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A Study of Ultrasonics by Magnetostriction Vibrators

By

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1. Introduction and summary

The present paper describes the results of some experimental and theoretical studies on underwater ultrasonics. In experimental work here described, use has been made of some appropriate magnetostriction vibrators for the radiation and reception of underwater ultrasonics (up to about 60 Kc) (**1**).

It is shown, in the first place, that in case when an exciting current of equal strength is applied, the efficiency of a vibrator made of particular Al-Fe alloy containing about 13 % Al (which has been termed "Alfer" by Dr. H. Masumoto) is of the same order of magnitude as the efficiency of an Ni vibrator, but when an appropriate exciting current is applied, the Alfer vibrator is more sensitive than the Ni vibrator (**2, 3**).

Next, it is shown that the efficiency of a modified vibrator made of ferro-magnetic substance with inserted thin plates of non-magnetic substance is far greater than that of a vibrator made of ferro-magnetic substance only. Thus, for example, the Ni vibrator with inserted Al thin plates is far more effective than the unmodified simple Ni vibrator.

In case when the sound waves of high power are radiated by a magnetostriction vibrator to the surrounding water, the medium becomes in a metastable state and consequently an abnormal absorption of sound waves occurs. Thus, the intensity of sound waves radiated from a vibrator is diminished abruptly in the very neighbourhood of the vibrator and then the damped sound waves propagate in the medium with the attenuation factor appropriate to the condition of the medium.

As is well known, the absorption coefficient of weak sound waves can be calculated by the aid of Stokes's formula, but the absorption coefficient of intense sound waves (ultrasonics) cannot be obtained by the same formula. Thus, bearing in mind the fact that air-bubbles are liable to be generated in the water in the vicinity of a vibrator, theoretical discussions are made on the absorption of intense sound waves due to air-bubbles in the water, in two cases when the air-bubbles are pulsating and when they are not pulsating (4). It is thus shown that the abnormal absorption of ultrasonics in the water is mainly due to the pulsating air-bubbles.

2. A magnetostriction vibrator made of Al-Fe alloy

The magnetic properties of Al-Fe system alloys were studied in detail by Dr. H. Masumoto. He found that in the external magnetic field of 1200 Oe, the rate of elongation of the length $\delta l/l$ amounts to 50×10^{-6} in the case of 12.68 % Al-Fe alloy, which Dr. Masumoto has termed " Alfer." The present writer has measured the electro-acoustical transformation efficiency of a vibrator made of " Alfer " and found that the transformation efficiency of such a vibrator is of the same order of magnitude as in the case of an Ni vibrator (2).

By carrying out measurements on three Alfer vibrators which are made up of plates of 0.05 cm, 0.04 cm and 0.03 cm thickness respectively, we have investigated the effect of the thickness of thin plates in Alfer vibrators, and it has been found that the efficiency of an Alfer vibrator becomes larger as the thickness of thin plates forming the vibrator becomes smaller.

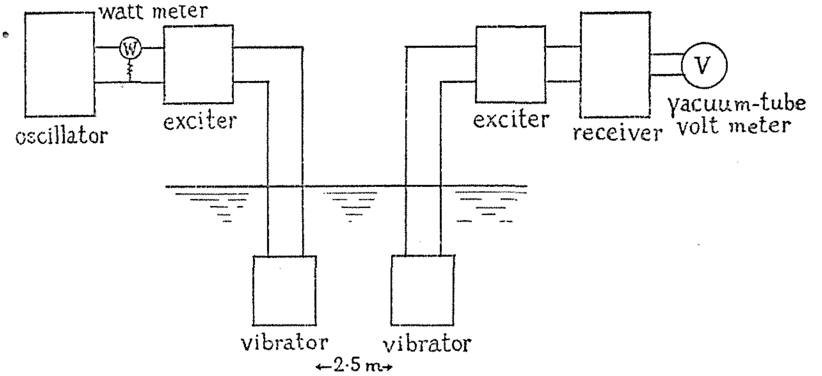

Fig. 1.

In order to compare the electro-acoustical transformation efficiency of the Alfer vibrator with that of the Ni vibrator, we have performed experiments by the device as illustrated in Fig. 1. The depth of the vibrators was 3 m and the exciting current was 15 ampere. The areas of radiating surfaces of both the vibrators were the same and were 188 cm^2.

The results obtained are shown in Fig. 2, where the input power in watts is taken as the abscissa, while the received voltage in receiver in μV is taken as the ordinate. From this figure, it is found that the Alfer vibrator is more effective than the Ni vibrator.

By placing the Alfer vibrator in the water at various depths ranging from 1 m to 5 m, we have also investigated the effect of the hydrostatic pressure upon the efficiency of the Alfer vibrator. The results thus obtained are shown in Fig. 3 (a), which shows clearly that when the large static pressure is applied to the surface of the vibrator, the sound intensity radiated from the vibrator to the surrounding medium becomes large.

The lowest curve in Fig. 2 gives the sound intensity radiated from the Alfer vibrator placed at a depth of 3 m in the water with disturbed surface. It is seen that there occurs an appreciable attenuation of sound intensity when there are disturbances in the medium which cause the decrease in the hydrostatic pressure.

The relationship between

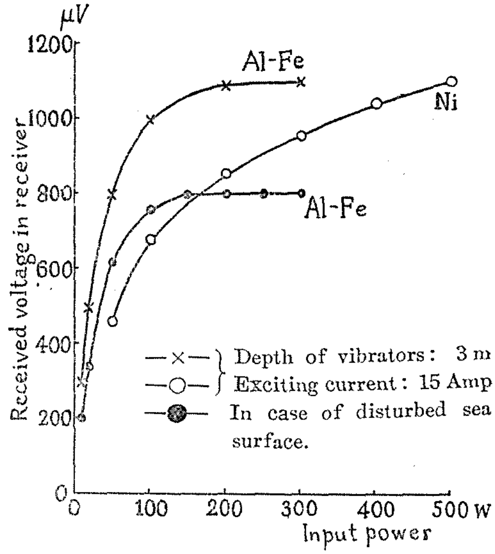

Fig. 2. Resonance frequency: 23.6 Kc
Density of water: 1.02
Temperature of water: 27° C

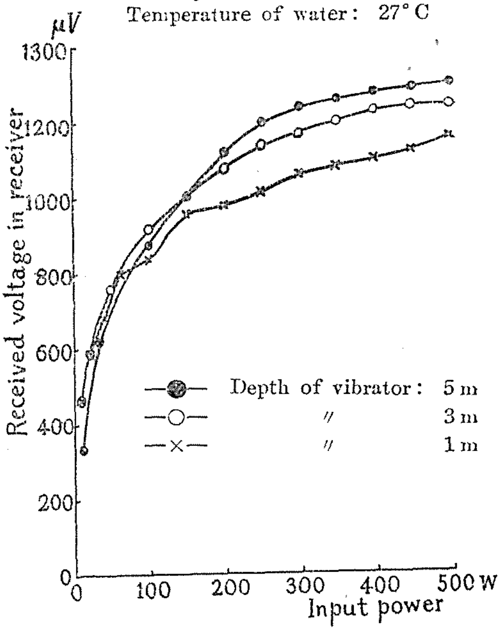

Fig. 3 (a). Resonance frequency: 19.2 Kc
Temperature of water: 26°.6 C
(at depth of 3 m)
Density of water: 1.02

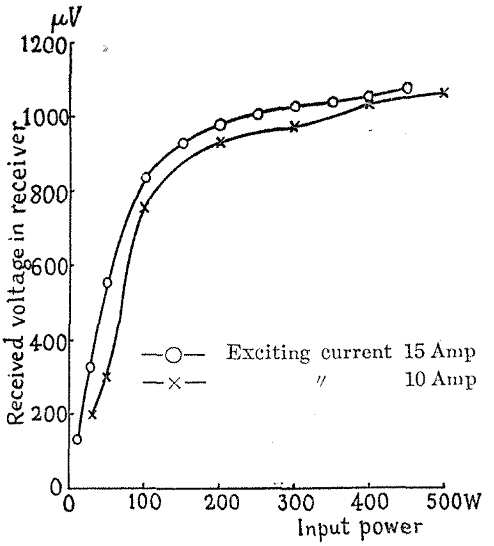

Fig. 3 (b). Resonance frequency: 23.6 Kc
Temperature of water: 25.6° C
Density of water: 1.0185
Depth of vibrator: 3 m

the efficiency of the vibrator and the exciting currents for the case of Alfer vibrator is shown in Figs. 3 (b) and 4. In the measurement for obtaining the results in Fig. 4, the radiating surface of the vibrator was of dimensions 5.0 cm × 2.65 cm and the number of turns of the coil was 18, while the resonance frequency was 22.9 Kc and the temperature of the water was 23.°8 C. From these figures we can find out the appropriate exciting current for obtaining the effective radiation of the Alfer vibrator.

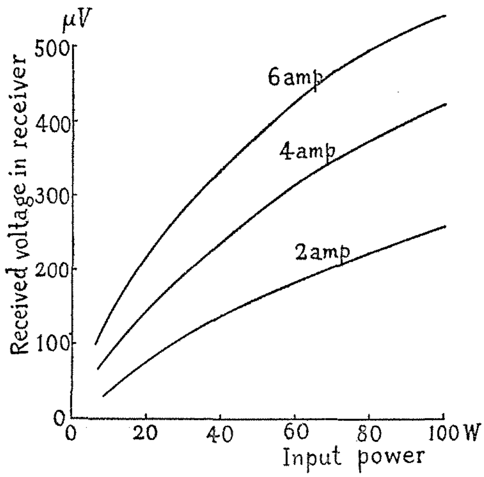

Fig. 4.

3. A magnetostriction vibrator with inserted non-magnetic substances

In order to obtain the effective sound radiation, it is necessary to give high hydrostatic pressure to the medium around a vibrator and consequently a modification of the vibrator becomes necessary. For the purpose of avoiding the interaction of eddy currents between thin plates forming the vibrator, we have inserted thin non-magnetic plates between thin Ni plates alternately.

When the vibrator is doing the magnetostriction and radiating the sound waves to the surrounding medium, the discontinuous damping appears, as mentioned already, in the surrounding medium. We shall now consider the damping effect in various substances which are commonly used as the material of a vibrator. For reference, the sound velocities in Ni, Alfer, Al and glass, together with the densities of these materials, are first shown in Table I.

TABLE I.

Material	Ni	Alfer	Al	Glass
Sound velocity, c (cm/sec)	4760	4720	5250	4500~5600
Density, ρ (g/cm^3)	8.75	6.55	2.65	2.5~5.9

When the sound waves are transmitted from the vibrator to the surrounding medium, the percentage transmission power varies according as the acoustic resistance ρc of the material of the vibrator varies, and it is found that the percentage transmission powers of mechanical vibration from solids to the surrounding water are respectively 14.1 %, 17.6 % and 35.2 % for Ni, Alfer and Al vibrators. These values have been obtained by applying the laws of reflection and refraction of sound waves to the solid and water.

Thus, if the arithmetic mean of the percentage transmission powers of the constituents be taken as the percentage transmission power of the combined vibrator, the Ni vibrator with inserted thin Al plates becomes 24.65 %. In like manner, the percentage transmission power of the Alfer vibrator with inserted thin Al plates is 26.40 %. It is found therefore that the percentage of the transmitted sound power for the Ni vibrator is smaller than that for the Ni vibrator with inserted thin Al plates, the ratio of the transmission power of the latter to that of the former being $24.65/14.10 = 1.748$. Thus, we see that the percentage of effective transformation in the water of the Ni vibrator with inserted thin Al plates is 1.748 times greater than the simple vibrator made of Ni only. This shows clearly that when thin plates of non-magnetic materials are inserted alternately between thin plates of magnetic substances, there occurs an effective radiation from the surfaces of the plates of non-magnetic substances to the surrounding medium.

Next, we shall consider the damping coefficients of the thin plates forming the vibrator. In case when the thin plates of ferro-magnetic substance are subjected to the driving periodic force $F_1e^{i\omega t}$, the equations of motion of the thin plates of ferro-magnetic substance and of the thin plates of non-magnetic substance are respectively

$$\left.\begin{aligned} m_{11}\ddot{x}_1 + r_{11}\dot{x}_1 + s_{11}x_1 + m_{12}\ddot{x}_2 + r_{12}\dot{x}_2 + s_{12}x_2 = F_1e^{i\omega t}\,, \\ m_{22}\ddot{x}_2 + r_{22}\dot{x}_2 + s_{22}x_2 + m_{21}\ddot{x}_1 + r_{21}\dot{x}_1 + s_{21}x_1 = 0\,, \end{aligned}\right\} \tag{3.1}$$

where

m_{11} : mass of the plates of ferro-magnetic substance,

m_{22} : mass of the plates of non-magnetic substance,

m_{12} : apparent mass of the plates of ferro-magnetic substance, generated from the plates of non-magnetic substance in steady-state motion,

m_{21} : apparent mass of the plates of non-magnetic substance, generated from the plates of ferro-magnetic substance in steady-state motion,

r_{12} : damping coefficient of ferro-magnetic substance as affected by the non-magnetic substance in steady-state motion,

r_{21} : damping coefficient of non-magnetic substance as affected by the ferro-magnetic substance in steady-state motion,

r_{11} : damping coefficient of ferro-magnetic substance only,

r_{22} : damping coefficient of non-magnetic substance only,

$s_{11}x_1$: force required to produce a small displacement from the equilibrium position in the vibrating plate of ferro-magnetic substance,

$s_{22}x_2$: force required to produce a small displacement from the equilibrium position in the vibrating plate of non-magnetic substance,

$s_{12}x_2$: force acting on ferro-magnetic substance by non-magnetic substance,

$s_{21}x_1$: force acting on non-magnetic substance by ferro-magnetic substance.

Neglecting the transient-state motion, we shall consider only the steady-state motion. Then, particular solutions of the above two equations of motion are given by

$$x_1 = Ae^{i\omega t}\,, \qquad x_2 = Be^{i\omega t}\,. \tag{3.2}$$

Putting these expressions in the equations of motion we have

$$\left.\begin{aligned} A &= \frac{F_1 Z_{22}}{i\omega(Z_{22}Z_{11} - Z_{12}Z_{21})}\,, \\ B &= \frac{-F_1 Z_{21}}{i\omega(Z_{22}Z_{11} - Z_{12}Z_{21})}\,, \end{aligned}\right\} \tag{3.3}$$

where

$$\left.\begin{aligned} Z_{11} &= r_{11} + i(m_{11}\omega - s_{11}/\omega)\,, \quad Z_{12} = r_{12} + i(m_{12}\omega - s_{12}/\omega)\,, \\ Z_{22} &= r_{22} + i(m_{22}\omega - s_{22}/\omega)\,, \quad Z_{21} = r_{21} + i(m_{21}\omega - s_{21}/\omega)\,. \end{aligned}\right\} \tag{3.4}$$

Thus, we have

$$x_1 + x_2 = \frac{Z_{22} - Z_{21}}{i\omega(Z_{22}Z_{11} - Z_{12}Z_{21})} F_1 e^{i\omega t}\,. \tag{3.5}$$

On the other hand, the displacement of a magnetostriction vibrator made of ferro-magnetic substance only is given by

$$\xi = \frac{F_1}{i\omega Z_{11}} e^{i\omega t}\,. \tag{3.6}$$

Therefore, in order that the vibration of the Ni vibrator with inserted non-magnetic thin plates should have larger effective efficiency than that of the vibrator made of Ni plates only, it must be that

$$\left.\begin{aligned} \frac{x_1}{\xi} &= \frac{Z_{22}Z_{11}}{Z_{22}Z_{11} - Z_{12}Z_{21}} \geqq 1\,, \\ \frac{x_2}{\xi} &= \frac{-Z_{21}Z_{11}}{Z_{22}Z_{11} - Z_{12}Z_{21}} \geqq 1\,, \\ \frac{x_1 + x_2}{2\xi} &= \frac{(Z_{22} - Z_{21})Z_{11}}{2(Z_{22}Z_{11} - Z_{12}Z_{21})} \geqq 1\,. \end{aligned}\right\} \tag{3.7}$$

From these relations, we have

$$-Z_{21} \geqq Z_{22}\,, \qquad Z_{12} \leqq 0\,. \tag{3.8}$$

In the state of resonance, it follows therefore that $-r_{21} \geqq r_{22}$ and $r_{12} \leqq 0$. On the other hand, $-r_{21}$ must be equal to r_{12} in the state of resonance. Thus, we see that r_{22} is of the negative value, and this means that the resistance term in the equation of motion of the plate of non-magnetic substance must take a negative value. Since, however, this is impossible in the practical case, r_{22} should take a small positive value.

When the efficiency of the vibrator with inserted non-magnetic substance is equal to the efficiency of the vibrator made of ferro-magnetic substance only, we have $x_1/\xi = x_2/\xi = 1$, from which it follows that $Z_{22} = -Z_{21}$ and $Z_{12} = 0$.

Consequently, it is seen that in order to obtain the most effective vibration in the Ni vibrator with inserted plates of non-magnetic substance, we must use such a non-magnetic substance of which r_{22}, r_{21} and r_{11} have the smallest positive values. Such a new vibrator is analogous to a vibrator whose non-magnetic material is loaded on the surface of ferro-magnetic vibrator. The mechanical Q of this new magnetostrictive transducer has a large value in comparison with the usual magnetostriction vibrator.

4. On large absorption of sound waves in the water

The abnormal absorption of the underwater sound waves seems to be due to the presence of air-bubbles in the water. We shall now discuss the effect of such air-bubbles in two cases when the air-bubbles are pulsating and when they are not pulsating.

In the first place, we shall calculate the absorption coefficient of sound waves due to the pulsation of air-bubbles, by assuming that they are of spherical form. The energy loss at the surface of a spherical air-bubble of radius r is given by the product of three quantities: the oscillating pressure of simple harmonic type, the area of the surface of the pulsating bubble and the radial velocity. Since the mean value of the sine term is $1/2$, the mean loss of sound energy per unit time becomes

$$W_1 = \frac{\rho \omega^2 A^2}{8\pi V}, \tag{4.1}$$

where ρ is the density of the water, ω the angular frequency of pulsation and V the velocity of propagation of sound waves, while A is the maximum rate of volume change of the air-bubble so that $A = 4\pi r^2 \omega \xi_0$, where ξ_0 is the maximum displacement of a point on the surface of the bubble. Thus, we have

$$W_1 = \frac{2\pi \rho r^4 \omega^4 \xi_0^2}{V}. \tag{4.2}$$

Now, the volume dilatation of the air-bubble is given by

$$\Delta_1 = \frac{3}{r}\,\delta r_1\,. \tag{4.3}$$

The variation of pressure due to sound waves is given by

$$\delta P = \rho V \dot{\xi} = \frac{\kappa\omega\xi}{V}\,, \tag{4.4}$$

where κ is the volume modulus of elasticity of the water.

If we denote the volume modulus of elasticity of the air by κ_1, the volume dilatation Δ_1 and the pressure variation δP are connected to each other by the relation $-\Delta_1 = \delta P/\kappa_1$, and therefore we have

$$-\Delta_1 = \frac{\kappa}{\kappa_1}\frac{\omega\xi}{V}\,. \tag{4.5}$$

Thus, by (4.3) and (4.5) we get

$$-\delta r_1 = \frac{r}{3}\frac{\kappa}{\kappa_1}\frac{\omega\xi}{V}\,. \tag{4.6}$$

On the other hand, if the sphere of radius r were filled with water instead of the air, we may have $\Delta_2 = \frac{3}{r}\delta r_2$ and $-\Delta_2 = \frac{\kappa}{\kappa}\frac{\omega\xi}{V} = \frac{\omega\xi}{V}$. Therefore, we have

$$-\delta r_2 = \frac{r}{3}\frac{\omega\xi}{V}\,. \tag{4.7}$$

In order to apply the above results to the practical case of air-bubbles in the sound field, we write $\xi_0 = \xi_{10} - \xi_{20} = \delta r_1 - \delta r_2$, where ξ_{10} is the displacement of a point on the surface of the air-bubble and ξ_{20} is the displacement of a point on the surface of a sphere with the same radius as the air-bubble, which is filled with water. Then, making use of the above expressions for δr_1 and δr_2 given by (4.6) and (4.7) respectively, we have

$$\xi_0 = \frac{r}{3}\left(1 - \frac{\kappa}{\kappa_1}\right)\frac{\omega\xi}{V}\,. \tag{4.8}$$

Substituting this into the right-hand side of (4. 2) and taking $\kappa_1 = 1.2\times10^6$, $\kappa = 2.25\times10^{10}$, $V = 1.5\times10^5$ and $\rho = 1.0$ in C. G. S. units, we have

$$W_1 = 0.746\times10^{-7}\,r^6\,\omega^6\,\xi^2\,.$$

Thus, the energy loss per unit volume of the underwater sound waves is given by

$$W = nW_1 = 0.746 \times 10^{-7} r^6 \omega^6 \xi^2 n\,, \tag{4.9}$$

where n is the number of air-bubbles contained in unit volume of the water.

On the other hand, if we denote the sound intensity at any point $x = x$ by I and the corresponding quantity at the origin $x = 0$ by I_0, we have

$$I = I_0 e^{-2\alpha x}\,, \tag{4.10}$$

where α is the coefficient of absorption.

Hence, we have

$$W = -\frac{\partial I}{\partial x} = 2\alpha I = \rho V \omega^2 \xi^2 \alpha\,, \tag{4.11}$$

from which we get

$$\alpha = \frac{W}{\rho V \omega^2 \xi^2}. \tag{4.12}$$

By the aid of (4.9) and (4.12) the values of the absorption coefficient α have been calculated, assuming various values for the radius r of air-bubbles as well as for the number n of air-bubbles contained in unit volume of the water. The results are shown in Table II.

TABLE II.

Radius, r, of air-bubbles (cm)	Number, n, of air-bubbles in unit volume	Absorption coefficient			Strength at a distance of 1 cm from source with unit intensity		
		10 Kc	20 Kc	30 Kc	10 Kc	20 Kc	30 Kc
10^{-3}	1	7.74×10^{-12}	1.24×10^{-10}	6.27×10^{-10}	1	1	1
10^{-3}	2.39×10^{8}	1.85×10^{-3}	2.96×10^{-2}	1.50×10^{-1}	0.996	0.943	0.741
10^{-2}	1	7.74×10^{-6}	1.24×10^{-4}	6.27×10^{-4}	1	1	0.999
10^{-2}	10^{3}	7.74×10^{-3}	1.24×10^{-1}	6.27×10^{-1}	0.985	0.780	0.285
10^{-2}	2.39×10^{5}	1.85	2.96×10	1.50×10^{2}	0.025	0	0
3×10^{-2}	1	5.64×10^{-3}	9.02×10^{-2}	4.57×10^{-1}	0.989	0.835	0.401
5×10^{-2}	1	1.21×10^{-1}	1.94	9.80	0.785	0.021	0
10^{-1}	1	7.74	1.24×10^{2}	6.27×10^{2}	3×10^{-7}	0	0
10^{-1}	2.39×10^{2}	1.85×10^{3}	2.96×10^{4}	1.50×10^{5}	0	0	0
3×10^{-1}	1	5.64×10^{3}	9.02×10^{4}	4.57×10^{5}	0	0	0
5×10^{-1}	1	1.21×10^{5}	1.94×10^{6}	9.80×10^{6}	0	0	0

Next, we shall discuss the absorption coefficient of the sound waves in case when the air-bubbles are not pulsating. The acoustic resistance of the mixture of water and air-bubbles is given by

$$R = \sqrt{\frac{EE_0\{v\rho_0 + (1-v)\rho\}}{vE + (1-v)E_0}}, \tag{4.13}$$

where ρ_0 and E_0 are the density and the elasticity of the air respectively, while ρ and E are the corresponding quantities of the clear water respectively. v is the total volume of non-resonant air-bubbles in unit volume of the medium.

If we assume that there exist surfaces of discontinuity in the medium, the reflection of sound waves may occur at these surfaces. Let R and R' be the acoustic resistances of the media on two sides of one of such surfaces of discontinuity. Then, the loss of sound power due to the reflection of sound waves at this surface is given by

$$-W\left(\frac{R-R'}{R+R'}\right)^2.$$

If we assume that the sound waves are reflected at the non-pulsating air-bubbles, the loss of sound power in passing through a distance dx is therefore given by

$$dW = -W\left(\frac{R-R'}{R+R'}\right)^2 dx. \tag{4.14}$$

If we put $R' = (\rho + \delta\rho)(V + \delta V)$, this equation becomes

$$dW = -W\left(\frac{\delta R}{2R}\right)^2 dx. \tag{4.15}$$

On the other hand, if the absorption coefficient of sound waves in the medium be denoted by α, the loss of sound power in a thickness dx is given by

$$dW = -\alpha W dx. \tag{4.16}$$

Thus, comparing the above two expressions we have

$$\alpha = \left(\frac{\delta R}{2R}\right)^2. \tag{4.17}$$

Taking $\rho_0 = 0.0012$, $\rho = 1.0$, $E_0 = 1.2 \times 10^6$ and $E = 2.25 \times 10^{10}$ in C. G. S. units, we have, from (4.13),

$$R/R_0 = (1 + 18750\,v)^{-\frac{1}{2}}, \tag{4.18}$$

where R_0 is the acoustic resistance of the clear water.

This equation gives immediately

$$\delta R = -9375\frac{R^3}{R_0^2}\,\delta v\,, \tag{4.19}$$

and if use is made of this, the expression for α becomes

$$\alpha = \left(\frac{\delta v}{2.133\times 10^{-4}+4v}\right)^2.$$

Denoting the radius of non-resonant air-bubbles by r and the number of bubbles in unit volume of the medium by n, we put $\delta v = v - v' = \frac{4}{3}\pi r^3$ and $v = \frac{4}{3}\pi r^3 n$. Then, we have

$$\alpha = \left(\frac{1}{5.094\times 10^{-5}r^{-3}+4n}\right)^2. \tag{4.20}$$

The values of the absorption coefficient α have been calculated by this formula, obtaining the results as shown in Table III.

TABLE III.

Radius, r, of air-bubbles (cm)	Number, n, of air-bubbles in unit volume	Absorption coefficient, α
10^{-3}	1	3.84×10^{-10}
10^{-2}	1	3.31×10^{-4}
10^{-2}	10^3	6.10×10^{-8}
10^{-2}	10^5	6.25×10^{-12}
3×10^{-2}	1	1.91×10^{-3}
5×10^{-2}	1	5.15×10^{-2}
10^{-1}	1	6.10×10^{-2}

From this table it will readily be seen that the value of the absorption coefficient increases as the number of air-bubbles decreases. It follows therefore that larger absorption of sound occurs in the region near the boundary surface between the bubbly water and the clear water, where there are only a few air-bubbles in unit volume of the medium, than in intermediate regions where there are many air-bubbles.

Comparing Tables II and III it is readily seen however that large attenuation of the sound waves is mainly due to the presence of pulsating air-bubbles.

Further, by making use of the results given in Tables II and III we have obtained Fig. 5 which shows the relationship between the sound intensity at a distance of 1 cm from the vibrator and the radius of air-bubbles.

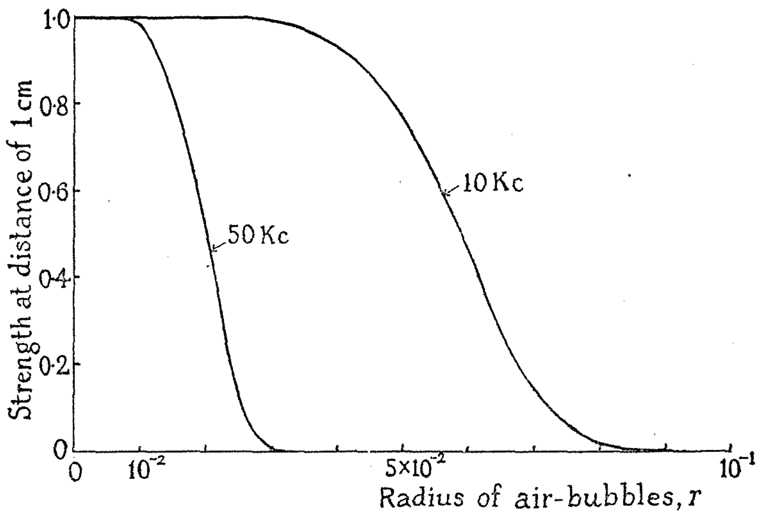

Fig. 5.

In conclusion, the writer wishes to express his cordial thanks to Professor M. Hasegawa and Assistant Professor Y. Tamura for their kind advices. Many thanks are also due to Professor S. Tomotika for his kind encouragement and valuable remarks.

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