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A CONSIDERATION ABOUT THE MECHANISM OF OCCURRENCE OF LARGE-SCALE EARTHQUAKES, II.

BY

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ABSTRACT

In this paper, the writer intends to re-examine the results reported in the preceding paper from the view-point of both dislocation of the Earth's crust and the energy of the seismic waves produced by a large-scale earthquake.

Generally speaking, the direction of the crustal dislocation accompanying an earthquake does not always coincide with that of the initial ground-motion of p-waves. But in such earthquakes as Kita-Tango, Kita-Izu and Fukui, it has been said that the above-mentioned two directions agree with each other. Now, the result we obtained is that the horizontal dislocation of the crust computed according to Honda-Miura's or Whipple's idea agrees approximately with the actually surveyed one. And the writer pointed out that the mechanism of the occurrence of the Fukui Earthquake could be explained as composed of a series of three cracks which grew up from north to south along the N 20° W nodal line, as described in the preceding paper. Then, when computed according to Whipple's forces-model, the resultant dislocation at each surveying station shows a satisfactory agreement with the actual dislocation determined by geodesic survey; the similar satisfactory results may be obtained as regards Kita-Tango and Kita-Izu Earthquakes. Moreover, we may certify the validity of the estimated length of a great crack accompanying each earthquake, by recognizing that the seismic energy calculated from the length of the crack coincides fairly well in the order with the seismic energy computed by the newly corrected Gutenberg-Richter's formula.

1. Introduction

In the preceding paper, the writer pointed out that the periods of the first maximum ground-motions of p-waves due to a large-scale earthquake (as observed at various seismological stations) show a systematic pattern of distribution with respect to the azimuthal angles of the stations as seen from the origin of the earthquake. The seismic crack, which is composed of a series of small cracks—this is estimated by analyzing the actual ground-motion of p-waves observed, for instance, at Abuyama,—and from which the seismic waves are generated, is developed with a certain definite velocity from one of its ends to the other along either of the so-called nodal lines of initial motions of p-waves, and the writer also pointed out that the periods of waves originating from the crack during its formation and being propagated into various directions are modified according to Doppler effect in connection with the movement of seismic source along the growing crack at the focus.

In the present paper, the writer intends to re-examine the above facts from the view-point of both the horizontal dislocation of the Earth's crust and the energy of the seismic waves produced by a large-scale earthquake.

2. Horizontal dislocation caused by a large-scale earthquake

Setting aside the question whether the deformation of the Earth's crust is the cause of a large-scale earthquake or not, we know well the fact that a remarkable dislocation of the crust is noticed near the epicentre before and after a large-scale earthquake. Furthermore, we often notice great faults when a shallow remarkable earthquake took place. Of course, we know the case in which the crustal dislocation reversely goes on before and after a large-scale earthquake. Generally speaking, we cannot always assert that the growing direction of the crustal displacement produced by an earthquake coincides with that of the initial ground-motion of p-waves accompanying it. As for such earthquakes as Kita-Tango (March 7, 1927), Kita-Izu (November 26, 1930), and Fukui (June 28, 1948), however, it has been recognized that the direction of the crustal dislocation agrees fairly well with that of the initial ground-motion of p-waves.

On the whole, many seismologists have already discussed the direction of deformation of displacement in a homogeneous, isotropic, semi-infinite elastic medium owing to forces-source applied to its boundary plane or its interior. They have theoretically calculated the special solutions of the problems in various cases. Recently N. Yamakawa (**1**) has summarily discussed the problem of this kind. Now, the writer intends to re-examine the horizontal dislocation of the Earth's crust accompanying a large-scale earthquake according to Honda-Miura's or Whipple's forces-model. According to Honda-Miura's model (**2**) about forces-source, the horizontal displacements U, V of the Earth's crust due to a shallow large-scale earthquake are given by

$$U=\frac{A\sin 2\varphi}{10\mu}\left[\frac{2}{3}\frac{1}{\tilde{\omega}^3}+\frac{5}{3}\frac{1}{b\tilde{\omega}(\tilde{\omega}^2+b^2)^{1/2}}-\frac{2}{3}\frac{b}{\tilde{\omega}^3(\tilde{\omega}^2+b^2)^{1/2}}-\frac{2b}{\tilde{\omega}(\tilde{\omega}^2+b^2)^{3/2}}-\frac{3}{2}\frac{b\tilde{\omega}}{(\tilde{\omega}^2+b^2)^{5/2}}\right],$$

$$V=-\frac{A\cos 2\varphi}{10\mu}\,\frac{1}{3}\left[\frac{2}{\tilde{\omega}^3}-\frac{2}{b\tilde{\omega}(\tilde{\omega}^2+b^2)^{1/2}}-\frac{2b}{\tilde{\omega}^3(\tilde{\omega}^2+b^2)^{1/2}}+\frac{b}{\tilde{\omega}(\tilde{\omega}^2+b^2)^{3/2}}\right],$$

assuming that Lamé's elastic parameters λ and μ are equal to each other.

Here, $\tilde{\omega}$, φ are the cylindrical coordinates, the origin of which is taken at a certain point on the boundary surface, the positive direction of the z-axis being taken downwards from the surface. U and V are the horizontal components of the crustal displacements parallel and perpendicular to $\tilde{\omega}$ respectively. The force at the focus is represented by

$$A\,\frac{\tilde{\omega}^3}{(\tilde{\omega}^2+b^2)^{7/2}}\sin 2\varphi$$

That is to say, when a force of the so-called four-quadrants type acts at the origin of coordinates along the boundary surface of a semi-infinite elastic medium, the horizontal components of displacement U and V on the surface are expressed by the above formulae, if we assume that Lamé's elastic parameters λ and μ are equal. In case when $b=10$, the calculated horizontal displacements at several points on the surface of a semi-infinite elastic solid become as shown in Fig. 1.

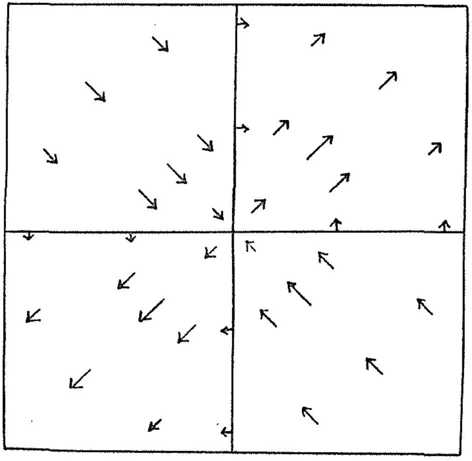

Fig. 1. Short arrows show the horizontal displacements of the crust according to Honda-Miura's model.
(After Drs. H. Honda and T. Miura)

On the other hand, F. J. W. Whipple (3) dealt with the crustal deformation produced by the so-called stress-nucleus situated in the interior of a semi-infinite elastic medium. He calculated the horizontal displacement-components U, V on the surface of the medium, as produced by the two "double forces with moment" (denoted by the symbol #) acting on the plane situated at a depth b, parallel to the surface. His expressions for U, V are given by

$$U = A\frac{\varpi \sin 2\varphi}{r^3}\left[\frac{\lambda+2\mu}{\mu}\frac{\varpi^2}{r^2} - \frac{\lambda+2\mu}{3(\lambda+\mu)}\frac{\varpi^2-2br}{(r+b)^2}\right],$$

$$V = A\frac{2}{3}\frac{\lambda+2\mu}{\lambda+\mu}\frac{\varpi\cos 2\varphi}{r(r+b)^2},$$

with $r^2=\varpi^2+b^2$, where, as before, ϖ, φ, z are cylindrical coordinates with the origin on the surface and the z-axis is taken downwards. Assuming $\lambda=\mu$, the calculated displacements by the above formulae become as shown in Fig. 2.

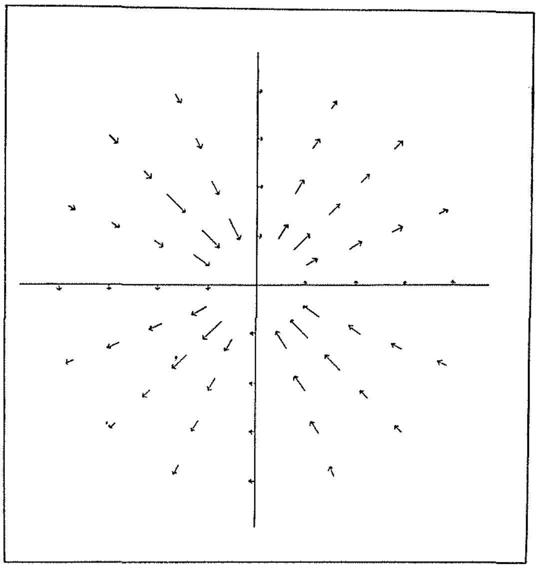

Fig. 2. Short arrows show the horizontal displacements of the crust calculated according to Whipple's model.
(After F.J.W. Whipple)

In case when the plane containing Whipple's forces is not horizontal, but has a certain inclination against the surface of a homogeneous, isotropic, semi-infinite medium, the expressions for the horizontal components U, V of the displacement at any point on the surface are given by (**4**) :

$$\begin{aligned} U = \frac{2}{3} A\Bigg[&\left\{3\,\frac{\lambda+2\mu}{\mu}\,\frac{\varpi^3}{r^5}-\frac{\lambda+2\mu}{\mu}\,\frac{\varpi^2-2br}{r^3(r+b)^2}\,\varpi\right\} \\ &\qquad\times(\cos\chi\cos\psi\cos\varphi+\sin\psi\sin\varphi)(\cos\psi\sin\varphi-\cos\chi\sin\psi\cos\varphi) \\ &-\frac{2}{3}\,\frac{\lambda+2\mu}{\mu}\,\frac{\varpi^2 b}{r^5}\left(\frac{1}{2}\sin 2\chi\sin 2\psi\cos\varphi-\sin\chi\cos 2\psi\sin\varphi\right) \\ &-\left\{3\,\frac{\lambda+2\mu}{\mu}\,\frac{\varpi b^2}{r^5}+\frac{\lambda+2\mu}{\mu}\,\frac{(2r+b)b\varpi}{r^3(r+b)^2}\right\}\left(\frac{1}{2}\sin^2\chi\sin 2\psi\right)\Bigg], \end{aligned}$$

$$V = \frac{2}{3} A\left[\frac{\lambda+2\mu}{\lambda+\mu}\,\frac{\varpi}{r(r+b)^2}\left\{\cos\chi\cos 2\psi\cos 2\varphi+\frac{1}{2}\sin 2\psi(1+\cos^2\chi)\sin 2\varphi\right\}\right],$$

where χ and ψ are the Eulerian angles between the plane containing Whipple's forces and the boundary surface of the elastic medium.

However, we deal here with such a case in which the plane containing Whipple's forces is, as in his paper, parallel to the horizontal plane so that $\chi=0$, $\psi=0$. Of course, the components of the horizontal displacement U, V as obtained by setting $\chi=0$, $\psi=0$ in the above formulae are in complete agreement with those computed by Whipple himself.

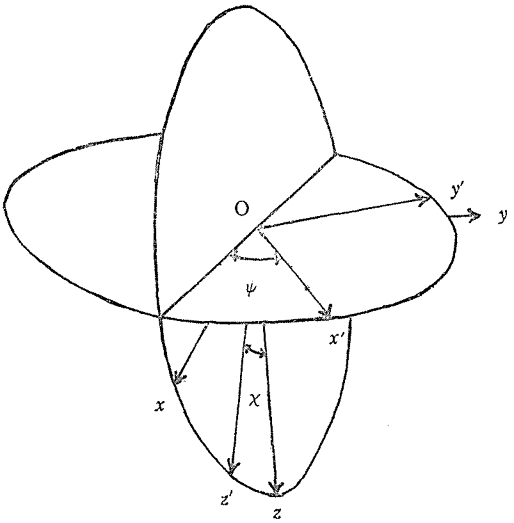

Fig. 3. The Eulerian angles χ, ψ representing the inclination between the plane parallel to the boundary surface (xy-plane) of a semi-infinite solid and the plane ($x'y'$-plane) containing Whipple's forces which act inside the solid.

If the above forces-source at the focus exists on the occasion of each crack-formation, it may be possible to compare the result of the actual survey with the resultant surface displacement of each station by organizing the vectors which show the distribution of our so-called four-quadrants type as regards the horizontal dislocation that each crack produces (Fig. 4). As already pointed out in the preceding paper, we may consider the mechanism of occurrence of the Fukui Earthquake as composed of three cracks. Namely, we suppose the first, second and third cracks C_1, C_2, C_3 were formed at the focus in such turns as indicated by suffixes 1, 2, 3 from north to south nearly along the N 20° W nodal line with such intervals of distance and time as already given. In this manner, the results obtained at each station by composing the vectors of the

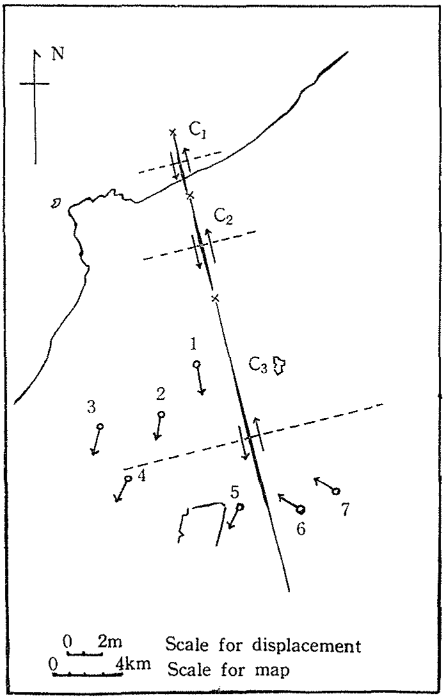

Fig. 5. Each short arrow shows the horizontal displacement calculated according to Honda-Miura's model near the focal region of the Fukui Earthquake of June 28, 1948. Each sign C_1, C_2, C_3 shows every crack formed in such turns as 1, 2, 3. The mark × shows its starting point, the unbroken line is drawn in order to correspond with its length, and the dotted line shows one of the nodal lines accompanying each crack.

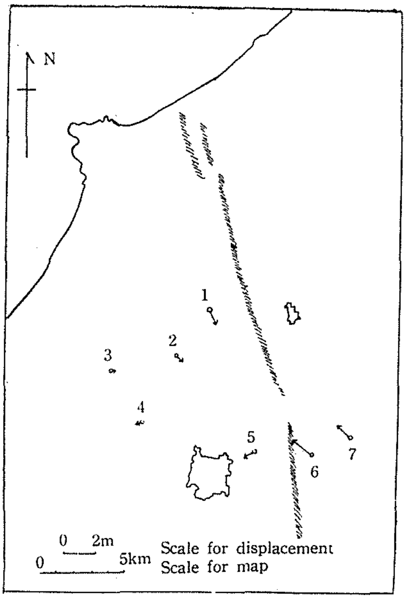

Fig. 4. Each short arrow shows the horizontal corrected dislocation of the crust, actually surveyed after the Fukui Earthquake of June 28, 1948, and shaded parts show the observed faults.

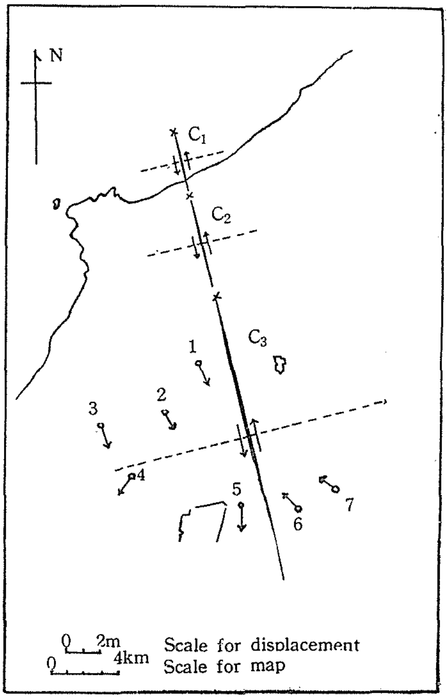

Fig. 6. The horizontal displacements of the crust calculated according to Whipple's model.

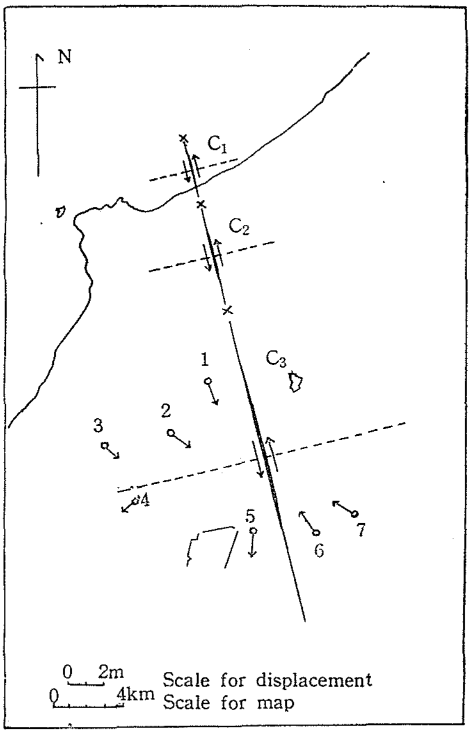

Fig. 7. The horizontal displacements of the crust are calculated just as they are proportional to the length of each crack, in accordance with Whipple's model.

horizontal displacement of the crust which show a distribution to be divided into four quadrants by two straight lines crossing at the centre of each crack are indicated in Figs. 5, 6 and 7.

In Figs. 5 and 6 are shown the above-mentioned results based on Honda-Miura's or Whipple's model, respectively. Furthermore, Fig. 7 shows the resultant displacement calculated according to Whipple's forces-model, assuming that each horizontal displacement is proportional to the length of each crack. So far, we cannot but say that it is of great interest to find that these calculated values of the horizontal dislocation of the crust are in satisfactory agreement with the actually surveyed ones. In these figures as well as in subsequent figures, each mark × represents the starting point of crack-formation. Now, when we compare these figures with the corrected one (**5**) (see Fig. 4) of the horizontal dislocation of the crust near the focus (as determined by geodesic survey after the Fukui Earthquake), we shall find a fairly good agreement between the trend of the horizontal dislocations as shown in Fig. 7 and that of actual dislocations shown in Fig. 4.

Considering that the standard points of survey are immovable, and various errors exist together with the inequality of geological structure, and that the horizontal dislocations are naturally added before and after the earthquake, it may be taken for granted that there exists such a disagreement as shown between Figs. 7 and 8.

Furthermore, let us apply the latter model (that is, Whipple's forces-model, horizontal displacement being proportional to the crack-length) to interpretations about the crustal dislocations after the Kita-Izu and Kita-Tango Earthquakes, respectively. Analyzing the ground-motion of p-waves due to the Kita-Izu Earthquake of November 26, 1930, such as observed at Kumagaya, we suppose the mechanism of its occurrence as composed of a series of three cracks C_1, C_2, C_3 which were horizontally formed in such turns as shown by suffixes 1, 2, 3 at the focal region with sizes (8, 15, 5 km), the directions, and the space intervals drawn in Fig. 8.

When we compose the displacement-vectors at each station which are estimated to be produced by each crack, we shall be able to obtain the result shown in the same figure. If we compare Fig. 8 with Fig. 9 (**6**) which represents the horizontal dislocations of the crust actually determined by a geodesic survey after the earthquake, we may be satisfied with good agreement in the trends of both the size and the direction of crustal dislocation.

Lastly, let us deal with the data of the Kita-Tango Earthquake of March 7, 1927. In the same way, analyzing the wave-form of the ground-motion of p-waves due to the earthquake, such as observed at Kôchi, we can grasp an outline of the horizontal dislocation of the crust near the focal region by dint of combining a series of three cracks C_1, C_2, C_3 along the N 29° W nodal line and a series of three cracks C_4, C_5,

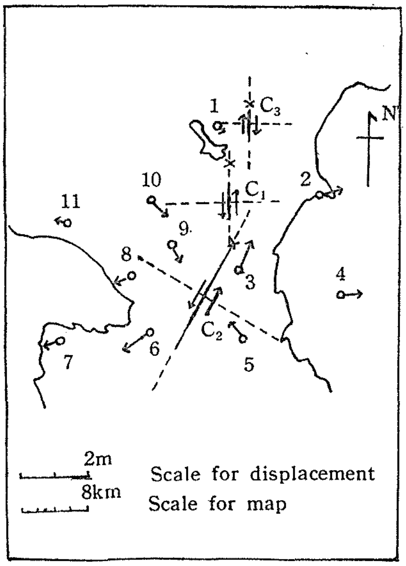

Fig. 8. The horizontal displacements of the crust calculated according to the same model as explained in Fig. 7. (The Kita-Izu Earthquake)

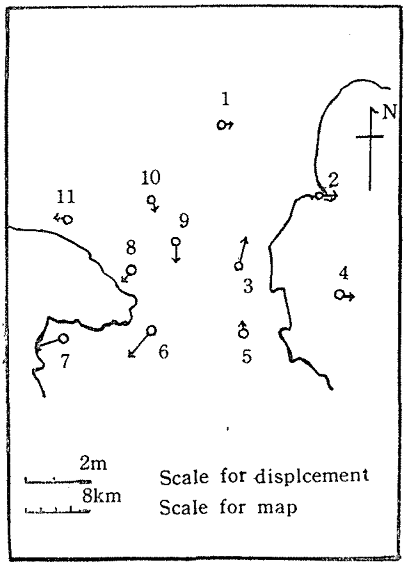

Fig. 9. The horizontal dislocations of the crust actually surveyed after the Kita-Izu Earthquake of November 26, 1930. (After Dr. C. Tsuboi)

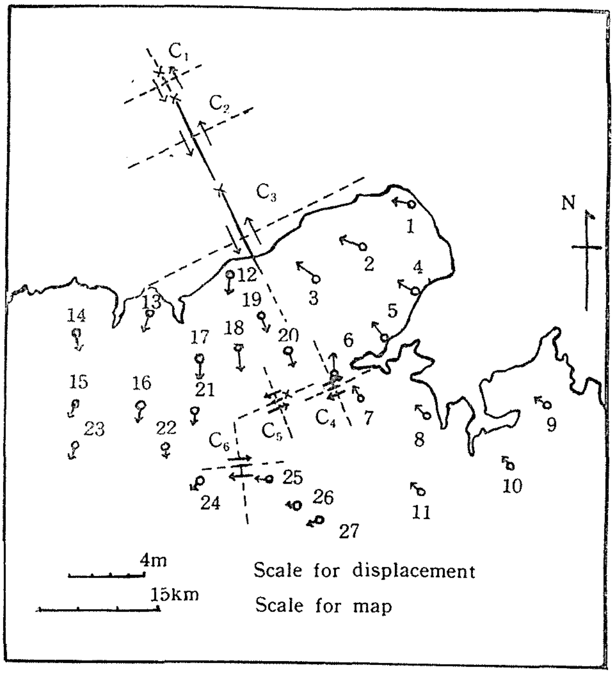

Fig. 10. The horizontal displacements of the crust calculated according to the same model as explained in Fig. 7. (The Kita-Tango Earthquake)

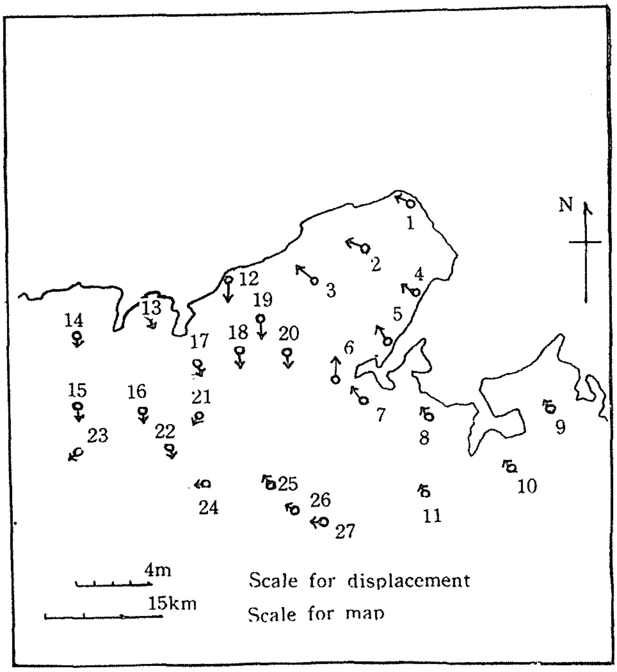

Fig. 11. This map shows the horizontal dislocations of the crust, actually surveyed after the Kita-Tango Earthquake of March 7, 1927. (After Dr. C. Tsuboi)

C_6, seemingly perpendicular to the former series; and all of them have such sizes, directions and intervals as mainly shown in Fig. 10. Thus, the result shown in Fig. 10 will be obtained by composing the horizontal displacement-vectors (by each crack) calculated according to the last forces-model.

If we compare this result with that (shown in Fig. 11) (7) of the actual dislocations determined by the geodesic method after the earthquake, we shall be unable to notice any remarkable difference between the computed result and the actual one.

3. Crack-length and seismic wave energy

Each crack-length estimated at the focal region of each large-scale earthquake reported in the preceding paper is described again in the third column of Table I.

Next, we shall calculate each order of seismic energy estimated from the crack-length. According to Jeffrey's or Bullen's consideration about the crack formed at the focus, the order of seismic energy released in the wave-form may be represented as Kl^2 ergs (8), when their so-called pseudotachylite is not formed along the shearing plane. Here l indicates the length of a crack in km, and K is taken as $K=4\times 10^{19}$. Of course, we must have a greater value for K than 4×10^{19} according to their

Table I.

Large-scale earthquake	Magnitude	Length of crack (horizontal)	Energy released in seismic waves in earthquake	
			Calculated from Kl^2	Calculated from formula (1)
Kita-Tango (Mar. 7, 1927)	7.6	34 km	4.6×10^{22} ergs	3.6×10^{22} ergs
Kita-Izu (Nov. 26, 1930)	7.3	28	3.1×10^{22}	1.3×10^{22}
Tottori (Sept. 10, 1943)	7.4	29	3.4×10^{22}	1.8×10^{22}
Tônankaidô (Dec. 7, 1944)	8.0	42	0.7×10^{23}	1.3×10^{23}
Nankaidô (Dec. 21, 1946)	8.2	46	0.9×10^{23}	2.2×10^{23}
Fukui (Jun. 28, 1948)	7.2	27	2.9×10^{22}	0.9×10^{22}
Tokachi-Oki (Mar. 4, 1952)	8.2_5	49	0.9×10^{23}	2.6×10^{23}

notes in case when the pseudotachylite is formed during the slipping. In other words, it means that the temperature during the slipping is more than 1000°C. The values of seismic energy of each large-scale earthquake calculated according to Kl^2 are given in the fourth column of the same table. On the other hand, the new relation (**9**) between the magnitude and the seismic wave-energy has been recently reported by Gutenberg and Richter as follows:

$$\log E = 9.1+1.75M+\log(9-M)\,. \tag{1}$$

When a large-scale earthquake occurs, the energy released in the form of seismic waves is calculated by using this formula (1). Thus, the computed results are shown in the fifth column.

It is of great interest that we can find, on the whole, the agreement in the order between the seismic wave energy calculated according to its estimated crack-length and the energy calculated according to its magnitude M. Then, especially, in such cases (Tônankaidô, Nankaidô, and Tokachi-Oki Earthquakes) when the seismic energy calculated according to Kl^2 is less than the energy obtained from its magnitude, we may be able to understand that the pseudotachylite was possibly formed in the slipping and the temperature of the shearing plane was much more than 1000°C. But, considering that the crack-length denoted by l which is determined by Doppler effect is the horizontal component of the actual one which is suitably inclined to the boundary surface (namely, here, the horizontal plane), we shall be able to estimate from Kl^2 the more approximate value of the seismic energy than that shown in the same table. On the other hand, some unavoidable inequality among these values of the seismic energy may be considered to result from the fact that the crack-length occurring at the focus may differ from that of the fault which took place at the Earth's surface. Anyhow, these problems concerning the inclination of crack-line at the focus as well as the mechanism of the occurrence of the above three large earthquakes of Pacific origins will be discussed later on.

4. Summary

In the present paper, the writer re-examined the results reported in the preceding paper, from both sides of the horizontal dislocation of the Earth's crust actually produced by an earthquake and the seismic wave-energy calculated by using an estimated crack-length, and he obtained the approximately satisfactory results. Thus, it may safely be said that the writer has somewhat succeeded in the phenomenological explanation concerning the mechanism of occurrence in a large-scale earthquake. First of all, the azimuthal distribution of the period of the first maximum ground-motion of *p*-waves due to an earthquake should be noticed to be regular and its regularity may be accounted for by considering that the period of the initial ground-motion is modified according to Doppler effect in connection with the movement of seismic source along the growing crack at the focus.

Secondly, the resultant vector obtained by composing (at each triangular point) each crack displacement at the focal region coincides fairly well with what was actually determined by the geodesic survey. Thirdly, it seems to be of great interest that the order of the seismic wave-energy calculated according to the crack-length estimable in connection with the regularity of azimuthal distribution of the above-mentioned period coincides well, on the whole, with that of the seismic energy computed according to Gutenberg and Richter's recently reported relation between the magnitude and the energy of an earthquake.

Judging from the above interpretations on the observed or surveyed results regarding the ground-motion, the crustal dislocation, etc., produced by a large-scale earthquake, the writer cannot but consider the mechanism of occurrence of an earthquake as composed of a series of small cracks.

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