

# THE VERTICAL DISTRIBUTIONS OF THE TEMPERATURE AND DENSITY IN THE LOWER IONOSPHERE\*

BY

Yuji INOUE

*Institute of Geophysics, University of Kyoto*

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## ABSTRACT

The vertical distributions of the temperature and density in the lower ionosphere, D, E and F<sub>1</sub> regions, are derived simultaneously by supposing the dissociative, thermal and mechanical equilibria. It is assumed that the atmosphere is well mixed and the chemical composition of the ionosphere is the same as that of the atmosphere at the ground. At the height of the ionosphere, the nitrogen molecule remains undissociated, while oxygen is dissociated by the solar ultraviolet radiation, which is supposedly of the Planckian radiation with the temperature 5713°K. The investigation on the photo-dissociation of molecular oxygen, based on the absorption coefficients inquired by R. Penndorf, enables us to secure some important result, namely the relation between the rate of the dissociative absorption and the optical depth of the dissociating radiation. The gas-kinetic energy in the lower ionosphere is mainly supplied by the excess energy made in the case of the photodissociation and is lost effectively by the inelastic collision with the molecule resulting in the vibrational excitation of the latter and the subsequent emission by the quadrupole transition. The formation of molecular oxygen is due to radiative association. Unfortunately, little is known about the rate of this process. Thus it was deduced in this paper from the ionospheric data. We have chosen as a reference level of height the level in the E region, where the total particle concentration can be roughly detected by the radio echos. However, with respect to the temperature and temperature gradient, their absolute values can be derived by the equilibrium considerations. The equilibrium equations were solved by the numerical method.

Some important results obtained by the present calculation are as follows:

- (i) There occur a steep temperature gradient and the almost abrupt decrease of the particle concentration of molecular oxygen in the E region.
- (ii) The theoretical possibility for the existence of the particle concentrations enough to account for the generation of the F<sub>1</sub> and F<sub>2</sub> layers has been shown.

It must be remarked that our theoretical pressure distribution is not in agreement with that of the ambient pressure measured during the rocket flights. However, the matter does not seem to be decisive yet if we reflect upon the uncertainties inherent in the theory on the one hand and the observation on the other.

The problems of the F<sub>2</sub> region related with the present problem are discussed briefly. Also, the effect of thermal conduction is examined.

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## 1. Introduction

Up to the present time, many investigations (1-5) have been made to determine the vertical distribution of the particle concentration in the ionosphere, by assuming the temperature distribution. However, the derivation of the vertical distribution of the particle concentration by assuming the latter seems to be unsatisfactory unless the latter has been firmly established. The particle concentration and the temperature at a level of the ionosphere are closely connected with each other. Hence, in order to get the accurate information of these distributions, it is much more desirable to find a method capable of deducing simultaneously the temperature and particle concentrations at every altitude. It seems relevant and desirable from the theoretical point of view to establish the distributions in the lower ionosphere satisfying the dissociative, thermal and mechanical equilibria simultaneously. Such an idea has first been proposed by H. E. Moses and Ta-You Wu. (6) The present calculations are at variance with them about the principal process of the loss of gas-kinetic energy in the lower ionosphere. D. R. Bates (7) has investigated precisely every atomic processes responsible for the heat gain and heat loss. He has presumed that oxygen in the ionosphere is completely in the dissociative condition, and in consequence he ascribed less importance, contrary to the present work, to the dissociative absorption of molecular oxygen on the gain of gas-kinetic energy. With respect to the loss of gas-kinetic energy, his inspection about the emissions of the vibrational energy of various ionospheric molecules was made by assuming the thermodynamical equilibrium. However, it seems much more plausible to suppose that the distribution of the vibrational levels under the ionospheric condition does not obey the Boltzmann distribution.

## 2. The mechanical equilibrium in the lower ionosphere

The principal constituents of the ionosphere are oxygen and nitrogen, as generally accepted from the evidence of the spectra of the night sky and the polar aurorae. As pointed out by various authors (1-6, 8), the absorption of the solar radiation in the continuum of the Schumann-Runge bands causes a heavy dissociation of oxygen in the ionosphere, and the oxygen is present mainly as molecules at the lower level, while as atoms at greater heights. The detailed calculations of Wulf and Deming (1), and, recently, of Penndorf (4) have shown that the transition takes place in a region near but slightly above 100 km level. While, the degree of dissociation of nitrogen in the ionosphere is at present a controversial question; G. Herzberg and L. Herzberg (9) have suggested that the nitrogen is dissociated to a much small extent by absorption in the forbidden bands  $a^1\Pi_g \rightarrow X^1\Sigma_g^+$  below 1250Å and subsequent predissociation. It may not be expected from this process that nitrogen is considerably dissociated in the conditions of the ionosphere, so that it probably

remains in the molecular state up to a very great height in the ordinary conditions. As pointed out by Chapman, this may be implied by the appearance of the band spectra attributed to molecular nitrogen in sunlit aurorae up to about over 1000 km level (10).

The rocket measurements have been made of the helium content in the air at a height more than 70 km, and they have shown that it is not appreciably different from that at the ground (11). It seems to be relevant to suppose that the atmosphere is considerably stirred up to this level. There is at present no knowledge about the mixing of the atmosphere above this level. While, it is generally accepted by radio measurements that in the region of the ionosphere there are motions of the air whose wind-velocities are as great as 100 m sec<sup>-1</sup> or more (12). As to the existence of such motions of the air, it may be supposed that by them the atmosphere is considerably stirred up to the top of the ionosphere.

The conceivable constituents other than oxygen and nitrogen seem to be so scarce that they probably have no direct influence upon the phenomena discussed in this paper.

Hence for the conclusion, it may hold good to presume that the gases in the lower ionosphere consist of nitrogen and oxygen in the same proportion as at the ground, provided that nitrogen is undissociated, while oxygen is heavily dissociated by the solar ultraviolet.

In view of this, assigning generally a parameter  $\eta$ , which is also independent of height, to express the relative proportion of oxygen and nitrogen, the particle concentrations of oxygen and nitrogen at any height may be expressed by the relation:

$$\eta \left[ n(\text{O}_2, z) + \frac{1}{2} n(\text{O}, z) \right] = n(\text{N}_2, z), \quad (1)$$

$n(\text{O}, z)$ ,  $n(\text{O}_2, z)$ , and  $n(\text{N}_2, z)$  being the particle concentrations per cm<sup>3</sup> at a height  $z$  of atomic oxygen, molecular oxygen, and molecular nitrogen respectively. Let  $N(z)$  be the total particle concentration per cm<sup>3</sup> at a height  $z$ , it follows that

$$N(z) = n(\text{O}, z) + n(\text{O}_2, z) + n(\text{N}_2, z). \quad (2)$$

Substitution of (1) into (2) yields

$$N(z) = n(\text{O}, z) \left[ 1 + \frac{\eta}{2} + (1 + \eta) \chi(z) \right], \quad (3)$$

where

$$\chi(z) = \frac{n(\text{O}_2, z)}{n(\text{O}, z)}.$$

Let  $M(z)$  be the mean molecular mass and let  $M_0$  be the mass of atomic oxygen. The variation of the mean molecular mass with height is described by

$$M(z) = \frac{\left(1 + \frac{7}{8}\eta\right)(1 + 2\chi)}{1 + \frac{\eta}{2} + (1 + \eta)\chi} M_0. \quad (4)$$

It may be supposed that velocities of air motions in the lower ionosphere are not so great as the mechanical equilibrium of the atmosphere is very far remote from that for a rest atmosphere, so that the hydrostatic equilibrium of the atmosphere may be approximately available for the lower ionosphere. Neglecting the variation of the gravitational acceleration  $g$  with height, we have

$$\frac{d}{dz} \log_e P(z) = -\frac{M(z)g}{KT(z)}, \quad (5)$$

where  $P(z)$  and  $T(z)$  are the pressure and the temperature at the height  $z$  respectively, and  $K$  Boltzmann's constant.

Table 1

$\lambda$ Å	$\log_{10} K_\nu$ , cm <sup>2</sup>	$h\nu - h\nu_0$ , eV	$\log_{10} K_\nu I_\nu(\infty)$ sec <sup>-1</sup>
1300	19.5403	2.44	7.0633
1350	18.8195	2.09	6.6196
1400	17.1367	1.76	5.1931
1450	17.2672	1.46	5.5611
1500	17.1673	1.17	5.6835
1550	17.0253	0.91	5.7466
1600	18.8420	0.66	5.7551
1650	18.6010	0.43	5.6937
1700	18.3010	0.21	5.5635
1750	19.5403	0.00	6.9605
1800	20.6375	1.77	6.2068
1850	21.2810	1.59	8.9886
1900	22.4440	1.41	8.2833
1950	23.6375	1.24	9.6021
2000	24.9395	1.09	9.0212
2050	24.6902	0.94	10.8848
2100	24.3424	0.79	10.6420
2150	24.1461	0.66	10.5465
2200	25.9395	0.52	10.4330
2250	25.5911	0.40	10.1775
2300	25.2553	0.28	11.9269
2350	26.9395	0.17	11.6924
2400	26.7404	0.06	11.5717
2450	24.0531	0.00	—
2500	25.7832	0.00	—

### 3. The photodissociation of molecular oxygen and the gain of gas-kinetic energy in the lower ionosphere

The photodissociation of molecular oxygen takes place by the absorption of the solar radiation in the continuum of the Schumann-Runge and of the Herzberg bands. The latter has no appreciable effect in the ionospheric region. The photodissociative absorption coefficient  $K_\nu$  of molecular oxygen in the range of wavelength concerned has been investigated by Pennedorf (4). His result as reproduced in Table 1 will be used in the present work.

When the photodissociation takes place, the absorption of radiation with energy

$h\nu$  exceeding the threshold energy necessary for dissociating molecular oxygen, contributes to the increase of the gas-kinetic energy in the lower ionosphere. Thus,  $h\nu - h\nu_0$  is the appropriate part of the original quantum  $h\nu$  transformed to the kinetic energy, in which  $h\nu_0$  means the threshold energy, that is, 7.047 eV for the Schumann-Runge transition, and 5.080 eV for the Herzberg transition (13). Values of  $h\nu - h\nu_0$  are given in Table 1. But all of these energies are, in general, not available for local heating: due to the dissociative recombination a certain fraction of it would be spent in converting into radiation or in exciting the molecular oxygen generated. This, however, is not necessary to be taken into account by the reasoning that the rate of recombination is very slow, as is generally accepted, and that the temperature in the lower ionosphere is supposed to be as low as about 300°K.

The solar ultraviolet radiation in the range of wavelengths capable of dissociating molecular oxygen is supposed to be of a diluted black-body radiation with the temperature 5713°K.

Let  $I_\nu(\infty) d\nu$  be the incident flux of the dissociating radiation in quanta per cm<sup>2</sup> per sec from the sun in frequencies within the range from  $\nu$  to  $\nu + d\nu$  received at the top of the earth's atmosphere. The numerical values of  $K_\nu I_\nu(\infty)$  are given in Table 1. Let  $I_\nu(z) d\nu$  be the flux of quanta per cm<sup>2</sup> and per second of the dissociating radiation in the frequency range between  $\nu$  and  $\nu + d\nu$  at the height  $z$  in the ionosphere. For shortness of the representation of solutions it is convenient to introduce  $\sigma_m(z)$ ,  $\tau_m(z)$ ,  $D(z)$ , and  $\varepsilon D(z)$  defined as:

$$\sigma_m(z) = \int_z^\infty n(\text{O}_2, z') dz', \quad (6)$$

$$\tau_m(z) = \int_z^\infty K_{1450\text{\AA}} n(\text{O}_2, z') dz', \quad (7)$$

$$D(z) = \int_\nu K_\nu I_\nu(z) d\nu, \quad (8)$$

and

$$\varepsilon D(z) = \int_\nu K_\nu (h\nu - h\nu_0) I_\nu(z) d\nu, \quad (9)$$

where

$$I_\nu(z) = I_\nu(\infty) \exp[-K_\nu \sigma_m(z)] \quad (10)$$

is the ordinary exponential solution of the equation for the pure absorption:

$$\frac{dI_\nu(z)}{dz} = -K_\nu n(\text{O}_2, z) I_\nu(z), \quad (11)$$

and the integrations in (8) and (9) cover the frequency range concerned.  $D(z) n(\text{O}_2, z)$

is the rate of the photodissociative absorption per  $\text{cm}^3$  and per second at the height  $z$ ,  $\epsilon D(z) n(\text{O}_2, z)$  is the rate of the gain of gas-kinetic energy in  $\text{erg cm}^{-3} \text{sec}^{-1}$  due to the photodissociation of the height  $z$ ,  $K_{1450\text{\AA}}$  is the maximum absorption coefficient in the frequency range concerned,  $\sigma_m(z)$  is the number of molecular oxygen in a column with a base of unit area of the atmosphere above the height  $z$ , and  $\tau_m(z)$  is the optical depth of the dissociating radiation as to the maximum absorption coefficient  $K_{1450\text{\AA}}$ , the suffix  $m$  distinguishing the molecular from the atomic oxygen. The integrations of  $D(z)$  and  $\epsilon D(z)$  have been carried out numerically step by

Table 2

$\log_e \tau_m(z)$	$\log_{10} \epsilon D(z)$ $\text{erg sec}^{-1}$	$\log_{10} D(z)$ $\text{sec}^{-1}$	$\lambda_{\text{max}} \text{\AA}$	$\lambda - \lambda_{\text{abs}} \text{\AA}$
-2.50	$\overline{16.3683}$	$\overline{4.2103}$	1600	1300-1850
-2.00	$\overline{16.3541}$	$\overline{4.2041}$	1600	1300-1850
-1.50	$\overline{16.3243}$	$\overline{4.1761}$	1600	1300-1850
-1.00	$\overline{16.2742}$	$\overline{4.1303}$	1600	1300-1850
-0.50	$\overline{16.1986}$	$\overline{4.0719}$	1600-1650	1300-1850
0.00	$\overline{16.0969}$	$\overline{5.9993}$	1650	1300-1850
0.50	$\overline{17.9385}$	$\overline{5.8819}$	1650	1300-1850
1.00	$\overline{17.7058}$	$\overline{5.7177}$	1650-1700	1300-1850
1.50	$\overline{17.3983}$	$\overline{5.5109}$	1700	1300-1850
2.00	$\overline{17.0449}$	$\overline{5.2624}$	1700	1450-1850
2.50	$\overline{18.7068}$	$\overline{6.9974}$	1700	1550-1850
3.00	$\overline{18.4548}$	$\overline{6.7539}$	1750	1600-1900
3.50	$\overline{18.3201}$	$\overline{6.4969}$	1750	1650-1950
4.00	$\overline{18.2393}$	$\overline{6.1938}$	1750	1700-1950
4.50	$\overline{18.1987}$	$\overline{7.9523}$	1750	1700-1950
5.00	$\overline{18.1408}$	$\overline{7.7839}$	1800	1750-1950
5.50	$\overline{18.0473}$	$\overline{7.6201}$	1800	1750-1950
6.00	$\overline{19.9009}$	$\overline{7.4579}$	1800	1750-1950

step in each interval of  $50\text{\AA}$ , and the computed results are shown in Table 2 and Fig. 1. Since, at present, there is little knowledge concerning the distribution of molecular oxygen with height,  $\log_e \tau_m(z)$  will be taken as a measure of height. The wavelength appropriate for the maximum photodissociative absorption,  $\lambda_{\text{max}}$ , and the range of wavelength of the appreciable absorption,  $\lambda - \lambda_{\text{abs}}$ , for each  $\log_e \tau_m(z)$  are given in Table 2. Inspection of Fig. 1 reveals that both  $\log_e D(z)$  and  $\log_e \epsilon D(z)$  show locally linearity against  $\log_e \tau_m(z)$ . This is not unexpected, but has a simple physical interpretation: taking (6), (7), (8) and (10) into account, equation (11) can be written as:

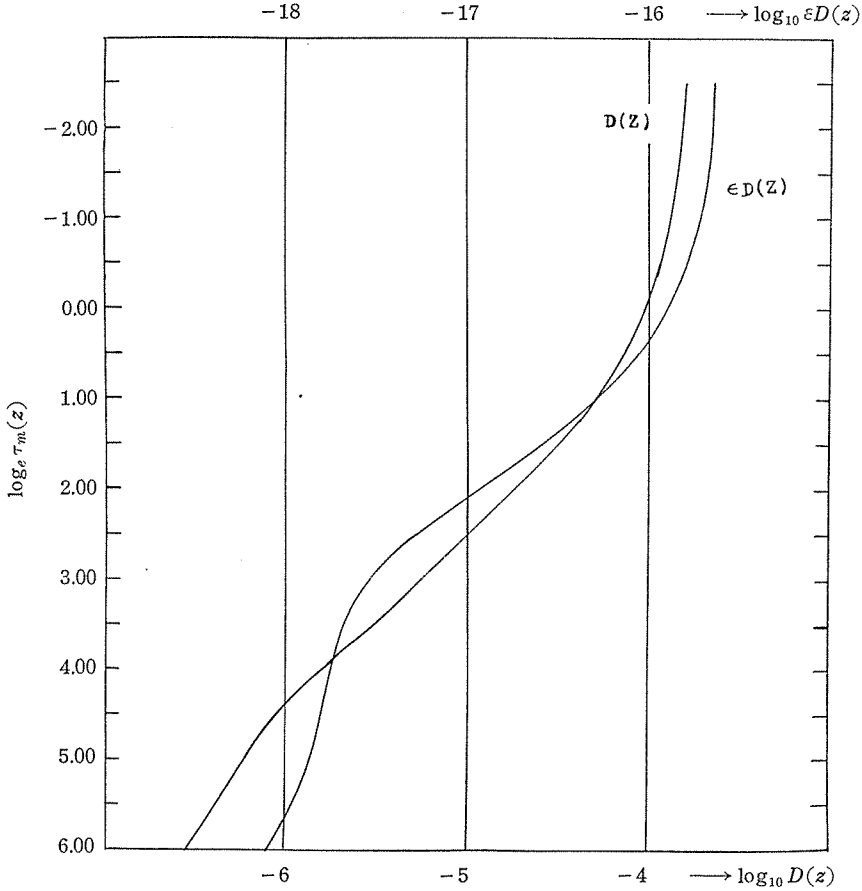


Fig. 1. Distributions of  $D(z)$  and  $\epsilon D(z)$  in relation to  $\log_e \tau_m(z)$ .

$$\frac{d \log_e D(z)}{d \log_e \tau_m(z)} = \frac{\int_{\nu} K_{\nu} I_{\nu}(z) (-K_{\nu} \sigma_m(z)) d\nu}{\int_{\nu} K_{\nu} I_{\nu}(z) d\nu} \quad (12)$$

Similarly, referring to (9) equation (11) can be written as:

$$\frac{d \log_e \epsilon D(z)}{d \log_e \tau_m(z)} = \frac{\int_{\nu} K_{\nu} I_{\nu}(z) (h\nu - h\nu_0) (-K_{\nu} \sigma_m(z)) d\nu}{\int_{\nu} K_{\nu} I_{\nu}(z) (h\nu - h\nu_0) d\nu} \quad (13)$$

$h\nu$ -quanta are absorbed effectively in a region where the optical depth for the radiation is nearly equal to unity. It may be supposed that the dissociating radiation mainly absorbed at the level  $z$ , corresponding to a value of  $\sigma_m(z)$ , is specified by the condition that  $K_{\nu} \sigma_m(z)$  is nearly equal to unity. Putting  $K_{\nu} \sigma_m(z) \approx 1$  in (12) and

(13), and integrating over a frequency-range in which  $K_\nu I_\nu(z)$  has an appreciable amount, it will be seen that

Table 3

$\log_e \tau_m(z)$	$f$	$f'$
-2.50	0.0654	0.0285
-2.00	0.137	
-1.50	0.231	0.129
-1.00	0.348	0.211
-0.50	0.468	0.269
0.00	0.729	0.334
0.50	1.071	0.541
1.00	1.416	0.757
1.50	1.627	0.952
2.00	1.557	1.144
2.50	1.160	1.221
3.00	0.620	1.121
3.50	0.372	1.184
4.00	0.187	1.396
4.50	0.266	1.112
5.00	0.431	0.775
5.50	0.674	0.754
6.00		0.747

and

$$\frac{d \log_e D(z)}{d \log_e \tau_m(z)} = -1,$$

$$\frac{d \log_e \epsilon D(z)}{d \log_e \tau_m(z)} = -1.$$

Now, since the dissociating radiation concerned is not monochromatic, these quantities slightly deviate from unity due to the superposition of absorption of the photons with adjacent frequency to that corresponding to  $K_\nu \sigma_m(z) \simeq 1$ . Let  $f$  and  $f'$  be

$$\frac{d \log_e \epsilon D(z)}{d \log_e \tau_m(z)} = -f, \quad (14)$$

and

$$\frac{d \log_e D(z)}{d \log_e \tau_m(z)} = -f' \quad (15)$$

respectively. In fact, numerical computations give the values different from unity for  $f$  and  $f'$  for each level of  $\log_e \tau_m(z)$ , as can be seen from Table 3 and Fig. 2.

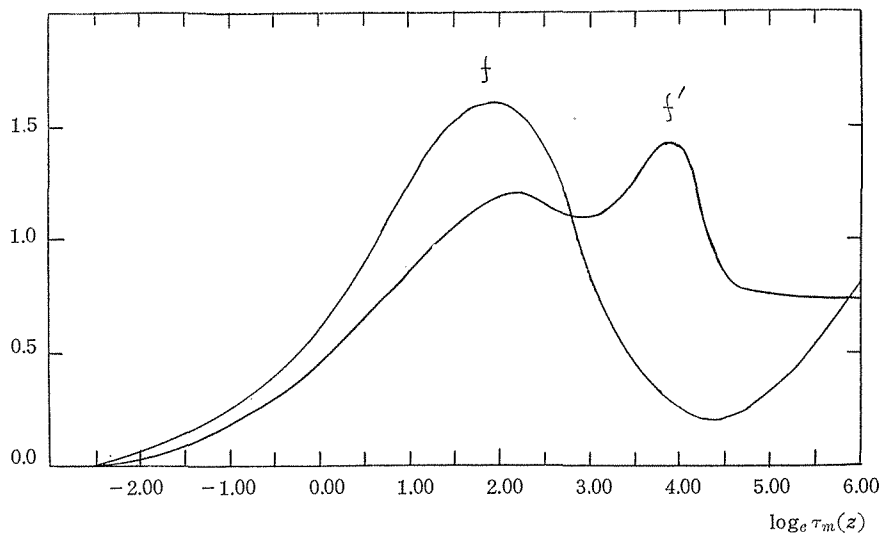


Fig. 2.  $f$  and  $f'$  plotted against  $\log_e \tau_m(z)$ .



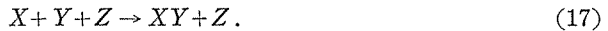
In conclusion, it is found, as a result of examination of the other conceivable processes of the gain of gas-kinetic energy in the lower ionosphere, that the most important process of the gain of gas-kinetic energy is the photodissociation of molecular oxygen. This rate is expressed by  $\epsilon D(z) n(\text{O}_2, z)$  in  $\text{erg cm}^{-3} \text{sec}^{-1}$ .  $\epsilon(z)$ , that is, the average gain of gas-kinetic energy due to the absorption of one photon at the height  $z$ , is given in Fig. 3.

The dissociative equilibrium in the lower ionosphere must now be investigated. In the atmosphere of such a low temperature as is generally supposed for the lower ionosphere, the dissociation of molecular oxygen by collisions may be unimportant. While, the photodissociation mentioned above decides the formation of atomic oxygen in the ionosphere.

Next, we must discuss the formation of molecular oxygen in the ionosphere. In general, free atoms form molecules by radiative association



or by the three-body collision (14, 15)



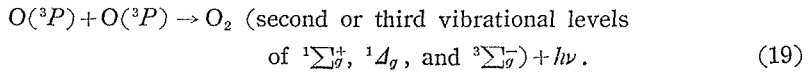
The rates of the former for various cases have been examined by Bates (14); the cross section of the latter may be expressed, with a considerably good approximation, by

$$Q_c \simeq \pi r^2 \cdot \frac{r}{l} \cdot n_z, \quad (18)$$

$r$  being the atomic radius,  $l$  the mean free path for effective collisions between the atom and the third body when the concentration of the latter is unity, and  $n_z$  the particle concentration of the third body. If it is possible that the third body is a molecule, and that the internal energy to be transferred is absorbed completely, by a resonance process, as a potential energy of the third body,  $l$  may take values down to  $10^{14} \text{cm}$ . Then taking  $r$  to be  $10^{-8} \text{cm}$ ,  $Q_c$  becomes as large as  $10^{-24} \text{cm}^2$  at the bottom of the ionosphere, where the particle concentration of the third body, capable of being of resonant absorption, may be supposed to be as great as  $10^{14} \text{cm}^{-3}$ . If resonance transfer is not possible, the excess energy must be partially taken up as kinetic energy of the relative motion of the atomic systems. Then  $l$  may be of the order of  $10^{18} \text{cm}$  or more, hence  $Q_c$  reduces to the values of the order of  $10^{-28} \text{cm}^2$  (16). At the lower gas density below the certain critical value, radiative association must be of the dominant process.

Atomic oxygen in the state  $^1D$  is yielded with a considerably high rate by the photodissociation of the Schumann-Runge transition, but it falls rapidly to the ground state  $^3P$  with emission of the forbidden radiation; under the ionospheric condition it

may be unimportant on the reaction of association. This may be concluded from the intensity of the night sky radiation  $6300\text{\AA}$ , corresponding to the transition  ${}^1D \rightarrow {}^3P$ . Hence, it is sufficient to consider the association between oxygen atoms in the ground state,  ${}^3P$ . Inspection of the configuration of the potential curves of molecular oxygen (17) shows that the radiative associations for molecular oxygen appear primarily to make the transitions from the  ${}^3\Sigma_u^+$  state to the second or third excited vibrational levels in the electronic states of  ${}^1\Sigma_g^+$ ,  ${}^1\Delta_g$ , and  ${}^3\Sigma_g^-$ , viz.



About the rates of the processes mentioned above, we have at present no experimental as well as theoretical knowledge. The probability of radiative association is very sensitive to the configuration of the potential curves of a molecule; in the most favourable case the effective cross section would become as great as  $10^{-21}\text{cm}^2$  (15), in the order of magnitude. Making a reciprocal deduction of the effective cross section from experimental values of the absorption coefficient, as to such slow encounters as in the ionosphere, the deduced values associated with both the Schumann-Runge transition of oxygen and the  ${}^3\Pi_{0u}^+ \rightarrow {}^1\Sigma_g^+$  transition of  $\text{Cl}_2$ -molecule (18) would be obtained to be of the order as small as  $10^{-26}\text{cm}^2$ . The actual value of the cross section associated with the reaction (19) would lie between the two extremities quoted above. The computations by Bates (14), though for other molecules, show the rate coefficients for a few specific reactions to be of the order of  $10^{-17}$  or  $10^{-18}\text{cm}^3\text{sec}^{-1}$ , which lies between the maximum acceptable value of the order of  $10^{-16}\text{cm}^3\text{sec}^{-1}$  and the values of the order of  $10^{-21}\text{cm}^3\text{sec}^{-1}$  deduced from the experimental results of absorption coefficient. If this order of magnitude were applied to our case (19), the radiative association would sufficiently surpass the association by the three-body collision even at the bottom of the ionosphere. After the discussion described above, though it leaves some uncertainties in the quantitative examination for this probability, it does not seem to be unreasonable to suppose that the radiative association (19) is the dominant process down to the bottom of the ionosphere. Though we shall use in the following this cross section as the standard value, it may be wise at our present stage to try another way to determine it from the structure of the lower ionosphere, as will be shown in §5.

Let  $Qv$  be the rate coefficient of the radiative association concerned, which may be supposed to be independent of temperature. The dissociative equilibrium of molecular oxygen is described by

$$Qvn(O, z) n(O, z) = \frac{1}{3} D(z) n(O_2, z), \quad (20)$$

the factor  $1/3$  having been introduced to express roughly the effect of no diurnal variation of the degree of dissociation; the reason for this lies in the fact that  $Qv$  is very small.

#### 4. The processes of loss of gas-kinetic energy and the thermal equilibrium in the lower ionosphere

In the lower ionosphere molecules play an important part in the loss of gas-kinetic energy. Band emissions are sensitive to a possible deviation from the thermodynamical equilibrium. As is well known, the temperatures of the upper atmosphere derived by different method of measurement are often of different values: from the intensity distribution of the rotational spectra of polar aurorae, Vegard (19) has proposed, for the negative nitrogen band  $4278\text{\AA}$ , that the auroral temperature comes out to be of the order  $343^\circ\text{K}$ ; from the intensity-distribution of the vibrational spectra in the ultraviolet negative and positive nitrogen bands, Rosseland and Steenshould (20) have derived the corresponding temperature to be very high, something between  $2000$  and  $3000^\circ\text{K}$ . These facts indicate that the condition under which the bands are emitted may differ appreciably from those of the thermodynamical equilibrium. Making allowance for the fact that the density in the ionosphere is extremely tenuous, there would be such spectral evidences as to the band emissions even in the ordinary ionosphere. It should be borne in mind that such deviation from the thermodynamical equilibrium can possibly exist in the ordinary ionosphere.

The rotational level of molecular oxygen and nitrogen can be easily excited by impacts of gas-particles; however, the reciprocal process of this, i. e. the exchange of energy by collisions from rotational to translational levels, also easily takes place, and the rate of the spontaneous emission of rotational energy is very small; such rotational excitations cannot play any important role in the process of the loss of gas-kinetic energy. The detailed discussion on this point has been made by Spitzer: it may be shown after a reasonably good approximation that the total energy radiated from the rotational excited states as a loss of gas-kinetic energy is always less than those emitted from molecules in which Boltzmann's distribution among the excited rotational states is established by collisions.

But emission from vibrationa llevels merits some detailed treatments. Molecular oxygen has first vibrational level lying at  $0.1950\text{ eV}$  above the zeroth, and in the case of nitrogen the first vibrational level lies at  $0.292\text{ eV}$  above the zeroth (22). Let these values be denoted by  $\varepsilon_r(\text{O}_2)$ , and  $\varepsilon_r(\text{N}_2)$  respectively. The values of the Einstein coefficients of the quadrupole emissions for oxygen  $\delta(\text{O}_2)$  and for nitrogen  $\delta(\text{N}_2)$  are unknown, but they are certainly very low, as quadrupole transitions are involved. If it were assumed that the moment-lengths of the quadrupole appropriate to molecular

oxygen and nitrogen were  $1.204\text{\AA}$  and  $1.094\text{\AA}$ , respectively, it would seem likely that the rates of the emission  $\delta(\text{O}_2) \simeq 7.04 \times 10^{-4} \text{ sec}^{-1}$  for  $h\nu$  of  $0.195 \text{ eV}$ , and  $\delta(\text{N}_2) \simeq 3.65 \times 10^{-3} \text{ sec}^{-1}$  for  $h\nu$  of  $0.292 \text{ eV}$ . These values may be adopted without risk of serious error (23).

The probability of transformation from translational to vibrational energy by collisions among gas-particles has been calculated by Jackson and Mott(24). To treat the ionospheric problems conveniently, the rate of such excitation in the  $0 \rightarrow 1$  transition, which alone needs to be taken into account, will be modified, making allowance for that the gas-particles have Maxwellian velocity distribution appropriate to some temperature  $T$ , to the following forms for each type of collision;

$$W_1(\text{O}_2|\text{O}) \simeq 4.31 \times 10^{-6} T^{1/6} \exp(-2.201 \times 10^2/T^{1/3}), \quad (21)$$

and

$$W_1(\text{N}_2|\text{O}) \simeq 1.21 \times 10^{-5} T^{1/6} \exp(-2.835 \times 10^2/T^{1/3}), \quad (22)$$

where the suffix 1 refers to the excitation of the first vibrational level of the molecules considered, and the designations  $(\text{O}_2|\text{O})$ , and  $(\text{N}_2|\text{O})$  are used to refer to the type of encounter: the formers within the round brackets represent the excited molecules, while the latters represent the colliding particles.

In the remainder of this paper it will be assumed that the velocities of the particles in the atmosphere are distributed according to Maxwell's law, and the parameter  $T$  in that law will be referred to as the kinetic temperature of the atmosphere.

The transfer of the relative kinetic energy between two molecules on impact to their vibrational energies is impossible unless the relative kinetic energy is so large that it can excite the vibrational states of both molecules together; this transfer is not necessary to be considered in the lower ionosphere.

The excitations to higher vibrational levels need not be taken into account in the process of the loss of gas-kinetic energy in the lower ionosphere. The excited vibrational energy can also reciprocally convert into translational energy by collisions. The rate of such de-excitation is of the same order as that of the excitation, because an encounter with a sufficient high speed is generally necessary to transform between vibrational and translational energy. Under the ionospheric conditions, the rates of the spontaneous quadrupole emissions for the vibrational energy are greater than those of exchange from vibrational to translational energy by collisions, viz.

$$\delta(\text{O}_2) \gg W_1'(\text{O}_2|\text{O}) n(\text{O}, z), \quad (23)$$

and

$$\delta(\text{N}_2) \gg W_1'(\text{N}_2|\text{O}) n(\text{O}, z), \quad (24)$$

where the prime refers to the de-excitations.

Hence, when the vibrational levels of a molecule are excited by the impacts of

gas-particle, a greater part of the excited vibrational energies are emitted in the form of radiation, because the atmosphere is transparent so that the radiation can freely escape out of the atmosphere. If the excitations to higher states can be neglected, the loss of gas-kinetic energy in this case is described by the following form:

$$\epsilon W n_m n_0 \frac{\delta n_m'}{\delta n_m' + W' n_m' n_0}, \quad (25)$$

where  $\epsilon$  is the excitation energy,  $W$  the rate of excitation by impacts,  $W'$  the rate of de-excitation,  $\delta$  the rate of spontaneous emission for the excited state,  $n_m$  the particle concentration of molecule in the ground levels,  $n_m'$  that of the excited state, and  $n_0$  that of the colliding gas-particles. According to (23) and (24), it may be said that the vibrational energies excited by the impacts of gas-particles are released in the forms of radiation. Thus, to a good approximation, the loss of gas-kinetic energy can be expressed as

$$\epsilon W n_m n_0. \quad (26)$$

In the case of the radiative association of oxygen, the relative kinetic energy between the two oxygen atoms turns out to be the radiation, and hence the atmosphere is cooled down, as was proposed by Moses and Wu. However, if this were a significant effect, by the condition of the thermal equilibrium between the heat gain due to the photodissociation of molecular oxygen:

$$\frac{1}{3} D(z) n(\text{O}_2, z) = Q v n(\text{O}, z) n(\text{O}, z),$$

and the heat loss:

$$\frac{1}{3} \epsilon D(z) n(\text{O}_2, z) = \epsilon_0 Q v n(\text{O}, z) n(\text{O}, z),$$

in which  $\epsilon_0$  means the average energy to be lost in one reaction, we are led to the result that

$$\epsilon(z) = \epsilon_0.$$

Considering that  $\epsilon_0 \simeq \frac{3}{2} KT$ , and that the minimum value of  $\epsilon(z)$  is about  $0.5 \times 10^{-2}$  erg, as can be seen from Fig. 3, the gas-kinetic temperature of the lower ionosphere would have been much higher than 2500°K, contrary to the observations. As is generally the case, when thermal equilibrium is established by the photodissociation with its reciprocal process, even if it is not completely reciprocal, the gas-kinetic temperature of the atmosphere settles to a great value near the temperature of the incident radiation.

From the discussion mentioned above, we take the following procedure as the most effective process responsible for the loss of gas-kinetic energy in the lower ionosphere: a molecule is excited vibrationally by impact with an atomic oxygen

with sufficiently large kinetic energy enough for the excitation. The excited vibrational energy is lost in the form of radiation, since the atmosphere is almost transparent for such infrared radiation.

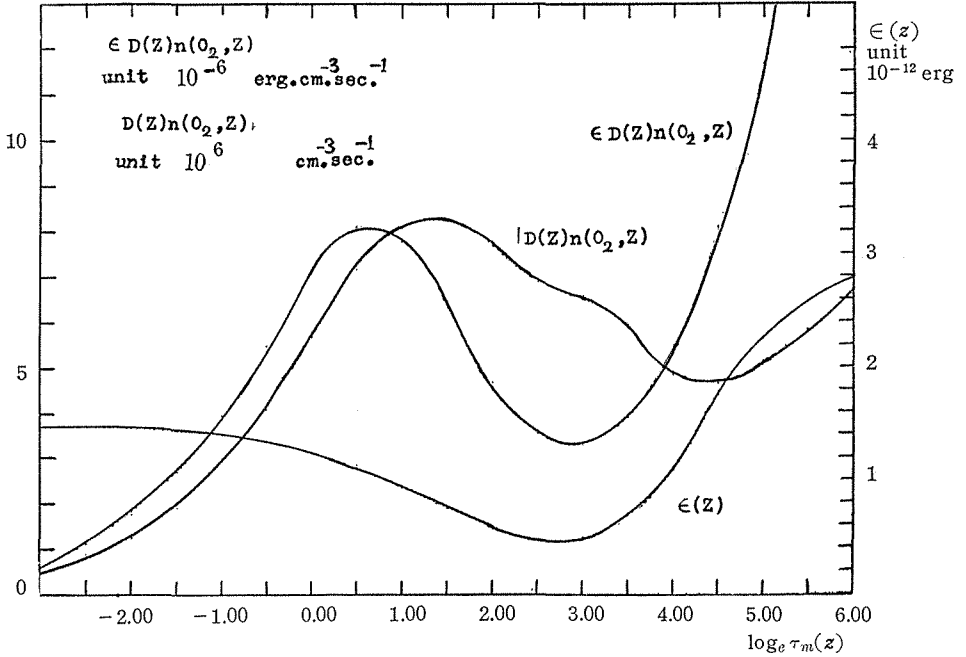


Fig. 3.  $\epsilon(z)$ ,  $D(z)n(O_2, z)$  and  $\epsilon D(z)n(O_2, z)$  plotted against  $\log_e \tau_m(z)$ .

It follows that using (21), (22) and (26), the rate of the loss of gas-kinetic energy in the lower ionosphere  $L_V(z)$  is expressed by

$$L_V(z) = \epsilon_V(O_2)W_1(O_2|O)n(O_2, z)n(O, z) + \epsilon_V(N_2)W_1(N_2|O)n(N_2, z)n(O, z), \quad (27)$$

where the designation  $L_V$  expresses the loss by the vibrational excitations; and  $(z)$  specifies explicitly that it is the function of the height.  $L_V$  does not contain  $\delta$ ; only some accurate information on  $W_1$  is required in order to get the loss of gas-kinetic energy.

We shall next turn to the comparative study of the two terms on the right-hand side of (27). The excitation energy for nitrogen is about 1.5 times that for oxygen. Below the dissociative transition layer of oxygen, near above 100 km, it may be supposed that the relative proportions of molecular oxygen to molecular nitrogen is equal to 1/4, and that the temperature is as low as 300°K. Under these conditions, it can be concluded from the difference between the excitation energies that oxygen

molecules are decidedly effective. In regard to the colliding particles, it can be said that the atomic oxygen is effective, because molecules are excluded from consideration. Above the dissociative transition layer, the particle concentration of molecular oxygen decreases rapidly owing to the strong photodissociation and the very slow association of oxygen; molecular nitrogen is obliged to play the role of excited molecule. Thus, the second term of (27) is more effective than the first above the level where

$$\frac{n(\text{O}_2, z)}{n(\text{N}_2, z)} \simeq 4.22 \exp(-6.34 \times 10/T^{1/3}), \quad (28)$$

as can be seen from (21) and (22). Table 4 gives the values of  $4.22 \exp(-6.34 \times 10/T^{1/3})$  in relation to temperature  $T$ . It is difficult, as yet, to determine where this critical level is to be placed because of the uncertainty of the temperature and the degree of the dissociation. However, as is implied, since the temperature is about  $700^\circ\text{K}$  in this critical level, it will be seen from Table 4 that  $n(\text{N}_2, z)$  there should become about 300 times as great as  $n(\text{O}_2, z)$ . It may be seen from the inspection of Fig. 1 that the absorption of the dissociating radiation begins sharply at certain level, and hence both  $\epsilon D(z)$  and  $D(z)$  may be regarded to be nearly constant above this level; the lower ionosphere may be separated into the two regions in this respect: one is the weak absorption region, and the other is the strongly absorbing region. On the other hand, the region in which the first term of (27) is greater than the second will be designated as the region A.

The region B will be designated as the region where the second term is effective. Since molecular oxygen is considerably abundant in the region A, the region A, of course, may also correspond to the strong absorption region; in the region B, molecular oxygen is less populated, hence the region B may correspond to the region of weak absorption.

Thus, the thermal equilibrium in the lower ionosphere can be expressed by

$$\frac{1}{3} \epsilon D(z) n(\text{O}_2, z) = L_V(z), \quad (29)$$

where the factor  $1/3$  refers to taking into account that the diurnal temperature-variation in the lower ionosphere is of a negligible amount, because  $L_V(z)$  is very small compared with the energy stored up in the atmosphere.

Table 4

$T$	$4.22 \times \exp(-6.34 \times 10/T^{1/3})$
200	$7.63 \times 10^{-5}$
250	$1.70 \times 10^{-4}$
300	$3.04 \times 10^{-4}$
350	$4.89 \times 10^{-4}$
400	$7.33 \times 10^{-4}$
450	$1.00 \times 10^{-3}$
500	$1.36 \times 10^{-3}$
550	$1.76 \times 10^{-3}$
600	$2.19 \times 10^{-3}$
650	$2.68 \times 10^{-3}$
700	$3.22 \times 10^{-3}$
750	$3.79 \times 10^{-3}$
800	$4.27 \times 10^{-3}$
860	$5.00 \times 10^{-3}$

### 5. Derivation of the particle concentrations and of the temperature in the lower ionosphere

It is relevant to suppose that in the lower ionosphere the vertical distributions of the particle concentrations of various constituents, and of the temperature should be determined simultaneously so as to satisfy the dissociative equilibrium of oxygen, the thermal equilibrium, as well as the mechanical equilibrium of the atmosphere. These equilibria have been reviewed in the previous sections.

Let us begin with the problem of the region A, which practically corresponds to the E and D regions of the ionosphere, as is easily seen from the discussion of the previous section. In this region the equation for the thermal equilibrium can be represented by

$$\frac{1}{3} \varepsilon D(z) n(\text{O}_2, z) = \varepsilon_V(\text{O}_2) W_1(\text{O}_2|\text{O}) n(\text{O}_2, z) n(\text{O}, z). \quad (30)$$

The equation for the dissociative equilibrium has been given by (20). As  $D(z)$  and  $\varepsilon D(z)$  cannot be expressed in terms of simple analytical functions, computation was performed by the numerical method.

Equations (30) and (20) lead to

$$n(\text{O}, z) = \frac{1}{3} \varepsilon D(z) / \varepsilon_V(\text{O}_2) W_1(\text{O}_2|\text{O}), \quad (31)$$

$$n(\text{O}_2, z) = \frac{1}{3} Qv \cdot \varepsilon D(z)^2 / D(z) \cdot \varepsilon_V(\text{O}_2)^2 \cdot W_1(\text{O}_2|\text{O})^2, \quad (32)$$

and

$$\chi(z) = Qv \varepsilon D(z) / D(z) \cdot \varepsilon_V(\text{O}_2) W_1(\text{O}_2|\text{O}). \quad (33)$$

Substituting (3), (4), (31), and (33) into the equation for the mechanical equilibrium of the atmosphere (5), and neglecting the unimportant term  $\frac{d}{dz} \log_e T(z)$ \*, equation (5) becomes

$$\begin{aligned} \frac{d}{dz} \log_e \varepsilon D(z) - \frac{d}{dz} \log_e W_1(\text{O}_2|\text{O}) - \frac{(1+\eta) \chi(z)}{1 + \frac{\eta}{2} + 2(1+\eta) \chi(z)} \frac{d}{dz} \log_e D(z) \\ = -\frac{1}{\mathfrak{H}_a} \frac{\left(1 + \frac{7}{8} \eta\right) (1 + 2\chi(z))}{1 + \frac{\eta}{2} + 2(1+\eta) \chi(z)}, \end{aligned} \quad (34)$$

where  $\mathfrak{H}_a = \frac{KT}{M_0 g}$ , namely, the scale height associated with atomic oxygen in a uniform atmosphere with temperature  $T$ .

The variation of the particle concentration of molecular oxygen with height may

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\* A rigorous treatment retaining this term is possible. The calculation runs parallel to the followings in spite of its greater complexity.



be described, taking equations (32) and (34) into account, by

$$\begin{aligned} \frac{d}{dz} \log_e n(\text{O}_2, z) = & -\frac{1}{\mathfrak{H}_m} \frac{\left(1 + \frac{7}{8} \eta\right)(1 + 2\chi(z))}{1 + \frac{\eta}{2} + 2(1 + \eta) \chi(z)} \\ & - \frac{1 + \frac{\eta}{2}}{1 + \frac{\eta}{2} + 2(1 + \eta) \chi(z)} \frac{d}{dz} \log_e D(z), \end{aligned} \quad (35)$$

where  $\mathfrak{H}_m = \frac{KT}{2M_0g}$ , namely, the scale height associated with molecular oxygen in a uniform atmosphere with temperature  $T$ . The vertical distribution of the particle concentration of molecular oxygen represented by (35) differs, of course, from that expressed by

$$\frac{d}{dz} \log_e n(\text{O}_2, z) = -\frac{1}{\mathfrak{H}_m}. \quad (36)$$

For convenience, we shall abbreviate the right-hand side of (35) as follows:

$$\frac{d}{dz} \log_e n(\text{O}_2, z) = -\frac{1}{H_m(z)}. \quad (37)$$

If we put

$$a(z) = \frac{\mathfrak{H}_m}{H_m},$$

it measures the deviation of the vertical distribution of molecular oxygen from that of a uniform atmosphere without dissociation.  $H_m$  is the local scale height expressing the vertical distribution of molecular oxygen. As can be seen from equation (35), the photodissociation exerts a strong influence upon the vertical distribution of molecular oxygen; it gives rise to such a deviation as is mentioned above.

Using the local scale height  $H_m$ , it approximately yields from the definitions (6) and (7) that

$$\frac{d}{dz} \log_e \tau_m(z) = -\frac{1}{H_m}, \quad (38)$$

and

$$\frac{d}{dz} \log_e \sigma_m(z) = -\frac{1}{H_m}, \quad (39)$$

respectively. Therefore the relations (14) and (15) can be written as:

$$\frac{d}{dz} \log_e D(z) = \frac{f'}{H_m}, \quad (40)$$

and

$$\frac{d}{dz} \log_e \varepsilon D(z) = \frac{f}{H_m} \quad (41)$$

respectively. Making substitution of (40), (41) and (37) into equation (35),  $a(z)$  can be transformed to

$$a(z) = \frac{\left(1 + \frac{7}{8}\eta\right)(1 + 2\chi(z))}{(1-f')\left(1 + \frac{\eta}{2}\right) + 2(1+\eta)\chi(z)}. \quad (42)$$

The vertical distribution of atomic oxygen is deduced by the similar process to that described above, viz.

$$\frac{d}{dz} \log_e n(\text{O}, z) = -\frac{1-f'}{2H_m}. \quad (43)$$

Let us turn to the discussion about  $a(z)$ . If we consider the case when the gases in the lower ionosphere consist of nitrogen and oxygen in the same proportions as the atmosphere at the ground,  $\eta$  is constant and equal to 4, and substituting this into (42), we are led to

$$a(z) = \frac{c(1+2\chi)}{A+\chi}, \quad (44)$$

where  $c = \frac{9}{20}$  and  $A = \frac{3}{10}(1-f')$ . If we consider the case of pure oxygen ionosphere,  $\eta$  vanishes, and (42) becomes

$$a(z) = \frac{c'(1+2\chi)}{A'+\chi'}, \quad (45)$$

where  $c' = \frac{1}{2}$  and  $A' = \frac{1}{2}(1-f')$ . The difference of  $a(z)$  between these two cases is small, so that it can be seen that  $a(z)$  depends little upon  $\eta$ , but the dissociative absorption strongly affects  $a(z)$ , especially in the region where  $\chi$  is small, that is, the particle concentration of atomic oxygen is greater than that of molecular oxygen. Hereafter, we will consider only the case  $\eta = 4$ .

On substitution of (40), (41) and (42) into equation (34), we obtain the equation for the vertical distribution of temperature, viz.

$$\frac{d}{dz} \log_e W_1(\text{O}_2|\text{O}) = \frac{a(z)}{\delta_a} (1-f'+2f), \quad (46)$$

which, on making use of (21), and regarding the slowly varying term  $T^{1/6}$  as a constant, can be written as:

$$\frac{B(\text{O}_2)}{3} \frac{1}{T^{1/3}} \frac{dT}{dz} = \frac{M_0 g}{K} a(z)(1-f'+2f), \quad (47)$$

where  $B(\text{O}_2)$  is  $2.201 \times 10^2$ . Since  $a(z)$  is a slowly varying quantity, the integration of (47) can be performed approximately as

$$\frac{B(\text{O}_2) K}{2M_0 g b(z)} (T^{2/3} - T_i^{2/3}) = z - z_i, \quad (48)$$

where

$$b(z) = a(z)(1-f'+2f), \quad (49)$$

and  $T_i$  is the temperature at an initial level  $z_i$ .

Provided we know the particle concentrations of molecular and atomic oxygen at a reference level, then by aid of the relations (37) and (43), we can determine those at any height measured in unit of  $H_m$  from the reference level; our knowledge is still in the unit of scale height and not in kilometers.

Now, we can derive the variation of  $\chi(z)$  with respect to  $\sigma_m(z)$ :  $\sigma_m(z)$  may be regarded as  $n(O_2, z)H_m(z)$  approximately, viz.

$$\sigma_m(z) = H_m(z) n(O_2, z), \quad (50)$$

which, taking account of  $H_m(z) = \frac{\mathfrak{H}_m(z)}{a(z)}$  and (44), can be written as:

$$\sigma_m(z)a(z) = n(O_2, z) \mathfrak{H}_m(z). \quad (51)$$

Then the equation for  $\mathfrak{H}(z)$  can be obtained, viz.

$$\chi^2 + (A - 2c\lambda_m)\chi - c\lambda_m = 0, \quad (52)$$

where

$$\lambda_m = \frac{\sigma_m(z)}{\mathfrak{H}_m n(O, z)}. \quad (53)$$

The solution of (52) can be obtained as follows:

$$\chi = c\lambda_m - \frac{A}{2} + \left[ \left( c\lambda_m - \frac{A}{2} \right)^2 + c\lambda_m \right]^{\frac{1}{2}}. \quad (54)$$

It can be easily shown that to adopt  $\log_e \sigma_m(z)$  or  $\log_e \tau_m(z)$  as a measure of height, as have been made in § 3, has the same meaning as to measure the height in the unit of  $H_m$ . Hence, it will readily be made to express any value of  $\sigma_m(z)$  or  $\tau_m(z)$  in terms of height measured in the unit of  $H_m$  from the reference level.

To push our calculation further we must determine where the reference level is to be taken, and must fix  $\log_e \tau_m(z)$ . For this purpose, let us discuss a few reliable observational facts.

(i) Attempts for the determination of the electron collision frequency in the E layer were made by Baily and Martyn (25), and their calculations on the interaction of radio waves led to the value  $2 \times 10^5$  per sec. The cross section of elastic collision of slow electron with atomic oxygen was evaluated by Yamanouchi (26) as  $2.41 \times 10^{-15} \text{cm}^2$ . Assuming the same order of magnitude for both the molecular oxygen and nitrogen,  $NT^{1/2}$  in this level becomes  $1.33 \times 10^{14} \text{cm}^{-3} \text{deg}^{\frac{1}{2}}$ , corresponding to  $\nu = 2.0 \times 10^5$  per sec.

(ii) The sporadic E ionization appears frequently about 10 km below the maximum level of the E layer; it is well known that they are in a non-uniform

state, and have the semi-transparent property for radio waves. G. J. Phillips (12) deduced from the fading observations by using the method of S. N. Mitra (27) that at a height of about 100 km the E layer had an irregular structure in which the smallest size of irregularity is about 200 m. The maximum level of the E layer is supposed to be at a height of 110 km or slightly above. Hence, the irregularity deduced by Phillips may be considered to correspond to that of the sporadic E ionization; the minimum dimension of irregularity of the sporadic E ionization may be supposed to be 200 m. From this fact it can be said that turbulent motions of the air are occurring in the region of the sporadic E. The velocity of the eddy system with the minimum dimension is nearly equal to the mean velocity of the gas-kinetic motion in the surrounding gases. If the state of turbulence persists to a dimension of 200 m., the Reynolds number of the eddies associated with this dimension must be as great as about 1000. On the other hand, the Reynolds number may be described by

$$R = 2.88 \frac{VL}{V_0} \pi \sigma^2 N(z),$$

where  $V$  and  $L$  are the velocity and the dimension of the eddy system under consideration respectively,  $\pi \sigma^2$  the mean cross section for the elastic collision of gas particles and  $V_0$  the mean velocity of gas particles in the surrounding. If, for example,  $R$  were taken to be 1000, which might well be the case, and the value of  $\sigma^2$  were assumed to be  $3.0 \times 10^{-16} \text{cm}^2$ , in the case in which the minimum dimension  $L$  equals to  $2.0 \times 10^4 \text{cm}$ , regarding  $V$  as  $V_0$ , the total particle concentration  $N(z)$  at the level where the sporadic E ionization is occurring would be deduced to be  $1.84 \times 10^{13} \text{cm}^{-3}$ . It would be concluded, therefore, that the level of this total particle concentration should be situated about 10 km below the maximum level of the E layer.

(iii) It can be shown that the formation of the D layer is attributed to the ionization of atomic oxygen by the scattered radiation produced in the ionosphere with the aid of the solar visible radiation, whose energy of the quantum is 9.48 eV or in wavelength 1302 Å. As is well known, the absorption coefficient appropriate to nitrogen near this wavelength is so small that the field of this scattered radiation is scarcely affected by absorption of nitrogen. But the scattering will be cut down by absorption of molecular oxygen in the lower boundary of the ionosphere. Thus, the field of this scattered radiation has a fairly sharp boundary. Because the absorption coefficient of molecular oxygen for this radiation is supposed to be about  $3.47 \times 10^{-19} \text{cm}^2$ , it corresponds to the level defined by

$$\tau_{1302\text{Å}} = 3.47 \times 10^{-19} \sigma_m(z) \simeq 1, \quad \text{i. e. } \sigma_m(z) \simeq 2.88 \times 10^{18} \text{cm}^{-2},$$

or  $\log_e \tau_m(z) \simeq 4.00$ . It may be supposed that the maximum level of the D layer is

situated at this level. Therefore,  $\log_e \tau_m(z) \simeq 4.00$  must be about 15 or 20 km below the maximum level of the E layer.

No serious inconsistencies exist among these three facts. Therefore, we shall take the level of  $NT^{1/2} \simeq 1.33 \times 10^{14} \text{ cm}^{-3} \text{ deg}^{1/2}$  as the reference level, whose height will be denoted by  $z_0$ .

Now, we shall evaluate  $\log_e \tau_m(z)$  at the reference level, and we shall determine the physical state at that level. For our purpose, we make use of equation (54). If we rewrite  $c\lambda_m(z)$  of (53) contained in (54), by using (31), it follows that

$$c\lambda_m(z) = \left( \frac{3c\varepsilon_V(O_2) M_0 g}{K} \right) \cdot \frac{\sigma_m(z)}{\varepsilon D(z)} \frac{W_1(O_2|O)}{T}, \quad (55)$$

where  $\frac{\sigma_m(z)}{\varepsilon D(z)}$  is a known function for any value of  $\log_e \tau_m(z)$ ,  $\frac{W_1(O_2|O)}{T}$  is a function of temperature only, and the remaining quantities are constants. Thus,  $c\lambda_m(z)$  depends on  $T$  with respect to the parameter of  $\log_e \tau_m(z)$ .  $\frac{A}{2}$  in (54) is, as is defined by (44),

$$\frac{A}{2} = \frac{1 + \frac{\eta}{2}}{4(1 + \eta)} (1 - f'), \quad (56)$$

where  $f'$  is known for any value of  $\log_e \tau_m(z)$ . Thus the right-hand side of equation (54) depends on  $T$  by taking  $\log_e \tau_m(z)$  as a parameter. On the other hand, the left-hand side of equation (54),  $\chi(z)$ , becomes, when (31) and (3) are taken into account,

$$\chi(z) = \frac{3\varepsilon_V(O_2) NT^{1/2}}{1 + \eta} \frac{1}{\varepsilon D(z)} \frac{W_1(O_2|O)}{T^{1/2}} - \frac{1 + \frac{\eta}{2}}{1 + \eta}. \quad (57)$$

This is, also, a function of temperature for each value of  $\log_e \tau_m(z)$ .

And  $NT^{1/2} = 1.33 \times 10^{14}$ . In general, assuming  $NT^{1/2}$  to be an appropriate constant, except for the non-conceivable value in the concerning region of the atmosphere,  $T$  and  $\chi$  for any  $\log_e \tau_m(z)$  can be found from the rewritten equation of (54): inserting (55), (56) and (57) into (54) it can be reduced to

$$\begin{aligned} \frac{3\varepsilon_V(O_2) NT^{1/2}}{1 + \eta} \frac{1}{\varepsilon D(z)} \frac{W_1(O_2|O)}{T^{1/2}} - \frac{1 + \frac{\eta}{2}}{1 + \eta} &= \frac{3c\varepsilon_V(O_2) M_0 g}{K} \frac{\sigma_m(z)}{\varepsilon D(z)} \frac{W_1(O_2|O)}{T} \\ - \frac{1 + \frac{\eta}{2}}{4(1 + \eta)} + \left[ \left( \frac{3c\varepsilon_V(O_2) M_0 g}{K} \frac{\sigma_m(z)}{\varepsilon D(z)} \frac{W_1(O_2|O)}{T} - \frac{1 + \frac{\eta}{2}}{4(1 + \eta)} \right)^2 \right. \\ &\quad \left. + \frac{3c\varepsilon_V(O_2) M_0 g}{K} \frac{\sigma_m(z)}{\varepsilon D(z)} \frac{W_1(O_2|O)}{T} \right]^{1/2}. \end{aligned} \quad (58)$$

From (58),  $T$  corresponding to any  $\log_e \tau_m(z)$  can be found.  $\chi$  for any obtained

value of  $T$  can be determined by using (57). Thus, under the condition that  $NT^{1/2} \approx 1.33 \times 10^{14}$ ,  $T$ -diagram with  $\log_e \tau_m(z)$  as a parameter is given in Fig. 4. A similar treatment is, of course, possible for the case when  $N$  is a constant.

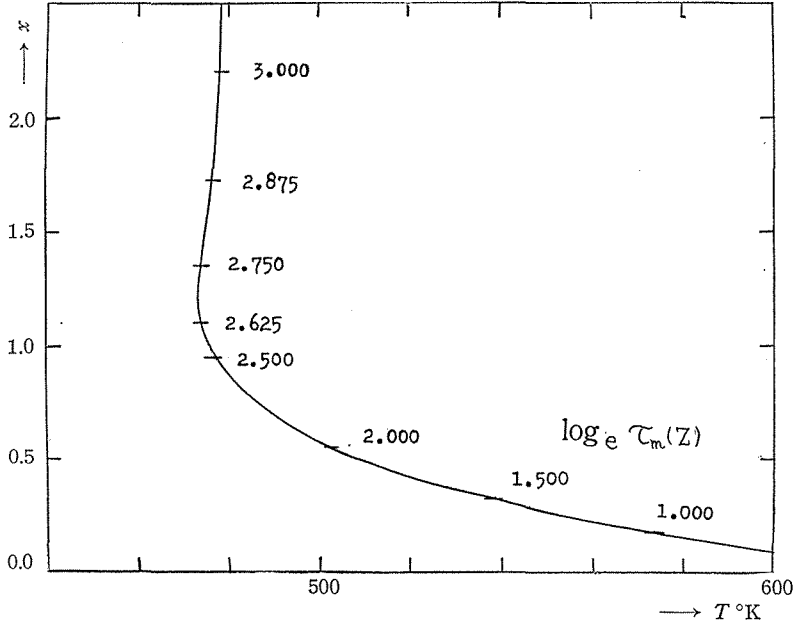


Fig. 4. The solution of (58) with  $\log_e \tau_m(z)$  as a parameter.

Now, we must determine to what place in the  $T-\chi$  diagram the reference level is to be located. To do this, selection is made so that the results of our computation are self-consistent: if we take a  $(T-\chi)$  pair corresponding to one of  $\log_e \tau_m(z)$ ,  $n(O, z)$  and  $n(O_2, z)$  appropriate for it can easily be obtained from (3) and (31). For the other  $\log_e \tau_m(z)$  than the selected  $\log_e \tau_m(z)$ ,  $n(O, z)$  and  $n(O_2, z)$  are found by the relations (37) and (43). Making allowance for (42), we examine whether the condition is satisfied for every value of  $\log_e \tau_m(z)$ . Such successive approximations can readily approach to a self-consistent selection of  $\log_e \tau_m(z)$ . The result is that  $\log_e \tau_m(z) \approx 2.625$ . It may be

Table 5

$\log_e \tau_m(z_0) = 2.625,$	$A = -0.0363,$
$\log_{10} D(z_0) = \bar{6}.9365,$	$n(O, z_0) = 7.28 \times 10^{11},$
$\log_{10} \epsilon D(z_0) = \bar{1}8.6438,$	$n(O_2, z_0) = 7.88 \times 10^{11},$
$\log_{10} \sigma_m(z_0) = 17.8728,$	$N(z_0) = 6.13 \times 10^{12},$
$T(z_0) = 474 K,$	$a(z_0) = 1.37,$
$\chi(z_0) = 1.10,$	$\bar{\Phi}_m(z_0) = 12.56 \text{ km.},$
$f' = 1.12,$	$H_m(z_0) = 9.16 \text{ km.},$

significant in view of energy consideration that the point  $\log_e \tau_m(z) \approx 2.625$  in Fig. 4 expresses the minimum of temperature.

The physical state at the reference level may be expressed by the following quantities. Be the difference of  $\log_e \tau_m(z)$  between arbitrary adjacent levels, and let  $\Delta z$  be the difference of height between these levels. Numerical computation is made with the step  $\Delta \log_e \tau_m(z) = 0.5$ . Equation (38) can be approximated as

$$\frac{\Delta \log_e \tau_m(z)}{\Delta z} = -\frac{1}{\bar{H}_m}.$$

When  $\Delta \log_e n(z)$  is taken sufficiently small,  $\bar{H}_m$  may be considerably accurately replaced by the mean value of  $H_m$  between two levels concerned. With this approximation, (37) becomes

$$\frac{\Delta \log_e n(O_2, z)}{\Delta \log_e \tau_m(z)} = 1.$$

It follows then that

$$n(O_2, z + \Delta z) = n(O_2, z) e^{\Delta \log_e \tau_m(z)},$$

and it can, in general, be said that

$$n(O_2, z) = n(O_2, z_0) \exp \left[ \log_e \tau_m(z) - \log_e \tau_m(z_0) \right].$$

Similarly, it yields from (43) that

$$\frac{\Delta \log_e n(O, z)}{\Delta \log_e \tau_m(z)} = \frac{1 - \bar{f}'}{2},$$

and

$$n(O, z) = n(O, z_0) e^{\sum \frac{1 - \bar{f}'}{2} \Delta \log_e \tau_m(z)}, \quad (60)$$

where  $\bar{f}'$  stands for the mean value of  $f'$  in each interval, and  $\sum$  replaces the integration for each interval.

Making use of (58) and (59)  $n(O, z)$  and  $n(O_2, z)$  at each value of  $\log_e \tau_m(z)$  can be derived. As is seen from (59),  $n(O, z)$  exhibits a maximum or minimum at the place where  $1 - f' = 0$ . Inspection of Fig. 2 shows that at the point where  $\log_e \tau_m(z) \simeq 1.50$ ,  $1 - f'$  steps into the negative side from the positive side, and at the point where  $\log_e \tau_m(z) \simeq 4.50$ ,  $1 - f'$  changes sign. The former corresponds to a maximum, and the latter corresponds to a minimum.

$\chi(z)$  at each level can be obtained. Further calculation of  $a(z)$  by (44) and  $b(z)$  by (49) can be made.

From the known value of  $n(O, z)$  and the relation (34), each value of  $W_1(O_2|O)$  is found.  $T(z)$  at each level can readily be seen. If (46) be transformed into

$$\frac{K\bar{T}(z)}{M_0 g \bar{b}(z)} \Delta \log_e W_1(O_2|O) = \Delta z, \quad (61)$$

where  $\bar{T}(z)$  and  $\bar{b}(z)$  are the mean values of  $T(z)$  and  $b(z)$  between the two levels

considered, the difference of height between any adjacent levels can be found. The local scale height  $H = \frac{KT}{Mg}$  as a gradient of the total particle concentration is known at each level. The computations described above are given in Table 6 according to sequence of reduction:

$Qv$  can be obtained from the dissociative equilibrium (20), with the result:

$$Qv = 4.28 \times 10^{-18} \text{ cm}^3 \text{ sec}^{-1}, \quad (62)$$

which is in general in agreement with Bates (14), who considered several cases other than oxygen.

Now, as is seen in Fig. 2,  $f$  and  $f'$  become very small when  $\log_e \tau_m(z) < -1.50$ : the gradients of  $\epsilon D(z)$  and  $D(z)$  with height become then very small. This means that the dissociative absorption of molecular oxygen does not appreciably affect the dissociating radiation field in the region where  $\log_e \tau_m(z) < -1.50$ . On the other hand, the particle concentration of molecular oxygen decreases and the temperature rises to about 800°K. Then molecular nitrogen becomes to have a considerable effect upon the loss of gas-kinetic energy. This region corresponds to the region B designated in § 4.

We shall now consider this region.  $\epsilon D(z)$  and  $D(z)$  together can be approximated to the constant values of  $\epsilon D(\infty)$  and  $D(\infty)$  respectively. We must take into account the loss of gas-kinetic energy by molecular nitrogen as well as by molecular oxygen. The equation for thermal equilibrium becomes then

$$\begin{aligned} \frac{1}{3} \epsilon D(\infty) n(\text{O}_2, z) &= \epsilon_{\mathcal{V}}(\text{O}_2) W_1(\text{O}_2|\text{O}) n(\text{O}_2, z) n(\text{O}, z) \\ &+ \epsilon_{\mathcal{V}}(\text{N}_2) W_1(\text{N}_2|\text{O}) n(\text{N}_2, z) n(\text{O}, z), \end{aligned} \quad (63)$$

and the equation for the dissociative equilibrium (20) is

$$\frac{1}{3} D(\infty) n(\text{O}_2, z) = Qv n(\text{O}, z) n(\text{O}, z). \quad (64)$$

Inserting (1) and (64) into (63) it yields that

$$\begin{aligned} \frac{1}{3} \epsilon D(\infty) - \frac{2}{3} \epsilon_{\mathcal{V}}(\text{N}_2) W_1(\text{N}_2|\text{O}) \frac{D(\infty)}{Qv} \\ = \left[ \epsilon_{\mathcal{V}}(\text{O}_2) W_1(\text{O}_2|\text{O}) + 4\epsilon_{\mathcal{V}}(\text{N}_2) W_1(\text{N}_2|\text{O}) \right] n(\text{O}, z). \end{aligned} \quad (65)$$

Comparison between (21) and (22) leads to the conclusion that for any conceivable temperature of the ionosphere,

$$\epsilon_{\mathcal{V}}(\text{O}_2) W_1(\text{O}_2|\text{O}) \ll 4\epsilon_{\mathcal{V}}(\text{N}_2) W_1(\text{N}_2|\text{O}),$$

so that the right-hand side of (65) becomes  $\epsilon_{\mathcal{V}}(\text{O}_2) W_1(\text{O}_2|\text{O}) n(\text{O}, z)$ , and we have



Table 6

$\log_e \tau_m(z)$	$D(z)$ sec <sup>-1</sup>	$\varepsilon D(z)$ erg sec <sup>-1</sup>	$\bar{F}$	$\bar{F}'$	$\log_{10} \frac{n'(O, z)}{n(O, z_0)}$	$\log_{10} \frac{n'(O_2, z)}{n(O_2, z_0)}$	$n(O_2, z)$	$n'(O, z)$	$N(z)$	$x(z)$
-2.50	$1.62 \times 10^{-4}$	$2.34 \times 10^{-16}$	0.0654	0.0285	-0.4768	-2.2257	$4.69 \times 10^9$	$2.43 \times 10^{11}$	$7.52 \times 10^{11}$	0.0193
-2.00	$1.60 \times 10^{-4}$	$2.26 \times 10^{-16}$	0.137	0.129	-0.3716	-2.0086	$7.73 \times 10^9$	$3.10 \times 10^{11}$	$9.68 \times 10^{11}$	0.0249
-1.50	$1.50 \times 10^{-4}$	$2.11 \times 10^{-16}$	0.231	0.211	-0.2770	-1.7914	$1.27 \times 10^{10}$	$3.85 \times 10^{11}$	$1.22 \times 10^{12}$	0.0331
-1.00	$1.35 \times 10^{-4}$	$1.88 \times 10^{-16}$	0.348	0.269	-0.1913	-1.5743	$2.10 \times 10^{10}$	$4.69 \times 10^{11}$	$1.51 \times 10^{12}$	0.0448
-0.50	$1.18 \times 10^{-4}$	$1.58 \times 10^{-16}$	0.468	0.334	-0.1120	-1.3571	$3.46 \times 10^{10}$	$5.63 \times 10^{11}$	$1.86 \times 10^{12}$	0.0615
0.00	$9.99 \times 10^{-5}$	$1.25 \times 10^{-16}$	0.729	0.541	-0.0397	-1.1400	$5.71 \times 10^{10}$	$6.65 \times 10^{11}$	$2.28 \times 10^{12}$	0.0859
0.50	$7.62 \times 10^{-5}$	$8.68 \times 10^{-17}$	1.071	0.757	0.0113	-0.9229	$9.42 \times 10^{10}$	$7.48 \times 10^{11}$	$2.71 \times 10^{12}$	0.126
1.00	$5.22 \times 10^{-5}$	$5.08 \times 10^{-17}$	1.416	0.952	0.03768	-0.7057	$1.55 \times 10^{11}$	$7.95 \times 10^{11}$	$3.16 \times 10^{12}$	0.195
1.50	$3.24 \times 10^{-5}$	$2.50 \times 10^{-17}$	1.627	1.144	0.04292	-0.4885	$2.56 \times 10^{11}$	$8.04 \times 10^{11}$	$3.69 \times 10^{12}$	0.318
2.00	$1.83 \times 10^{-5}$	$1.11 \times 10^{-17}$	1.557	1.221	0.02725	-0.2714	$4.22 \times 10^{11}$	$7.76 \times 10^{11}$	$4.44 \times 10^{12}$	0.544
2.50	$9.94 \times 10^{-6}$	$5.09 \times 10^{-18}$	1.160	1.121	0.00328	-0.05428	$6.96 \times 10^{11}$	$7.34 \times 10^{11}$	$5.68 \times 10^{12}$	0.948
2.625	$8.70 \times 10^{-6}$	$4.40 \times 10^{-18}$	1.160	1.121	0.00000	0.00000	$7.88 \times 10^{11}$	$7.29 \times 10^{11}$	$6.13 \times 10^{12}$	1.082
3.00	$5.68 \times 10^{-6}$	$2.85 \times 10^{-18}$	0.620	1.184	-0.00981	0.1629	$1.15 \times 10^{12}$	$7.12 \times 10^{11}$	$7.87 \times 10^{12}$	1.611
3.50	$3.14 \times 10^{-6}$	$2.09 \times 10^{-18}$	0.372	1.396	-0.02975	0.3800	$1.89 \times 10^{12}$	$6.80 \times 10^{11}$	$1.15 \times 10^{13}$	2.780
4.00	$1.56 \times 10^{-6}$	$1.74 \times 10^{-18}$	0.187	1.112	-0.07273	0.5971	$3.12 \times 10^{12}$	$6.16 \times 10^{11}$	$1.74 \times 10^{13}$	5.060
4.50	$8.96 \times 10^{-7}$	$1.58 \times 10^{-18}$	0.266	0.775	-0.08490	0.8143	$5.14 \times 10^{12}$	$5.99 \times 10^{11}$	$2.75 \times 10^{13}$	8.580
5.00	$6.08 \times 10^{-7}$	$1.38 \times 10^{-18}$	0.431	0.754	-0.06051	1.0314	$8.48 \times 10^{12}$	$6.33 \times 10^{11}$	$4.43 \times 10^{13}$	13.374
5.50	$4.17 \times 10^{-7}$	$1.12 \times 10^{-18}$	0.674	0.747	-0.03384	1.2486	$1.40 \times 10^{13}$	$6.74 \times 10^{11}$	$7.19 \times 10^{13}$	20.736
6.00	$2.87 \times 10^{-7}$	$7.96 \times 10^{-19}$		0.747	-0.00636	1.4657	$2.30 \times 10^{13}$	$7.18 \times 10^{11}$	$1.17 \times 10^{14}$	32.092

$a(z)$	$T(z)$ °K	$D(z) n(O_2, z)$ cm <sup>-5</sup> sec <sup>-1</sup>	$\varepsilon D(z) n(O_2, z)$ erg cm <sup>-5</sup> sec <sup>-1</sup>	$\bar{a}(z)$	$\bar{b}(z)$	$\Delta \log_e W_1(O_2 O)$	$\Delta z$ km	$\Delta z - z_0$ km	$H_m$ km	$H$ km
1.47	845	$7.63 \times 10^5$	$1.10 \times 10^{-6}$	1.52	1.67	0.2751	7.24	48.57	14.48	28.8
1.57	817.5	$1.24 \times 10^6$	$1.74 \times 10^{-6}$	1.64	1.87	0.2864	6.52	41.33	13.04	28.2
1.70	789	$1.91 \times 10^6$	$2.69 \times 10^{-6}$	1.75	2.19	0.3127	5.87	34.81	11.74	27.3
1.79	759	$2.84 \times 10^6$	$3.95 \times 10^{-6}$	1.83	2.61	0.3564	5.38	28.94	10.76	25.9
1.87	727.5	$4.09 \times 10^6$	$5.47 \times 10^{-6}$	1.96	3.15	0.4009	4.78	23.56	9.56	23.7
2.06	693	$5.69 \times 10^6$	$7.13 \times 10^{-6}$	2.24	4.32	0.4819	3.99	18.78	7.98	22.9
2.42	655	$7.17 \times 10^6$	$8.17 \times 10^{-6}$	2.52	6.01	0.5966	3.35	14.79	6.70	22.0
2.62	611.5	$8.10 \times 10^6$	$7.89 \times 10^{-6}$	2.52	7.26	0.7202	3.09	11.44	6.18	19.8
2.42	563	$8.30 \times 10^6$	$6.40 \times 10^{-6}$	2.17	6.75	0.7776	3.30	8.35	6.60	18.0
1.92	518	$7.72 \times 10^6$	$4.67 \times 10^{-6}$	1.69	4.88	0.7234	3.93	5.05	7.86	15.9
1.45	480	$6.92 \times 10^6$	$3.55 \times 10^{-6}$	1.41	3.09	0.1375	3.59	0.00	9.43	14.9
1.36	473.5	$6.87 \times 10^6$	$3.47 \times 10^{-6}$	1.29	2.83	0.4126	3.59	0.00	10.15	14.4
1.21	453	$6.52 \times 10^6$	$3.27 \times 10^{-6}$	1.16	1.23	0.2643	5.08	-3.59	10.15	13.4
1.11	441	$5.94 \times 10^6$	$3.95 \times 10^{-6}$	1.06	0.369	0.0872	5.00	-8.67	11.00	13.2
1.00	438	$4.87 \times 10^6$	$5.42 \times 10^{-6}$	0.976	0.256	0.0655	5.91	-14.17	11.82	12.9
0.950	433.5	$4.62 \times 10^6$	$8.13 \times 10^{-6}$	0.939	0.711	0.1893	6.08	-20.08	12.15	12.8
0.928	427	$5.16 \times 10^6$	$1.17 \times 10^{-5}$	0.923	-1.02	0.2768	6.05	-26.15	12.10	12.5
0.918	415	$5.83 \times 10^6$	$1.56 \times 10^{-5}$	0.915		0.4002	5.88	-32.20	11.76	12.00
0.912	398.5	$6.60 \times 10^6$	$1.83 \times 10^{-5}$	0.915	1.47			-38.08		

$$\frac{1}{3} \varepsilon D(\infty) - \frac{2}{3} \varepsilon_{\nu}(\text{N}_2) W_1(\text{N}_2|\text{O}) \frac{D(\infty)}{Qv} = \varepsilon_{\nu}(\text{O}_2) W_1(\text{O}_2|\text{O}) n(\text{O}, z). \quad (66)$$

The left-hand side of (66) depends on the temperature only through  $W_1(\text{N}_2|\text{O})$ . Defining  $F(T)$  as follows:

$$F(T) = \frac{\varepsilon D(\infty) Qv}{2D(\infty) \varepsilon_{\nu}(\text{N}_2) W_1(\text{N}_2|\text{O})}, \quad (67)$$

which is a function of temperature only, we find, from (66),

$$n(\text{O}, z) = \frac{1}{3} \frac{\varepsilon D(\infty)}{\varepsilon_{\nu}(\text{O}_2) W_1(\text{O}_2|\text{O})} \frac{F(T) - 1}{F(T)}. \quad (68)$$

From (68),  $n(\text{O}, z)$  can be expressed in terms of a function of temperature only.

Since  $n(\text{O}_2, z)$  is negligible compared with  $n(\text{O}, z)$ , (3) and (4) can be replaced by the approximate relations

$$N(z) = \left(1 + \frac{\eta}{2}\right) n(\text{O}, z), \quad (69)$$

and

$$M(z) = \frac{1 + \frac{7}{8} \eta}{1 + \frac{\eta}{2}} M_0. \quad (70)$$

Making transformation of the equation for the hydrostatic equilibrium (5) by the similar treatment to that described in the case of the region A, we obtain

$$G(T) \frac{dT}{dz} = \frac{1 + \frac{7}{8} \eta M_0 g}{1 + \frac{1}{2} \eta K}, \quad (71)$$

where

$$G(T) = \frac{B(\text{N}_2)}{3(F-1) T^{1/3}} + \frac{B(\text{O}_2)}{3T^{1/3}} - 1, \quad (72)$$

and

$$B(\text{N}_2) \simeq 2.835 \times 10^2,$$

$$B(\text{O}_2) \simeq 2.201 \times 10^2.$$

It is possible from (71) to find step of height  $\Delta z$  corresponding to the step of temperature  $\Delta T$  at any height, viz.

$$\frac{K}{M_0 g} \frac{1 + \frac{1}{2} \eta}{1 + \frac{7}{8} \eta} \overline{G(T)} \Delta T = \Delta z, \quad (73)$$

where  $\overline{G(T)}$  is some mean value of  $G(T)$  in each step. Provided that  $\Delta T$  is

taken sufficiently small, we get  $\Delta z$  at each height with good approximation. We have divided into the 5°K step where the temperature gradient is steep, and the 1°K step where the temperature gradient is very small. We determine successively upward from the level  $\log_e \tau_m(z) = -1.50$  so far as the solution is obtainable. Thus, computation was possible up to about 165 km above the reference level. But above 100 km from the reference level we must take into account the gain of gas-kinetic energy by the ionization of the F<sub>1</sub> and F<sub>2</sub> layers. On this account, we must simultaneously make allowance for the problem of the distribution of electron concentration. We shall leave over the problem above 100 km from the reference level without further consideration. The main quantities of the computed results are tabulated in Table 7 according to the sequence of reduction.

Table 7

$T$ °K	$F(T)$	$G(T)$	$W_1(O_2 O)$	$n(O, z)$	$n(O_2, z)$	$N(z)$	$(z-z_0)$ km	$\epsilon D(z) n(O_2, z)$ erg cm <sup>-3</sup> sec <sup>-1</sup>	$D(z) n(O_2, z)$ cm <sup>-3</sup> sec <sup>-1</sup>
780	4.329	10.054	$5.396 \times 10^{-16}$	$3.58 \times 10^{11}$	$1.01 \times 10^{10}$	$1.12 \times 10^{12}$	34.81	$2.36 \times 10^{-6}$	$1.64 \times 10^6$
800	3.342	11.249	$6.628 \times 10^{-16}$	$2.65 \times 10^{11}$	$5.55 \times 10^9$	$8.23 \times 10^{11}$	42.35	$1.29 \times 10^{-6}$	$9.01 \times 10^5$
820	2.601	13.144	$8.128 \times 10^{-16}$	$1.90 \times 10^{11}$	$2.86 \times 10^9$	$5.84 \times 10^{11}$	50.98	$6.67 \times 10^{-7}$	$4.63 \times 10^5$
840	2.041	16.396	$9.782 \times 10^{-16}$	$1.31 \times 10^{11}$	$1.35 \times 10^9$	$3.98 \times 10^{11}$	61.38	$3.15 \times 10^{-7}$	$2.19 \times 10^5$
860	1.614	22.899	$1.185 \times 10^{-15}$	$8.06 \times 10^{10}$	$5.14 \times 10^8$	$2.44 \times 10^{11}$	75.17	$1.20 \times 10^{-7}$	$8.34 \times 10^4$
880	1.286	41.148	$1.407 \times 10^{-15}$	$3.97 \times 10^{10}$	$1.25 \times 10^8$	$1.20 \times 10^{11}$	96.69	$2.91 \times 10^{-8}$	$2.02 \times 10^4$
900	1.038	323.87	$1.676 \times 10^{-15}$	$5.55 \times 10^9$	$2.44 \times 10^6$	$1.67 \times 10^{10}$	165.78	$5.69 \times 10^{-10}$	$3.95 \times 10^2$

We shall evaluate the vertical distribution of the optical depth with respect to the ionizing radiation of atomic oxygen, which is of importance in the consideration of the ionization of the E region. Let  $K_\alpha \simeq 4.5 \times 10^{-18} \text{cm}^2$  be the absorption coefficient of the ionizing radiation for atomic oxygen. Let  $\sigma_\alpha(z)$  be the number of atomic oxygen in a column of unit base above a height  $z$ , viz.

$$\int_z^\infty n(O, z') dz' = \sigma_\alpha(z), \tag{74}$$

and further let  $\tau_\alpha(z)$  be this optical depth. Then, according to definition, it follows that

$$K_\alpha \sigma_\alpha(z) = \tau_\alpha(z). \tag{75}$$

$\tau_\alpha(z)$  and  $n(O, z)$  are reproduced in Fig. 7.  $T(z)$  is expressed in Fig. 5.  $n(O_2, z)$  and  $N(z)$  are indicated in Fig. 6.  $\epsilon D(z) n(O_2, z)$  and  $D(z) n(O_2, z)$  in Fig. 3 are plotted by using the results of computation. They show together to exhibit a flat maximum and a gradual change in the E region.

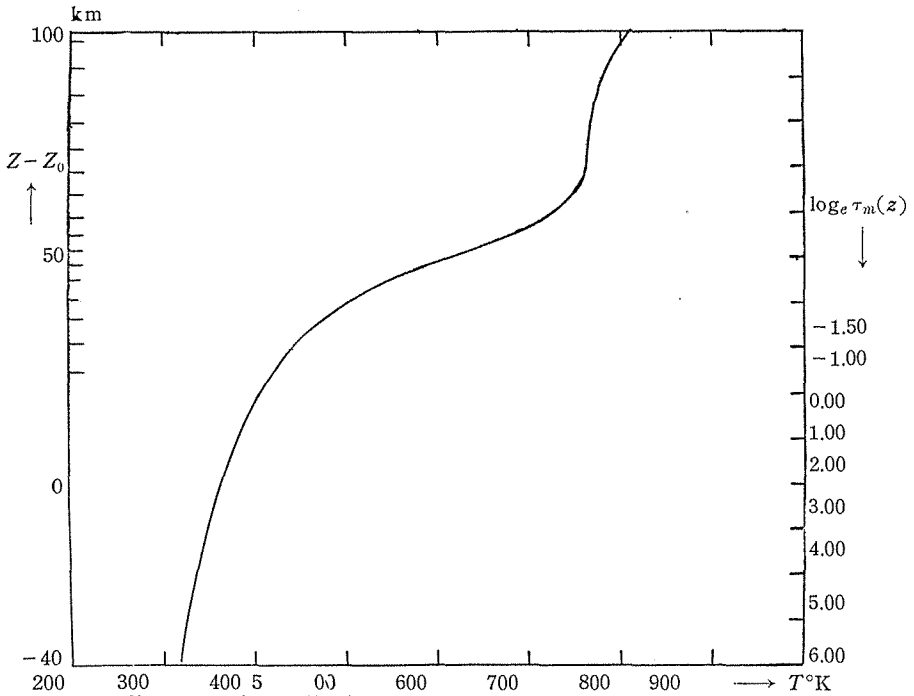


Fig. 5. The vertical distribution of the temperature in the lower ionosphere.

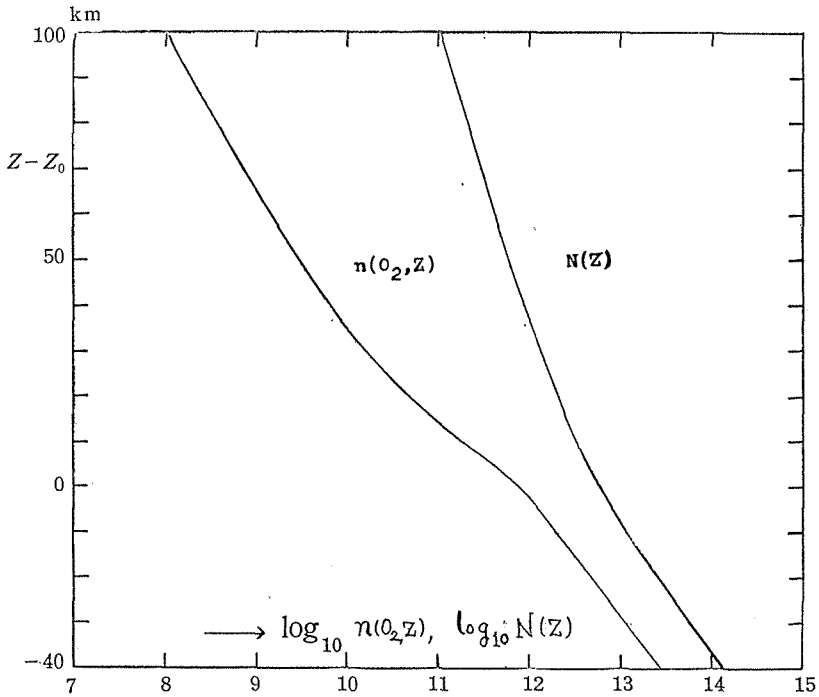


Fig. 6. The vertical distributions of the particle concentration of molecular oxygen and of the total particle concentration in the lower ionosphere.

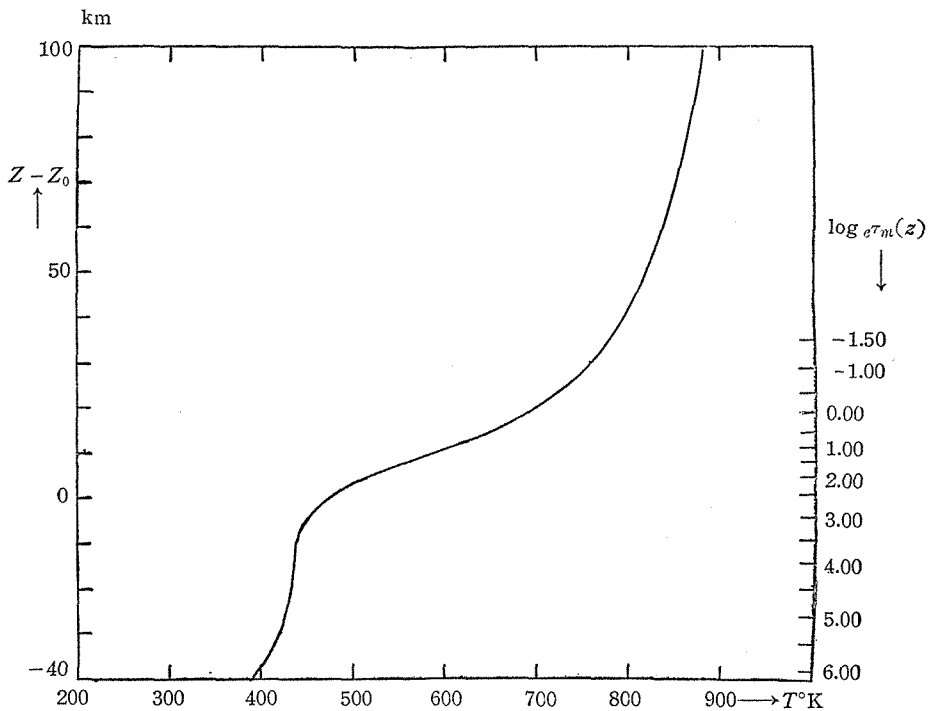


Fig. 5 The vertical distribution of the temperature in the lower ionosphere.

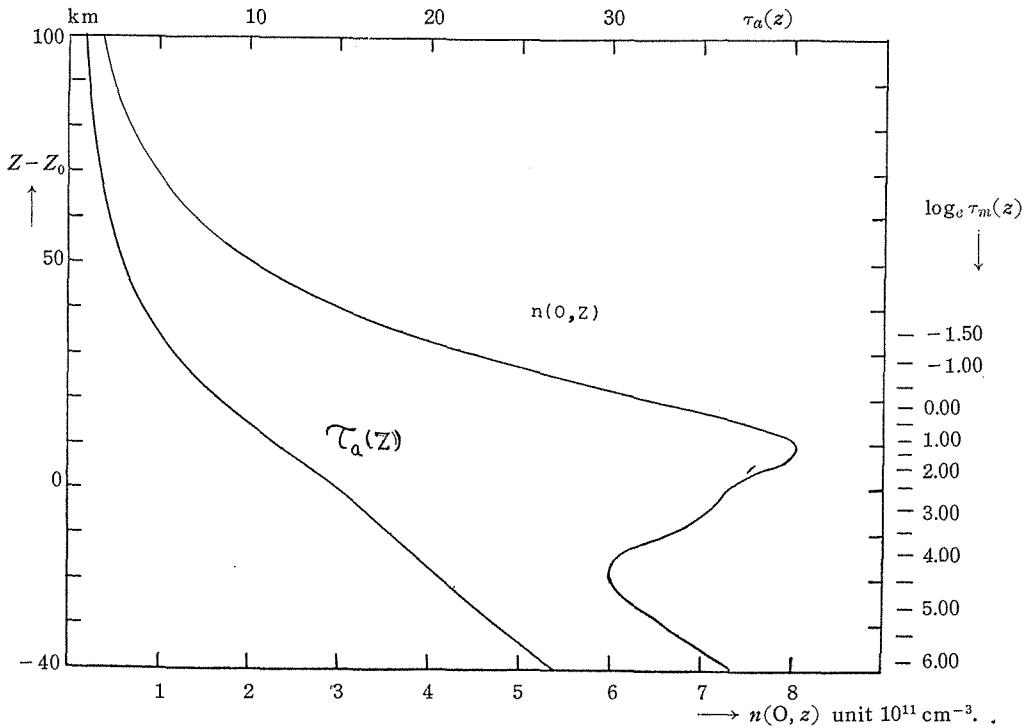


Fig. 7. The vertical distribution of the particle concentration of atomic oxygen and the optical depth  $\tau_a(z)$  of the ionizing radiation in the lower ionosphere.

## 6. Discussion, conclusion and further problem

There is a considerable difference between the distribution of pressure from rocket measurements (28) and the theoretical results derived here. This might be partially attributed to the deduction of pressure from rocket measurements. On the other hand, our result depends on the distribution of the dissociative absorption coefficient with wavelength. There would be some uncertainties on the coefficient over some range from the Schumann-Runge limit toward Herzberg region. The accuracy of information on the coefficient in this range of wavelength is of importance in determining the temperature and the particle concentrations in the region below 100 km, which is in close connection with the free oscillation of the atmosphere as has been suggested by Pekeris (29). Thus, it will be desirable to re-examine closely the dissociative absorption coefficient in this range. As the absorption in the Schumann-Runge region, where the values of its coefficient are to be correct, takes a significant part for our results above the reference level, the results in this region may be reliable. To determine the actual height of the reference level, we must make

allowance for the ionization of the E region. It would, however, not be unnatural to locate the reference level at the height 120 km from the ground level.

There seems to be a resemblance between the temperature curve in Fig. 5 and the  $\epsilon D(z)$ -curve described in logarithmic scale in Fig. 1. This resemblance may be interpreted as follows: As is seen from Fig. 7,  $n(O, z)$  in the E region varies slowly. Hence it may be written approximately as:

$$\epsilon D(z) \propto W_1(O_2|O),$$

and assuming only the principally variable part of the right-hand side, it follows that

$$\log_e \epsilon D(z) \propto -\frac{B(O_2)}{T^{1/3}},$$

from which the resemblance mentioned above may be accounted for.

A study is made of the effect of the ionization in the F regions on the gain of gas-kinetic energy. As far as its effect is negligible, the temperature in the  $F_1$  region remains constant. Inspection of  $\epsilon D(z) n(O_2, z)$  in Table 7 gives a knowledge on the variation of the dissociative gain of gas-kinetic energy with height. On the other hand, to determine the gain by ionization, we must know the mechanism for the ionization in the F regions. It may be supposed that the line emissions from the solar corona are attributed to the ionization associated with the formation of both the  $F_2$  and  $F_1$  layers together. Any possibility of unobservable ionization that has been suggested by Bates(7) might not be considered. Hence, we tentatively assume that the average gain to the gas-kinetic energy seems likely to be about 13 eV photon. It does not seem unreasonnable to assume that the electron recombination coefficient is  $3 \times 10^{-10} \text{ cm}^3 \text{ sec}^{-1}$  for the  $F_2$  layer and  $7 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1}$  for the  $F_1$  layer. The maximum electron concentration is  $1.8 \times 10^6 \text{ cm}^{-3}$  for the  $F_2$  layer and  $3.3 \times 10^5 \text{ cm}^{-3}$  for the  $F_1$  layer. Thus, the gain of gas-kinetic energy will be about  $2.0 \times 10^{-8} \text{ erg cm}^{-3} \text{ sec}^{-1}$  for the  $F_2$  layer and about  $1.6 \times 10^{-8} \text{ erg cm}^{-3} \text{ sec}^{-1}$  for the  $F_1$  layer. On comparing with  $\epsilon D(z) n(O_2, z)$  in Table 7, it may be said that the gain of gas-kinetic energy made by ionization predominates over the dissociative gain in the  $F_2$  region, but the latter increases rapidly with decreasing height and in the  $F_1$  region the former becomes smaller than the latter. Making allowance for the fact that the scattered ionizing radiation contributes to the formation of the  $F_1$  layer, and that because the  $F_1$  layer lies in great optical depth, the flow of the direct incident radiation to the  $F_1$  level is remarkably dependent upon the solar zenith angle, the true gain due to ionization in the  $F_1$  region will be further reduced from  $1.6 \times 10^{-8} \text{ erg cm}^{-3} \text{ sec}^{-1}$ . Summarizing, it may be supposed that in the  $F_1$  region the gain made by ionization has no appreciable effect. Since we cannot avoid the con-

clusion that the diurnal variation of temperature in the  $F_2$  region cannot be great, we shall average the gain by ionization through the day and assume the above value multiplied by 0.3 for the average value. Adding the dissociative gain to it, we get  $6.60 \times 10^{-9}$  erg  $\text{cm}^{-3}$   $\text{sec}^{-1}$  for the total gain of gas-kinetic energy, for which a temperature of  $1250^\circ\text{K}$  is appropriate.

We must now investigate the effect of thermal conduction in the ionosphere, which was suggested by Bates as one of the most effective losses of gas-kinetic energy. There is in the E region an abrupt temperature gradient of about  $10 \text{ K}^\circ/\text{km}$ . The temperature in the  $F_1$  region remains constant unless a great amount of heat is transported by thermal conduction to the  $F_1$  level from the upper region. The gain or loss of heat by thermal conduction  $Q$  is expressed by

$$Q = \frac{d}{dz} \left( \lambda \frac{dT}{dz} \right), \quad (76)$$

where (30)

$$\lambda = \frac{75}{64\sigma^2} \left( \frac{K^3 T}{\pi M_0} \right)^{\frac{1}{2}} \approx 5.16 \times 10^2 T^{1/2}. \quad (77)$$

First, we shall begin with the evaluation of the thermal conduction in the E region. We shall inquire about a place that the temperature changes most abruptly, at the height 10 km above the reference level, where  $\log_e \tau_m(z)$  ranges from 1.00 to 1.50. Inserting (47) and (77) into (76), it yields that

$$\begin{aligned} \frac{d}{dz} \left( \lambda \frac{dT}{dz} \right) &= \frac{d}{dz} \left[ \frac{3M_0 g \lambda}{KB(O_2)} T^{1/3} b(z) \right], \\ &\approx 1.33 \times 10^{-3} \frac{d}{dz} [T^{5/6} b(z)]. \end{aligned} \quad (78)$$

Substituting the values appropriate for that level, we obtain

$$\frac{d}{dz} \left( \lambda \frac{dT}{dz} \right) \approx 8.84 \times 10^{-8} \text{ erg cm}^{-3} \text{ sec}^{-1}. \quad (79)$$

In comparing with  $\varepsilon D(z) n(O_2, z)$  given in Table 6, taking account of the factor 1/3 for no detectable diurnal variation of temperature, the effect of thermal conduction becomes only about 1/27 of the dissociative gain at the level concerned. The thermal equilibrium in the E region is little affected by the thermal conduction.

We shall discuss briefly the effect of thermal conduction for the  $F_1$  region. The temperature of the  $F_2$  level, at about 300 km height, would be about  $1250^\circ\text{K}$ , as has been considered before. Assuming that the temperature between the  $F_1$  and  $F_2$  levels rises linearly, its gradient would become  $3.5^\circ\text{K}/\text{km}$ . Taking into consideration that the isothermal region of  $900^\circ\text{K}$  is found below the  $F_1$  level, the flow of heat by thermal conduction from the upper region will be stopped there. In consequence, the



temperature rises in some degree making a temperature gradient and then heat flows to lower region. It seems relevant to suppose that above the  $F_2$  level the temperature gradient is extremely small, so that the heat generation to affect the  $F_1$  region must take place at the region between  $F_1$  and  $F_2$ . The maximum gain of gas-kinetic energy in this region will be about  $6.60 \times 10^{-9}$  erg  $\text{cm}^{-3}$   $\text{sec}^{-1}$ , as has been studied just before. If it were supposed, as an upper limit, that all the gain in this region flows into the  $F_1$  level by the thermal conduction, and that the heat flow is spent on raising the temperature in a range of altitude as great as one scale height there, the gain by the thermal conduction in the  $F_1$  level would be about  $1.65 \times 10^{-8}$  erg  $\text{cm}^{-3}$   $\text{sec}^{-1}$ . As is seen from  $\varepsilon D(z) n(\text{O}_2, z)$  given in Table 7, this is comparable with the dissociative gain at 100 km above the reference level. Since the quantities quoted in the above discussion would be regarded as an extremely over-estimated, the thermal conduction from the upper region would not have an appreciable effect on the thermal equilibrium in the region below the  $F_1$  level, but it might have some effect in the region above the  $F_1$  level.

It is interesting that the particle concentrations  $n(\text{O}, z)$  and  $n(\text{N}_2, z)$  deduced from the present research are favourable for the formation of the  $F_1$  and  $F_2$  layers.

We have shown that the temperature of  $F_2$  region will be about  $1250^\circ\text{K}$ . It must be said that this temperature is intimately connected with the degree of the stirring of the atmosphere: If the stirring of the atmosphere halts at a certain height, the atmosphere above it will be distributed by a diffusive equilibrium and the relative populations of molecular nitrogen to atomic oxygen there will reduce to a much small value. As regards the loss of gas-kinetic energy, the most effective process would not be the vibrational excitation by impacts of atomic oxygen, which have been quoted in the present work, but it would become the excitation to the metastable states  $^1D$  and  $^1S$  of atomic oxygen by electron impacts (31). Inspection of the ionospheric observational data suggests that the stirring of the atmosphere ceases, if possible, between the  $F_1$  and  $F_2$  layers. Would it be the case, the temperature at great height would rise up over  $2000^\circ\text{K}$ . However, the existence of  $\text{N}_2^+$ -spectra at great height seems to show that this is not the case. In the  $F_2$  region, the temperature is about  $1250^\circ\text{K}$ . It is not meaningless to add here that the observed intensity of the oxygen line  $5577\text{\AA}$  in the night sky light cannot be accounted for by the excitation by the electron impact.

In conclusion, it may be emphasized that the quantitative results of the present work depend much on the accuracy of information about the dissociative absorption coefficients of molecular oxygen. By the future establishment of these molecular data, the properties of the upper atmosphere could be researched theoretically with sufficient accuracy.

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