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DETERMINATION OF ULTRASONIC SOUND VELOCITIES AND ELASTIC CONSTANTS OF SOLID

BY

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1. Introduction

We have measured the variation of transmission of ultrasonic sound beam through a solid plate immersed in liquid, when either the thickness of the plate or the incident angle is changed. From the above data, we have determined the velocities of two elastic waves, longitudinal and transversal, in the solid, applying the results developed from the theory of Rayleigh(1) and that of Reissner(2). From these sound velocities, we have calculated the elastic constants of the solid, using the well-known formulae.

In order to measure the variation of transmission of ultrasonic sound beam, we used the method utilizing the ultrasonic light diffraction phenomena found by Debye and Sears. In the light diffraction generated in the liquid by an ultrasonic sound beam which has passed through the solid plate, the number of the highest order of diffraction which can be observed, is proportional to the intensity of ultrasonic sound at the point passed by light beam. By observing the variation of the number of the highest order, we can determine the incident angles, at which the intensity of ultrasonic sound which has passed through the solid plate, becomes maximum or minimum.

From the experimental results which we have got about aluminium, applying the above method, we have determined the two sound velocities and the elastic constants of aluminium.

2. The results developed from the theory of Rayleigh and that of Reissner

A. The results from the theory of Rayleigh

Rayleigh has studied the velocity potential of the reflected wave, when the longitudinal elastic wave is incident to the plane parallel medium [1] of density ρ_1 and thickness d, from the medium [0] of density ρ. From this result, we get the following formulae:

a) The conditions for transmission to be maximum or minimum in the case of normal incidence are:

$$V\rho = V_L \rho_1 , \qquad \text{for maximum only,} \tag{1}$$

where V and V_L are longitudinal velocities in both media, and

$$\left.\begin{aligned} d &= m\frac{\lambda_L}{2} && \text{for maximum,} \\ d &= (2m+1)\frac{\lambda_L}{4} && \text{for minimum,} \end{aligned}\right\} \tag{2}$$

where m is 0 or a positive integer, and λ_L is the longitudinal wavelength in the midium [1].

b) The conditions for transmission to be maximum or minimum in the case of oblique incidence are:

$$\left(\frac{\rho_1^2}{\rho^2} - \frac{V^2}{V_L^2}\right)\cot^2\theta_i = \frac{V^2}{V_L^2} - 1 \quad \text{for maximum only,} \tag{3}$$

where θ_i is the intromitting angle as Rayleigh named, at which the whole wave energy can pass through the medium [1], and this angle is independent of the thickness of the medium [1] and

$$\sin^2\theta = \left(\frac{V}{V_L}\right)^2 - \left(\frac{m\lambda}{2}\right)^2\frac{1}{d^2} \qquad \text{for maximum,} \tag{4}$$

and

$$\sin^2\theta = \left(\frac{V}{V_L}\right)^2 - \left(\frac{2m+1}{2}\right)^2\left(\frac{\lambda}{2}\right)^2\frac{1}{d^2} \qquad \text{for minimum.} \tag{4'}$$

B. The results from the theory of Reissner

Reissner has calculated the transmission on the assumption that the longitudinal and transversal waves are induced in the solid plate by the longitudinal wave in the surrounding liquid. From this result, we get the following conditions, for the transmission to be maximum,

$$\sin^2\theta = \frac{1}{2}\left(\frac{V}{V_T}\right)^2, \tag{5}$$

$$\sin^2\theta = \left(\frac{V}{V_T}\right)^2 - \left(\frac{m\lambda}{2}\right)^2\frac{1}{d^2}, \tag{6}$$

and

$$\sin^2\theta = \left(\frac{m\lambda}{2}\right)^2\frac{1}{d^2}, \tag{7}$$

where V, λ are respectively the sound velocity and wavelength in the liquid and V_T is the transversal velocity in the solid.

It is interesting that the condition (4) in the case of longitudinal wave has the same form as the condition (6) in the case of transversal wave. In practice, it is difficult to determine the number of order m of the observed maximum corresponding to (4). In Reissner's case, we have, besides (6), the condition (7) which is helpful to determine m. And moreover, we can find the angle θ which satisfies equations (5), (6) and (7) simultaneously and determine the transversal wave velocity V.

3. Experimental procedure

The experimental arrangement is shown in Fig. 1. An ultrasonic sound beam is generated by a quartz crystal in the vessel filled with water. A solid plate can be rotated about the axis perpendicular to the plane of the figure. The number of the highest order of diffraction and the intensity of each spectral line vary with the thickness of the plate and the incident angle.

Relations between the sound intensity at the point passed by light on one hand,

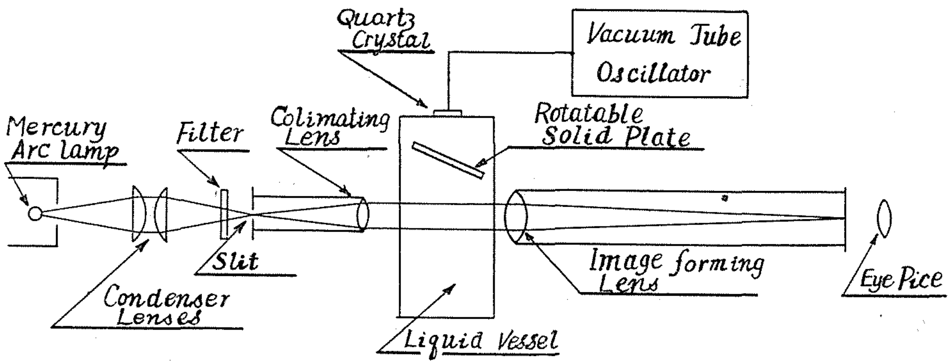

FIG. 1. Experimental arrangement.

and the number of the highest order of diffraction and the intensity of each spectral line on the other, are the following:

1. The stronger the sound intensity is, the higher order appears.
2. The stronger the intensity of the highest order is, the larger is the sound intensity.

According to the above principle, when we observe the transmission intensity of ultrasonic beam, by rotating the solid plate whose thickness is constant, we can find that the intensity of the transmitted beam changes periodically with the angle rotated. The positions of maximum and minimum which appear at that time, can be determined within the error of 0.1°. In the case of aluminium, we get the transmission patterns as shown in Fig. 2. The ordinate of this diagram is the highest order of diffraction which shows the measure of the sound intensity.

A. Determination of longitudinal velocity

We have found the thickness, at which the transmission becomes maximum or minimum for normal incidence, by changing continuously the thickness of the Al-plate by means of roller pressing. From these data, we can determine, using formula (2) the longitudinal velocity V_L, as shown in Table 1.

TABLE 1. Longitudinal wavelength for normal incidence (frequency 3940 Kc, temp. 25.8 ± 1.5°C)

thickness d	$1.92 = \frac{5}{4}\lambda_L$	$1.63 = \lambda_L$	$1.19 = \frac{3}{4}\lambda_L$	$0.85 = \frac{1}{2}\lambda_L$
wavelength λ_L	1.53	1.63	1.59	1.70
mean value of λ_L: $\overline{\lambda_L} = 1.61$ mm				

From $\overline{\lambda_L}$, we get

$$V = \overline{\lambda_L} f = 6350 \pm 350 \text{ [m/sec]}.$$

For oblique incidence, we calculated V_L from the first maximum or minimum in Fig. 2, using formula (4) or (4′). The result is shown in Table 2.

TABLE 2. Longitudinal wavelength for oblique incidence

thickness d	incident angle θ		order m	λ_L [mm]	V_L [m/sec]
1.92	8.4	max.	2	1.54	6070
1.63	10.2	min.	1	1.53	6030
1.19	10.3	max.	1	1.58	6230
0.85	9.4	min.	0	1.93	7550

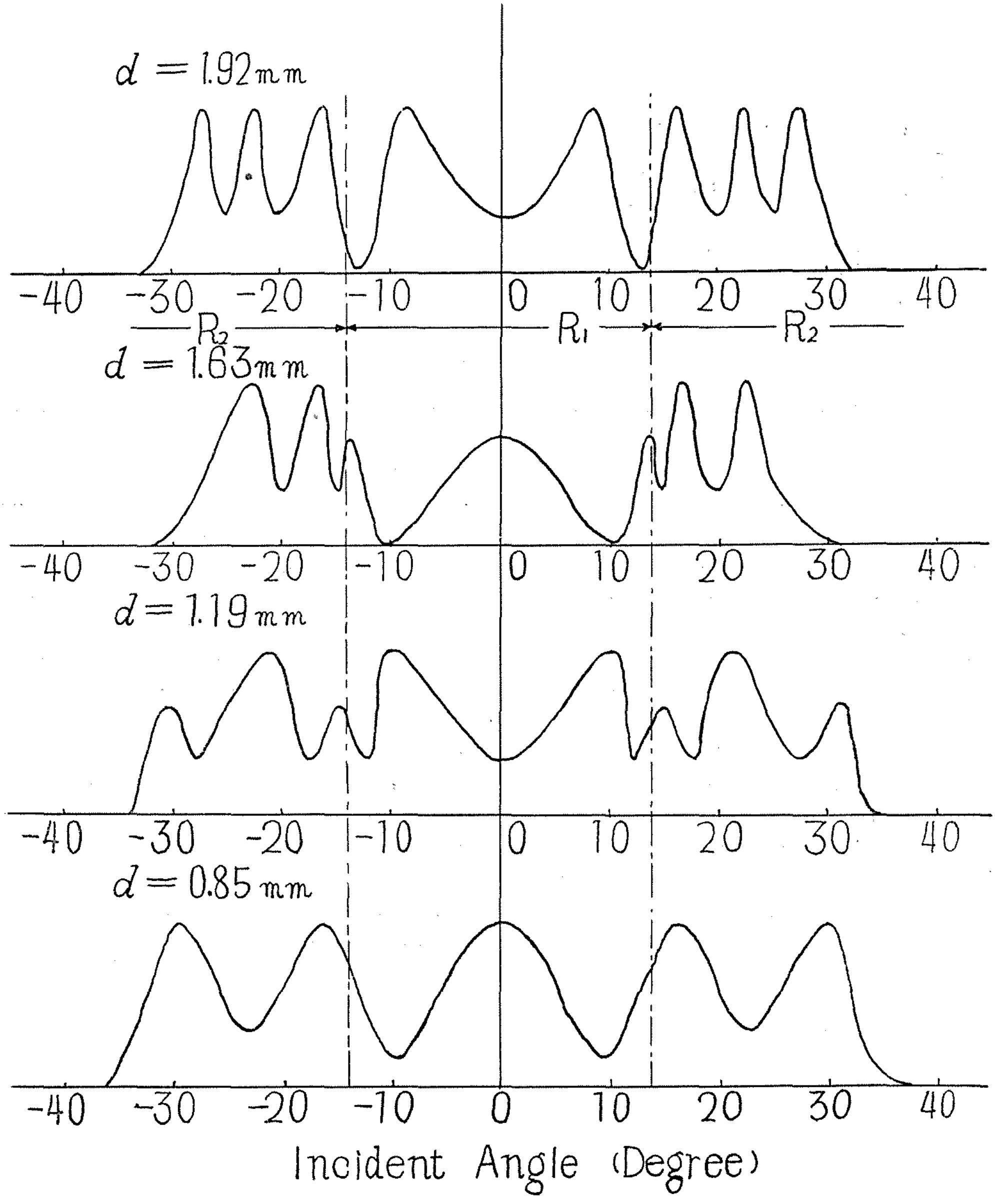

FIG. 2. Transmission patterns of aluminium.

The values of λ_L and V_L in the first three rows show rough coincidence, but the values in the last row is considerably apart from others.

We have calculated the total reflection angle $\theta_C = 13.4°$, according to $\sin\theta_C = V/V_L$, using the values $V_L = 6350$ [m/sec] and $V = 1500$ [m/sec]. This angle is shown by broken line in Fig. 2. We get two regions R_1 and R_2 on one side separated by this line. And in R_2, longitudinal wave and transversal one coexist, but in R_2 only transversal wave exists. We have expected that the transmission minimum will appear at the angle

θ_C for any thickness, because this angle is independent of the thickness. But in our experiment about aluminium, the transmission minimum has not appeared at this angle. From this fact, we have deduced that in region R_1 the transversal wave is co-existing.

B. Detemination of transversal velocity

The determination of the transversal velocity can be performed by finding such θ which satisfies (5), (6) and (7), by using maximum points in R_2. Applying formulae (5), (6) and (7) to these points summarized in Table 3, we determine the transversal velocity diagrammatically. If we take $\sin^2\theta$ as ordinate, $1/d^2$ as abscissa and m as parameter in Fig. 3, the formula (6) corresponds to a group of straight lines passing through a common point on the ordinate axis, and the formula (7) to a group of straight lines passing through the origin.

We plot on this graph the measured points shown in Table 3, and, selecting adequate several points, we draw such straight lines as pass through a common point on the ordinate axis, which are called (a_1, a_2, $\cdots$, a_6), corresponding to $m=1, 2, \cdots 6$ and as the whole group (a). Straight lines of group (a) satisfy the formula (6). Next, λ in formula (7) can be computed to 0.381 mm, from the sound velocity in water and the frequency. Giving m integral values ($1, 2, \cdots 6$) successively in (7), we get a group of straight lines b ($b_1, b_2, \cdots b_6$), corresponding to $m=1, 2, \cdots, 6$, which satisfies formula (7). In the third place, we must find the angle θ independent of the thickness. As easily seen from the forms of the formulae (5), (6) and (7), such θ ought to be on the intersections (P_1, $P_2, \cdots P_6$) of two straight lines of (a) and (b) with the same suffix. In practice, we get $\sin^2\theta=0.169$ as the mean value of ordinates of these points. We

TABLE 3. Maximum points in region R_2

d [mm]	θ	$\sin\theta$	$\sin^2\theta$	$1/d^2$
3.016	16.1	0.2773	0.077	0.11
3.016	19.3	0.3305	0.109	0.11
3.016	23.0	0.3907	0.152	0.11
3.016	25.5	0.4305	0.185	0.11
1.92	16.1	0.2773	0.077	0.271
1.92	22.0	0.3746	0.140	0.271
1.92	26.9	0.4524	0.204	0.271
1.63	13.6	0.2351	0.055	0.377
1.63	16.3	0.2807	0.079	0.377
1.63	22.0	0.3746	0.140	0.377
1.19	14.6	0.2521	0.063	0.708
1.19	20.9	0.3567	0.127	0.708
1.19	30.4	0.5060	0.256	0.708
1.035	18.0	0.3090	0.095	0.935
1.035	23.4	0.3971	0.157	0.935
1.035	30.2	0.5030	0.252	0.935
0.85	16.0	0.2756	0.076	1.39
0.85	29.2	0.4879	0.237	1.39
0.72	14.0	0.2419	0.058	1.94
0.72	28.8	0.4818	0.230	1.94

use this value and the sound velocity in water $V=1500\,[\mathrm{m/sec}]$ in formula (5). We get

$$V = 2580 \pm 140\,[\mathrm{m/sec}]$$

as the transversal velocity.

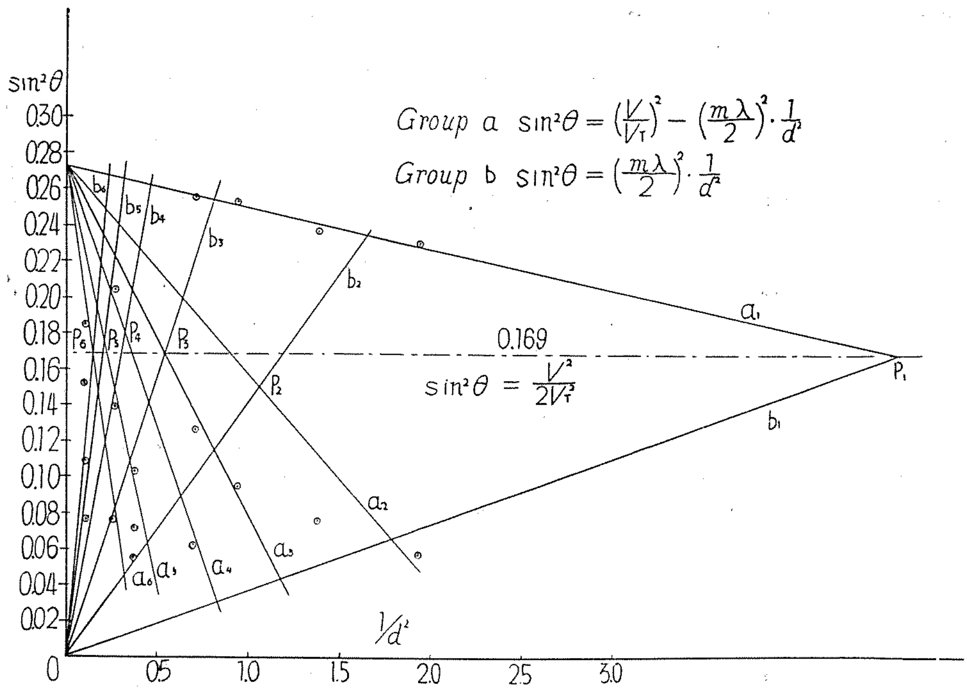

FIG. 3. Diagrammatical determination of transversal velocity.

4. Calculation of elastic constants

In order to calculate the elastic constants, we use following formulae:

$$V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \tag{8}$$

$$V_T = \sqrt{\frac{\mu}{\rho}}, \tag{9}$$

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}, \tag{10}$$

and

$$\sigma = \frac{\lambda}{2(\lambda + \mu)}, \tag{11}$$

where λ and μ are Lamé constants, E Young's modulus and σ Poisson's ratio.

Our results, together with those obtained by other writers, are summarized in Table 4.

TABLE 4. Elastic constants of aluminium

V_L [m/sec]	V_T [m/sec]	$E\times10^{-11}$ [dyne/cm^2]	$\mu\times10^{-11}$ [dyne/cm^2]	σ	experimenter
6350 ± 350	2580 ± 140	5.00	1.78	0.400	Present authors
7050	2820	6.45	2.14	0.400	W. C. Schneider & C. J. Burton
5104					Masson (Handbook)

It will be seen that there is a great difference between the value of V_L found by Schneider and Burton (3) and the corresponding value obtained by Masson and that our value lies between these two.

REFERENCES

1. LORD RAYLEIGH, Theory of Sound, Vol. 2, 86.
2. H. REISSNER, *Helv. Phys. Acta*, **11** (1938), 140.
3. W. C. SCHNEIDER & C. J. BURTON: *J. Appl. Phys.*, **20** (1949), 48.