4 - 2j\omega T_1 + 2j\omega T_2) e^{-j\omega(T_1+T_2)}\Big]. \end{aligned} \tag{23}$$

We assume ωT_1 to be sufficiently small compared with unity and expand $e^{-j\omega T_1}$ and $e^{-j\omega T_1}$ in powers of $-j\omega T_1$, and then we express (23) in an ascending power series of $-j\omega T_1$.

Then, it will be seen that (23) begins with the second power of $-j\omega T_1$. This corresponds to the factor $1/d^2$, and therefore the approximate expression for Y_1 becomes

$$Y_1 = \frac{ne^2}{mu_0^2}\left[e^{-j\omega T_2} + \frac{1}{2} j\omega T_2\, e^{-j\omega T_2}\right], \tag{24}$$

whose real part is

$$G_1 = \frac{ne^2}{mu_0^2}\left[\cos\omega T_2 + \frac{1}{2}\omega T_2 \sin\omega T_2\right]. \tag{25}$$

The expression (25) of G_1 takes its minima (negative values) in the vicinity of

$$\omega T_2 = 2\pi\left(n + \frac{3}{4}\right), \quad n = 0, 1, 2, \cdots\cdots\cdots. \tag{26}$$

Thus, we can demonstrate the possibility of the oscillation in the reflex krystron. The relation (26) gives the well-known optimum phase between the bunched current and the gap potential in the reflex klystron.

If we retain the term with the factor $\alpha_2 R$ in the expression of Y, we have the following additional term Y_2 to the expression (23) for Y_1:

$$Y_2 = \frac{ne^2}{mDd\omega^2}\Big[1 - e^{-j\omega T_2} - j\omega T_2 e^{-j\omega T_2} - e^{-j\omega T_1} + e^{-j\omega(T_1+T_2)} + j\omega T_2 e^{-j\omega(T_1+T_2)} - \frac{T_1}{T_2}\left\{(-2 + j\omega T_2)\,e^{-j\omega T_1} + (2 + j\omega T_2)\,e^{-j\omega(T_1+T_2)}\right\} + \frac{1}{j\omega T_2}\left\{(-2 + j\omega T_2) + (2 + j\omega T_2)\,e^{-j\omega T_2}\right\}(1 - e^{-j\omega T_1})\Big]. \quad (27)$$

When we expand Y_2 in powers of $-j\omega T_1$, it will be seen that Y_2 begins with the first power of $-j\omega T_1$, corresponding to the factor $1/d$ in this expression and we get approximately

$$Y_2 = \frac{ne^2}{mD\omega u_0}\left[j\{1 - e^{-j\omega T_2} - j\omega T_2 e^{-j\omega T_2}\}\right]. \quad (28)$$

Therefore, in order to justify our neglection of the term with the factor $\alpha_2 R$ against the term with the factor α_1 in Y, we must compare $|\alpha_1(-j\omega T_1)^2|$ with $|\alpha_2 R(-j\omega T_1)|$. This ratio is $\omega T_1 \alpha_1 : \alpha_2 R = \omega T_1 : d/D$.

If ωT_1 is sufficiently larger than d/D, we are justified to adopt (24) as the electronic admittance.

If, however, ωT_1 and d/D are of comparable order of magnitude, (28) cannot be neglected against (24). Then, the electronic admittance is

$$Y = Y_1 + Y_2 = \frac{ne^2}{mu_0^2}\left\{\left[e^{-j\omega T_2} + \frac{1}{2} j\omega T_2 e^{-j\omega T_2}\right] + \kappa\left[j(1 - e^{-j\omega T_2} - j\omega T_2 e^{-j\omega T_2})\right]\right\}, \quad (29)$$

with $\kappa = \dfrac{d/D}{\omega T_1}$, and the corresponding conductance is

$$G = \frac{ne^2}{mu_0^2}\left\{\left[\cos\omega T_2 + \frac{1}{2}\omega T_2 \sin\omega T_2\right] + \kappa\left[-\sin\omega T_2 + \omega T_2 \cos\omega T_2\right]\right\}. \quad (30)$$

For the expression (30) of G, we can expect the production of the minimum (negative value) by simple calculation, in the vicinity of $\omega T_2 = \frac{3}{2}\pi$.

For the finite values of ωT_1, we can demonstrate numerically the existence of the minimum, which is negative, of the real part of the function $Y = Y_1 + Y_2$ in the vicinity of $\omega T_2 = \frac{3}{2}\pi$.

REFERENCES

1. I. Takahashi: Über die Dielektrizitätskonstante des Magneto-aktiven ionisierten Gases. This Memoirs, Vol. XXIII, No. 4, (1941).
2. I. Takahashi: On the possibility of Barkhousen-Kurz Oscillation I. This Memoirs, Vol. XXV, No. 1, (1947).
3. H. Hollmann: Ultrakurzwellengeräte mit Laufzeitkompression, ZS. d. H. T. (1941).
4. H. Hollmann: Zur Theorie der Triftröhren. ZS. f. H und E, **56**, (1940).
5. E. U. Condon: Principle of Microwave Radio. Rev. Mod. Phys. Vol. 14, No. 4, (1942).
6. Recknagel : Schwingungsanfachung durch einen Elektronenstrahl im Feld des Plattenkondensators. ZS. f. tech. Phys. Ns. 3, S. 74, (1938).
7. J. C. Slater : Microwave Electronics. Rev. Mod. Phys. Vol. 18, No. 4, Oct. (1946).