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ON A NEW METHOD OF MEASURING SPECIFIC HEAT OF METALS*

BY

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ABSTRACT

The direct heating method of measuring the true specific heat of metals has been improved. The details of apparatus and experimental procedure are given. The accuracy of the present method is shown in detail with the experimental data for copper. The advantages of the present method lie in the considerable precision and speediness.

1. Introduction

For the determination of specific heat of metals, a method has been adopted by Jaeger and Diesselhorst (1) and others (2–5) in which specimens in the form of a rod or a wire are heated directly by an electric current. In this method the temperature is measured usually by thermo-couples or resistance thermometers. The writer has tried to develop a new method in which the temperature is determined by measuring the linear expansion of the specimen. The advantages of the present method lie in its precision and speediness.

2. Theory of the method

Now consider the ideal case of a uniform wire of metal exposed to the air held at constant temperature θ_0. Let ρ and r be the mass and the resistance of the wire per unit length respectively, and also let p, c and h be the perimeter, the specific heat and the surface emissivity of the wire respectively. Further, let an electric current of intensity I flow through the wire. Then the temperature θ of the wire at any time t will be given by the equation:

$$\rho c d\theta = \frac{I^2 r}{J} dt - ph(\theta - \theta_0)\, dt\,, \tag{1}$$

where J is the mechanical equivalent of heat. In the following lines, the dimensions, temperatures, times and heat quantities are expressed in centimeters, degrees, Centigrade, seconds and calories respectively.

On integration the solution of equation (1) may be written as:

$$\theta - \theta_0 = \frac{I^2 rt}{J\rho c}\left\{1 - \frac{1}{2}\frac{pht}{\rho c} + \frac{1}{6}\left(\frac{pht}{\rho c}\right)^2 - \cdots\right\}. \tag{2}$$

If the time t is small and the surface of the wire is polished sufficiently in order to diminish the emissivity, the term $\frac{1}{6}(pht/\rho c)^2$ in equation (2) may be neglected in

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comparison with the first two terms. For example, if the wire is of copper, and $t = 8$, $h = 0.0003$ and $p = 0.9$, the value of $\frac{1}{6}(pht/\rho c)^2$ becomes 0.0001. Thus, equation (2) may now be written as:

$$\theta - \theta_0 = \frac{I^2 rt}{J\rho c}\left(1 - \frac{pht}{2\rho c}\right). \tag{3}$$

Therefore it is seen that the temperature-excess $(\theta - \theta_0)$ is represented by a quadratic function of the time t. Since, over a short range of temperature, the magnitude of thermal expansion is proportional to the increase of temperature, the linear expansion l of the wire per unit length at the time t may be expressed as follows:

$$l = At + Bt^2,$$

where A and B are constants. If the thermal expansions at t_1 and t_2 are denoted by l_1 and l_2 respectively, we have

$$\left|\frac{dl}{dt}\right|_0 = A = \frac{l_1 t_2^2 - l_2 t_1^2}{t_1 t_2 (t_2 - t_1)}, \tag{4}$$

where $|dl/dt|_0$ is the ratio at $t = 0$ of the linear expansion per unit length to the time, at which the heat-loss from the surface of the wire is nil. The temperature-rise $\Delta\theta$ per unit time is equal to A/α, where α is the coefficient of linear expansion, provided that the heat-loss is nil. Therefore the specific heat c is calculated from the formula:

$$c = \frac{I^2 r}{J\rho\Delta\theta} = \frac{I^2 r\alpha}{J\rho A}. \tag{5}$$

3. Apparatus and experimental procedure

The arrangement of the apparatus is shown in Fig. 1. S is the specimen in the form of a uniform wire, the both ends of which being bent rectangularly and immersed in mercury cups. The specimen rests upon a knife edge K and upon a roller with a mirror, the roller resting upon a horizontal flat plate too. S′ is a wire of the same

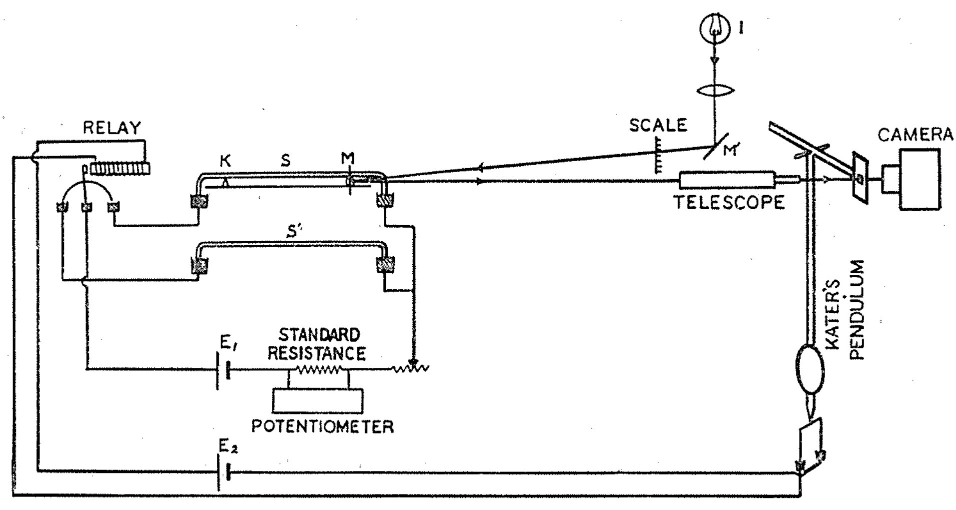

FIG. 1

material and shape as S.

The electric circuits connecting the battery E_1, the standard resistance, the potentiometer, S and S′ are shown in Fig. 1. The electric current flowing through S′ is capable of being exchanged to S by means of a relay which works at an instant at which Kater's pendulum comes to the vertical position and passes through the mercury contact. The intensity of electric current can be determined by means of the standard resistance and the potentiometer. The light started from the lamp I passes through the scale and then is reflected at the mirror M and again passes through the telescope with a cross-wire and through a hole in a plate which is attached to Kater's pendulum, and enters finally into the camera. The light can pass through the hole when and only when the pendulum comes to the vertical position.

At the beginning of the observation, the pendulum is inclined so that the light is obstructed to enter into the camera, and the electric current flows through S′. After a few minutes, when the electric current has become steady, the pendulum is started. When it comes to the vertical position, the light enters into the camera through the hole and the images of the scale and the cross-wire are photographed, and at the same time the electric current is exchanged from S′ to S. After t_1 and t_2 seconds, the light is made to enter into the camera separately, photographing the displacement of the image of the scale at each time, and thus the thermal expansions l_1 and l_2 for the time intervals t_1 and t_2 respectively are calculated. Thus the value of $|dl/dt|_0$ is obtained by equation (4). At the same time the intensity of the electric current is measured, while the resistance and the coefficient of thermal expansion are determined by separate experiments.

4. Sources of error

(1) If the length of the wire of specimen is too short, the temperature-rise at the centre becomes larger than the one at the end, and heat flows along the length of the wire, which gives rise to an error, especially when the specimen is of a good conductor. The appropriate length of the wire is so determined experimentally that the temperature at any point on the specimen may rise equally when it is heated by electric current.

(2) When the temperature-excess is too large and the radius of the wire is too small, Newton's law will not be applicable.

(3) When the time interval of heating is too long and the radius of the wire is too small, the heat-loss from the surface becomes large in comparison with the heat quantity used to raise the temperature of the wire, as is evident from equation (3), and consequently the experimental error may increase.

(4) If the temperature of the surrounding atmosphere is not constant, a correction is needed. For example, when the temperature of the surrounding atmosphere is ascending, the thermal expansion due to its effect for the time t_1 must be subtracted from l_1 in equation (4).

5. Precision of the method

The precision of the present method will be shown, taking, as an example, the measurements for electrolytic copper. The specimens in the form of uniform wire of 2.9 mm in diameter were annealed in vacuum at the temperature 550° for 30 minutes and then cooled down slowly to the room temperature. With three specimens, eighteen measurements were carried out. The detailed experimental data for one typical measurement are shown below. The results of all our measurements are given in Table II.

Typical experimental data

Mean temperature of the specimen during the experiment: 30.5°,
Distance between the mirror and the scale: 78.65 cm,
Length of the specimen between the knife-edge and the axis of the roller: 17.51 cm,
Diameter of the roller: 0.2643 cm,
Period of Kater's pendulum (T): 2.379 sec,

$t_1 = 4T$, $t_2 = 8T$, $l_1 = 0.9448 \times 10^{-5}$ cm, $l_2 = 1.8437 \times 10^{-5}$ cm,
$A = 1.0170 \times 10^{-6}$ cm/sec, $I = 22.57$ ampere, $\alpha = 0.00001651$,
$r = 0.00002733$ ohm/cm, $J = 4.185$, $\rho = 0.5868$ gr/cm, $\Delta\theta = 0.06159°$,
$c = I^2r/(J\rho\Delta\theta) = 0.09205$.

TABLE I

Specimen No. 1		Specimen No. 2		Specimen No. 3	
Temp.	Specific heat	Temp.	Specific heat	Temp.	Specific heat
26.9°	0.09186	26.7°	0.09201	26.1°	0.09206
26.3	9177	26.8	9160	26.7	9191
28.0	9201	27.4	9166	28.2	9172
28.9	9180	28.2	9189	29.0	9183
30.5	9164	31.6	9206	31.1	9233
30.5	9205	31.3	9202	30.2	9171
Mean 28.5	0.09186	28.7	0.09187	28.6	0.09193

The specific heat at 28.6° is 0.09189 ± 0.00004, the experimental error being 0.03%. The true values of specific heat for copper obtained by other authors are summarized in Table II, where the "specific heat at 18°" is the modified value obtained from the observed one by assuming that the temperature coefficient of specific heat is 0.00041 (**6**).

The present writer measured also the specific heats of constantan and brass by making use of the same method. The experimental error of these measurements was about 0.05% which shows better precision of the present method in comparison with those obtainable by the method of mixture or cooling.

TABLE II

Experimenter	Specific heat observed	Specific heat at 18°	
Gaede (7)	0.09108 at 16.7°	0.09113	0.09143
Griffith (8)	0.09159 at 14.21°	0.09174	
Harper (2)	0.0915 at 20°	0.09142	
Present writer	0.09189 at 28.6°	0.09145	

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