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ON THE GLIDE TWINNING IN SINGLE CRYSTALS OF TIN

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ABSTRACT

By measuring the energy and the critical shearing stress simultaneously, the mechanism of glide twinning in single crystals of tin has been investigated experimentally by the use of a method first devised by B.Chalmers and somewhat improved by the present writers.

The results obtained varied from specimen to specimen, ranging from 0.9×10^6 to 4.4×10^6 ergs/cc for the energy and from 23 to 61 kg/cm² for the critical shearing stress. Plotting the energy against the critical shearing stress, it has been found that there holds a relation $E = \frac{1}{2}eS$, where E is the energy for twinning per unit volume, e the magnitude of shear (being 0.12 in our experiment), and S the critical shearing stress for twinning.

1. Introduction

In 1935, Chalmers (1) made some researches on the mechanical twinning of single crystals of white tin, and estimated the energy necessary for twinning by a method which consisted in comparison of the amplitude of a pendulum of the specimen when it twinned on impact with that when it did not twin.

The present writers have made some improvements upon his method, and a simple relationship between the energy necessary for twinning and the critical shearing stress for twinning by impact has been obtained.

2. Description of the experiments

i) *Experimental procedure*

A number of single crystals of white tin, cylindrical in form, 6 mm in diameter and about 3 cm in length, were prepared by the method of drawing up a seed crystal from the melt. The orientation of the seed crystal was so arranged that there was only one kind of twinning plane in the specimen. The purity of the specimen was only of commercial, although pure enough to make single crystals of it. After the orientation of the crystal was checked by the optical method, the specimen was set in the pendulum of specimen-holder and struck on one end by a striking pendulum of the same weight, as was done in Chalmer's experiment (Fig. 1).

From the amplitude measured on a scale set closely to the both pendulums the velocities just before and after the collision were estimated. An example of the results is shown in Fig. 2, where the abscissa represents the velocity of the striker, while the ordinate denotes the velocity of the specimen.

As the velocity of the striker is increased, the velocity of the specimen increases linearly, and when the velocity of the striker reaches a certain definite value, say V_0,

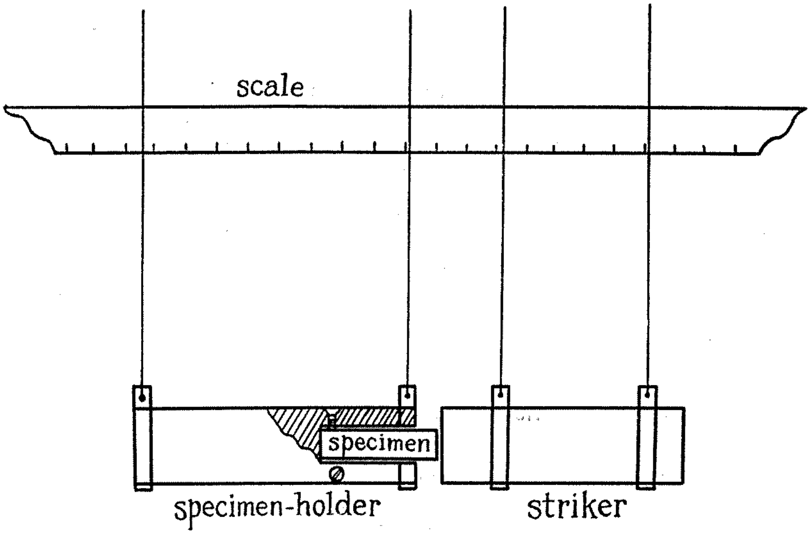

FIG. 1

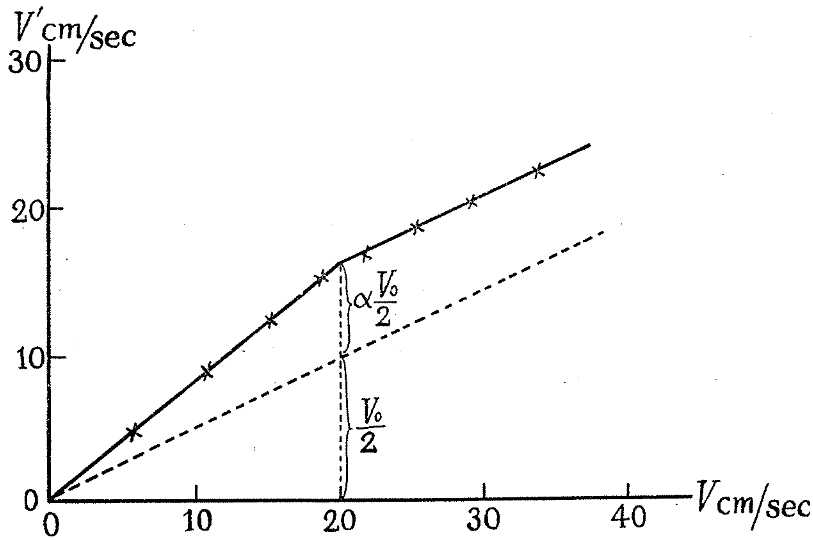

FIG. 2 The relation between the velocities of the striker and the specimen-holder

twinning begins to take place, after which the velocity of the specimen is seen to increase at the gradient of ½ to the velocity V of the striker, as will be seen from the figure.

ii) *Velocity changes by collision*

In order to explain such a relation, we shall for the time being consider a case in which both the pendulums collide on one another with equal speed of $V/2$, which is half the velocity of the striker in our experiment. This is equivalent to a case in which we are observing the collision in our experiment from a coordinate-system moving with the velocity $V/2$ parallel to the velocity of the striker.

In such a case, the two pendulums will repulse one another at the speed $\alpha V/2$, where α is the coefficient of repulsion in the elastic range. As the speed of the two pendulums, approaching one another, is increased, the collision will become stronger, and at a certain definite velocity, say $V_0/2$, the collision will become inelastic, and twinning will begin to take place. While the twinning is proceeding, the residual kinetic energy, which will exist if the velocity $V/2$ is larger than $V_0/2$, will be consumed in the work of twinning, and when all of the kinetic energy have been consumed, twinning will stop, and the elastic energy stored in the pendulums will be turned into the kinetic energy of repulsion: the velocity of repulsion after twinning will correspond to the elastic energy at its elastic limit.

Turning now to our case in which the velocity of the striker is V and the specimen-holder is situated at its lowest position, the velocity of the specimen V' after collision will be represented by adding $V/2$ to the velocity of the specimen in the case considered above. Thus,

$$V' = \frac{1}{2}V + \alpha\frac{1}{2}V = \frac{1}{2}(1+\alpha)V \tag{1}$$

in the elastic range,

$$V' = \frac{1}{2}V_0 + \alpha\frac{1}{2}V_0 = \frac{1}{2}(1+\alpha)V_0 \tag{2}$$

at the elastic limit, and

$$V' = \frac{1}{2}V + \alpha\frac{1}{2}V_0, \tag{3}$$

when twinning occurs.

On the other hand, the velocity of the striker will be equal to the difference of the initial velocity and the velocity of the specimen after collision, preserving the law of conservation of momentum.

iii) *The energy loss by twinning*

From the relations thus obtained, the energy necessary for twinning may be estimated as follows:

The energy-loss by collision will be equal to the difference of the kinetic energies of the system before and after collision, but the energy-loss which exists in the course of the elastic process, i. e., in the elastic compression before twinning and elastic expansion after twinning, should be subtracted from the total energy-loss, in order to estimate the energy-loss by twinning.

If we assume that the energy-loss in the elastic process is equal to that by the collision at V_0, i. e., at the elastic limit, this will be expressed as

$$\frac{1}{2}MV_0^2 - \frac{1}{2}M\left\{(1+\alpha)\frac{V_0}{2}\right\}^2 - \frac{1}{2}M\left\{(1-\alpha)\frac{V_0}{2}\right\}^2,$$

in which the first term is the energy of the striker before collision, while the second and third terms are those of the specimen and the striker after collision respectively.

Then the energy-loss E by twinning is given by

$$E = \frac{1}{2}MV^2 - \frac{1}{2}M\left\{\left(\frac{V}{2}+\alpha\frac{V_0}{2}\right)^2 + \left(\frac{V}{2}-\alpha\frac{V_0}{2}\right)^2\right\}$$
$$- \frac{1}{2}MV_0^2 + \frac{1}{2}M\left\{(1+\alpha)\frac{V_0}{2}\right\}^2 + \frac{1}{2}M\left\{(1-\alpha)\frac{V_0}{2}\right\}^2$$

$$= \frac{1}{4}M(V^2 - V_0^2). \qquad (4)$$

In the earlier experiment by Chalmers, the energy-loss was estimated to be the difference between the amplitude of the specimen when it twinned and that when it did not twin, the initial amplitude having been the same in both cases.

Since, however, the cofficient of repulsion is nearly constant, the energy-loss by the elastic repulsion will increase rapidly with the incident velocity of the striker, so that the result will become less than our result which is obtained under the assumption that the energy-loss is estimated to be the difference of the initial energy of the striker and the energy of the specimen after twinned.

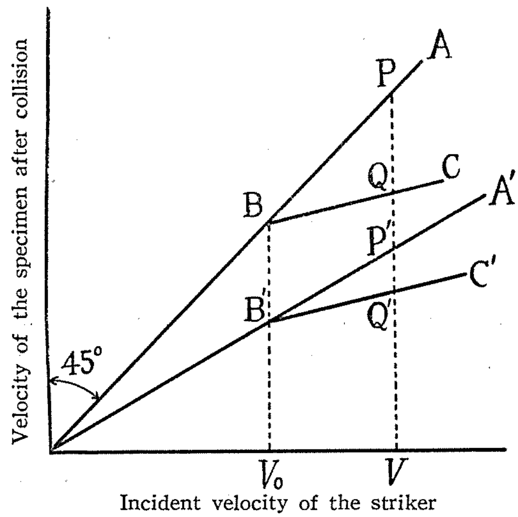

FIG. 3

Fig. 3 explains such circumstances, where the abscissa represents the incident velocity of the striker, while the ordinate is the velocity of the specimen after repulsion. The velocity of the specimen after a perfectly elastic repulsion will be represented by the straight line OA, while the velocity after the actual repulsion by OA'. The velocity after twinning is represented by BC in the perfect case, and by B'C' in the actual case, and when twinning have occurred at the incident velocity V, the total energy-loss by collision will correspond to the velocity difference PQ' in the figure, and since we have assumed in the above that the energy-loss in the elastic process by twinning is equal to that at the critical point V_0, this will correspond to QQ' (= BB'), and thus the energy-loss by twinning will correspond to PQ (= PQ' − QQ') in our method.

On the other hand, according to the earlier method the energy-loss by twinning will correspond to P'Q'.

Quantitatively, according to the earlier method the energy-loss will become

$$E = \frac{1}{2}M\left[\left\{(1+\alpha)\frac{V}{2}\right\}^2 + \left\{(1-\alpha)\frac{V}{2}\right\}^2\right]$$
$$-\frac{1}{2}M\left[\left\{\frac{V}{2} + \alpha\frac{V_0}{2}\right\}^2 + \left\{\frac{V}{2} - \alpha\frac{V_0}{2}\right\}^2\right]$$

$$= \frac{1}{4} M\alpha^2 (V^2 - V_0^2) , \tag{5}$$

since the velocity of the specimen will be $(V + V_0/2)$ when the specimen twinned and $(1 + \alpha)V/2$ when it did not twin.

The value of α observed in the experiment is approximately 0.6, so that the result obtained from (5) will become about one-third of that obtained from (4). As will be seen in Table I, this is in fact the case, since the mean value of the energy-loss obtained by Chalmers was 8×10^5 ergs/cc, while it ranges from 0.9×10^6 to 4.4×10^6 ergs/cc in our experiment.

iv) *The critical shearing stress*

The critical shearing stress for twinning will be estimated on the ground that the elastic energy stored in the pendulum is changed into the kinetic energy of repulsion.

We may assume that most of the elastic energy is stored in the tiny specimen of single crystal of tin attached to the front of the specimen-holder, because the cross-section of the specimen is about one-tenth of the cross-section of both the specimen-holder and the striker, and the elastic constants are smaller than that for iron, so that the compression will take place mainly in the specimen.

If we denote the volume of the specimen by V_s, the normal pressure at the elastic limit by P and the compressibility of the specimen along the direction of the specimen axis by k, the energy stored in the specimen at the elastic limit will be equal to $V_s P^2/(2k)$, and since this should be equal to the kinetic energy of the two pendulums, as is considered in ii), we obtain the relation:

$$\frac{V_s P^2}{2k} = 2 \times \frac{1}{2} M \left(\frac{\alpha}{\alpha'} V_0 \right)^2 ,$$

or

$$P = \sqrt{\frac{2M}{kV_s}} \frac{\alpha}{\alpha'} V_0 , \tag{6}$$

where M is the mass of each pendulum and α' is a correction factor due to the fact that a part of the elastic energy must have been dissipated during the elastic expansion and that some of the elastic energy stored both in the specimen-holder and the striker must contribute to the velocity after collision.

The critical shearing stress which is denoted by S_c will be obtained from P by the relation:

$$S_c = P \cos\theta \cos\psi , \tag{7}$$

where θ is the angle between the axis of the specimen and the normal to the twinning plane and ψ the angle between the specimen-axis and the direction of twinning.

The compressibility k will be obtained from the elastic parameters for white tin as

$$k = (l^4 + m^4)c_{11} + n^4 c_{33} + l^2 m^2 (2c_{12} + c_{66}) + n^2 (l^2 + m^2)(2c_{13} + c_{44}) , \tag{8}$$

where l, m and n are the direction-cosines of the specimen-axis referred to the a, b and c axes of the crystal.

As to the correction factor α', it will be adequate to estimate it to be nearly 0.8, if we neglect the effect of the striker and the holder, because the value of α', being a part of the coefficient of repulsion concerning only the elastic expansion, will not be far from $\sqrt{\alpha}$, where α is 0.6 or 0.65 in our experiment. Taking account of the effect of the striker and the holder, the value of α' would be a little larger than 0.8.

The critical shearing stress by static compression may not be utilized, since it might depend upon the speed of compression, at any rate.

3. Results of experiments

i) *The energy-loss and the critical shearing stress*

As already mentioned, the mean value obtained for the energy was approximately 3×10^6 ergs/cc and varied from 0.9×10^6 to 4.4×10^6 ergs/cc. On the other hand, the critical shearing stress for twinning also varied from specimen to specimen, ranging from 23 to 61 kg/cm^2. The details are shown in Table I below.

TABLE I

No.	$E \times 10^5$ ergs	V_t cc	$E/V_t \times 10^6$ ergs/cc	$\alpha' P$ kg/cm^2	$\alpha' S_c$ kg/cm^2	orientation θ	ϕ
1	10.9	0.4	2.5	102	39	57	47
2	4.7	0.3	1.6	60	23	〃	〃
3	2.6	0.17	0.9	61	28	48	47
4	9.4	0.26	3.6	114	52	〃	〃
5	3.6	0.12	2.9	93	42	〃	〃
6	6.7	0.24	2.8	94	42	〃	〃
7	5.1	0.12	4.2	135	61	〃	〃
8	7.8	0.23	3.6	105	47	〃	〃
9	7.8	0.18	4.2	108	49	〃	〃
10	6.3	0.15	4.2	130	59	〃	〃
11	7.0	0.37	1.9	110	51	〃	〃
12	16.6	0.38	4.4	108	49	〃	〃

E is the total energy-loss by twinning and V_t is the volume twinned for each specimen. In the fifth and sixth columns, α' is not removed, because it is not yet determined exactly.

ii) *A relation between the energy-loss and the critical searing stress*

As is shown in Fig. 4, a linear relation is obtained by plotting the energy-loss against

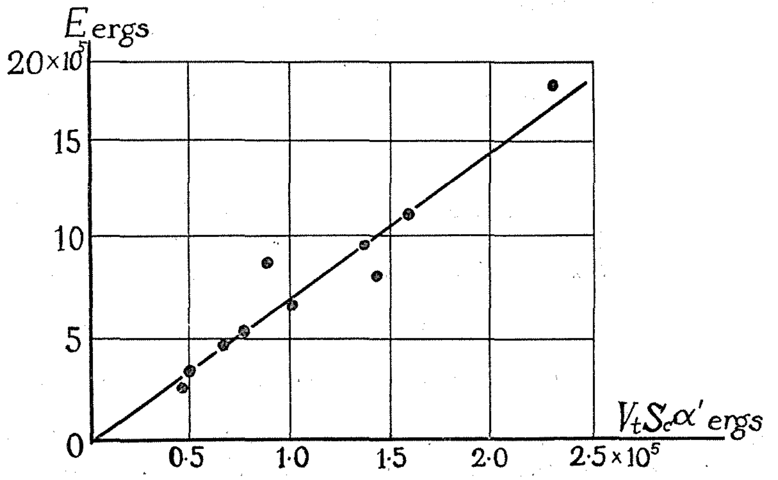

FIG. 4 Experimental results representing the linear relation between the energy-loss by twinning and the product of the volume twinned V_t, the critical shearing stress S_c.

the product of V_t and $\alpha' S_c$. This will be expressed as:

$$E = 0.07 V_t \alpha' S_c ,$$

and, since the magnitude of shear e is 0.12 for the twinning of white tin, this becomes

$$E = \frac{1}{2} e V_t \left(\frac{\alpha'}{0.9}\right) S_c ,$$

or approximately

$$E = \frac{1}{2} V_t S_c , \tag{9}$$

for, as was already mentioned in iv), α' is equal to, or a little larger than, 0.8.

If, on the other hand, we consider that the energy is consumed against the friction between the gliding plane and the underlying plane, the energy-loss will be expressed as:

$$E = e V_t S , \tag{10}$$

where S represents the friction per unit area, since the product eV_t is equal to the product of total area of twinning lattice planes and its displacement.

From (9) and (10) we obtain a relation that

$$S = \frac{1}{2} S_c . \tag{11}$$

Returning to (9), if ε is the energy-loss, f the critical shearing stress each for one atom twinned, and d the distance between the original and its twinned lattice point, it is easily found that the relation (9) is translated into

$$\varepsilon = \frac{1}{2} f d , \tag{12}$$

which shows that the friction during the process of twinning is one-half of the critical shearing stress.

If the above relation (12), which is directly translated into microscopic terms, is to be correct, this may be interpreted schematically in such a way that there is a potential barrier of height ε between the original and the twinned sites and a straight potential gradient f from the original site to the top of the potential barrier. But this should not be accepted directly, since ε is much less than the energy of thermal vibration kT, which is about 4×10^{-14} ergs at room temperatures, while ε is nearly equal to 8×10^{-17} ergs.

An alternative description may be made by assuming that ε is again equal to the energy of activation: the rate of twinning will be roughly proportional to

$$\left[\exp\left\{-\left(\varepsilon - \frac{1}{2} d f\right)\Big/ kT\right\}\right]^n ,$$

or

$$= \exp\left\{-n\left(\varepsilon - \frac{1}{2} d f\right)\Big/ kT\right\} ,$$

if we assume that n atoms will twin simultaneously. Since n will be a large number, the exponential will be infinitesimal when $df/2 < \varepsilon$, but will become enormously large as soon as $df/2$ exceeds ε. This seems to explain the step-wise characteristics of mechanical twinning as well as the above relation (12).

The examinations on the temperature dependency of the results and on the variation of the critical shearing stress for twinning are now being continued, of which we may report shortly.

In conclusion, the writers express their sincere thanks to Professor K. Tanaka for his kind guidance throughout the present experiments. The present study is supported by a grant-in-aid of the Ministry of Education.

REFERENCE

1. B. CHALMERS, *Proc. Phys. Soc.* **47** (1935), 733.