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ELASTIC PROPERTIES OF ROCKS WITH RELATION TO THE EARTH'S INTERIOR

BY

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ABSTRACT

With respect to the value of the pressure coefficient of rock-elasticity hitherto discussed, the remarkable discrepancy between the values obtained from rock-experiments and from the velocity distribution of seismic waves in the Earth's interior was fairly reconciled by an analysis of the rock-experiment data by considering the relation between the elasticity and density rather than the direct treatment of the pressure.

The pressure coefficient of the bulk modulus and the velocity of the dilatational elastic wave with respect to dunite determined from the high pressure experiment was ascertained to be in a good agreement with those obtained from the velocity distribution of the seismic wave in the upper mantle. In the present article particularly the effect caused by micro-pore space in the rocks upon the character of rock-elasticity in a range of comparatively low pressure is discussed in detail.

1. Introduction

The study of the structure and property of the Earth's interior is an important and attractive field of research in geophysics. Recent developments in geophysical techniques have greatly facilitated the observational approach even to the deep central part of the Earth. The Earth structure with respect to the seismic wave velocity has been investigated more and more in detail by the analysis of seismograms obtained at many well-equipped observatories all over the world. And, moreover, our knowledge on the elastic property of the Earth has also gradually advanced with that of the phenomena of Earth tides and latitude variation. Concerning the density distribution of the Earth's interior the values as derived from various theoretical and observational treatments have shown a fairly good agreement in recent years. But problem of temperature distribution and the thermal state of the Earth's interior still remains unsolved due to the lack of knowledge on the internal distribution of radioactive elements and to other reasons as well. Even if the structure and such properties of the Earth's interior as the existence of discontinuity and the physical quantities of density, pressure, temperature, elasticity, and seismic wave velocity were fully clarified, another important problem may yet remain, i.e., of what is the Earth's interior composed and how does this core material under high pressure and temperature behave against both internal and external stress? In this point ordinary geophysical field observation is generally powerless for examining the internal dynamics of the Earth. Laboratory experiments and theoretical considerations of materials under high pressure and temperature, are certainly powerful means for this type of investigation especially on the phenomena of plasticity and rupture in the Earth. The properties of plasticity and the modes of rupture of the material in the Earth may doubtlessly play an important role in the process of earthquake occurrence, accumulation of seismic energy, crustal deformation, orogeny and epirogeny, continental drift, core mobility and other important telluric phenomena.

A great advance in the experimental study of the effect of high pressure and temperature on property changes in rocks has been made by the elaborate experiments in U. S. A., such as those of L. H. Adams at the Geophysical Laboratory, and P. W. Bridgman and F. Birch at Harvard University. The compressibility of rocks have early been measured up to some tens of thousands bars of pressure under several hundreds degrees centigrade, but it is only recently that the velocity of elastic waves in rocks could be measured by the supersonic method up to only several thousands bars under several hundreds degrees as was done by D. S. Hughes and J. H. Cross at the University of Texas. As the pressure of 10^3 bars, 10^4 bars, and 10^5 bars correspond roughly to those prevailing at the depths of 3 km, 30 km, and 300 km respectively below the Earth's surface, their respective pressure values tell us also the pressure state at the foci of local shallow earthquakes, large destructive earthquakes, and deep plutonic earthquakes. For the purpose of studying the nature and occurrence mechanism of large destructive earthquakes it is desirable to examine by laboratory experiment the change in rock properties under conditions of more than several ten thousands bars of pressure and one or two thousands degrees centigrade, and property to be examined should contain the density change, elastic property, seismic velocity, thermodynamical element, electric and magnetic property, plastic deformation and mechanism of rupture. These demands will be fulfilled as experimental technique advances. But the artificial arrangement of rocks in the laboratory in the state of more than several hundreds of thousand or several million bars of pressure and several thousands degrees centigrade to correspond to the state of material in the lower mantle and core of the Earth's deep interior is technically almost, if not quite, impossible in spite of the astonishing advance of modern scientific techniques. On the other hand, the behaviour of materials under these conditions of very high pressure and temperature such as more than several tens of millions bars of pressure and degrees of temperature can be rather simply and easily treated from the theoretical side of modern thermodynamics, namely for study on the internal state and energetics of the Sun and other stars, the optically and magnetically variables, the pulsating stars, super-novae and other astrophysical phenomena, the purely theoretical treatment can be and has been successfully applied. From the circumstances above stated it may be concluded, if we may be permitted an over-simplification, that pressure and temperature in the Earth's deep interior are too high to be experimented in the laboratory and too low to be treated by the purely theoretical thermodynamics of recent development. Therefore, a combined method is naturally introduced for investigating the nature and behaviour of the material, that is a method of extrapolation to a higher position of the experimental data obtained at the lower pressures and temperatures or to a lower position of the theoretical deductions of modern thermodynamics for the higher pressures and temperatures along with a suitable method of combining the results.

Fortunately, the values of some physical quantities such as density, pressure, seismic wave velocity and elasticity, even in the deeper part of the Earth can be fairly accurately determined by various geophysical observations on the Earth's surface, so that the validity of prospection assumed in various theoretical treatments of the material in the Earth's deep interior can safely be checked by comparing these theoretically deduced values with those of physical properties directly observable. However, we should not confine our studies to this aspect alone or remain satisfied with a good agreement between theoretically derived geophysical values and those obtained from geophysical field observation, but we should advance towards the ideal of deriving a theory and model of matter which would explain the reason for discontinuity and the mode of behaviour of material in the Earth's interior under high pressure and temperature against the internal stress and external force. After these considerations the nature of earthquakes and their source of energy, the existence of the central core of the Earth and its mobile property, and even the original formation of the Earth itself, may possibly be revealed to us in process of time.

Concerning studies hitherto made on the Earth's interior along the lines above stated, i.e., the application of the modern theory of matter to the material in the Earth under high pressures and temperatures in consideration of data based on experimentation and direct geophysical observation, a brief account will be given: Early in 1923 E. D. Williamson and L. H. Adams (**1**) and others discussed the density distribution of the Earth's interior by taking the pressure dependency of density and compressibility of rock into account, while, on the other hand, H. Haalck (**2**) introduced, for the first time, the concept of atomic potential in a simple form of interatomic repulsive force into the calculation of the relation between the compressibility and pressure of the material in the Earth. And these two pioneering works, owing to their different standpoints, are also considered to represent the establishment of the two important and distinct ways of research on the Earth's interior according to the modern theoretical treatment of state of matter. Namely, the former is the extrapolation of rock properties obtained in laboratory experiment to the deeper part of the Earth, while the latter examines the validity of the theory postulated by comparing its deduced values with those derived from geophysical observation.

On the most attractive and essential problem of the Earth's core, W. Kuhn and A. Rittman (**3**) published in 1941 their opinion on the large hydrogen content in the core, a concept which differs greatly from the ordinary one. K. Sezawa and K. Kanai (**4**) treated the problem of the core in a thermodynamical way in 1940 and 1941, and several researchers such as J. Lynch (**5**), H. Lorent (**6**), A. Eucken (**7**), A. Mercier (**8**), have also taken up the problems of the core in various ways. Since 1948 W. H. Ramsey (**9, 10, 11, 12, 13**) treated the formation and existence of the core of the Earth and other planet from the very particular standpoint of the phase transition of matter by pressure only, and discussed the existential condition of the central core in each solar planet and the difference of their mean densities. In 1952 H. Miki compared (**14**) the degrees of stability between the three phases of solid, liquid and gas as being the state of the Earth's core from the standpoint of statistical mechanics. On the property of the mantle F. Birch applied since 1935 (**15, 16, 17, 18**) the F. D. Murnaghan theory of finite strain to the Earth's interior with the consideration of date derived from laboratory experiments under high pressure, and treated various problems of the mantle. Y. Shimazu and H. Takeuchi also treated the problem of the Earth's interior since 1949 (**19, 20**) according to the Murnaghan theory of finite strain, and Y. Shimazu (**21, 22**) published comprehensive papers for study of state of materials composing the Earth's interior from the standpoints of the finite strain theory and the comparative method with meteorite constituents.

On the density distribution of the Earth's interior, K. E. Bullen has been very actively developing his theory since 1936 (**23, 24, 25, 26, 27, 28, 29, 30**) and W. M. Elsasser (**31**), H. Miki (**32**), and others have also been treating the problem of density distribution from approach of statistical mechanics. In connection with Bullen's density distribution a brilliant work on the theory of Earth tides was made by H. Takeuchi in 1950 (**20**).

Concerning the problem of temperature distribution within the Earth, many studies have been made from various standpoints, and here the researchers directly connected with the Earth's interior may be mentioned. F. Birch (**33**), H. P. Coster (**34**), W. D. Urry (**35**), A. E. Benfield (**36**), E. C. Bullard (**37**), A. E. Verhoogen (**38**), R. J. Uffen (**39**), T. Rikitake (**40**), J. A. Jakobs (**41, 42**), H. Miki (**43**) and others have been making great contributions to the study of thermal states within the Earth. Lastly, in the recent development in theories on the origin of the main geomagnetic field and its secular variation, we are greatly indebted to W. M. Elsasser (**44, 45, 46, 47**) and

E. C. Bulland (**48**, **49**) whose theories were fairly well ascertained by H. Takeuchi and Y. Shimazu (**50**, **51**) from the standpoint of a self-exciting process in the magneto-hydrodynamics within the core. In connection with geomagnetism, the electric properties of the Earth's interior were, recently investigated in detail by T. Rikitake (**52**).

Under the circumstances above stated, the various problems concerning the state and nature of material in the Earth's interior will be taken in our series of investigations and will be treated from the theoretical point of view in consideration of the data obtained in laboratory experiments and geophysical field observations. In the present article the data obtained in laboratory experiments on the elastic properties of rock are discussed, comparing with those derived from the observation of natural earthquakes. And the conclusions are applied to the problem of the elastic state of the Earth's mantle, along with some considerations on its constitution.

2. Compressibility of rocks and the Earth's interior

The velocities of seismic waves in the Earth's interior have been investigated more and more in detail since the days of E. Wiechert (**53**) ; and, except in some particular points, the values recently obtained by H. Jeffreys (**54**), B. Gutenberg and C. F. Richter (**55**), and B. Gutenberg (**56**) generally show a fairly good agreement.

From the data obtained by the analysis of seismic waves, we can derive both density and the elastic constants at any point in the Earth, under the assumption that the material in the Earth is in a state of compression by the weight of the overlying material ; that is,

$$\frac{d\rho}{dR}=\frac{\rho}{k}\frac{dP}{dR}, \tag{1}$$

$$\frac{dP}{dR}=-g\rho, \tag{2}$$

where P, k, ρ, and g are the pressure, bulk modulus, density and gravity acceleration respectively at a point of distance R from the Earth's centre.

The above relations were first applied by L. H. Adams and E. D. Williamson (**1**) and recently by K. E. Bullen (**28**).

And k/ρ in equation (1) is, on the other hand, expressed by the velocities of the seismic P and S waves as follows :

$$V_p=\sqrt{\frac{k+4\mu/3}{\rho}}, \tag{3}$$

$$V_s=\sqrt{\frac{\mu}{\rho}}, \tag{4}$$

$$\frac{k}{\rho}=V_p^2-\frac{4}{3}V_s^2, \tag{5}$$

where V_p, V_s, and μ denote the velocity of the P wave, S wave and modulus of rigidity respectively. Therefore, the density, pressure and elastic constants at any depth in the Earth's interior will be expressed as some function of radius R when the temperature of the Earth is not high.

On the other hand, many laboratory experiments were made to examine the effect of pressure upon the elasticity of rocks and the results thus obtained were compared with those of the seismic data.

In this case, there always remains the problem as to what sort of rock each layer of crust and the mantle are composed of. From the geological standpoint, rock of the olivine system which contains a great quantity of Mg strongly combined with isolated SiO_4–tetrahedron is considered to form the lower part of the Earth's crust, and, as the layers go up in the crust, rock abundant of SiO_4 compounds with Ca and Fe as well as pyroxene groups (mainly of di- and tri-silicates) are found to prevail. The upper part of the crust is generally composed of granitic rock containing Na and K. With regard to the material underlying the crust, some geologists insist on the existence of basaltic rock under the granitic crust, while others assert eclogite as the under crustal material. It is natural to suppose that the rock under the crust may be considerably deformed by pressure; therefore, the material under the crust may possibly be treated as in a pseudo-crystalline or vitreous, glassy state. On the constituents and system of rocks composing the layers of the crust and the upper part of the mantle some detailed treatment will be made in a succeeding article in relation to the geophysical (geodetic, seismic and meteoritic) investigations of the nature and behaviour of the crust.

In the present article the problem will be treated under the simple assumptions that the material of the crust is mainly granitic and gabbroic, and that underlying the crust is dunite. These assumptions are generally adopted in various aspects of seismology, geology, mineralogy and the study of meteorites. H. Jeffreys pointed out the fact in his paper in 1952 (**57**) that the increasing degree of seismic wave velocity observed in the upper mantle differs greatly from that deduced in laboratory experiments on the effects of high pressure on the property of rocks. Near the surface of the mantle at some 30 km depth and under the pressure of 10^4 bars the value of dk/dP calculated from the data of seismic wave velocity obtained by H. Jeffreys (**54**) and B. Gutenberg (**56**) are

$$\frac{dk}{dP}=4\sim6\,. \tag{6}$$

On the other hand, the value of dk/dP obtained frcm the laboratory experiments on dunite (mainly composed of olivine) by L. H. Adams (**58**) is as the mean value between the pressure condition of 2000~12,000 bars

$$\frac{dk}{dP}\approx 20\,. \tag{7}$$

This discrepancy is not attributable to the difference between the effect of temperature distribution usually assumed in the Earth's interior upon values obtained from the seismic waves and the effect of room temperature upon the laboratory experiment. Jeffreys suggested the following three effects as the possible cause of the discrepancy: (i) A rapid decrease in the increasing rate of elasticity with increasing pressure; (ii) An anomalous increase in the effect of temperature upon elasticity with depth; (iii) An increase in the degree of admixture of low velocity material with depth. In this article this discrepancy will be treated below.

On laboratory experiments on the effects of pressure and temperature on the elasticity of rock, many elaborate works have been made by L. H. Adams and his collaborators (**58, 59**), Zisman (**60**), Ide (**61**), Bridgman (**62, 63**), Birch and his collaborators (**16**, **17**, **18**), and Hughes (**64**, **65**). Through their experiments, many important properties of rock under high pressure have gradually been found, such properties as the rapid increase in rock-elasticity by pressures up to 2,000 bars and over that pressure a somewhat slow increase, and a hysteresis phenomenon of rock deformation under a low pressure. Zisman examined the difference of property in direction, the hysteresis effect and the relation between the elastic constant and the frequency of applied force. Ide reported a considerable effect of hysteresis in his experiment on the change of dilatational velocity with temperature. Recently Hughes and Cross (**65**), investigated the rock-elasticity by applying the supersonic wave in the range of up to 5,000 bars-pressure and 300°C–temperature. In his experiment it was clearly observed that the mode of change of elastic wave velocity with pressure and temperature is classified into two parts. Namely, the velocity of elastic wave of rock increases rapidly with pressure up to nearly 2,000 bars and then slowly and linearly increases in the pressure range over 2,000 bars as seen in Fig. 1.

They explained the rapid increase of wave velocity in the low pressure range as derived from the effect of a compression of the pore spaces in rock and the slow and linear increase in high pressure range as the normal and reversible change of wave velocity with pressure. In his paper it is reported that the frequency effect above described is negligibly small.

It is easily understood from the above-mentioned complexity of rock under high pressure and temperature that it is a very difficult task to compare the data obtained by laboratory experiments with seismometric and other observational data from the Earth's interior. In other words, the application and extension of the knowledge on rocks derived from laboratory experiments to the interpretation of any property of the real existing matter in the Earth's deep interior should cautiously be made under the consideration of difference of structure, chemical composition, physical state and other various circumstances in both cases.

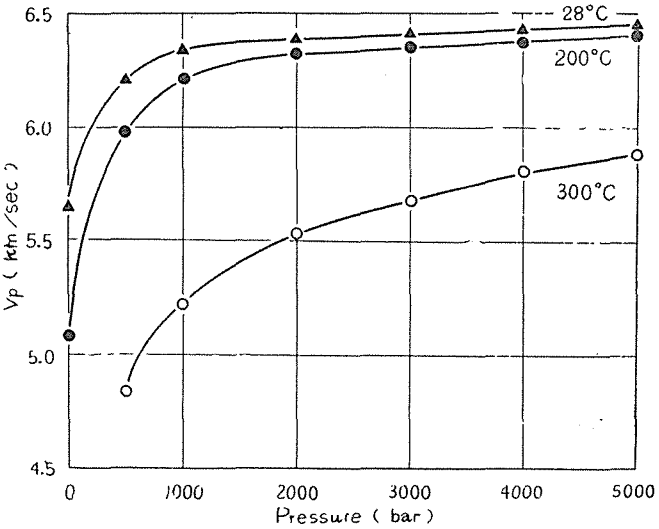

Fig. 1a. Relation between the dilatational wave velocity of granite and the exerted pressure. (After Hughes and Cross)

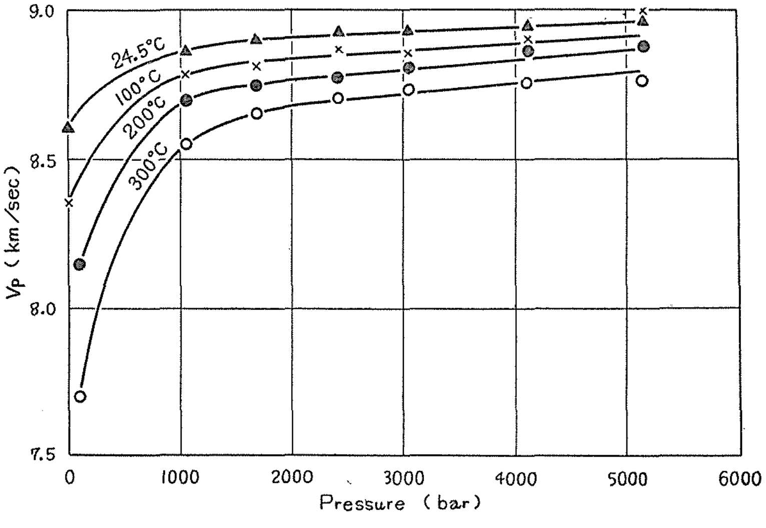

Fig. 1b. Relation between the dilatational wave velocity of dunite and the exerted pressure. (After Hughes and Cross)

From Fig. 1b, we see that at high pressure $dV_p/dP \approx 0.2\,\mathrm{km/sec}\cdot 10^4$ bars for dunite, whereas from the data by seismic observation, we find $dV_p/dP \approx 0.1\,\mathrm{km/sec}\cdot 10^4$ bars at the depth of 300 km (Jeffreys and Gutenberg). Therefore the discrepancy pointed out by Jeffreys was almost disappeared. But, as D. S. Hugdes and J. H. Cross pointed out, there still remains the effect of porosity. In the present article,

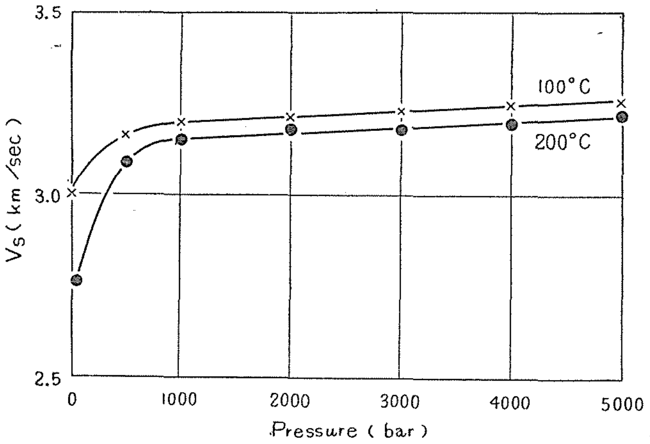

Fig. 1c. Relation between the shear wave velocity of granite and the exerted pressure. (After Hughes and Cross)

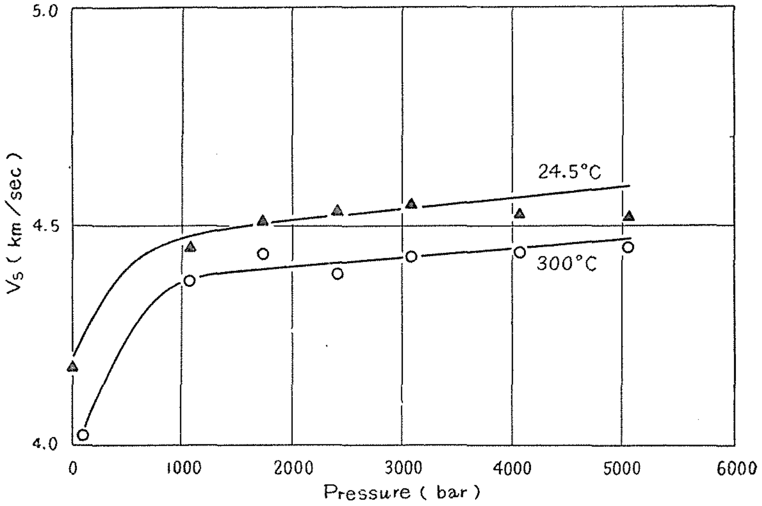

Fig. 1d. Relation between the shear wave velocity of dunite and the exerted pressure. (After Hughes and Cross)

the writer tries to examine the variation of elastic constants due to pressure only, eliminating the effect of porosity and temperature.

Among the various conditions affecting the elasticity of rock, the existence of porosity is generally considered and experimentally ascertained as one of the most essential. The pore spaces in rock are considered to originate from the circumstances in the epoch of rock formation. Namely, of the sedimentary rock, a considerable amount of pore spaces is inevitable as the sedimentary rock is formed under very low pressure, and also the igneous rock may have derived from the ejection and pool of gas contained in the liquid state during the period of rock-solidification. Even in a single crystal of natural quartz many cracks and pore spaces are frequently found. Thus all rocks are generally considered to be more or less porous material and their elasticity depends largely on the amount and manner of distribution of porosity. In the process of rock compression under increasing hydrostatical pressure, the pore space would, at first, become deformed and the main substance would suffer little compression. Therefore, the compressibility of rock measured in the laboratory is considered to show apparently a considerably large value in the range of initial low pressure and gradually to approach the small real value after the stage of pore space disappearance. These effects are shown in Hughes' experiment on the variation of elastic wave velocity with pressure and, in his case, the effect of pore space was estimated to be negligibly small over the range of pressure of 2,000 bars. And, on the other hand, what are the practical effects of pore space on the elastic property of the material in the Earth's interior? To what depth of layer in the Earth is estimated the existence of influential pore space? As a natural consequence of Hughes' and other laboratory experiments, the effect of pore space upon various properties of rock is roughly considered to be influential upon the crustal layer down to nearly 5 km in hydrostatic pressure of 2,000 bars, and ineffective for the deeper part of crust and mantle. But, from another point of view, the enclosed volatile constituents in the deep crustal layer and the upper mantle may exert the same influence as the pore space upon various properties of the material in the Earth's interior. This effect is moreover complicated by the condition of high temperature, and renders the problem of the properties of material in the Earth more intricate.

Next, the phenomena of hysteresis observed in the laboratory experiment of rock will be mentioned. Ide and Zisman reported the conclusion that the hysteresis of rock under changes in pressure of up to 2,000 bars and temperatures within several hundreds degrees may mainly be attributable to the existence of pore space, contrary to the normal hysteresis caused by the so-called plastic flow. The deformation by pressure of cavity artificially prepared in glass and granite was experimentally examined by Bridgman (**63**) and a certain kind of rupture was observed around the cavity. The same kind of rupture of micro-cracks as in Bridgman's case would probably be found around the pore spaces in rock in the high pressure experiment. On this point the following rough estimation may be useful. From the laboratory experiment, the amount of pore spaces in rock is ascertained to be greatly decreased at the pressure of some 2,000 bars. And then the deformation of rock cavities is calculated from the standpoint of the theory of perfect elasticity. In this case the mutual distance of small cavities in the perfectly elastic rock is assumed to be very much large compared with the radius of cavity. And then a spherical surface of suitably large radius which contains only one cavity at its centre can easily be imagined. And the pressure exerted on the spherical surface thus imagined is safely assumed to be the same as that hydrostatically applied to the rock-sample. In this case the radial displacement u at the point of distance r from the centre in the sphere is given by the following (cf. A. E. H. Love (**66**)):

$$u=\frac{r_0^3p_0}{(3\lambda+2\mu)(r_i^3-r_0^3)}r+\frac{r_i^3r_0^3p_0}{4\mu(r_i^3-r_0^3)}\frac{1}{r^2}, \tag{8}$$

where p_0 denotes the external pressure, λ, μ Lame's elastic constants, r_i and r_0 the radius of cavity and radius of the sphere respectively. In case when $r_0 \gg r_i$, the terms higher than $(r_i/r_0)^2$ can be neglected in a first approximation, and in this case the volume change of cavity does not depend upon the value of the radius. Taking the bulk modulus and rigidity of rock as some 10^{12} and 5.10^{11} c.g.s., the volume change of cavity by the pressure of 2,000 bars is estimated to be about 2 percent of the original volume of cavity. Under these circumstances the pressure of some 10^5 bars should be applied for reducing the cavity-volume to one half. From these calculations it is safely concluded that the formation of small ruptures and micro-cracks around the pore spaces in rock play an important role in the process of compression by high pressure and the phenomena of hysteresis in the laboratory experiment, especially within the range of pressure up to some 2,000 bars.

Concerning the hysteresis of rock caused by temperature-change, Ide (**61**) proposed a theory that the hysteresis of rock is caused by the formation of pore spaces in rock due to the directional difference of thermal expansion of constituent minerals. From Ide's interpretation and the above-mentioned consideration, the hysteresis of rock under the change of pressure and temperature should generally be related to the formation and deformation of pore spaces in rock in the experimental range of pressures up to 2,000 bars and temperatures up to 300°C. Irreversible phenomena other than hysteresis caused by the effect of viscosity, etc., have not yet been investigated in detail in laboratory experiments and these are necessary for clarifying the nature and behaviour of rock under high pressure and temperature. On the other hand, the property of irreversibility of the material in the Earth's interior has been studied by some researchers and it was reported by H. Jeffreys (**54**) from the study on the attenuation of energy of seismic waves that the irreversibility phenomena were scarcely observed in the mantle in case of wave propagation of short period such as seismic waves. Therefore, also on the problem of hysteresis and other irreversible phenomena, the direct application of the results obtained in the laboratory experiment of rock under high pressure and temperature to the interpretation of the nature and behaviour of material in the Earth's interior should be made more and more cautiously.

Next, with regard to the important problem of effect of the frequency of the applied force upon the value of rock properties measured in the laboratory experiment, the measurement on changes of rock properties by applying a force of variable frequency on the same sample of rock could hardly be found, especially under the conditions of varying pressures and temperatures. It is very important and essential in the investigation of the nature and behaviour of materials in the Earth's interior to clarify the difference of rock properties measured by, for example, supersonic waves, natural seismic waves, static or long continuing and one-directional forces, and to operate the experiment under high pressures and temperatures. Such experiments are urgently needed, for the development of Earth science.

As above stated there remain many important and critical unsolved problems such as the hysteresis, the frequency-effect, and others in the research field of the application of rock-experiment results to the Earth's interior. But in the present treatment we shall confine ourselves to the effect of pore space upon the elasticity of rock and put aside the other effects until they are fully measured by laboratory experiments in a near future.

From the standpoint of statistical mechanics in the theory of solids, the bulk modulus of single crystals is generally supposed to be expressed by two terms. One term is derived from the mutual action of atoms at their mean position, and the other from the vibration in thermal agitation of atoms around their mean positions. The former depends only on the specific volume of substance, and the latter on both temperature and specific volume. The contribution of the former to the bulk modulus of solids is much larger than the latter at low temperatures such as room temperature, and under such circumstances, the bulk modulus of a single crystal depends only upon its specific volume and is independent of its temperature. Of the polycrystalline substances, the effect upon the bulk modulus by the mosaic structure should be added to the above-described terms. In this case, as the effect by the mosaic structure, the effects derived from the directionality of constituent single crystals, the impurity at the grain boundary, the lattice-defect and other will be considered. These effects are supposed to have little influence upon the bulk modulus of such substances as the metal, for example, whose structure is undirectional and compact in grain-boundary. But in the case of rocks of complex structure, the effect of the mosaic structure should not be neglected. These effects are considered to be also related, in an approximation, to the specific volumes of rocks as deduced from their energetical consideration if not irrevesible.

The effect of the existence of pore space upon the bulk modulus of rock has certainly no simple relation to the specific volume of the rock as the changes of pore space comprise an irreversible process. Therefore, from the reasoning mentioned above it is possible to state that the effect of temperature and pore space upon the bulk modulus of rocks which are supposed to be very complex in nature will certainly be investigated by analysing the deviation from the normal and simple relation of the bulk modulus with its specific volume obtained from laboratory experiment. Under these assumptions, the relations between the bulk modulus and density (reciprocal of specific volume) were investigated with the data of elastic wave velocity of granite and dunite obtained from laboratory experiments under high pressures and temperatures by D. S. Hughes and J. H. Cross (**65**). In Fig. 2 the relations of the bulk modulus with the density are shown with respect to granite and dunite which were calculated from Hughes' results. It ought to be mentioned that the values of density and bulk modulus shown in this figure were calculated from the following relations:

$$\frac{dP}{d\rho}=\frac{k}{\rho}\,, \tag{9}$$

$$\frac{k}{\rho}=V_p^2-\frac{4}{3}V_s^2\,. \tag{10}$$

As in Hughes' paper only the values of pressure, temperature and their corresponding values of elastic wave velocity were specified, the values of ρ were obtained by calculating first the values of k/ρ from the values of V_p and V_s by (10) and then integrating equation (9). And the value of k was calculated by (10) for the known value of ρ. Moreover in the calculation the coefficients of the cubical thermal expansion (α) of rock at zero pressure were to be assumed and, in this case, the values of α of 2.4×10^{-5} and 2.7×10^{-5} were adopted for granite and dunite respectively (cf. Birch (**67**) and Landolt-Börnstein (**68**)). It should be remarked here that the above two equations (9) and (10) are safely applied to the case of isotropic and perfect elastic material, and its application to the real rock, especially to the porous and imperfect elastic rock under low pressure, should be made very cautiously. But in the present treatment, the above equations were tentatively and suitably used for the purpose of determining the pressure-limit of the effect of pore space for the rock-elasticity.

As seen in Fig. 2 the bulk moduli of granite at various temperatures are simple and linearly related to the density under high pressure. In granite the bulk modulus depends only on the density over the pressure of 2,000 bars. In dunite the curves of bulk modulus at various temperatures also have a common asymptotic line under the high pressure range of over 5,000 bars, and these asymptotic lines in both cases of granite and dunite are certainly supposed to express the behaviour of bulk modulus which is a function of only one parameter of density. Namely, taking the pressure of 2,000 bars for granite and 5,000 bars for dunite as a critical pressure, it is roughly stated that the behaviour of rock with respect to its bulk modulus may conventionally be divided into two parts, namely, the low pressure range affected mainly by pore

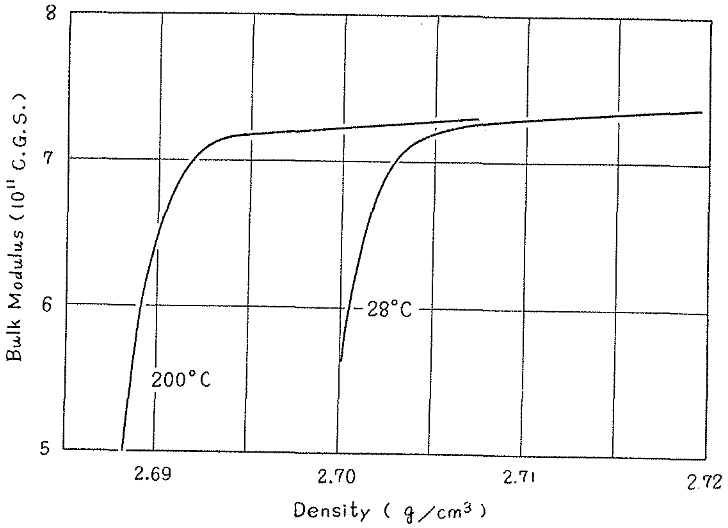

Fig. 2a. Relation between the bulk modulus and the density for granite.

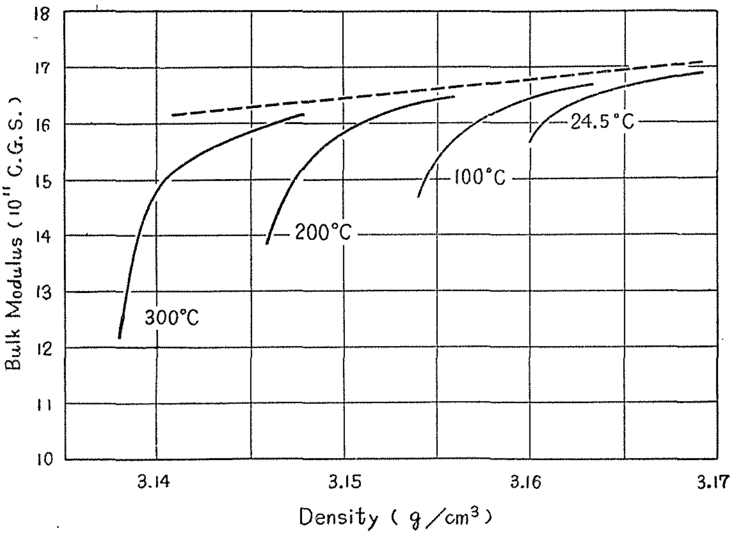

Fig. 2b. Relation between the bulk modulus and the density for dunite.

space and the high pressure range depending upon the interatomic potential. Practically, an asymptotic line will easily be found in the data over the pressure of 2,000 bars in the case of granite, but for dunite it is somewhat difficult to draw such a definite line as in the case of granite. In the case of dunite a line was tentatively drawn by connecting each value of k at their uppermost pressure (5,000 bars). From these considerations the values of $dk/d\rho$ for both rocks were calculated as follows:

$$\frac{dk}{d\rho} \approx 2.0 \cdot 10^{12} \text{ c.g.s.} \qquad \text{(for granite)}, \tag{11}$$

$$\frac{dk}{d\rho} \approx 2.2 \cdot 10^{12} \text{ c.g.s.} \qquad \text{(for dunite)}. \tag{12}$$

From these values the values of dk/dP are calculated as follows:

$$\frac{dk}{dP} \approx 7 \qquad \text{(for granite)}, \tag{13}$$

$$\frac{dk}{dP} \approx 4 \qquad \text{(for dunite)}. \tag{14}$$

It ought to be remarked that these values may be altered by some twenty percent by applying different values of the thermal expansion coefficient of rock, the measured values of which are greatly diverse from each other even for rocks with the same name of granite or dunite.

From the above argument it will safely be said that the bulk modulus of rocks under a certain high pressure beyond the critical point may approximately be treated as that governed by an ordinary theory of perfect elasticity. Namely, the bulk modulus of rock in this case is considered to be derived mainly from the effect of interatomic or interionic action and partly from that caused by mosaic structure. From these reasons the relation of the bulk modulus of rock to its pressure will conveniently be compared with those for single crystalline metarials. Lazarus (**69**) measured the adiabatic bulk modulus of single crystal of alkali-halide under the pressure up to 10,000 bars and obtained the following relations:

$$\frac{dk}{dP} \approx 6 \qquad \text{(for NaCl)}, \tag{15}$$

$$\frac{dk}{dP} \approx 8 \qquad \text{(for KCl)}. \tag{16}$$

Bridgman (**70**) also measured the isothermal compressibility of single crystal of α-quartz up to the pressure of 100,000 bars and obtained the result that

$$\left(\frac{dk}{dP}\right)_{P=0} \approx 5 \qquad \text{(for quartz)}. \tag{17}$$

The following value for the single crystal of MgO was calculated under the assumption that the bulk moduls depends only upon the density, as deduced from the data obtained by Durand (**71**) in his experiment on the temperature dependency of bulk modulus of MgO.

$$\frac{dk}{dP} \approx 4 \qquad \text{(for MgO)}. \tag{18}$$

The above values of dk/dP are not influenced by the effect of mosaic structure and they will be useful, to a certain degree, for the investigation of the bulk modulus of granite and dunite in which the contribution from the mosaic structure is considered to be partial. The mean value of dk/dP above obtained is

$$\frac{dk}{dP}=4\sim8\,. \tag{19}$$

In these cases the bulk modulus is derived only from the interionic action, and the obtained values are nearly of the same order of magnitude as those of granite and dunite. From these comparisons it will correctly, in the present approximation, be interpreted that the bulk moduli of granite and dunite in the pressure range beyond the critical point shown in Fig. 2 are mainly contributed by the interionic potential. In particular, as the bulk modulus of dunite is explained to be derived from the mutual action of Mg–O and Si–O, the accordance of the value of dk/dP of dunite with those obtained from the single crystals of MgO and quartz is favourable for the interpretation.

From the standpoint of the theory of solids, the interatomic potential ω can be assumed as:

$$\omega=\frac{A}{s^m}+\frac{B}{s^n}\,, \tag{20}$$

where s is the interatomic distance and A, B, m, n are certain constants proper to the material. Ramsay (**12**) obtained the following relation as deduced from the above potential:

$$\frac{dk}{dP}=\frac{1}{3}(m+n+6)\,. \tag{21}$$

For the case of dunite in which the mutual action of Mg–O, Si–O is assumed, the numerals are reasonably assigned as $m=1$ for Coulomb potential and $n=7$ after Born (**72**) for exchange force of Ne-nucleus in both cases of Mg and Si, and the calculated value is

$$\frac{dk}{dP}=4\sim5\,. \tag{22}$$

This value is nearly equal to that obtained for dunite. And on the other hand, it is to be emphasized that nearly the same value is also obtained from the finite strain theory of Murnaghan (**73**) as extended by Birch (**15**). From these considerations it is approximately concluded that the bulk modulus over the critical pressure shown in Fig. 2 is derived from the main contribution of the mutual action between ions and partially from the effect of the mosaic structure, and in this case the effects by hysteresis and temperature are negligibly small.

It is an important fact that both the bulk modulus and $dk/d\rho$ have a single relation with the density at the pressure of over 2,000 bars in the case of granite as seen in Fig. 2a, and, on the other hand, in the case of dunite only the bulk modulus depends approximately on the density at the pressure of over 2,000 bars but not so for $dk/d\rho$. Namely, the asymptotic lines for the values of bulk modulus at various temperatures for dunite do not coverge so well to a common line even at the pressure of over 5,000 bars as shown in Fig. 2b. This tells us that the effect of pore space upon the values of bulk modulus and $dk/d\rho$ becomes negligibly small at the pressure of over 2,000 bars in the case of granite, but it persists at higher pressures in the case of dunite, especially for the value of $dk/d\rho$. And as the diminution of the pore space is caused by some processes of rupture, the effect above stated for every rock is intimately related with their own breaking strength. In simple calculation, the maximum stress difference at the boundary of a small spherical pore is roughly estimated as $3p_0/2$ when the hydrostatic pressure p_0 is exerted on the mother body (cf. Love (**66**)). From the calculation, the stress difference around the pore space is roughly estimated as 3,000 bars in the case of external pressure of 2,000 bars. The so-called breaking strength of ordinary rocks is less than 3,000 bars, but this strength represents a value of breaking strength at the weakest point and besides the exerting pressure in the measurement is not hydrostatical. As the shape of a pore space is actually never spherical, the above argument is not necessarily accurate quantitatively, but it may be concluded that it necessitates a very much higher stress difference than the breaking strength to crush the all pore spaces and eliminate its effect for the elasticity of rock. The breaking strength for granite is estimated to be generally lower than that for dunite, so that the pore space will be reduced greatly by a comparatively lower pressure in the case of granite, but in the case of dunite the higher pressure, perhaps up to 10,000 bars, will be required for reducing the pore space. As stated above, it is not only natural to examine the relation between the bulk modulus and density in order to eliminate the effect of temperature and porosity, but there is another advantage. Namely, as seen in Fig. 2, the density of dunite in this diagram covers a range four times as large as that in the $k \sim P$-diagram. And really the value of $dk/d\rho$, which is little influenced by the effect of porosity and temperature, is more accurately calculated from the $k \sim \rho$-diagram compared with that derived from the $k \sim P$-diagram, because the inclination ($dk/d\rho$) of an asymptotic line to any curve in the $k \sim \rho$-diagram is uniquely determined under the condition of no intersection of the asymptote with the other curves.

Concerning the problem of temperature, the coefficient of thermal expansion is a subject of importance. From the values of the bulk modulus of dunite at various temperatures shown in Fig. 2b, it is experimentally concluded that the bulk modulus at a high temperature, in case of the same density, has a larger value than that at a low temperature. This means that the decreasing rate of thermal expansion coefficient with pressure is smaller than the increasing rate of bulk modulus with pressure. That is, the product of the thermal expansion coefficient by bulk modulus is an increasing function of pressure. On the contrary, the coefficient of thermal expansion decreases with pressure more rapidly than does compressibility for ordinary substances, as shown by Bridgman (**62**). This tendency will be mainly due to porosity and this is the reason why it is more reasonable to determine dk/dP from the relation of density-bulk modulus than from the relation of pressure-bulk modulus.

In summarizing the present section, it may be concluded that, in the determination of the pressure coefficient (dk/dP) for the bulk modulus of rock in the range of high pressure of more than several thousands bars, the use of a $k\sim\rho$-diagram is more natural and accurate than the direct application of a $k\sim P$-diagram and that the value of dk/dP thus obtained from the data of rock-experiments is in a good agreement with that derived from the observed data of seismic waves.

3. Rigidity of rocks and the Earth's interior

As in case of the bulk modulus, the rigidity of rocks will greatly be influenced by the existence of micro-pore spaces, especially in the range of low external pressure. The same method, as in the case of the bulk modulus, was applied to the experimental rock data for the purpose of determining the pressure coefficient of rigidity of rocks ($d\mu/dP$). Namely, instead of the direct determination of $d\mu/dP$, the relation between the rigidity and density ($d\mu/d\rho$) was conveniently calculated, but in the present case it was not so successfully and accurately determined as in the case of the bulk modulus. The cause of failure may partly be attributable to the greater uncertainty in the determination of the shear wave velocity (V_s) compared with that of dilatational wave (V_p) in the case of rock-experiments, and partly to the stronger influences by temperature, viscosity, porosity, etc. upon the value and behaviour of the rigidity of rock compared with those upon the bulk modulus. From the modern theory of solids, the nature and state equation with regard to the rigidity is more complicatedly connected with the temperature, viscosity, and other complex quantities compared with those in the case of the bulk modulus, and for these reasons, the problem on the nature and behaviour of the rigidity of the material in the Earth's interior and their relation to the experimental data of rocks will be studied separately in a forthcoming paper from a special standpoint reasonably assumed.

In the present section some problems on the ratio of bulk modulus to rigidity and on the pressure coefficient of the dilatational wave velocity (dV_p/dP) will be treated. The ratio of bulk modulus to rigidity (k/μ) is simply derived from the values of velocities of both dilatational and shear waves as follows:

$$V_p=\sqrt{\frac{k+4\mu/3}{\rho}}, \quad V_s=\sqrt{\frac{\mu}{\rho}}, \tag{23}$$

$$\frac{k}{\mu}=\left(\frac{V_p}{V_s}\right)^2-\frac{4}{3}. \tag{24}$$

Thus, in the treatment of isotropic and perfect elasticity, the value of k/μ is simply derived from the ratio V_p/V_s and can be conveniently discussed without any knowledge of the density distribution of the Earth's interior. Practically, the value of k/μ in the range of the Earth's mantle is shown in Fig. 4 from the materials by H. Jeffreys (**54**). The curves were previously published in 1951 (**74**) by the writer and discussed in some detail. And on the other hand, the values of k/μ plotted against pressure, which were calculated from the experimental data by D. S. Hughes and J. H. Cross (**65**), are shown in Fig. 3.

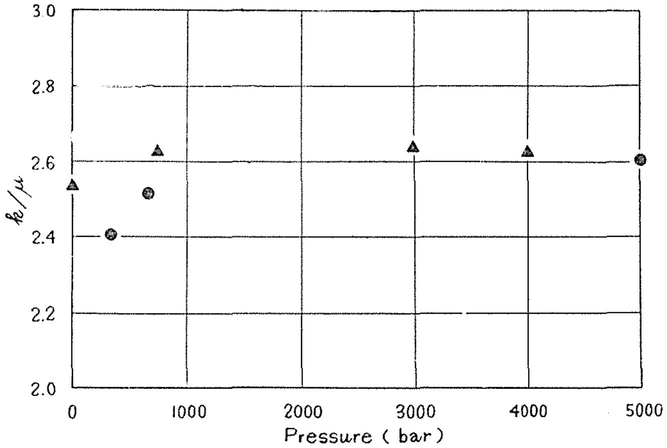

Fig. 3a. Relation between the value of k/μ of granite and the exerted pressure.
▲ (at 28°C), ● (at 200°C)

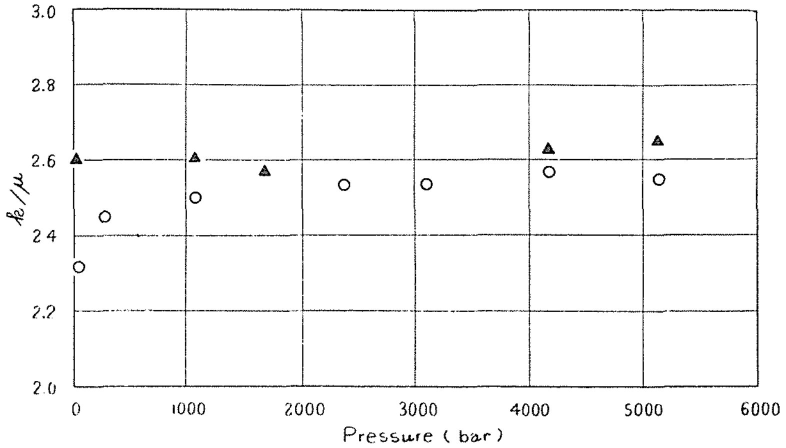

Fig. 3b. Relation between the value of k/μ of dunite and the exerted pressure.
▲ (at 24.5°C), ○ (at 300°C)

As seen in Fig. 3, the value of k/μ increases in the range of low pressure, but beyond the pressure of 2,000 bars, it becomes nearly constant or rather slightly decreases with the increase of pressure for both cases of granite and dunite. In the range of high pressure the values of k/μ determined from the rock-experiment are estimated to be nearly

$$\frac{k}{\mu}\approx 2.5 \quad \text{(for granite),} \tag{25}$$

$$\frac{k}{\mu}\approx 2.5 \quad \text{(for dunite).} \tag{26}$$

On the other hand, the value of k/μ obtained from Jeffreys' velocity distribution is

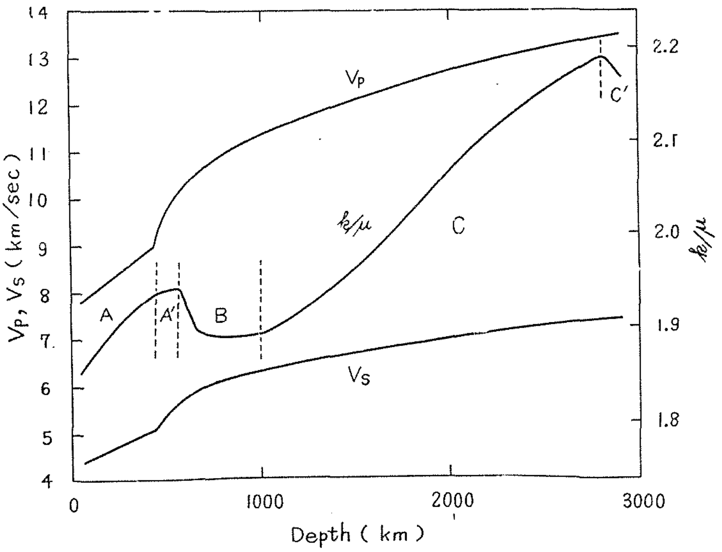

Fig. 4. Distribution of the values of V_P, V_S, and k/μ in the Earth's mantle.

estimated to be nearly 1.8 at the mantle surface as shown in Fig. 4. The inconsistency between the values of k/μ obtained from the rock-experiment and seismic observation is considered to be caused from various sources and its solution will offer an important key towards clarifying the problem on the character of rigidity distribution of the Earth's interior and especially the existence of a central core of zero rigidity. Recently P. W. Bridgman (**75**) has found a large increase in rigidity at pressures of about 85,000 atm. in the course of rock-experiments on dunite. The pressure of 85,000 atm. corresponds to that at the depth of about 250 km in the Earth, and the so-called 13°-discontinuity is also estimated to lie at that depth in the Earth's mantle. From the reasons above described and other reasons the problem on the character of rigidity in the Earth's interior is considered to be an important and essential item of research, and it will be treated separately in a near future in a forthcoming article.

In addition, the pressure coefficient of V_p will be discussed to some extent in the following, putting aside the problem of V_s. In Fig. 5, the relation between V_p (the dilatational wave velocity) and ρ (density) which are derived from Hughes' rock-experiment previously mentioned (**65**). This figure corresponds to Fig. 2 which represents the relation between bulk modulus and density. All curves at different temperatures converge to an asymtotic line which is approximately represented by a line connecting the points at 5,000 bars. This fact is reasonably interpreted, as in case of the bulk modulus, from the effect that the repulsive forces between the con-

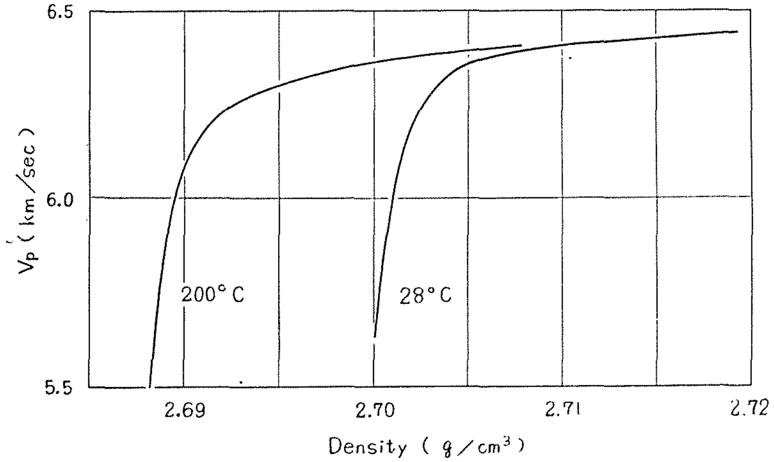

Fig. 5a. Relation between the dilatational wave velocity of granite and the density.

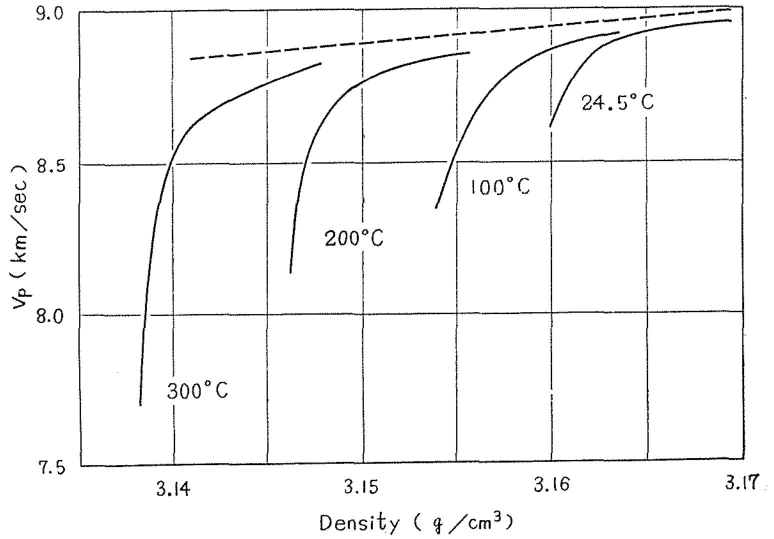

Fig. 5b. Relation between the dilatational wave velocity of dunite and the density.

stituent ions in the rocks become more and more predominant compared with the effects of porosity and temperature. From the gradient of the asymptote, we obtain:

$$\frac{dV_p}{d\rho} \approx 5\ \text{km/sec. g/cm}^3 \quad \text{(for granite)}, \tag{27}$$

$$\frac{dV_p}{d\rho} \approx 6\ \text{km/sec. g/cm}^3 \quad \text{(for dunite)}, \tag{28}$$

which correspond respectively to

$$\frac{dV_p}{dP} \approx 0.2\ \text{km/sec. } 10^4\ \text{bars} \quad \text{(for granite)}, \tag{29}$$

$$\frac{dV_p}{dP} \approx 0.1\ \text{km/sec. } 10^4\ \text{bars} \quad \text{(for dunite)}. \tag{30}$$

It ought to be remarked that the values above obtained may be altered by somewhat 20 percent, owing to the uncertainty of the thermal expansion coefficients, as in case of the bulk modulus. On the other hand, from the observed velocity of seismic waves by H. Jeffreys and by B. Gutenberg, the following value is estimated at the depth of 300 km:

$$\frac{dV_p}{dP} \approx 0.1\ \text{km/sec. } 10^4\ \text{bars}. \tag{31}$$

The value is in a good agreement with that obtained in (30). Accordingly the pressure coefficient of the velocity of dilatational seismic wave is correctly determined from the data of rock-experiments by the process of calculating first the value of $dV_p/d\rho$ and then converting it to the value of dV_p/dP, as in case of the determination of pressure coefficient of bulk modulus. In both cases, the influences of temperature, porosity, viscosity, etc. upon the determination of the pressure coefficient of the bulk modulus and the dilatational wave velocity of rocks are considered to be favourably minimized by the above-described method of analysis. In fact, the value of dV_p/dP obtained from Fig. 1b is twice larger than that obtained from Fig. 5b for dunite, and for the case of granite, the values obtained from Fig. 1a and Fig. 5a are in a good agreement.

4. Summary

The change with pressure of elasticity of rock obtained from laboratory experiments was discussed with reference to those derived from the analysis of velocity distribution of seismic waves in the Earth's interior. Putting aside the problem of the rigidity of rocks, the pressure coefficient of the bulk modulus and the dilatational wave velocity, which were determined from the data of rock-experiment by the present process of treatment of the data on the bulk modulus and dilatational wave velocity of rock with respect to the density instead of the pressure itself, were ascertained to be in a fairly good agreement with the values derived from the observation of seismic waves. It can safely be said that the various effects caused by temperature, porosity, viscosity, etc. upon the characters of rock-elasticity were minimized in the present method of analysis, and in this way, the marked discrepancy with respect to the pressure coefficient of elasticity hitherto discussed, between the values obtained from the rock-experiment and the values obtained from the seismic waves in the Earth's interior was certainly reconciled. On the other hand, the character of rigidity is considered to be very complicated compared with that of bulk modulus. Since its investigation is certainly important for the solution of many essential problems of the Earth's interior, the problem on rigidity will be discussed separately in a forthcoming paper.

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