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STUDY ON CHANGE OF GRAVITY WITH TIME
PART I. ON THE TIDAL VARIATION OF GRAVITY

BY

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ABSTRACT

In this article, a newly devised gravimeter of the double bifilar suspension type was treated theoretically and experimentally for the purpose of precise and continued observation of tidal and secular variation in gravity. It was proved theoretically and experimentally that the sensitivity of the gravimeter of this type may easily be raised to a degree as high as 10^{-9}g/mm/5m. A test observation with the double bifilar gravimeter has been operated continuously for about three years at the deep-seated observation room of the Ikuno Copper Mine, and the value for the tidal factor of gravity for the M_2-component was tentatively calculated as 1.08. Furthermore, in the present treatment, minimizing the gravimeter drift proved to be a most difficult but important problem for a perfect and precise gravimetric observation.

1. Introduction

The problem of gravitation and of terrestrial gravity in particular has been an essential and everlastingly attractive field of research in geophysics, ever since Newton's time. The mode of surface distribution of gravity on the earth depends on the shape of the earth and the density distribution of the earth's interior. The continuous and serious efforts of many geodesists for the precise and world-wide survey of gravity, even in the ocean areas, has made great contributions to the development of our knowledge on the figure and internal mass distribution of the earth. Under such circumstances as above-mentioned, the mode of gravity distribution on the earth's surface has been investigated more and more in detail, especially with the recent developments of survey technique by use of the static gravimeter of small size and easy manipulation. The gravity anomaly thus obtained informs us of the regional and local complexity of the structure of the earth's crust, and provides us with many interesting problems of a geological and geodetic nature. Contrary to the great progress of our knowledge on the mode of gravity distribution on the earth's surface, the problem of change of gravity with time, shortly expressed as time variation of gravity, may be said to remain an uncultivated field of research. Two ways of change of gravity with time are generally supposed, the one being periodic variation and the other non-periodic. The periodic variation of gravity caused by the tide-producing forces due to the Sun and Moon is mainly treated by the gravimetric observation of earth tides concurrently with tiltmetric and extensometric observations. The periodic gravity change of this kind is usually referred to as 'tidal variation of gravity' and its investigation gives us important information as to the elastic property of the earth.

As to non-periodic gravity change, however, only little definite knowledge has been ascertained as yet, but the observation on anomalous changes in gravity, if they exist, of short duration (rapid change) or of long duration (secular variation) may greatly serve for the development of our knowledge on the change of state or motion of the deep underground material. In a succeeding series of articles, some problems on the time variation of gravity will be treated from the standpoint of geodesy and seismology. In the present article, some considerations are made on the phenomena of the tidal variation of gravity, especially on a newly designed gravimeter of high sensitivity and its application.

On the problem of earth tides Lord Kelvin first discussed the phenomena in 1863 (**1**) from the theoretical treatment of the earth as an elastic sphere. Since the first successful observation of earth tides with the tiltmeter of the horizontal pendulum type by E. v. Rebeur-Paschwitz in 1892 (**2**), a great many tiltmetric observations of earth tides have been made at various places in the world. And the number of orthodox observations of earth tides with the tiltmeter amounted to nearly twenty five since the day of Rebeur-Paschwitz to the present time. On the contrary, the gravimetric observation of earth tides may correctly be said to have remained untouched, because of the technical difficulty of observation with this instrument. Under such circumstances the problems of intensity variation of gravity vector caused by the tide-producing forces of the Sun and Moon are little solved at the present time, in contrast to the great advance in knowledge on the tidal change of gravity direction by tiltmetric observation, and we are eagerly awaiting future developments in research by the method of gravimetric observation.

After the first but provisional observation of tidal variation in gravity with the gravimeter of the bifilar suspension type by W. Schweydar in 1914 (**3**), a precise and orthodox gravimetric observation was made by R. Tomaschek and W. Schaffernicht in 1932 with a bifilar gravimeter of high sensitivity. And after a long interruption during the Second World War many but provisional gravimetric measurements of tidal variation have been made with portable gravimeters which were largely developed for the purpose of geophysical exploration for oil fields and other subterranean resources. The findings obtained by these gravimetric measurements of tidal variation will be listed in Table 1 at the end of this section.

On the theoretical treatment of earth tides, A. E. H. Love (**4**), in 1909, greatly developed and generalized the pioneering and brilliant work originated by Lord Kelvin, and recently in 1950, H. Takeuchi (5) fully discussed the problem of earth tides on the compressible earth of variable density and elasticity taking into consideration the density distribution of K. E. Bullen (6). According to this theory the quantity Δg of the tidal variation of gravity at the earth's surface is expressed by the formula:

$$\Delta g = G\left(\frac{\partial U_2}{\partial r}\right)_{r=a}, \qquad G \equiv 1 - \frac{3}{2}k + h\,,$$

where U_2 denotes the tide-producing potential expressed by a spherical solid harmonic of the second degree, r the radius vector from the earth's centre, a the mean radius of the earth, and h and k are dimensionless constants called the Love-numbers which are correlated with the elasticity and density distribution of the earth's interior. In case of a perfectly rigid earth, h and k are to become zero and then

$$\Delta g = \left(\frac{\partial U_2}{\partial r}\right)_{r=a}.$$

Therefore, in the case of a real deformable earth, the tidal variation of gravity shall be observed on the earth's surface G times as large as that in case of the hypothetical rigid earth. With such an interpretation the symbol G—the ratio of the observed amplitude of tidal variation of gravity to that of the theoretical force— is conveniently called the 'tidal factor of gravity' or 'gravimetric factor'. Corresponding to G of the tidal factor of gravity in the gravimetric observation of earth tides, the diminishing factor D should be introduced in the tiltmetric observation of earth tides which is also derived as the ratio between observed and theoretical amplitudes. Since D is expressed by the formula $D \equiv 1 + k - h$, the values of the Love-numbers h and k will directly be determined by the concurrent gravimetric observation with a tiltmeter at any place on the earth's surface. But the determination of the Love-numbers by the combination of gravimetric and tiltmetric observation is, in practice, troublesome for the following reasons. The tiltmetric observations of earth tides for determining the D-factor are greatly disturbed by the secondary effect of ocean tides at any place, especially near the open sea, and they are also spoiled by the large tilting motion of ground caused by rain fall, sun shine and other meteorological effects at some stations, especially at shallow-seated observation rooms. And with regard to the gravimetric observation, the precise and long continuous observation itself is very troublesome mainly on account of technical difficulty, namely the problem of raising the sensitivity of instrument to a sufficiently high degree, maintaining the instrument in a high sensitive and stable condition during the several years' continuous observation, and above all the most unmanageable problem of the so-called 'drift' or 'zero shift' of the instrument. But in spite of these difficult circumstances it is certain that the observation of earth tides and general crustal change concurrently with the tiltmeter, gravimeter, and extensometer will greatly increase our knowledge on the nature of earth tides and general crustal deformation. And in the present and succeeding articles some problems on the gravimetric observation on earth tides and rapid or secular changes of gravity will be treated in combination with the tiltmetric and extensometric observation.

Returning to the problem of the tidal factor of gravity, the values obtained by R. Tomaschek and W. Schaffernicht are greatly diverse from those obtained by other researchers, as will be seen in Table 1. The cause of its diversity in spite of their cautious observation with a highly sensitive gravimeter and at a deep and good conditioned room has yet remained uninterpreted. Other observations listed in Table 1 show various numerals from 0.9 to 1.6 as the tidal factor of gravity, and it is an essential and attractive problem on the gravimetric study of earth tides to seek the cause of the diversity of their values. The observation periods in Table 1 are for the most part two or four weeks duration and moreover the observation rooms are on the ground surface. Therefore the error in process of analysis for the data of short period or the instrumental error caused by temperature change and other disturbances might have vitiated the obtained G-value and caused their diversity. Or the effect caused by the difference of surface or subterranean geological structure or the perturbation effect of the ocean tides and other unknown forces should be examined as the source of diversity. Some detailed consideration will be made on these points in a succeeding article. In the present article several points will be treated on 'How to raise the sensitivity of gravimeter to a sufficiently high degree?', 'By applying what kind of material to the instrumental construction can we put the gravimeter in a stable condition?', 'At what sort of observation room should the gravimeter be set up and how should observation be conducted for the purpose of the precise and trustworthy study of earth tides?'. On these points both the theoretical and experimental aspects in our investigation will be described below.

The following list in Table 1 are the data of observation for tidal variation of gravity hitherto obtained by several investigators. In this table 'Total' and 'M_2, S_2, K_1, O_1' refer to the resultant tide and each component tide respectively, and G and κ denote respectively the ratio of the observed amplitude to the theoretical one and the phase lag between the observed and the theoretical phases. The plus sign means that the observed phase is behined the theoretical one, while the minus sign shows that the observed phase is in advance of the theoretical one. The data in this table are mainly taken from the papers of W. D. Lambert (7), P. J. Melchior (8) and other original papers.

Table 1

Author	Year	Period of harmonic analysis	Station	Room depth	Instrument	Value of the tidal factor of gravity: M_2		S_2		K_1		O_1		Total	
						G	κ	G	κ	G	κ	G	κ	G	κ
W. Schweyder (**9**)	1913		Potsdam	25 m.	Bifilar	1.20	$-2°.8$								
R. Tomaschek and W. Schaffernicht (**10**)	1932	3 months	Marburg	25 "	Bifilar	0.55	41.7	0.35	97°.0	0.58	16°.2	0.51	14°.4		
			Berchtesgaden	140 "		0.57	42.0								
R.D. Wyckoff (**11**)	1935		Pittsburg		Galitzin									1.10	50°
O.H. Truman (**12**)	1936		Houston		Truman	1.13	4.4								
E.A. Eckhardt (**13**)	1939		Santa Barbara		Gulf									1.14	
			Varde		Gulf									1.24	
M.S. Reford (**14**)	1949	29 days	Toronto		North American	1.08	2.6	1.10	8.0	0.97	−2.0	1.03	0.9		
A.J. Hoskinson (**15**)	1949	14 "	Muna		Mott Smith	1.19	5.3	1.29	10.9	1.78	−0.9	1.28	−2.9		
		15 "	Honolulu		La Coste	1.24	11.2	1.36	17.0	1.30	−10.4	1.56	−0.8		
		14 "	Panuco		Mott Smith	1.14	1.9	1.17	13.9	1.35	−2.4	1.16	2.9		
		15 "	Houston		La Coste	1.16	1.5	1.17	−7.6	0.89	3.8	1.15	−1.6		
		15 "	Houston		Worden	1.15	−4.0	1.16	−8.7	1.21	−4.5	1.16	−6.6		
		15 "	Beaumont		La Coste	1.17	−0.6	1.32	−2.8	1.24	−7.2	1.26	0.3		
		14 "	Austin		La Coste	1.15	2.1	1.10	6.3	1.16	1.1	1.05	−0.3		
		29 "	Pasadena		La Coste	1.19	7.4	1.26	5.1	0.93	3.8	1.14	0.6		
		15 "	Albuquerque		Humble										
		15 "	Tulsa		La Coste	1.23	2.9	1.54	−11.1	1.36	−19.3	1.11	−3.1		
		15 "	Washington		La Coste	1.24	−2.3	1.07	1.7	1.18	−6.2	1.17	0.8		
		15 "	Fort Morgan		North American	1.17	3.9	1.01	−4.3	1.13	−2.6	1.17	−5.1		
		14 "	Salt Lake City		Worden	1.25	0.0	1.07	8.9	1.39	8.4	1.06	−21.2		
		15 "	Toronto		North American	1.08	3.2	1.11	6.8	0.97	−8.8	1.04	−1.6		
		14 "	Ottawa		North American	1.28	4.8	1.42	5.2	1.17	2.6	1.26	2.9		
		15 "	Edmonton		North American	1.01	5.5	1.37	−6.4	1.13	−0.3	1.15	−7.6		
		15 "	Edmonton		North American	0.94	−2.4	1.47	4.7	1.06	−7.1	1.32	−1.5		
L.H. Tarrant and R. Tomaschek (**16**), (**17**)	1949	14 "	Peebles		Frost	1.09	352	1.09	348	1.06	359	1.13	12		
		14 "	Kirklington		Frost	1.24	349	(1.30)	354	1.08	3	1.11	6		
J.T. Pettit, L.B. Slichter and L. La Coste (**18**)	1949	2 "	Attu											1.17	52.5
		29 "	Pasadena											1.32	20
		90 "	Los Angels											1.35	20
		36 "	Hawaii											1.54	45
R. Tomaschek (**19**)	1951	8 "	Winsford	143 "	Frost	1.26	7	1.26	7	1.17	7	1.17	7		

2. Double bifilar gravimeter

(1) *Gravimeter of bifilar suspension type*

The diversity of G-values hitherto obtained was mentioned in the preceding section, the range of their extreme values having been estimated to be 0.9 to 1.6, which can not simply be attributed to observational and instrumental errors. W. D. Lambert (**20**) has attributed its sources to the following three main causes: (i) secondary effects of oceanic tides, (ii) effects of local geological structure and (iii) departures from statical theory, such as those due to the existence of the liquid core of the earth. In order to remove such disturbing factors and obtain an accurate value for G, we must make our observations (1) at various stations under various conditions, (2) during a sufficiently long period and (3) with the tiltmeter and extensometer concurrently. The gravimeter conforming to these three requirements must be a self-recording apparatus, structurally stout and easily manipulated. Under such circumstances, many portable gravimeters as used in these days for geophysical exploration cannot be regarded as the most suitable ones for these requirements. On the contrary, the 'bifilar gravimeter' or gravimeter of Perrot-Schmidt is considered to be the one that satisfies these requirements, and its structure is schematically shown in Fig. 1. A. Schmidt (**21**) has discussed the sensitivity of a bifilar gravimeter which he has called the 'trifilar gravimeter'; however, it appears that he did not make use of his gravimeter in practice. W. Schweydar first observed with a bifilar gravimeter and discussed the value of G. Long after the pioneering observation by Schweydar, R. Tomaschek and W. Shaffernicht greatly improved the bifilar gravimeter of the Schweyder type and made a precise and long continuous observation at a deep-seated underground room. The main points of the improvements were that 'elinvar' was used for the spring, and electrostatic force was utilized for calibration of sensitivity, along with other minor improvements. And they succeeded for the first time in taking a photographic record of the tidal variation of gravity. This type of gravimeter was also studied in detail by A. Berroth (**22**). Recently, H. Ellenberger (**23**) has fundamentally reanalyzed the bifilar suspension principle and has devised a new type of gravimeter named the 'Doppelbifilargravimeter'. His idea is very interesting, and it is easily supposed that its sensitivity will show a great increase over that of the ordinary bifilar gravimeter, though its adjustment will be considerably more troublesome; and we are awaiting for the observational results of tidal variation with his gravimeter. It ought to be mentioned here that, in the point of name of 'double bifilar gravimeter', the gravimeter designed by the writer and that by Ellenberger are entirely the same, but they are quite different in the principle of construction. According to experiments, it is considerably difficult to raise the sensitivity of the ordinary bifilar gravimeter to a degree sufficiently high to record the tidal variation of gravity. The source of the difficulty lies in the fact that the sensitive range of the instrument is very narrow, and furthermore, it lies near a labile equilibrium point. The writer has examined the above-mentioned circumstances and devised a new type of highly sensitive gravimeter, which was designated as 'double bifilar gravimeter'. Its principle of construction and some test experiments were reported at the Autumnal Meeting of the Seismological Society of Japan in 1949 and published in Japanese in 1950 (**24**) and in 1951 (**25**).

(*2*) *Theoretical considerations on the sensitivity of an ordinary bifilar gravimeter.*

An ordinary bifilar gravimeter is schematically shown in Fig. 1. If the instrument is in a state of equilibrium for the present value of gravity g, the conditions of equilibrium are expressed by

$$\left.\begin{aligned} s(l-l_0)+p\sin\alpha-Mg&=0\,,\\ f(\omega-\varphi)-p\frac{ab\sin\varphi}{m}&=0\,,\end{aligned}\right\} \qquad (1)$$

and geometrically

$$\left.\begin{aligned} &l=h_0+h\,,\\ &\sin\alpha=h/m\,,\\ &h^2=m^2-(a^2+b^2-2ab\cos\varphi)\,,\end{aligned}\right\} \qquad (2)$$

where $\frac{1}{2}p$ is the tension of the string, s and f the elastic constants of the spring with regard to elongation and torsion respectively, ω and φ the turning angle of the spring head and the weight respectively, M the mass of the weight system, and other symbols are as shown in the figure. If φ varies by $d\varphi$ correspondingly to the variation of gravity dg, the sensitivity $\frac{d\varphi}{dg}$ is calculated from (1) and (2) as follows:

$$\frac{d\varphi}{dg}=-\frac{ab\sin\varphi}{mf+abp\cos\varphi}\cdot\frac{m}{h}\left\{M-\left(s+\frac{p}{m}\right)\frac{dh}{dg}\right\}. \qquad (3)$$

Putting $m\simeq h$, $\frac{dh}{dg}\simeq 0$ in a first approximation, we have

$$\frac{d\varphi}{dg}=-\frac{abM\sin\varphi}{mf+abp\cos\varphi}. \qquad (4)$$

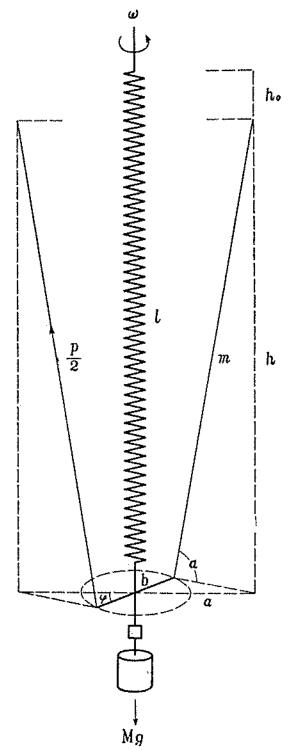

Fig. 1. Schematic figure of an ordinary bifilar gravimeter.

Next, let us calculate the correlation between the free oscillating period of rotation and the sensitivity. Assuming that φ is virtually varied by ε as a result of some minute disturbing force, then the restoring moment R is as follows:

$$R=f\{\omega-(\varphi+\varepsilon)\}-p\frac{ab\sin(\varphi+\varepsilon)}{m}.$$

Putting $\cos\varepsilon=1$, $\sin\varepsilon=\varepsilon$ approximately, since ε is small, we have

$$R=\left\{f(\omega-\varphi)-p\frac{ab\sin\varphi}{m}\right\}-\left\{f+p\frac{ab\cos\varphi}{m}\right\}\varepsilon.$$

By (1) the first term on the right-hand side is equal to zero, and therefore the equation of motion becomes

$$I\frac{d^2\varepsilon}{dt^2}=-\left\{f+p\frac{ab\cos\varphi}{m}\right\}\varepsilon, \tag{5}$$

where I is the moment of inertia of the weight. From this equation, the oscillating period P is expressed by

$$P=2\pi\sqrt{\frac{I}{f+p\frac{ab\cos\varphi}{m}}}. \tag{6}$$

Inserting (6) into (4) with the substitution of $I=r^2M$, we obtain

$$\frac{d\varphi}{dg}=-\frac{ab\sin\varphi}{4\pi^2mr^2}P^2. \tag{7}$$

In this expression, a, b, m and r are constants fixed to the instrument. Therefore, it will be understood that if φ is held constant, the sensitivity is proportional to the square of period. This relation is usually applied, in practice, to estimating the sensitivity of the gravimeter. In the expression (4), $\frac{d\varphi}{dg}$ is a function of p and φ. Fig. 2 shows the relation between p, φ and $\frac{d\varphi}{dg}$, in which the adopted numerical values of the constants are as follows: $a=6$, $b=3$, $m=130$, $f=1000$, $M=50$, $g=980$ c.g.s. units respectively, and $p=\frac{8.5}{50}Mg$ for curve I and $=\frac{15}{50}Mg$ for curve II.

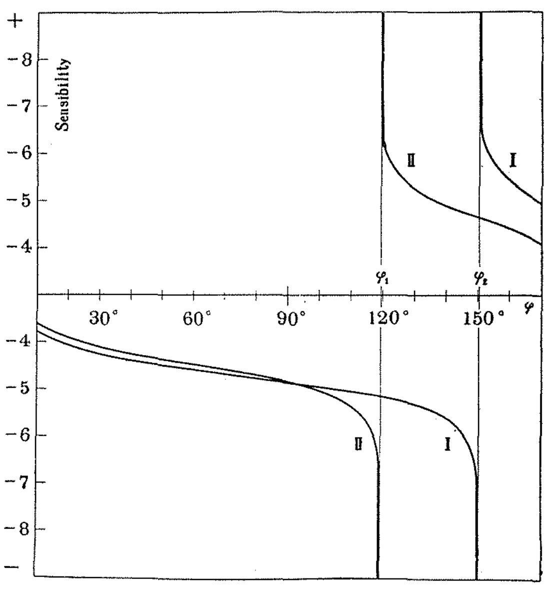

Fig. 2. Sensitivity curve of an ordinary bifilar gravimeter.

The numerals on the sensitivity-axis indicate the common logarithms of $\frac{dg}{g}$ corresponding to 1 mm. displacement of the optical image on the recording paper of which optical distance is 5 meters.

As clearly seen in Fig. 2, $\frac{d\varphi}{dg}$ suddenly varies from minus infinity to plus infinity at φ_1 or φ_2, and a sensitive domain is limited within the very narrow neighbourhood of φ_1 or φ_2. And furthermore, on account of the sensitivity curve being very abrupt at that point, the sensitivity is very labile, consequently the gravimeter itself is liable to become unstable. This is the cause of the difficulty of raising the sensitivity of the ordinary bifilar gravimeter to such a high degree.

For the purpose of removing the source of the difficulty and raising the sensitivity, let us again examine the expression (4). We learn from the expression (4) that in order to increase the absolute value of $\frac{d\varphi}{dg}$ over the whole range of φ, the first term mf in the denominator of the right-hand side may need to be minimized because p and φ in the second term are at our option. While, in the case of shortening the length of the string m, $\frac{dh}{dg}$ becomes large and is not neglected as above treated in a first approximation. From the relation (3), it is understood that the increase of $\frac{dh}{dg}$ results in a way to decrease the absolute value of $\frac{d\varphi}{dg}$. Therefore we must abandon this manner of shortening the length of string m but endeavour to reduce the value of the torsional elastic constant f.

The most direct method of reducing the torsional constant f is to use a feeble spring. While, the use of a feeble spring implies the decrease of the mass M of the weight for which the spring is endurable. According to the expression (4), the decrease of M results in the decrease of the absolute value of $\frac{d\varphi}{dg}$. Therefore, after all, the problem of raising the sensitivity of the bifilar gravimeter to a sufficiently hign degree comes to a point as to whether or not there exists a spring which would tolerate a considerably heavy load and yet have a low-enough torsional elastic constant. For the purpose of investing the spring with such quality as above required, the writer has devised the following construction of the instrument. The construction is simply to connect the lower end of the main spiral spring and the weight with a fine string as shown in Fig. 3. The finer the string is, the more favourable the condition, as long as it endures a considerably heavy load. Such a combination of a fine string with the main spring of a bifilar gravimeter makes it possible to gain a single spring with a small torsional constant and a large tenacity for load.

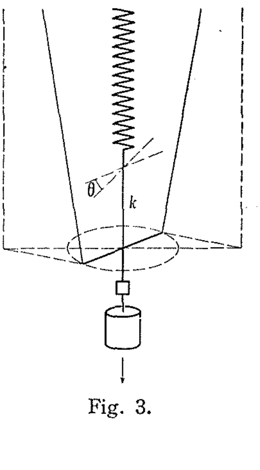

Fig. 3.

Now, let the torsional constant of string τ be equal to one-hundredth of that of spring f, then the sensitivity of the bifilar gravimeter equipped with such a string and spring is as follows:

$$\frac{d\varphi}{dg}=-\frac{ab\sin\varphi}{m\frac{\tau f}{\tau+f}+abp\cos\varphi}\cdot\frac{m}{h}\left\{M-\left(s+\frac{p}{m}\right)\frac{dh}{dg}\right\}. \tag{8}$$

Putting $m\simeq h$, $\frac{dh}{dg}\simeq 0$ as in the previously assumed approximation, and since $\tau\ll f$, we have approximately

$$\frac{d\varphi}{dg}=-\frac{abM\sin\varphi}{m\tau+abp\cos\varphi}. \tag{9}$$

The last expression is indentical with that in which f is replaced by τ in the expression (4). And it indicates that if only we appropriately control the tension of the bifilar strings p, the absolute value of $\frac{d\varphi}{dg}$ increases to the extent of a hundred times as large as that of the ordinary bifilar gravimeter over the whole range of φ. Namely, we can easily raise the sensitivity of a gravimeter a hundred times as high as that of the ordinary bifilar gravimeter by such a simple modification as the string connection. The writer has experimentally ascertained that the above condition is practically satisfied. Thus it may safely be said that this method of connecting a string might to some extent serve as one solution for the problem of raising the sensitivity of the bifilar gravimeter. However, it was experimentally proved that the gravimeter thus modified also has many points of inconvenience which should be examined for the purpose of precise and long continuous observation of tidal variation of gravity. Namely, the drift of the zero position of the gravimeter caused by the inevitable flow of the spring and the undesirable effect of temperature change upon the function of the instrument are generally the most troublesome problem for every gravimetric observation, and the case of present gravimeter cannot be an exception. Since the string connecting the spring and the weight yields to torsional force, the above-mentioned manner is disadvantageous for the drift and the temperature change. For the purpose of avoiding the foresaid inconvenience, the writer has further devised the following construction. It is the construction of connecting the spring and the weight with two strings so fine that the torsional moment is negligible, in place of one elastic string, as seen in Fig. 4. The writer has given a name of 'double bifilar gravimeter' to the bifilar gravimeter improved in this way. In the next, the function of a double bifilar gravimeter will be discussed in some detail.

(3) *Function of the double bifilar gravimeter*

If a double bifilar gravimeter is in a state of equilibrium for the present value of gravity g, neglecting the torsional moment of strings, the conditions of equilibrium are as follows: dynamically, in the vertical direction

$$T\sin\beta + t\sin\alpha - Mg = 0 \tag{10-1}$$

$$s(l-l_0) - T\sin\beta = 0\,, \tag{10-2}$$

in the rotational direction

$$T\frac{ce\sin\theta}{n} - t\frac{ab\sin\varphi}{m} = 0 \tag{10-3}$$

$$f(\omega-\theta-\varphi) - T\frac{ce\sin\theta}{n} = 0\,; \tag{10-4}$$

and geometrically

$$l+k = h_0 + h \tag{11-1}$$

$$\sin\alpha = \frac{h}{m} \tag{11-2}$$

$$\sin\beta = \frac{k}{n} \tag{11-3}$$

$$h^2 = m^2 - (a^2+b^2-2ab\cos\varphi) \tag{11-4}$$

$$k^2 = n^2 - (c^2+e^2-2ce\cos\theta)\,, \tag{11-5}$$

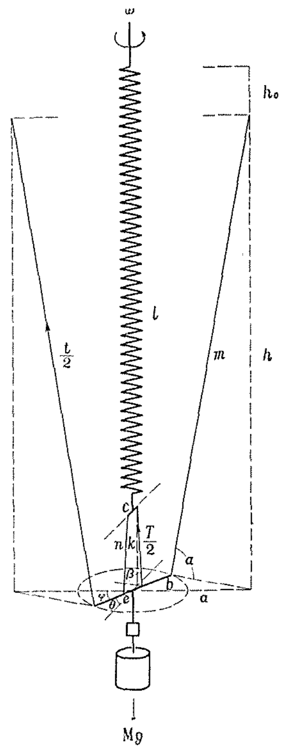

Fig. 4. Schematic figure of a double bifilar gravimeter.

where t and T are the tensions of the outer and inner bifilar strings respectively; s and f the elastic constants of the spring with regard to elongation and torsion respectively; ω, θ and φ the turning angle of the spring head, the angle between the lower end of the spring and the weight, and the turning angle of the weight; M the mass of the weight system; and other symbols are as shown in Fig. 4.

Introducing a new quantity q by the relation:

$$t = qT\,, \tag{12}$$

we can obtain the sensitivity formula from these conditions in the following way. From (10-1), (11-2), (11-3) and (12)

$$T = \frac{Mg}{\dfrac{k}{n} + q\dfrac{h}{m}}\,. \tag{13}$$

Inserting (13) into (10-4) and differentiating we get

$$-f(d\theta+d\varphi) = \frac{ceM}{n}\left[\frac{\left(\dfrac{k}{n}+q\dfrac{h}{m}\right)(\sin\theta dg + g\cos\theta d\theta) - g\sin\theta\left(\dfrac{dk}{n}+\dfrac{h}{m}dq+\dfrac{q}{m}dh\right)}{\left(\dfrac{k}{n}+q\dfrac{h}{m}\right)^2}\right]. \tag{14}$$

On the other hand, from (10-3) and (12)

$$\left.\begin{aligned} \sin\theta &= q\frac{abn}{cem}\sin\varphi \\ \cos\theta d\theta &= \frac{abn}{cem}\sin\varphi dq + \frac{abn}{cem}q\cos\varphi\, d\varphi \\ d\theta &= \pm\frac{\sin\varphi dq + q\cos\varphi d\varphi}{\sqrt{\left(\frac{cem}{abn}\right)^2 - (q\sin\varphi)^2}}\,, \end{aligned}\right\} \tag{15}$$

where the plus sign corresponds to the case $0 \leqq \theta \leqq \frac{\pi}{2}$, and the minus sign corresponds to the case $\frac{\pi}{2} < \theta \leqq \pi$.

Inserting (15) into (14) gives

$$-f\left\{d\varphi \pm \frac{\sin\varphi dq + q\cos\varphi d\varphi}{\sqrt{\left(\frac{cem}{abn}\right)^2 - (q\sin\varphi)^2}}\right\} = \frac{ceM}{n} \times \frac{1}{\left(\frac{k}{n}+q\frac{h}{m}\right)^2} \times \left[\left(\frac{k}{n}+q\frac{h}{m}\right)\left\{q\frac{abn}{cem}\sin\varphi dg\right.\right.$$
$$\left.\left.+g\left(\frac{abn}{cem}\sin\varphi dq + \frac{abn}{cem}q\cos\varphi\, d\varphi\right)\right\} - gq\frac{abn}{cem}\sin\varphi\left(\frac{1}{n}dk + \frac{h}{m}dq + \frac{q}{m}dh\right)\right].$$

Taking out $d\varphi$ and regulating, we get

$$-\left[f\left\{1 \pm \frac{q\cos\varphi}{\sqrt{\left(\frac{cem}{abn}\right)^2 - (q\sin\varphi)^2}}\right\} + \frac{abqMg\cos\varphi}{m\left(\frac{k}{n}+q\frac{h}{m}\right)}\right]d\varphi$$
$$= \frac{abqMg\sin\varphi}{m\left(\frac{k}{n}+q\frac{h}{m}\right)}dg + \left[\frac{M}{\left(\frac{k}{n}+q\frac{h}{m}\right)^2}\left\{\frac{abg\left(\frac{k}{n}+q\frac{h}{m}\right)}{m} - \frac{abqg}{m}\cdot\frac{h}{m}\right\}\right.$$
$$\left.\pm\frac{f}{\sqrt{\left(\frac{cem}{abn}\right)^2 - (q\sin\varphi)^2}}\right]\sin\varphi dq - \frac{abqMg\sin\varphi}{m\left(\frac{k}{n}+q\frac{h}{n}\right)^2}\left(\frac{1}{n}dk + \frac{q}{m}dh\right). \tag{16}$$

Next, dq in the expression (16) is calculated from the conditions in the following way. From (11-1)

$$dl = dh - dk\,. \tag{17}$$

Inserting (11-3) and (13) into (10-2) gives

$$s(l-l_0) - \frac{Mg}{\frac{k}{n}+q\frac{h}{m}}\cdot\frac{k}{n} = 0\,.$$

Differentiating this and substituting dl in the result with (17), we get

$$s(dh-dk)-\frac{M}{m}\cdot\frac{\left(\frac{k}{n}+q\frac{h}{m}\right)(kdg+gdk)-gk\left(\frac{1}{n}dk+\frac{1}{m}dq+\frac{q}{m}dh\right)}{\left(\frac{k}{n}+q\frac{h}{m}\right)^2}=0\,.$$

After rearrangement, we get

$$dq=-\frac{m}{h}\left[\frac{1}{gk}\left(\frac{k}{n}+q\frac{h}{m}\right)\left\{\frac{ns}{M}\left(\frac{k}{n}+q\frac{h}{m}\right)(dh-dk)-(kdg+gdk)\right\}+\left(\frac{1}{n}dk+\frac{q}{m}dh\right)\right]. \tag{18}$$

If necessary, we can express dh and dk by $d\varphi$ and dg by using the remaining conditions (11–4) and (11–5). However, the process of the calculation is very troublesome in spite of the circumstances that dh and dk are negligible quantities and do not affect the estimation of $\frac{d\varphi}{dg}$. Therefore it is useless to discuss in detail the values of dh and dk in general for the present treatment. Combining (16) with (18), we obtain the final formula for the sensitivity:

$$\left.\begin{aligned}\frac{d\varphi}{dg}=&-\frac{\sin\varphi}{f\left(1\pm\frac{q\cos\varphi}{F}\right)+\frac{abqMg\cos\varphi}{m\left(\frac{k}{n}+q\frac{h}{m}\right)}}\times\left[\left\{\frac{abMg}{\frac{mn}{k}\left(\frac{k}{n}+q\frac{h}{m}\right)^2}\pm\frac{f}{F}\right\}\frac{dq}{dg}\right.\\&\left.+\frac{abqM}{m\left(\frac{k}{n}+q\frac{h}{m}\right)}\left\{1-\frac{g}{\left(\frac{k}{n}+q\frac{h}{m}\right)}\left(\frac{1}{n}\frac{dk}{dg}+\frac{q}{m}\frac{dh}{dg}\right)\right\}\right],\\ \text{with}&\\ \frac{dq}{dg}=&-\frac{m}{h}\left[\frac{1}{gk}\left(\frac{k}{n}+q\frac{h}{m}\right)\left\{\frac{ns}{M}\left(\frac{k}{n}+q\frac{h}{m}\right)\left(\frac{dh}{dg}-\frac{dk}{dg}\right)-\left(k+g\frac{dk}{dg}\right)\right\}+\frac{1}{n}\frac{dk}{dg}+\frac{q}{m}\frac{dh}{dg}\right],\\ F=&\sqrt{\left(\frac{cem}{abn}\right)^2-(q\sin\varphi)^2}\,.\end{aligned}\right\} \tag{19}$$

We can simplify the formula (19) by putting

$$m\cong h,\quad n\cong k\quad\text{and}\quad\frac{dh}{dg}\cong\frac{dk}{dg}\cong 0\,. \tag{20}$$

Then, since $q\ll 1$, we get

$$\frac{dq}{dg}=\frac{1}{g} \tag{21}$$

Inserting (20) and (21) into (19), we obtain the approximate sensitivity formula as:

$$\left.\begin{aligned}\frac{d\varphi}{dg}=&-\frac{\left(abM\pm\frac{mf}{gF}\right)\sin\varphi}{mf\left(1\pm\frac{q\cos\varphi}{F}\right)+abqMg\cos\varphi}\,,\\ \text{with}&\\ F=&\sqrt{\left(\frac{cem}{abn}\right)^2-(q\sin\varphi)^2}\,.\end{aligned}\right\} \tag{22}$$

The formula (22) corresponds to the formula (4) for the sensitivity of the ordinary bifilar gravimeter. Next, let us obtain the correlation between the free oscillating period of rotation and the sensitivity. Consider that φ and θ are virtually varied by ε and δ respectively by some minute disturbing force. Then, according to the condition (10-4), the following relation ought to be established:

$$f\{\omega-(\theta+\delta)-(\varphi+\varepsilon)\}-T\frac{ce\sin(\theta+\delta)}{n}=0\,,$$

which, after transformation, becomes

$$\left\{f(\omega-\theta-\varphi)-\frac{Tce\sin\theta}{n}\cos\delta\right\}-\left\{f(\delta+\varepsilon)+\frac{Tce\cos\theta}{n}\sin\delta\right\}=0\,. \tag{23}$$

Since ε and δ are both small angles, we may put $\sin\delta=\delta$ and $\cos\delta=1$. Then, the terms in the first bracket of (23) vanish owing to (10-4), and we have

$$f(\delta+\varepsilon)+\frac{Tce\cos\theta}{n}\delta=0\,,$$

whence we get

$$\delta=-\frac{f}{f+T\frac{ce\cos\theta}{n}}\varepsilon\,. \tag{24}$$

Therefore, by (10-3) and (12), the restoring moment R caused by the angular variation ε becomes as follows:

$$R=T\left\{\frac{ce}{n}\sin\left(\theta-\frac{f}{f+T\frac{ce\cos\theta}{n}}\varepsilon\right)-q\frac{ab}{m}\sin(\varphi+\varepsilon)\right\} \tag{25}$$

As proved by numerical estimation,

$$\left|\frac{Tce}{fn}\cos\theta\right|\ll1\,,$$

so that

$$\frac{f}{f+T\frac{ce\cos\theta}{n}}\varepsilon=0(\varepsilon)\,.$$

Therefore we may put approximately as follows:

$$\sin\left(\frac{f}{f+T\frac{ce\cos\theta}{n}}\varepsilon\right)=\frac{f}{f+T\frac{ce\cos\theta}{n}}\varepsilon \quad\text{and}\quad \cos\left(\frac{f}{f+T\frac{ce\cos\theta}{n}}\varepsilon\right)=1\,.$$

Consequently, (25) is reduced to the following formula:

$$R=T\left(\frac{ce}{n}\sin\theta-q\frac{ab}{m}\sin\varphi\right)-T\left(\frac{ce}{n}\cos\theta\frac{f}{f+T\frac{ce\cos\theta}{n}}+q\frac{ab}{m}\cos\varphi\right)\varepsilon\,.$$

The first term on the right-hand side vanishes according to (10-3). Consequently,

$$R=-T\left(\frac{ce}{n}\cos\theta\frac{f}{f+T\frac{ce\cos\theta}{n}}+q\frac{ab}{m}\cos\varphi\right)\varepsilon\,. \tag{26}$$

Therefore, the equation of motion for the rotation becomes

$$I\frac{d^2\varepsilon}{dt^2}=-T\left(\frac{ce}{n}\cos\theta\frac{f}{f+T\frac{ce\cos\theta}{n}}+q\frac{ab}{m}\cos\varphi\right)\varepsilon\,, \tag{27}$$

where I is the moment of inertia of the weight. From (27) we get the oscillating period P as

$$P=2\pi\sqrt{\frac{I}{T\left(\frac{ce}{n}\cos\theta\frac{f}{f+T\frac{ce\cos\theta}{n}}+q\frac{ab}{m}\cos\varphi\right)}}\,. \tag{28}$$

On the other hand, from (10-3)

$$\left.\begin{aligned}\sin\theta&=\frac{qnab}{mce}\sin\varphi\,,\\ \cos\theta&=\pm\sqrt{1-\left(\frac{qnab}{mce}\sin\varphi\right)^2}\,,\end{aligned}\right\} \tag{29}$$

and from (10-1), we have approximately

$$T=Mg\,. \tag{30}$$

Inserting (29), (30) and $I=r^2M$ into (28), and squaring both sides we have

$$P^2=4\pi^2mr^2\frac{M\pm\frac{mf}{abg}\cdot\frac{1}{\sqrt{\left(\frac{mce}{nab}\right)^2-(q\sin\varphi)^2}}}{mf\left\{1\pm\frac{q\cos\varphi}{\sqrt{\left(\frac{mce}{nab}\right)^2-(q\sin\varphi)^2}}\right\}+abqMg\cos\varphi}\,, \tag{31}$$

where r is the radius of gyration of the weight. Combining (22) and (31), we obtain the following correlation:

$$\frac{d\varphi}{dg}=-\frac{ab\sin\varphi}{4\pi^2mr^2}P^2\,. \tag{32}$$

The expression (32) is indentical with the period-sensitivity relation (7) of ordinary bifilar gravimeter. Fig. 5 shows the relation between q, φ and $\frac{d\varphi}{dg}$. The numerical values of constructional constants adopted in the calculation are as follows: $a=6$, $b=3$, $c=e=0.1$, $m=130$, $n=10$, $f=1000$, $M=50$, $g=980$ c.g.s. units respectively, and $q=\frac{7.0}{1000}$ for curve I and $=\frac{7.2}{1000}$ for curve II. In Fig. 5, the full-line curves correspond to the plus sign of the formula (22) and the broken-line curves to the minus sign. The numerals on the sensitivity axis, similarly to Fig. 2, indicate the common logarithms of $\frac{dg}{g}$ corresponding to 1 mm. displacement of the optical image on the recording paper of which optical distance is 5 meters. The chain-line curves show the relation between P and $\frac{d\varphi}{dg}$ in the case of $r^2=2$ c.g.s. unit. The sensitivity of the double bifilar gravimeter thus theoretically obtained should be experimentally ascertained, and it will be described in some detail in the next section.

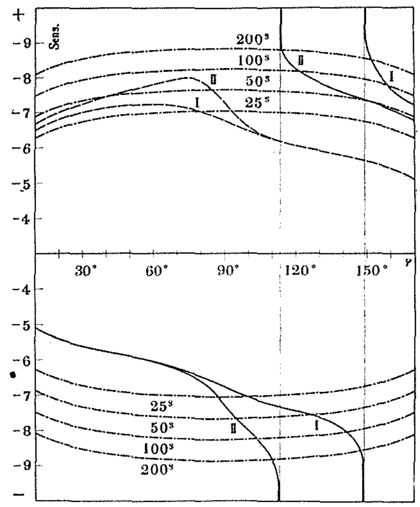

Fig. 5. Sensitivity curve of a double bifilar gravimeter.

(4) *Some experiments on the function of the double bifilar gravimeter.*

The writer has tentatively constructed a double bifilar gravimeter whose constructional constants are nearly the same as the above-mentioned numerical values. The method of appending a small mass to the gravimeter's weight is adopted for the measurement of the sensitivity. This is the most simple and sure method, but on the other hand, it also has a weak point in that the gravimeter cannot be used in so sensitive state on account of the lower limit of the dimension of the appendant mass. Therefore the experiment for sensitivity calculation had to be carried out in the condition of somewhat low sensitivity.

At first, the experiment on the period-sensitivity correlation will be mentioned. Since $\frac{d\varphi}{dg}$ is proportional to $\sin\varphi$ and the square of P by the expression (32), $\left(\varphi\text{-}\frac{d\varphi}{dg}\right)$ correlation ought to be expressed by a sine curve when P is kept constant, and $\left(P\text{-}\frac{d\varphi}{dg}\right)$ correlation by a parabola when φ is kept constant. Figs. 6 and 7 show those correlations respectively. The three dots in Fig. 6 and the four dots in Fig. 7 represent the experimental values respectively. These two figures indicate that the experimental results coincide perfectly with the theoretical ones, allowing for some experimental errors. Extraporating the experimental results obtained in the state of low sensitivity to the high sensitivity, we get the results as given in Table 2.

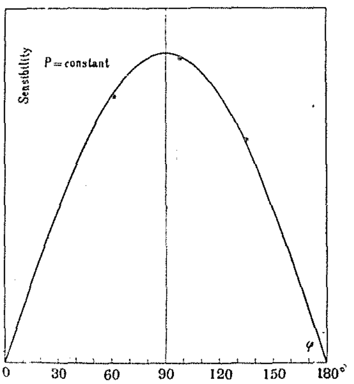

Fig. 6. Correlation between φ and $\frac{d\varphi}{dg}$.

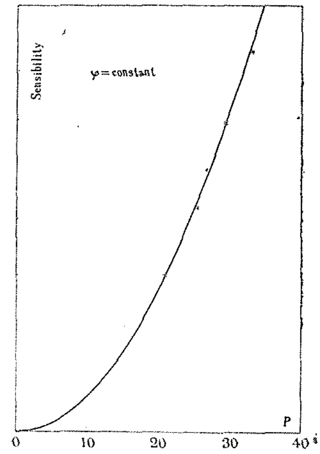

Fig. 7. Correlation between P and $\frac{d\varphi}{dg}$.

Table 2

φ	Optical distance	Period	Sensitivity
120°	5 meters	25 seconds	178.0×10^{-9}g/mm
"	"	50 "	44.4 "
"	"	100 "	11.1 "
"	"	150 "	4.93 "
"	"	200 "	2.78 "

Fig. 8 gives one example of gravitogram obtained by the above-stated double bifilar gravimeter constructed for a trial with a period of 160 seconds at the Abuyama Seismological Observatory of Kyoto University (λ=135°34′E, φ=34°52′ N, h=200 m). The subsequent restlessness is caused by the microseisms of the ground and the convection current of the air. Therefore, even if we could make the gravimeter more highly sensitive, it would be useless unless some devices were applied to reduce the effect of microseisms to negligibly small degree. Fig. 9 gives also another example of gravitogram obtained during one week by the same gravimeter as that in Fig. 8 with a period of 82 seconds at the same Observatory. The semi-diurnal periodic

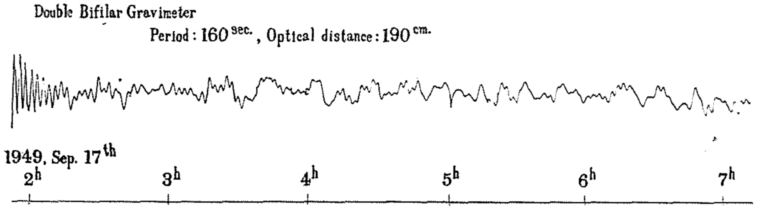

Fig. 8. Gravitogram in rapid recording at Abuyama.

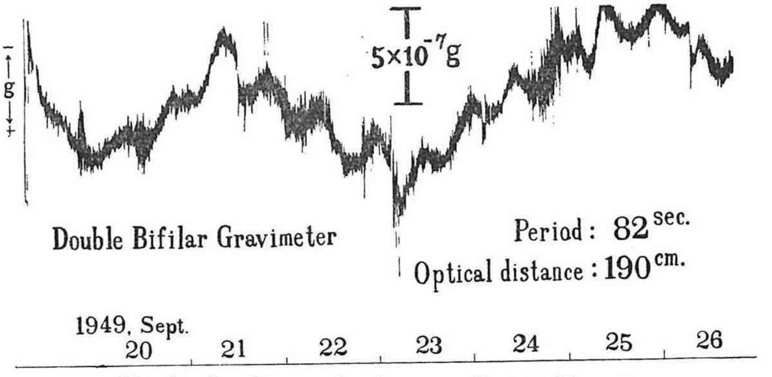

Fig. 9. Gravitogram in slow recording at Abuyama.

variation in the figure was proved to be the tidal variation of gravity, and the large swell was found to be caused by the variation of atmospheric pressure.

In conclusion, the problem of raising the sensitivity of gravimeter may be said to be settled to a certain degree by adopting the principle of double bifilar suspension, but in practical case other undesirable disturbances such as the earth's tremors, the change of meteorological elements and above all the unmanageable plastic flow of the main spring will greatly vitiate the gravimetric observation, and would really impede the aim of our research. In the next section, the experimental treatment on the problem of plastic flow of the gravimeter's main spring will shortly be reported.

3. Some experiments on the plastic flow of the spiral spring

The experiment on the plastic flow of the spiral spring of the bifilar gravimeter was made with the spring materials available in our laboratory. Since the plastic flow of the spring or, in other words, the drift of the gravimeter is the most essential and troublesome problem in all gravimetric observation, a detailed investigation should be made from every possible approach, and it will be discussed at other opportunity. In the present section a short account will be mentioned.

The materials used for the experiment are 'Tungsten', 'Steel', 'Tōhoku-elinvar' (manufactured by the Metal Research Institute, Tōhoku University), 'Sumitomo-elinvar A, B, C' (manufactured by the Sumitomo Metal Corporation), 'Fused silica' (manufactured by the Houston Technical Laboratories, U. S. A.), and 'Isoelastic alloy' (manufactured by the North American Geophysical Co., U. S. A.). These materials except 'Fused silica' and 'Isoelastic alloy' were formed into nineteen spiral springs whose annealing conditions were respectively different. The annealing conditions of these samples are as inserted in Table 3.

These samples were tested by the method shown in Fig. 10. The drift speed obtained from the experiment is shown in Figs. 11, 12, 13 and 14. In these figures,

Table 3

Number of group	Number of spring	Material	Annealing condition			
			Annealing temperature	Heating time	Holding time	Cooling method
I	1	Fused silica				
	2	Sumitomo-elinvar				N. C.
	3	Sumitomo-elinvar				N. C.
	4	Tungsten				N. C.
	5	Steel A				N. C.
II	6	Sumitomo-elinvar C	580°C	44 min.	0 min.	N. C.
	7	Sumitomo-elinvar C	580	40	0	Q. O.
	8	Sumitomo-elinvar C	580	39	0	Q. W.
	9	Sumitomo-elinvar B	580	52	0	N. C.
	10	Sumitomo-elinvar B	580	30	0	Q. O.
III	11	Sumitomo-elinvar C	700	62	10	Q. O.
	12	Sumitomo-elinvar C	800	73	10	Q. O.
	13	Sumitomo-elinvar C	900	58	0	Q. O.
	14	Sumitomo-elinvar C	800	7	0	Q. O.
	15	Sumitomo-elinvar C	580	40	0	Q. O.
IV	16	Steel B				
	17	Sumitomo-elinvar A				
	18	Sumitomo-elinvar A				
	19	Tōhoku-elinvar				
	20	Tōhoku-elinvar				
	21	Isoelastic alloy				

N.C.: Natural cooling, Q.O.: Quenching in cotton-seed oil, Q.W.: Quenching in water

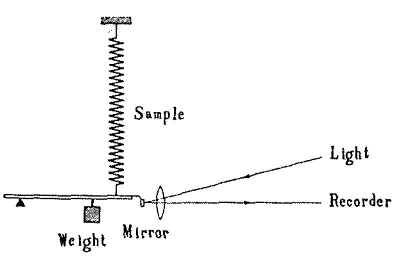

Fig. 10. Schematic diagram of drift experiment.

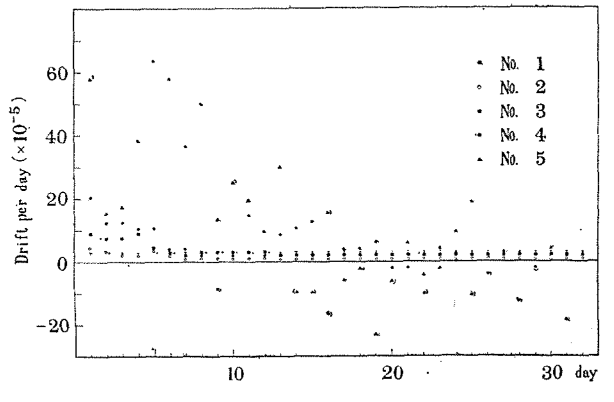

Fig. 11. Change in rate of drift with time.

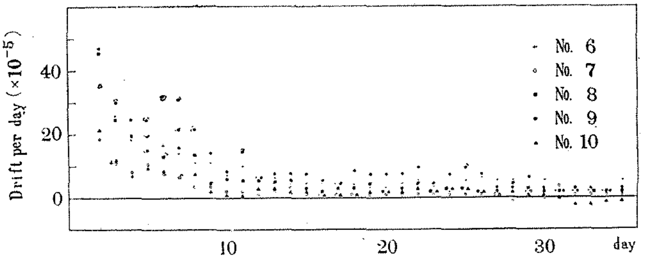

Fig. 12. Change in rate of drift with time.

the numerals on the abscissae denote the number of days from the beginning of the suspension of springs, except for the spring made of 'Fused silica' whose value is that of about fifteen months after suspension.

From these experiments, the following may be concluded:

1. Among them, the spring made of 'Isoelastic alloy' is considered to be the best in the point of drift, and no drift of the spring was observed in a range of the present measurement. Among the other materials except 'Isoelastic alloy', 'Sumitomo-elinvar C' is comparatively excellent.

2. With regard to the cooling methods, qnenching in cotton-seed oil produces a good effect on springs with respect to drift.

3. With regard to the heating conditions, the following case was the best as far as 'Sumitomo-elinvar' was concerned:

a. Heating temperature is about 600°C,

b. Heating time is between from 8 to 10 minutes,

c. Holding time in maximum temperature is about 10 minutes. But it is to be remarked that, in any case, the heating was carried out in air.

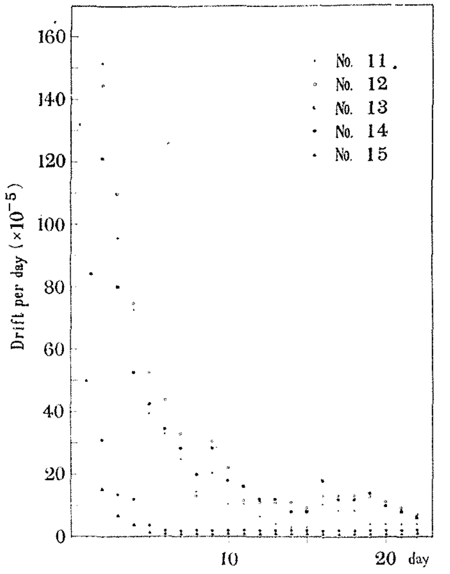

Fig. 13. Change in rate of drift with time.

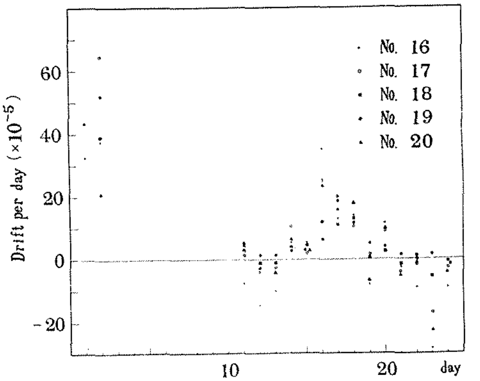

Fig. 14. Change in rate of drift with time.

4. Regarding drift, the putting of 'Carbon' in the furnace for the purpose of preserving the materials from oxidation seems to be effective.

Among the all springs except that of 'Isoelastic alloy', as seen in the figures, the spring of No. 2 has the smallest drift, even which amounts to 0.76×10^{-5}/day. Converting it into gravity change, this value is equivalent to about thirty times as large as the total amplitude of tidal variation. In practice, such an enormously large plastic flow of the gravimeter spring as above described greatly hinders the gravimeter from increasing its sensitivity to exceedingly high degree, and the elimination of secular drift of the gravimeter is the most important as well as the most unmanageable and troublesome matter, the solution of which should necessarily be found in near future for the purpose of precise and continuous secular observation of the gravity change.

4. Observation of tidal variation of gravity with the double bifilar gravimeter

The first long period observation of the tidal variation of gravity by the use of a double bifilar gravimeter was carried out at the adit of the Ikuno Copper Mine. The situation of the observation room in the mine is as follows: $\lambda=134°50'$E, $\varphi=$

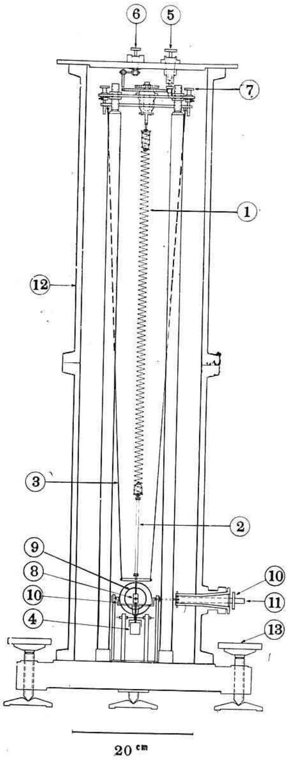

Fig. 15. Plan of the double bifilar gravimeter at Ikuno.

1: Main spiral spring (fused silica)
2: Inner bifilar string (30 microns super invar)
3: Outer bifilar string (30 microns super invar)
4: Weight (brass)
5: Fine torsion adjuster of main spring
6: Fine vertical adjuster of main spring
7: Vertical adjuster of outer bifilar string
8: Reflecting mirror
9: Lens
10: Calibration apparatus
11: Exhaust-opening
12: Cover (glass)
13: Level adjuster

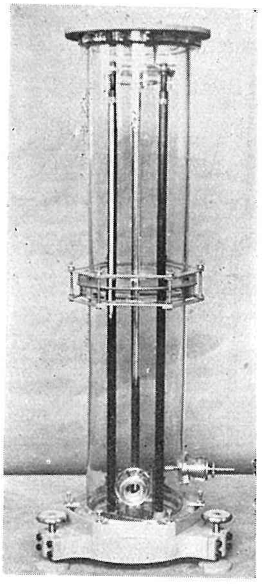

Fig. 16. Photograph of the double bifilar gravimeter at Ikuno.

35°10′ N, $h=440$ m above the sea level, and 237 m depth under the ground surface, in which mine the tiltmetric observations had continued since 1943. The temperature of the observation room is 17°C, and it maintains almost the same temperature through the year. The gravimeter had been set on April 3rd, 1952, and the observation was commenced on April 26th, 1952, it being continued to the present. The gravimeter used for the observation at Ikuno is somewhat smaller compared with that described in § 2. The structure of the gravimeter and its exterior view are as illustrated by the plan of Fig. 15 and the photograph of Fig. 16. The main spring was made of 'fused silica' which was No. 1 in Fig. 11 and manufactured by the Houston Technical Laboratories, U. S. A., the maximum durable load and the extension of which are 40 grams and 1.8 cm/gr respectively. The inner and outer bifilar strings are both composed of super-invar wires of 30 microns in diameter. The materials of the other main constructions are as explained in the figures. The gravimeter has no temperature compensater, for temperature variation in the observation room is very small. The gravimeter has an air-evacuating apparatus, but it was not used in the present observation. The rotating speed of the recording paper is 22.0 mm. per day, and the recording cylinder revolves once in two weeks. The optical distance is 398 cm. As mentioned in § 3, the spring has a considerable drift. Therefore, in order to prevent the light image of the gravimeter from getting out of the recording photographic paper, the sensitivity of the gravimeter should be kept at undesirably low degree. This restraint of sensitivity greatly reduced the efficiency of the double bifilar gravimeter and practically vitiated the present observation. Fig. 17 gives one example of gravitogram obtained during present long period observation. The calibration was done by means of the method of appending a small mass to the weight, similar to the method described in § 2. The numerical values of the quantities concerned with the calibration are as inserted in Table 4.

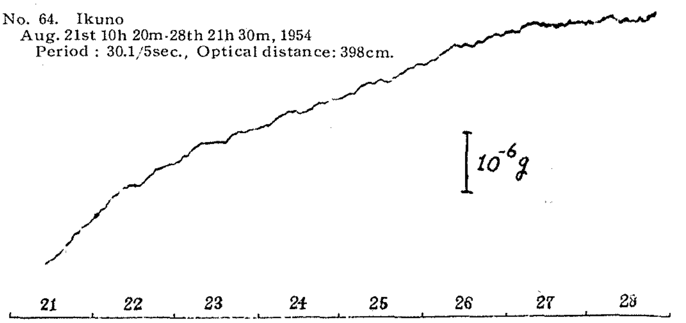

Fig. 17. Gravitogram with the double bifilar gravimeter at Ikuno.

Table 4

φ	120°
Period	6 sec
Optical distance	398 cm
Mass of the weight system Weight, containing a mirror and a cross-shaped body A half of mass of the main spring	26.690 gr 21.789 gr 4.901 gr
Mass of the appendage	0.63 mgr
Displacement of the light image on the recording paper	57.5 mm

The motion of the light image when the appendage was put on the weight and removed was as shown in the record of Fig. 18. From the record, we lean that the gravimeter does not leave any hysteresis caused by such a procedure. In consequence of the calculation by Table 4, we obtain the following value as the sensitivity:

$$\text{Sensitivity}: \quad 4.105\times10^{-7}\ g/\text{mm},$$

This is almost twice the sensitivity of Schweydar's gravimeter, but it is too low to be applied for the precise observation of tidal variation of gravity. From these

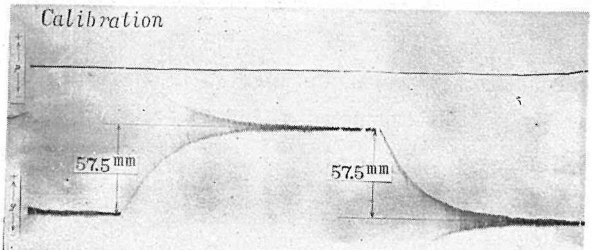

Fig. 18.

circumstances it is naturally understood that the most urgent problem in the precise gravimetric observation is to minimize the drift of the gravimeter as much as possible for the purpose of keeping sufficiently high degree of sensitivity in the gravimeter. The experiments on the effect of ground-tilt for the gravimeter was made by means of the method of really tilting the triangular base on which the gravimeter was set up. According to the results of the experiments, the changes becaused by tilting of 1″ angle are expressed in terms of variation of gravity as follows:

In the longitudinal direction: $-2.67\times10^{-8}\ g$/second angle,

In the transversal direction: $+30.05\times10^{-8}\ g$/second angle.

According to the tiltmetric studies of E. Nishimura (**26**), the observed tidal variations of tilting of M_2-component at Ikuno are as follows;

In the N–S direction: $0''.00240 \cos(2t-264°)$,
In the E–W direction: $0''.00458 \cos(2t-88°)$.

On the other hand, the expected amplitude of the tidal variation of gravity of the M_2-component was about $5\times10^{-8}g$. Therefore, we usually did not take any account of the corrections for tilting as far as the present tentative observation was concerned. The duration of the observation was about thirty-one months, but the period which was made subject to harmonic analysis was about three months during July 17th to October 17th, 1953, for various reasons such as interruptions in recording. Since the drift speed was not uniform through the period of analysis, the drift was reduced by subtracting from the corresponding observed values the running averages of the observed values over periods of 25 hours. As a result of a harmonic analysis of the data drift-removed, the following tidal values of gravity were obtained:

M_2(lunar semi-diurnal component): $5.43\times10^{-8}g\cos(2t-145°.03)\pm0.16\times10^{-8}g$
S_2(solar semi-diurnal component): $13.42\times10^{-8}g\cos(2t-17°.24)$
O_1(lunar diurnal component): $8.17\times10^{-8}g\cos(t-193°.38)$

As seen at a glance, these results are very curious, for the amplitudes of the S_2 and O_1-components are larger than the M_2-component, notwithstanding the fact that the former should theoretically be smaller than the latter. Detailed discussions on the cause of this curious result will be made later on. The theoretical value of the tidal variation of gravity, in other words, the tidal variation of gravity in the case that the earth be perfectly rigid is calculated as follows:

$$M_2(\text{theor.}):\ 5.01\times10^{-8}g\cos(2t-180°).$$

The tidal factor G calculated by dividing the value of observed amplitude by that of theoretical amplitude is

$$G\equiv1-\frac{3}{2}k+h=1.08\pm0.03\,,$$

and the phase lag κ is as follows:

$$\kappa=325°.03\ \ (\text{or}\ \ -34°.97)\,.$$

The value of $G=1.08$ thus obtained is considered to be right in approximation, but its accuracy is not sufficient to discuss the small difference, if it exists, of the G-value, because the present observation at Ikuno had to be operated with an undesirably low sensitivity in the gravimeter because of the large drift of the instrument. And, on the other hand, the G-value recently obtained by the one-month's precise observation with a portable Worden Gravimeter at Kyoto (**27**) was estimated as $G=1.18_5$, $\kappa=1°.96$. The difference between these values at both stations of Ikuno and Kyoto seems to be too large, even if the effects of geological structure, influences of oceanic tide and others might be different for both stations. The detailed treatment of the problem of the local character of the G-value, if it exists, will be reported in a succeeding article after the accomplishment of precise gravimetric observations at many stations under various conditions. Returning to the present observation at Ikuno the cause of the anomalous fact that the observed amplitude of the S_2-component is larger than M_2-component may be interpreted as follows. The curve of the S_2-component (including the S_1-component) bears a good resemblance to that of the diurnal variation of atmospheric pressure except phase difference of about three hours as seen in Fig. 19. By reason of the similarity between the forms of both curves, it is concluded that

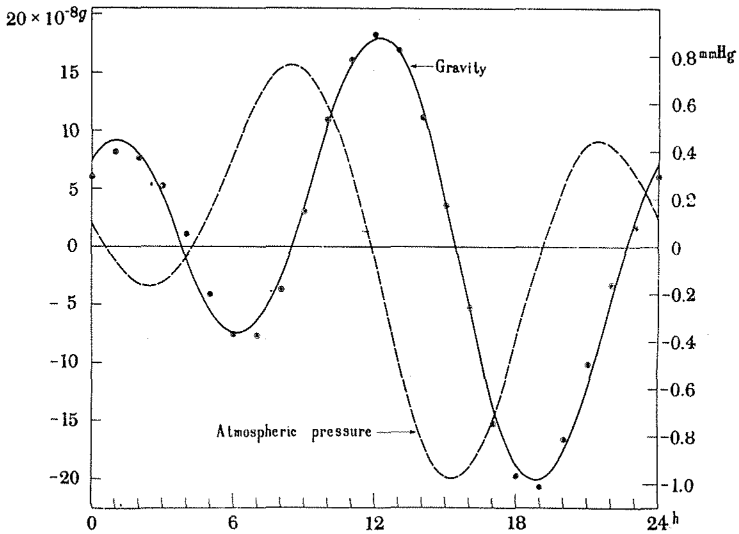

Fig. 19. Solar term in gravity variation and change of atmospheric pressure.

the abnormality of the amplitude of the S_2-component would certainly originate in the variation of atmospheric pressure. The similarity between the gravity-curve and the pressure-curve is also clearly recognized at the time of the passing of a typhoon, as in Fig. 20 which is a typical example among the many which indicate its similarity. In this case, contrary to the case of the S_2-curve, the phase difference is almost undetectable. Two ways as to how the variation of atmospheric pressure has an influence on the gravimeter may be supposed: the one is by way of the variation of buoyancy for the weight, and the other the variation of upperward attraction of air mass. By whichever manner, the decrease of pressure should apparently present itself as the increase of gravity, but the actual state is contradictory to this. In the present time it cannot be interpreted through what mechanism the variation of pressure has an influence as is shown in Figs. 19 and 20 upon the gravimeter. This question must be further examined in the future.

No. 38. IKuno, (partial), Sept. 1953.

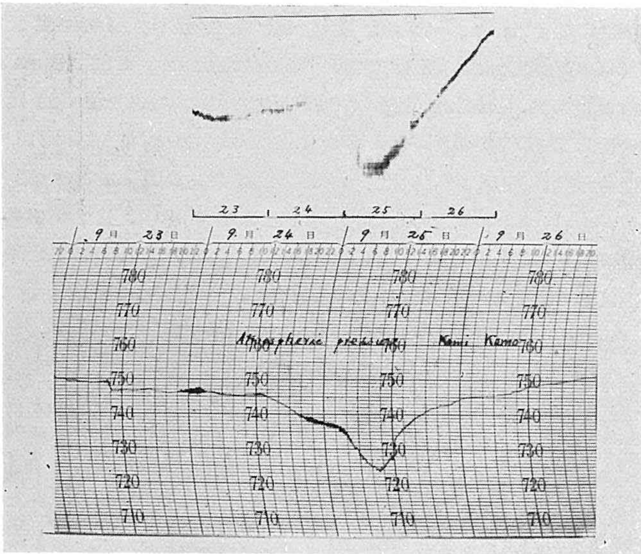

Fig. 20. Anomalous change of gravity at the time of the passing of a typhoon.

In concluding this section it was ascertained from the present observation at Ikuno that the problems of high sensitivity of the gravimeter and the effect of temperature change upon the instrument might, to a certain degree, be solved by the application of a double bifilar suspension and by setting the instrument in a deep underground room. But it is also quite evident that the remaining and most difficult problem for a long continuous and precise observation of tidal and secular variation of gravity is how to minimize gravimeter drift. The problem of drift will persistently be treated in succeeding articles.

Summary and acknowledgement

All the contents described in the present article may be summarized as follows. For the purpose of raising the sensitivity of the gravimeter, a double bifilar gravimeter was devised by the improvement of an ordinary bifilar gravimeter. Also, the function of double bifilar gravimeter was ascertained from both the theoretical and experimental sides. It became very easy to raise the sensitivity of the gravimeter to a sufficiently high degree by the application of a double bifilar suspension, but, to our regret, it was also proved that its efficiency was greatly reduced by the large drift of the gravimeter in the practical observation. A long period observation with the double bifilar gravimeter had been carried out, as a trial, at a deep underground room of the Ikuno Copper Mine, and a value of 1.08 was obtained as the tidal factor of gravity, referred to as the M_2-component. Although the accurate value of the tidal factor was not obtained by the present observation, it was proved that a gravimeter of the double bifilar suspension type is certainly useful for a long continuous and precise gravimetric observation with the condition of a suitably small instrumental drift.

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