Memoirs of the College of Science, University of Kyoto, Series A,
Vol. XXX, No. 3, Article 1, 1964.

ON THE DEVIATION FROM KNUDSEN'S FORMULA OF THE DENSITY OF SEA WATER AND ITS BEARING ON THE PRODUCTIVITY OF THE SEA

BY

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(*Received June 4, 1962*)

ABSTRACT

In oceanography, the salinity and density of sea water are usually obtained from Knudsen Tables by the chlorinity titration. This method is based on the fact that the relative composition of major constituents in sea water is quite uniform. In coastal areas, however, it may be considered that the uniformity of the relative composition is impaired by the addition of materials due to the influx of land water and the absorption of materials due to the biological production.

The author measured the density σ_0 of coastal water by Knudsen's method and the actual density σ_0' of the same water by a newly constructed hydrometer (accurately within $\pm 3\times 10^{-6}$) at the same time, and calculated the difference $\Delta\sigma_0 \equiv \sigma_0 - \sigma_0'$ between these two densities, which was termed as "salinity deficit".

The exchange of water in Akashi Straits and Tanabe Bay was discussed with reference to the correlation diagram between the chlorinity and salinity deficit, and then it was found that the salinity deficit was a valuable index for water mass analysis and was closely related with the stagnation of sea water.

Furthermore, considering the factors influencing the variations of the salinity deficit and biological production, two equations representing the increasing rates of the plankton population and salinity deficit were derived under various assumptions. These equations were regarded as the simultaneous differential equations representing the relationship between the plankton population and salinity deficit. The coefficients in these simultaneous differential equations were calculated by the method of least squares in every month, using the values of tidal range, chlorinity, salinity deficit and plankton population which were observed in Tanabe Bay from July, 1958 to July, 1959, and then, these simultaneous equations were solved numerically about the plankton population and salinity deficit by Runge-Kutta's method using the calculated coefficients. The result of numerical calculation agreed fairly well with observed variation.

The differential equation of only the plankton population, which was often seen in the theory of vibration, was obtained by eliminating the salinity deficit in the simultaneous differential equations. It was found that this differential equation had an unstable solution in November, 1958 and February, 1959 when the abrupt blooming of plankton population occurred and that the mode of oscillation on the right-hand side of this equation, which corresponds to external force, was roughly similar to the variation of plankton population.

Finally, the production in Tanabe Bay was estimated from the increasing rate of salinity deficit proportional to plankton population.

1. Introduction

Sea water is a solution containing various salts, and nowadays more than forty elements in the Periodic Table are identified and the relative composition of its major constituents is quite uniform. The uniformity of the relative composition, which has been examined and confirmed by many research workers since W. Dittmar, is a fact valuable to oceanography because it affords a means of estimating the concentration of all of the major constituents.

In oceanography, the total amounts of salt and chlorine in grams contained in one kilogram of sea water are called "salinity" and "chlorinity" respectively. The uniformity indicates a constant relationship between salinity and chlorinity. Empirical relationship, as established by the International Council for the Exploration of the Sea, is given by

$$\text{Salinity} = 0.03 + 1.805 \times \text{Chlorinity}. \tag{1}$$

M. Knudsen (1901) published "Hydrographical Tables" according to Eq. (1), which is called Knudsen's formula. The Cl-ratio is defined by the ratio of salinity to chlorinity and, in this term, the sea water has the constant Cl-ratio. (In the following, the sea water which has the relative composition of major constituents prescribed by Eq. (1) will be termed as "the normal water".) The uniformity of relative composition in the oceans is the result of circulation and mixing. These operations are continuous, and tend to eliminate regional differences in composition.

As is well known, in the coastal area, the inflow of river water causes a lowering of chlorinity in the sea water and the biological production is higher. Plankton population is more dense, algae grows more abundantly and the cultivations of lavers, oysters and pearls thrive. This fertility is due to the widespread organic and inorganic substances brought in large quantities by the influx of land water. The river water has a higher Cl-ratio than the sea water as seen in Table 1. A. Collier (**1**) reported in 1953 that organic matter content up to 50 mg in one litre of water may be found in the Gulf of Mexico. Recently, D. E. Carritt and J. H. Carpenter (**2**) discussed the relations among the chlorinity, salinity and density of sea water in various oceanic regions, with reference to Hydrographical Tables. It is questionable whether the normal composition is always present in the sea or not.

M. Ishibashi and T. Yamamoto (**3**) reported that sea weeds in Tomogashima Straits contained 10~20% of ashes which contain Na (1~10%), K (1~20%), Ca (3~25%), Mg (1~25%) and so on. H. Yamamoto (**4**) noted that both the ratios Ca/Cl and Mg/Cl measured in Urado Bay (in the Prefecture of Kōchi)

Table 1.

Element	Sea water	River water*	River water**
Na	100.	100.	100.
K	3.6	21.6	30.5
Ca	3.8	405.0	157.2
Mg	12.0	111.0	54.9
Cl	180.0	64.5	107.4
SO_4	25.0	102.0	181.5
CO_3	0.7	471.0	413.0
Total/Cl	1.8	19.7	9.7

*: G. E. Hutchinson, "*A Treatise on Limnology*".
**: Y. Miyake, "*Chikyū-Kagaku*"

showed appreciable changes and also the values of Ca/Cl and Mg/Cl of water in about 133°E, 20°N～30°N exceeded the values of those in the normal water. It is seen in "*The Oceans*" (H. U. Sverdrup, M. W. Johhson and R. H. Fleming) that the composition of plankton organisms is considerably different from the normal composition of sea water.

K. Hidaka and J. Yasui (5) reported, by measuring the density of sea water samples collected in Osaka Bay, Kii Channel and Pacific Ocean, by Knudsen's method and Nansen's hydrometer of total immersion, that the two values of density obtained by such methods were not consistent and the significant distribution of the density difference was found.

The various elements of marine living bodies are most likely supplied originally from the sea water. Inferring from the little fixation of chlorine, the uniformity is probably impaired and the Cl-ratio should be decreased by the biological production, although it is not yet clearly known in what means plants assort various salts. It is suggested that the biological activity may play a fair role in turning the Cl-ratio of land water into the lower and more stable Cl-ratio of sea water.

In oceanography, the specific gravity of water is determined from the chlorinity using "Hydrographical Tables" and usually expressed in $\sigma_t=1000\times(\rho_t-1)$ (ρ_t: specific gravity at t°C). However, the actual specific gravity σ_t' of sea water mixed effectively with land water may be larger than the specific gravity σ_t calculated from Table, since the Cl-ratio of land water is higher than that of normal water. On the other hand, the actual specific gravity σ_t' of water influenced by biological production may be lower than σ_t. So, if the difference $\Delta\sigma_0\equiv\sigma_0-\sigma_0'$ is introduced, $\Delta\sigma_0$ may be considered as a significant parameter reflecting the physical, chemical and biological processes in the sea. It is considered that the chlorinity represents the total amount of salt in normal water and $\Delta\sigma_0$ expresses the deficit of salts from normal water (The negative value of $\Delta\sigma_0$ will express the excess.) In this sense, $\Delta\sigma_0$ will be called "salinity deficit".*

Salinity deficit may not exist in major regions of open sea, but in coastal water, salinity deficit will increase by the consumption of salt due to biological production and will decrease by the addition of salt due to the influx of land water. Also, this salinity deficit is diffused by mixing and is carried away by the current.

It is worth noticing that the use of salinity deficit has the following advantages: in the first place, the value of $\Delta\sigma_0$ tells us the total amount of deficit or excess of salt, which may not be easily obtained by the chemical analysis of individual elements, and, in the second place, $\Delta\sigma_0$ is calculatable only from the measurements of chlorinity and specific gravity without any extra load on field work. Salinity deficit is, however, a small quantity and we are obliged to use a sensitive hydrometer to investigate its variations.

Part I

CONSTRUCTION OF HIGHLY SENSITIVE HYDROMETER

The specific gravity of sea water at 0°C changes by about 15×10^{-6} for the increment of 0.01‰ of chlorinity. Using the automatic potential titrimeter, we can accurately measure the chlorinity within ±0.003‰. The chlorinity of ±0.003‰ nearly corresponds to the specific gravity of $\pm5\times10^{-6}$. So, we require a hydrometer of accuracy of $\pm10^{-6}$. Nansen's submerged hydrometer and pycnometer fulfill this requirement, but they take too long a time to measure the specific gravity. We must measure a large number of samples to observe the complicated variation of salinity deficit. For this purpose we have constructed a hydrometer which operates accurately and quickly.

2. The principle of the newly constructed hydrometer

In Fig. 1, an arm is horizontaly fitted at a point B on a fine wire, both ends of which are fixed at A and C. When a force F is vertically acted upon P, the wire AC is twisted by the torque due to F. The torsional angle φ is proportional to the torque by F in the definite range of φ. Then

$$aF = c\left(\frac{\varphi}{l_1}+\frac{\varphi}{l_2}\right), \quad \text{or} \quad \varphi = \frac{l_1 l_2}{l_1+l_2}\frac{a}{c}F, \tag{1.1}$$

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* In the author's previous papers, $\Delta\sigma_0$ had been called "mass defect", but the same term is also used in Nuclear Physics. So, in order to avoid confusion, the term "salinity deficit" is used throughout this paper.

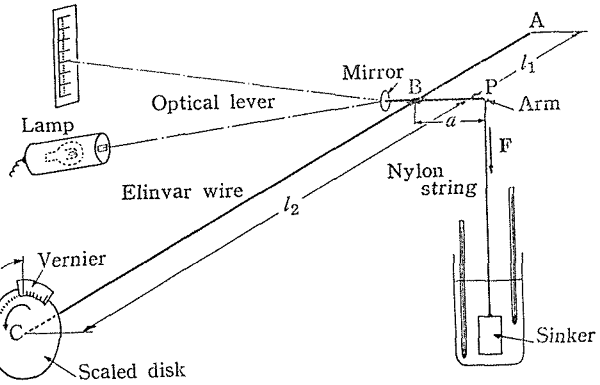


Fig. 1. Schematic view of a hydrometer.

where

$$c \equiv \frac{1}{2}\pi\mu r^4, \tag{1.2}$$

and l_1 is the length of $\overline{\mathrm{AB}}$, l_2 that of $\overline{\mathrm{BC}}$, a that of $\overline{\mathrm{PB}}$, r the radius of wire, and μ the rigidity of wire.

If the wire is twisted by the angle θ at C in the opposite sense to the torque of F, the corresponding torsional angle φ' at B is given by

$$\varphi' = \frac{l_1}{l_1+l_2}\theta. \tag{1.3}$$

The angle θ which makes φ' at B equal to φ is given by

$$\frac{l_1}{l_1+l_2}\theta = \frac{l_1 l_2}{l_1+l_2}\frac{a}{c}F,$$

which gives

$$\theta = \frac{a l_2}{c}F. \tag{1.4}$$

Now, a sinker (a weight submerged into the sample water) is hooked at P through a string. Then the force F is given by

$$F = g(M - V_t\rho_t) + (mg + T - B), \tag{1.5}$$

where M is the mass of sinker, V_t the volume of sinker at t°C, ρ_t the density of sample water at t°C, m the mass of suspension string, T the surface tension acted upon the string, B the buoyancy caused by a part of the string in water, and g the acceleration of gravity.

Substituting from (1.5) into (1.4), we have

$$V_t\rho_t = \left(M - \frac{c}{a l_2 g}\theta\right) + \left(m + \frac{T-B}{g}\right). \tag{1.6}$$

When the torque of the force given by (1. 5) is balanced to the torsional moment of wire caused by the angle θ at C,—the balance being assured in use of the optical lever by the zero-point method as seen in Fig. 1,—the water temperature t°C and the torsional angle θ are read at the same time.

The specific gravity of water is obtained by Eq. (1. 6) from the readings of temperature t°C and the angle θ.

3. The design of the hydrometer

Using a fine Nylon string of 0.02 mm in diameter and 20 cm in length as the suspension string, and adjusting the length of the string in water to be about 6 cm, we get

$$T \fallingdotseq 0.41\,\text{dyne} \fallingdotseq 0.41\,\text{mg weight (if the contact angle} = 30^\circ)$$
$$B \fallingdotseq 0.02\,\text{mg weight}$$
$$m \fallingdotseq 0.07\,\text{mg (the density of Nylon} = 1.12\,\text{g/cc}),$$

for the density of water between 1.00 and 1.03 and the temperature of water between 0°C and 4°C. Then, the second term on the right-hand side of Eq. (1. 6) is regarded as a constant value of 0.5 mg. Including $\left(m+\frac{T-B}{g}\right)$ in M, we get

$$\rho_t = \frac{M}{V_t} - k\frac{\theta}{V_t}, \tag{1. 7}$$

where

$$k \equiv \frac{\pi\mu r^4}{2a\,l_2 g}.$$

The error of ρ_t due to each element is given by

$$(\delta\rho)_M \equiv \frac{\partial\rho}{\partial M}\delta M = \frac{1}{V_t}\delta M, \tag{1. 8}$$

$$(\delta\rho)_{V_t} \equiv \frac{\partial\rho}{\partial V_t}\delta V_t = \left(-\frac{M}{V_t} + k\frac{\theta}{V_t}\right)\frac{\delta V_t}{V_t}, \tag{1. 9}$$

$$(\delta\rho)_\theta \equiv \frac{\partial\rho}{\partial\theta}\delta\theta = -k\frac{1}{V_t}\delta\theta, \tag{1. 10}$$

$$(\delta\rho)_{l_2} \equiv \frac{\partial\rho}{\partial l_2}\delta l_2 = k\frac{\theta}{V_t}\frac{\delta l_2}{l_2}, \tag{1. 11}$$

$$(\delta\rho)_a \equiv \frac{\partial\rho}{\partial a}\delta a = k\frac{\theta}{V_t}\frac{\delta a}{a}, \tag{1. 12}$$

$$(\delta\rho)_r \equiv \frac{\partial\rho}{\partial r}\delta r = -k\frac{\theta}{V_t}\frac{4\delta r}{r}. \tag{1. 13}$$

The value of δM is provided by the accuracy of the balance. The accuracy of precise chemical balance is ±0.1 mg. In order that $(\delta\rho)_M \lesssim 10^{-6}$, $V_t \gtrsim 100$ cc is specified. Since δl_2, δa and δr are caused only by thermal expansion after the construction of the hydrometer, the main part of the instrument was made from super-invar, but the wire was made from elinvar in order to keep the change of rigidity as small as possible.

The linear coefficient of thermal expansion of invar $= 1{\sim}2\times10^{-6}/°\mathrm{C}$;
The linear coefficient of thermal expansion of elinvar $= 1{\sim}2\times10^{-6}/°\mathrm{C}$;
The rigidity of elinvar $= 7\times10^{11}$ dyn/cm^2.

If we take $M=100$ g, $V=100$ cc, $l_2=36$ cm, $a=6$ cm and $r=0.01$ cm, the value of k is given by

$$k\equiv\frac{\pi\mu r^4}{2al_2g}\fallingdotseq5.2\times10^{-2}\qquad(\mathrm{g/rad}).\tag{1.14}$$

Then, for $\theta\leqq\pi$

$$k\frac{\theta}{V_t}\leqq1.6\times10^{-3}.\qquad(\mathrm{g/cc})$$

When the temperature is moderate, $(\delta\rho)_{l_2}$, $(\delta\rho)_a$, and $(\delta\rho)_r$ can be neglected.

From (1.9), we have

$$\delta V\leqq10^{-4}\ \mathrm{cc}.$$

Since it is not easy to measure directly the volume of sinker within $\pm10^{-4}$ cc, it is measured indirectly (see Section 4, b)

From (1.10), we get

$$\delta\theta\leqq1.9\times10^{-3}\ \mathrm{rad}\fallingdotseq0.1°.$$

The disk D is equipped with vernier in order to measure an angle within ±0.1°.

The densities of distilled water and sea water change at the rate of about 0.5×10^{-4} g/cc and 10^{-4} g/cc per 1°C respectively between 0°C and 2°C. Then, the water temperature must be read within an accuracy of ±0.01°C. The water sample is cooled in the bath controlled at about 0°C automatically and the temperature of the sample is read by two Beckmann thermometers put into the sample, one in the upper layer and the other in the lower layer.

4. Measurement of hydrometer constants

In the apparatus actually constructed, a special device is made for rapid measurement. A sample of water is poured into a pyrexglass beaker of about 1.2 litres in capacity which is put into a bath kept at about 0°C by an electric refrigerator equipped with thermostat. In the bath, ten beakers are arranged on a circular disc which is rotated around a vertical axis through its center. By rotating the disc successively, the density is measured one by one. The density of water is measured after all the suspended solids have settled down. The view and sketch of the apparatus are shown in Photos. 1~5 and Fig. 2 respectively.

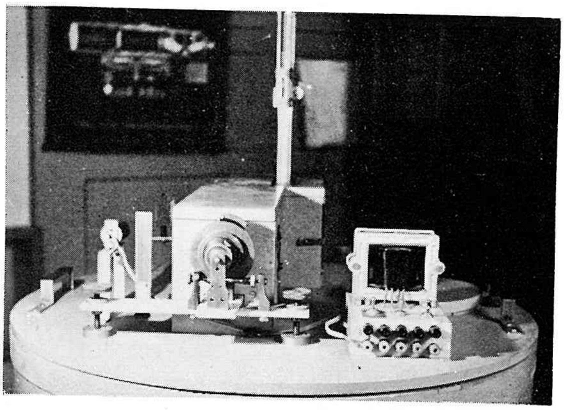


Photo 1. Front view

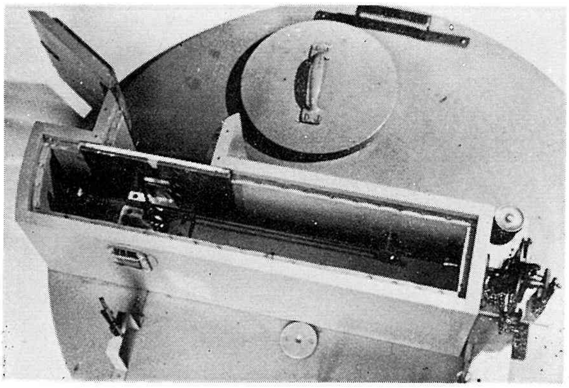


Photo 2. Upside view

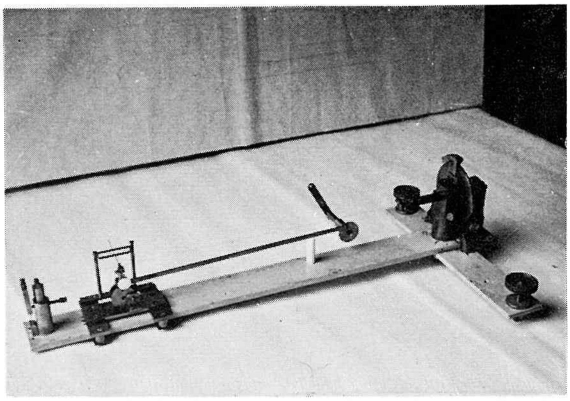


Photo 3. Elinvar wire. Arm and mirror. Scaled disc and vernier. Cramp

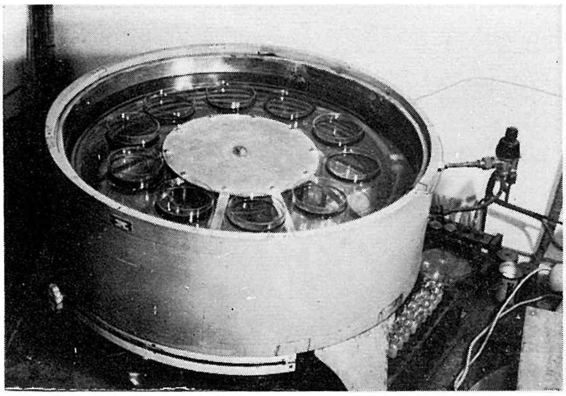


Photo 4. Beakers put on circular disc rotating in the bath. Cooling pipe

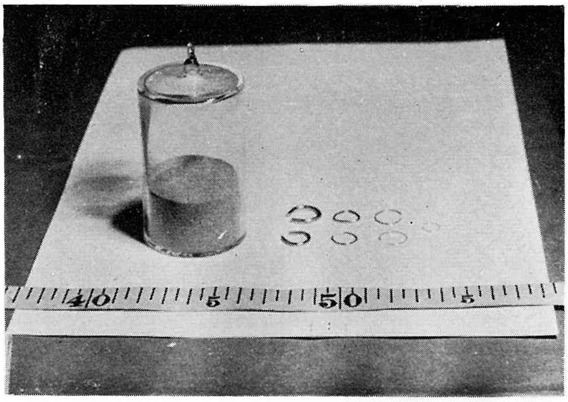


Photo 5. Sinker and rings

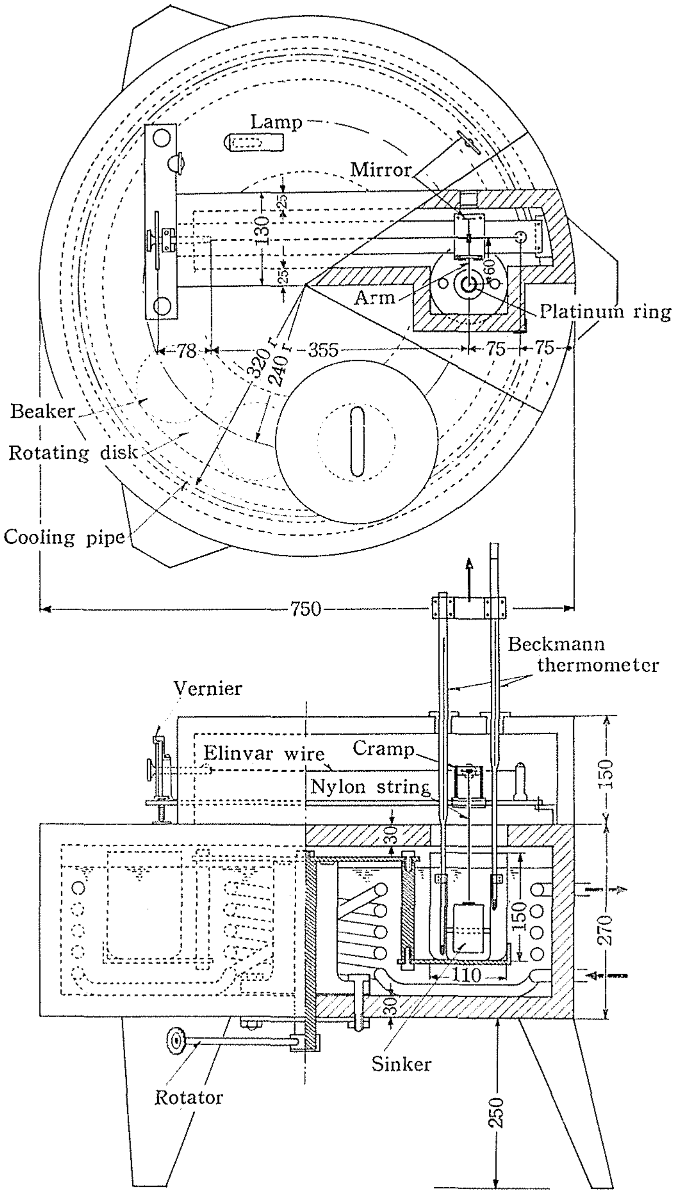


Fig. 2. Sketch of hydrometer.

a) *Determination of sensitivity*

Sensitivity is given by $\partial\theta/\partial\rho = -V_t/k$. To determine the sensitivity, various test weights hanged on the point P and the torsional angles at C were measured and the value of k was calculated. The result is shown in Fig. 3. The value of k seems to become slightly small as θ increases, and is satisfactorily expressed as a linear function of θ. Using the method of least squares, we obtained

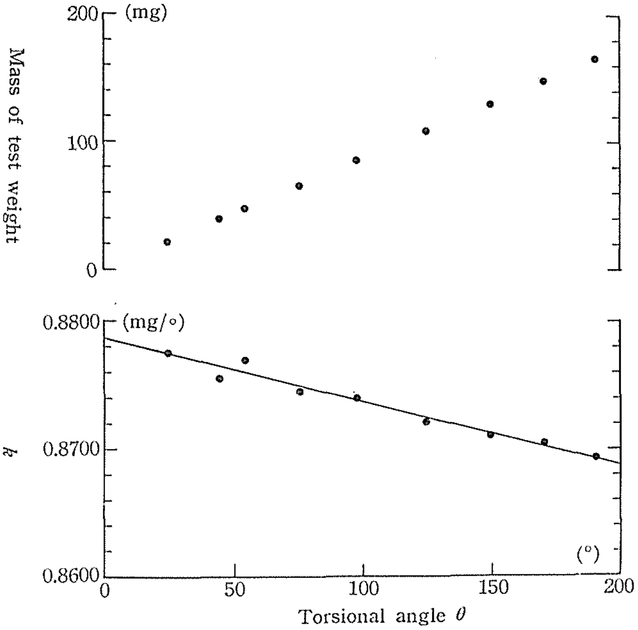


Fig. 3. Relation between the torsional angle and the mass of test weight hanged on the point P.

$$k = 0.8787 - 0.5\times10^{-4}\theta \qquad (\text{mg}/\circ)\,. \tag{1.15}$$

It is seen that the accuracy within 0.1° in θ is sufficient to obtain the density to the accuracy of $\pm10^{-6}$ g/cc.

b) *Determination of mass and volume of sinker*

Sinker is made from fused silica and has a cylindrical form with about 4.5 cm in diameter, 7 cm in height, and the value of V_t is approximately 110 cc. For the measurement of the mass of sinker to the accuracy of ±0.1 mg, the correction of air bouyancy is required. Taking $V_t = 110$ cc as a first approximation, we obtain $M = 110.2672$ gr including $m + (T-B)/g$. Substituting $M = 110.2672$ gr and the value of k into (1.6), we get

$$110.2672 - \int_0^\theta (0.8787 - 0.5\times10^{-4}\theta)\times10^{-3}d\theta = V_t\rho_t\,. \tag{1.6'}$$

The density of distilled water is dependent only on the temperature (N. E. Dorsey, "*Properties of Ordinary Water-Substance*"). From (1. 6′), the second approximate value of V_t is calculated if a torsional angle θ is measured at a certain temperature for distilled water.

The second approximate value of V_t is 110.196 cc. This value is sufficient to estimate the air buoyancy of sinker to 0.1 mg (air density≒1 mg/cc). The air buoyancy is calculated using $V=110.196$ cc and we get $M=110.2677$ gr. Substituting $M=110.2677$ gr and the value of k again into (1. 6′), we get

$$110.2677-\int_0^\theta (0.8787-0.5\times 10^{-4}\theta)\times 10^{-3}d\theta = V_t\rho_t\,. \qquad (1.6'')$$

If the torsional angles at C are measured at various temperatures for distilled water, the values of the volume V_t of sinker at various temperatures will be calculated from (1. 6″). The result is shown in Fig. 4. V_t is given by $V_0\times(1+\beta t)$,

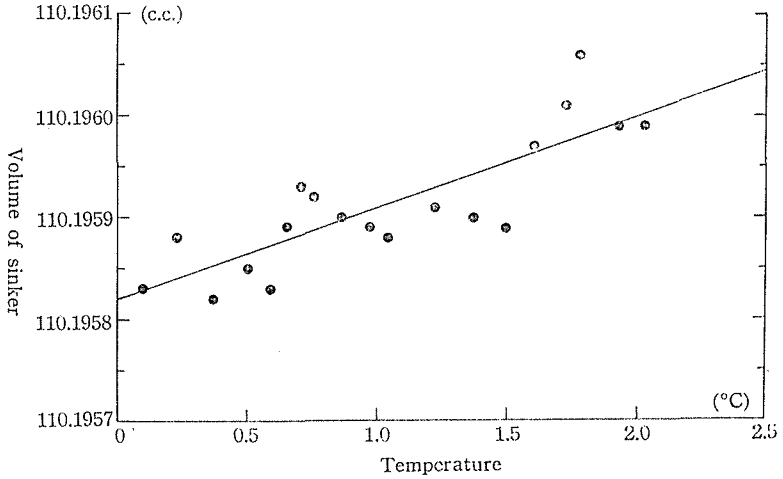


Fig. 4. Thermal expansion in volume of sinker.

where V_0 is the volume of sinker at 0°C and β is the volume coefficient of thermal expansion. V_0 and β are determined by the method of least squares as:

$$V_0 = 110.1958 \text{ (cc)} \quad \text{and} \quad \beta = 0.85\times 10^{-6} \text{ (/°C)}$$

This value of β is adequate as the coefficient of thermal expansion for the fused silica.

Putting the values of M, V_0, β and k in (1. 7), we obtain

$$\rho_t = \frac{1}{110.1958(1+0.85\times 10^{-6}t)}\left[110.2677-\int_0^\theta (0.8787-0.5\times 10^{-4}\theta)\times 10^{-3}d\theta\right]. \qquad (1.16)$$

This is the final formula for the density determination by our hydrometer.

c) *Examination of accuracy*

For given value of ρ_t, we can calculate the torsional angle from (1.16). Comparing the calculated value of θ with the observed one for distilled water, we can estimate the accuracy of the hydrometer. In Fig. 5, the solid line shows the calculated curve. The accuracy is given by $\pm 3\times10^{-6}$ g/cc. This accuracy was examined from time to time, and it was found that the accuracy was constantly kept.

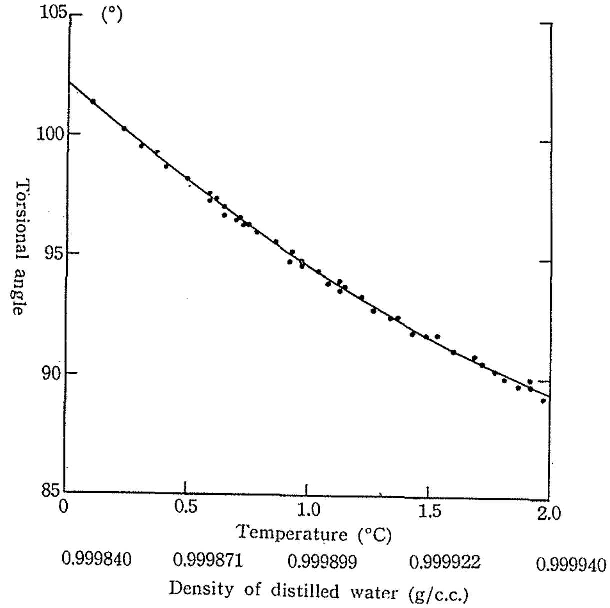


Fig. 5. Comparison of the observed and calculated torsional angles for the distilled water.

5. Method of practical calculation of density

Since Eq. (1.16) is not available for a density greater than 1.0007 g/cc, several rings made from platinum are added on sinker like Nansen's submerged hydrometer and the range of measurement is enlarged. Rings are ranked from No. 1 to No. 7 as seen in Table 2. By the adequate combination of these rings, we can measure the density up to about 1.03 g/cc.

Table 2.

No.	Mass (g)	Volume (cc) at 0°C
1	2.4958	0.1170
2	1.2243	0.0574
3	0.5164	0.0242
4	0.2999	0.0141
5	0.2116	0.0099
6	0.1039	0.0049
7	0.0554	0.0026

The density of platinum=21.335 at 0°C (*International Critical Tables*, II, p. 456)

When the density of a water sample is measured by using these rings,

$$\begin{aligned} s_t &= \frac{1}{V_t+v_{i\cdot t}}\left[M+m_i-\int_0^\theta k\,d\theta\right] = \frac{1}{V_t+v_{i\cdot t}}\left[\left\{M-\int_0^\Theta k\,d\theta\right\}+\left\{m_i+\int_\theta^\Theta k\,d\theta\right\}\right] \\ &= \left[\rho_t+\frac{m_i}{V_t}+\frac{1}{V_t}\int_\theta^\Theta k\,d\theta\right]\left[1-\frac{v_{i\cdot t}}{V_t}+\left(\frac{v_{i\cdot t}}{V_t}\right)^2-\cdots\right], \end{aligned} \tag{1.17}$$

where s_t is the density of sample water at t°C, m_i the mass of the rings added on sinker, $v_{i\cdot t}$ the volume of the rings added on sinker at t°C, ρ_t the density of distilled water at t°C, θ the torsional angle of sample water at t°C and Θ the torsional angle of distilled water at t°C.

The rings Nos. 1, 2 and 3 are combined for the maximum density of 1.03.

$$\begin{aligned} \text{Maximum of } \left(\frac{v_{i\cdot t}}{V_t}\right) &= \frac{v_{1\cdot t}+v_{2\cdot t}+v_{3\cdot t}}{V_t} = \frac{(v_{1\cdot 0}+v_{2\cdot 0}+v_{3\cdot 0})}{V_0}\,\frac{(1+\varepsilon t)}{(1+\beta t)} \\ &= 1.4980\times10^{-3}[1+(0.263\times10^{-4}-0.85\times10^{-6})t], \end{aligned}$$

where ε is the volume coefficient of thermal expansion of platinum. For the temperature between 0° and 4°C,

$$\rho_t\left[1+\frac{v_{i\cdot t}}{V_t}\right]^{-1} \risingdotseq \rho_t-\frac{v_{i\cdot 0}}{V_0}+\left(\frac{v_{i\cdot 0}}{V_0}\right)^2,$$

$$\frac{m_i}{V_t}\left[1+\frac{v_{i\cdot t}}{V_t}\right]^{-1} \risingdotseq \frac{m_i}{V_0}\left[1-\frac{v_{i\cdot 0}}{V_0}+\left(\frac{v_{i\cdot 0}}{V_0}\right)^2\right],$$

and

$$\frac{1}{V_t}\int_\theta^\Theta k\,d\theta\cdot\left[1+\frac{v_{i\cdot t}}{V_t}\right]^{-1} \risingdotseq \frac{1}{V_0}\int_\theta^\Theta k\,d\theta\,.$$

Consequently,

$$\begin{aligned} s_t = \rho_t &+ \left[\frac{m_i}{V_0}-\left(\frac{m_i}{V_0}+1\right)\left(\frac{v_{i\cdot 0}}{V_0}+\frac{v_{i\cdot 0}^2}{V_0^2}\right)\right] \\ &+0.7974\times10^{-5}(\Theta-\theta) \\ &-2.3\times10^{-10}(\Theta^2-\theta^2)\,. \end{aligned} \tag{1.18}$$

For the easy calculation of the density, the following tables and graphs are prepared for the right-hand side of Eq. (1.18).

1) Table of the density of distilled water.
2) Table of the value of second term for the adequate combination of the rings so as to cover the density between 1.00 and 1.03.
3) Table of the torsional angle Θ at t°C for distilled water.
4) Graph for the calculation of $(0.7974\times10^{-5}\theta)$.
5) Graph for the calculation of $(2.3\times10^{-10}\theta^2)$.

Eq. (1.18) gives the density. In order to convert the density into the specific gravity, the value obtained by Eq. (1.18) is divided by the density of distilled water at 4°C. Practically, it is sufficient to add a numerical constant 28×10^{-6} to s_t.

6. Reduction of density

As mentioned in Section 3, the temperature of sample water should be read within ±0.01°C to determine the density to $\pm 10^{-6}$ g/cc. Since it is practically impossible to carry out measurement of all samples at a constant temperature kept accurately within ±0.01°C, the reduction of density to the specified temperature is required. The minimum rates of the density change occur at 4°C and −2°C for distilled water and sea water respectively. Taking into account the easy control of the temperature of water samples, we specify 0°C as the standard temperature.

To obtain the method of reduction, the density variation due to temperature was measured by the hydrometer for some adequate samples. In Fig. 6, the

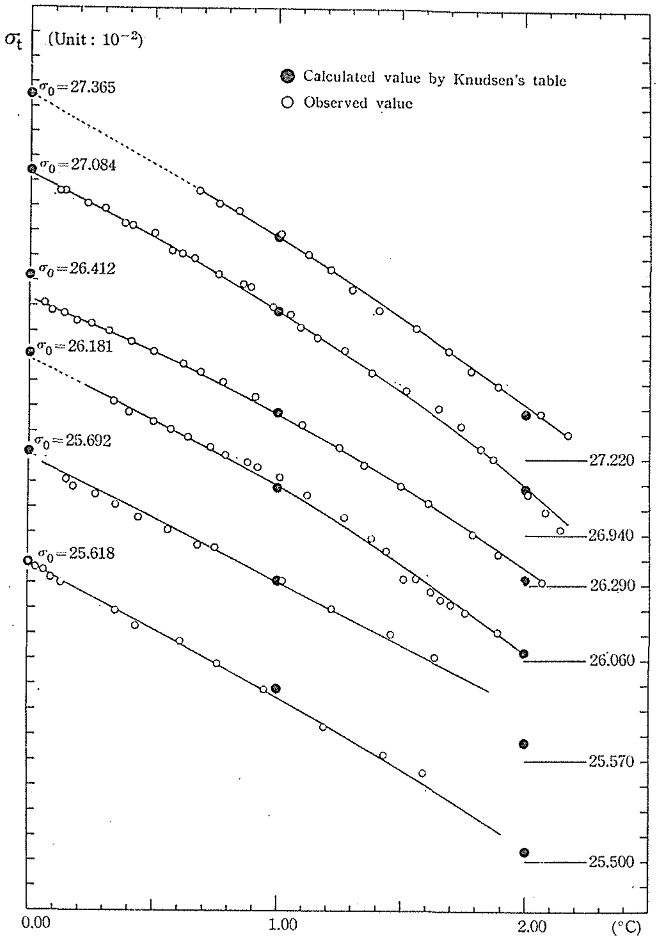


Fig. 6. Density variation of sea water sample by direct measurement using hydrometer constructed.

results are shown by smooth curves which naturally pass through the observed values marked by ○ of each sample. On each smooth curve, the actual density σ_1' just at 1°C is read (for example, 27.308 in top curve). The normal water which has the value σ_1' at 1°C is sought in "Knudsen Tables", and the densities σ_0 and σ_2 which this normal water has at 0°C and 2°C are plotted on the smooth curve respectively by the symbol ◍. The points ◍'s fall almost on the respective smooth curve. This means that, although the density of the sample water may not coincide with the density of normal water, the increment of the density due to a temperature change is the same.

To reduce exactly, more of such smooth curves should be drawn, but this reduction referred to Knudsen Tables may be sufficiently accurate between 0° and 2°C.

7. Preservation of sample water

Sample water was preserved into glass bottle and was sealed to avoid evaporation. Aged bottles were carefully selected lest the silicate and others should melt into water. To make sure of this point, it was examined whether the density of samples varied in the cause of preservation or not, by the successive measurements of 9 samples which were dipped in large quantities at the same time and at the same place and poured into the sampling bottles. The result is shown in Table 3. Their mean density is 1.022566. This result may ensure that the density of a sample is not practically varied during the preservation of nine months.

Table 3.

Date of measurement	Specific gravity
1958 Apr. 5	1.022564
" 12	1.022563
" 22	1.022563
May 17	1.022566
" 31	1.022570
Jun. 29	1.022565
Nov. 25	1.022567
Dec. 10	1.022570
" 26	1.022570

Sampled on 28, Mar. 1958, in Kaida Bay, Hiroshima Prefecture.

8. Measurment of chlorinity

The chlorinity was measured by Knudsen's method. Titration was carried out by the automatic potential titrimeter (with calomel electrode and salt bridge K_2SO_4), which has the potential difference of 0.187 volt and the sensitivity of 1 volt/‰ at the equivalent point. The chlorinity was obtained to the accuracy of 0.003‰ practically.

The correction k due to α, which is obtained from the titration of standard sea water, was graphically read with 0.001‰ referring to J. W. McGary's paper (**6**).

Part II

SALINITY DEFICIT AND EXCHANGE OF SEA WATER

9. Exchange of water through Akashi Straits

The water in Kii Channel inflows into Osaka Bay and Harima Sea through Kitan Straits and Naruto Straits respectively, being influenced by the influx of land water from Shikoku and Kii Peninsula (Fig. 7). Osaka Bay is connected with Harima Sea through Akashi Straits.

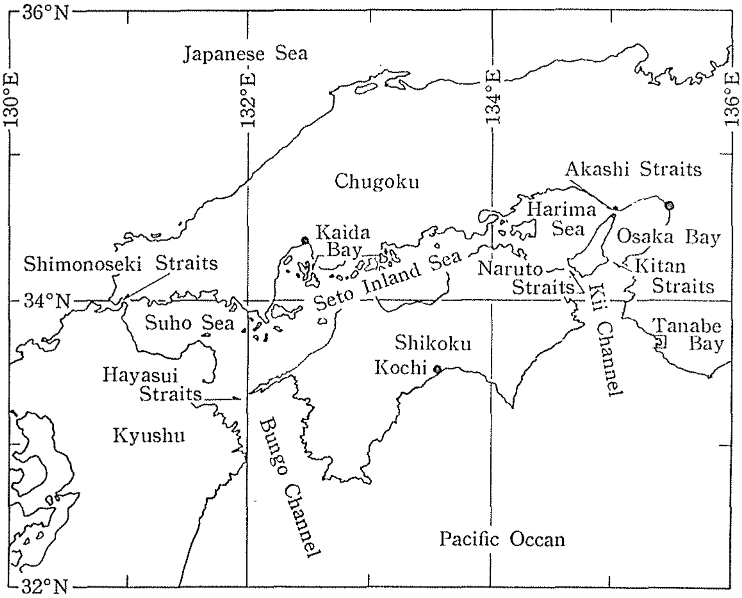


Fig. 7. Map of Seto Inland Sea.

In August 1955, an investigation of the exchange of sea water through Akashi Straits was carried out. Mean surface waters were taken on the survey line stretched from Akashi to Iwaya, which is shown by AB in Fig. 8. Sampling was made hourly ten times a day, and every day from August 2 to 17. The chlorinity and salinity deficit of samples were measured. These variations are shown in Fig. 9 with the change of tidal current and sea level at the same time.

Tidal variations in Cl and $\Delta\sigma_0$ are clearly seen in Fig. 9 and indicate that the exchange of water through the straits are governed by tidal currents. From the results of investigations, it is seen that (i) according as the tidal current becomes strong westerly or easterly, chlorinity increases or decreases respectively and (ii) as the westerly current becomes strong, salinity deficit decreases in major

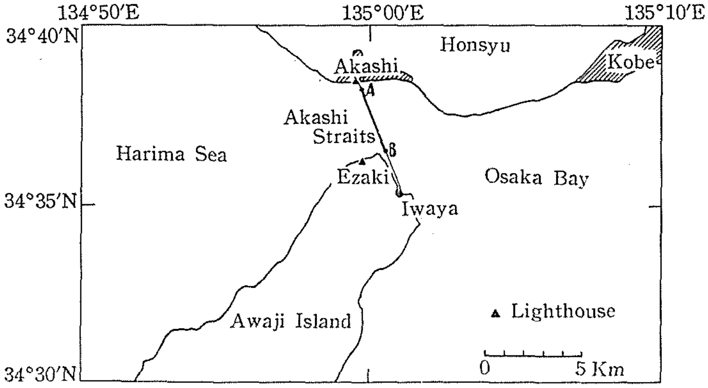


Fig. 8. Map showing the survey line (Aug. 1955).

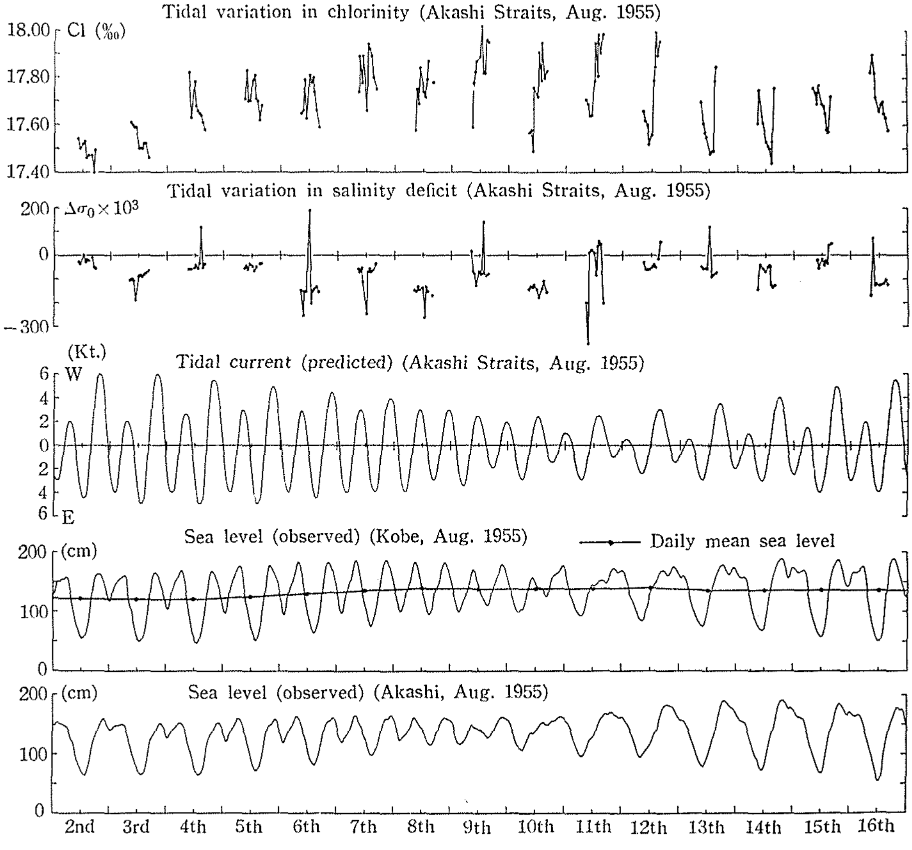


Fig. 9. Variations in chlorinity, salinity deficit, tidal current at Akshi Straits and sea level changes at Kobe and Akashi.

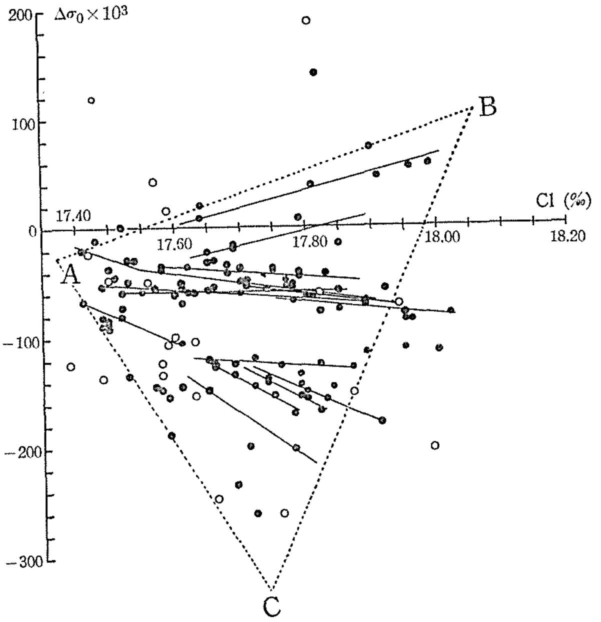


Fig. 10. Correlation diagram between Cl and $\Delta\sigma_0$.

cases. These relations are more clearly shown in the Cl–$\Delta\sigma_0$ diagram (Fig. 10).* In this diagram, full lines indicate regression lines day by day and white circles represent the values near the time of slack water. It seems that regression lines converge to one point denoted by A, diverge in the reverse direction and their envelope forms a triangle ABC in which major portion of points are contained. Among the points which scatter outside the triangle, many points occur at or near the time of slack water. From these results, it is inferred that the point A represents the water type of Harima Sea and the water which appears in the eastern part of Akashi Straits is the mixture of two water types represented by the points B and C.

Moreover, the points in Cl–$\Delta\sigma_0$ diagram are arranged close to a straight line from slack to slack and they have a tendency to scatter at and near the time of slack. This fact may suggest that (a) during the tidal current flows in one

---

* In a paper published in 1956 (7), the values of salinity deficit shown in all figures are not accurate because the reduction of water density to 0°C was referred to the decrement of pure water density due to temperature changes. Those values were corrected by the method of reduction explained in § 6, and Figs. 9 and 10 correspond to Figs. 2 and 6 in the previous paper respectively.

However, the conclusions in the previous paper still hold good, because the corrections merely make the incorrect values of salinity deficit lower almost uniformly and the configurations of variations remain unchanged.

direction, the space patterns of Cl and $\Delta\sigma_0$ are conserved and are carried by tidal current without appreciable mixing; and (b) the mixing occurs mainly at the time of slack, and new space patterns of Cl and $\Delta\sigma_0$ are made up. This concept is analogous to the kinetic theory of gas. The slack water corresponds to the mutual collision of gas particles and the tidal path from one slack to the next slack (tidal excursion) corresponds to the mean free path.

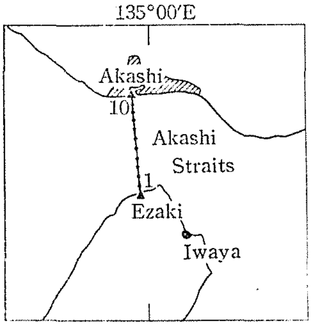


Fig. 11. Locations of 10 stations (from Aug. 31 to Sept. 1, 1956).

In Fig. 9, the variation of 6-hour-period is seen both in Cl and $\Delta\sigma_0$, but it is not clear because sampling was only made 10 times a day. To obtain more informations, an investigation was carried out from August 31 to September 1, 1956, at 10 stations set on the previous survey line. Surface water was successively taken every hour 30 times at each station. The weather was fine during the observation. The position of stations is shown in Fig. 11.

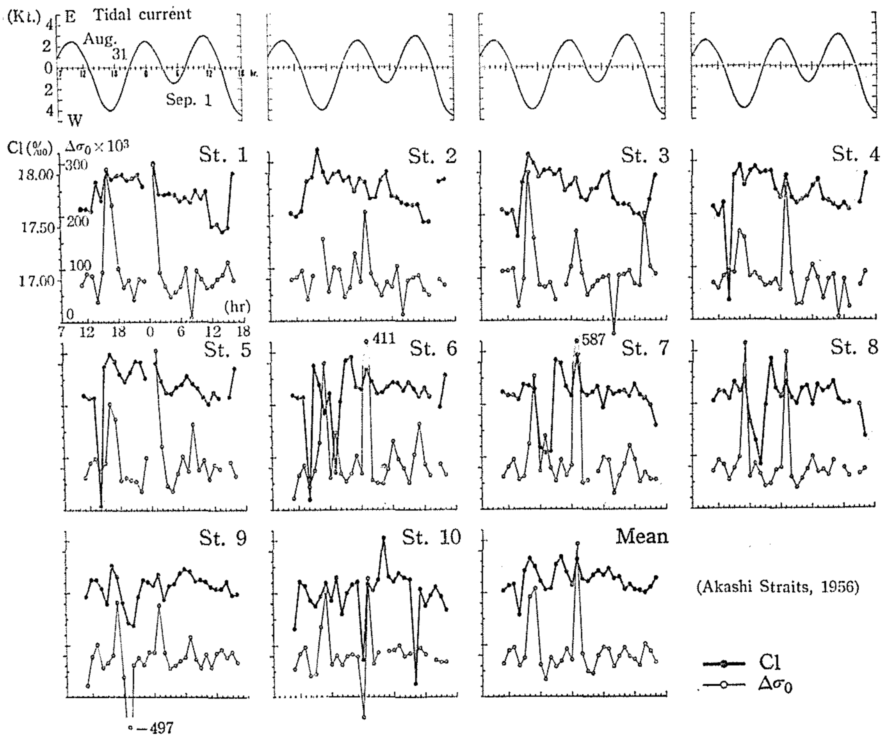


Fig. 12. Change of tidal current and variations in chlorinity and salinity deficit in Akashi Straits.

The variation in *Cl* and $\Delta\sigma_0$ obtained is shown in Fig. 12.

At each station, the variations of 6-hour-period of *Cl* and $\Delta\sigma_0$ are clearly seen. In Akashi Straits, the tidal current of 6-hour-period has been verified by observation. The component of 6-hour-period of tidal current in the present investigation is shown in Fig. 13, but it has not the velocity strong enough to account for the variation of 6-hour-period in *Cl* and $\Delta\sigma_0$.

It seems that at stations 1~5 near Awaji Island, the phases of the variations in *Cl* and $\Delta\sigma_0$ are the same, and at stations 6~10 near Akashi, they are contrary to each other. In order to make these relations clearer, the running means of *Cl* and $\Delta\sigma_0$ were calculated at 7-hour-intervals. Their running means are drawn in Fig. 13. From these, it may be considered that the water in the northern part of Akashi Straits has the character represented by the point C and the water in the southern part of the same Straits has the character represented by the point B.

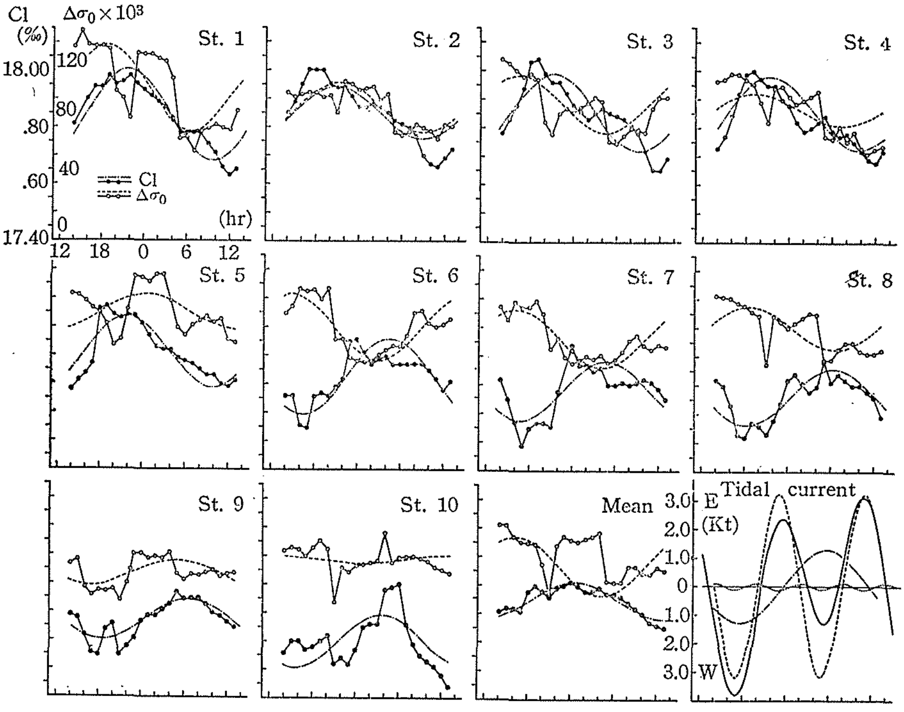


Fig. 13. Variations in running means of chlorinity and salinity deficit at 7 hrs. intervals at each station.

Fig. 14 shows the result of similar investigations in Hayasui Straits. Hayasui Straits is another channel communicating the water of Seto Inland Sea with open

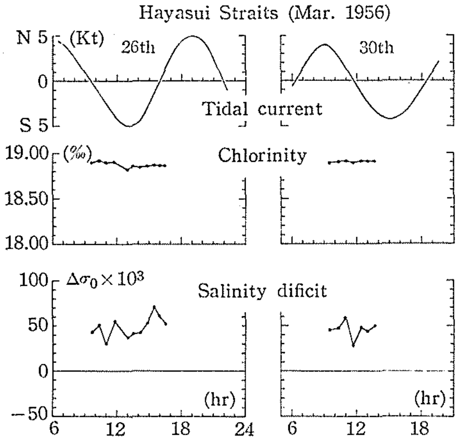


Fig. 14. Variations in tidal current, chlorinity and salinity deficit on Hayasui Straits.

sea. Also in this strait, tidal variations in $\Delta\sigma_0$ are seen and it is probably that as the water of Inland Sea southerly flows, the value of $\Delta\sigma_0$ increases and that a variation of 6-hour-period of $\Delta\sigma_0$ also exists, although it is not definite because of a short observation.

In 1951, H. Stommel and A.B. Arons (8) proposed a model of a large scale horizontal turbulence made up of tidal currents and, applying the concept of mixing length, they discussed the horizontal mixing by tidal currents as a sort of diffusion phenomenon.

Expressing tidal excursion by l, the mixing coefficient η will be given by a formula:

$$\eta = \alpha u l \quad \text{with} \quad l = uT, \tag{2.1}$$

where u is the mean velocity of tidal current, α a proportional factor representing the degree of mixing and T the half-period of a tidal current. Now we shall imagine a working model such that two semi-infinite water bodies of chlorinity Cl_0 and Cl_1 are initially ($t \leq 0$) separated by a vertical boundary plane ($x=0$) and then suddenly mixes by tidal currents in the direction of x, positive eastward, the distribution of chlorinity along the x-direction at any moment $t>0$ will be given by

$$Cl = \frac{Cl_1+Cl_0}{2} + \frac{Cl_1-Cl_0}{2} \operatorname{erf}\left(\frac{x}{2\sqrt{\eta t}}\right). \tag{2.2}$$

The expression (2.2) may be transformed into a dimensionless form:

$$f\left(\frac{x}{l}\right) = \operatorname{erf}\left(\frac{x/l}{2\sqrt{\eta t/l}}\right), \quad \text{where} \quad f\left(\frac{x}{l}\right) = \frac{2Cl-Cl_1-Cl_0}{Cl_1-Cl_0}. \tag{2.3}$$

In the present observation, t is given by T in the sense of average.

The observed variations in chlorinity are time distributions, but on the concept of mixing length, they may be transformed into space distributions by a substitution:

$$x = x_0 - \int_{t_0}^{t} u_0 \sin\left(\pi \frac{t-t_0}{T}\right) dt, \tag{2.4}$$

where u_0 is the maximum velocity of a tidal current, x is the position where a volume of water observed at (x_0, t) is expected to appear at the time t_0 of the preceding slack.

This model was applied to the variation of chlorinity in Akashi Straits. The chlorinities at the point A and points of intersection of daily regression lines and BC-line in Fig. 10 were assigned to the values of Cl_0 and Cl_1 respectively. The values of u_0 and l were calculated from the predicted tidal current in Fig. 9. The formula (2.4) gives only the relative value of x. Its absolute value must be estimated from the origin of the x-axis, that is, the point where error function in (2.3) vanishes. In the actual state, the position of this point is not stationary but fluctuates eastward and westward according to the advective current caused by the long-period fluctuations of the daily mean sea level. It will be difficult to determine such a position. Nevertheless, if the present model is applicable, it will be approximately determined for each half-period from slack to slack by interpolating or extrapolating the graph of error function which fits best to the relative distribution of the points observed during the same period.

The values of the function $f(x/l)$ in (2.3) were then plotted against x/l, the result of which is shown in Fig. 15. In this figure, white circles represent average values of $f(x/l)$ for every interval of x/l equal to 0.1. The full line in Fig. 15 is the graph of error function which fits best to the white circles. The fitness is moderate, but will be sufficient to suggest that the relation (2.3) may be valid in the average of large numbers. From Fig. 15 we get

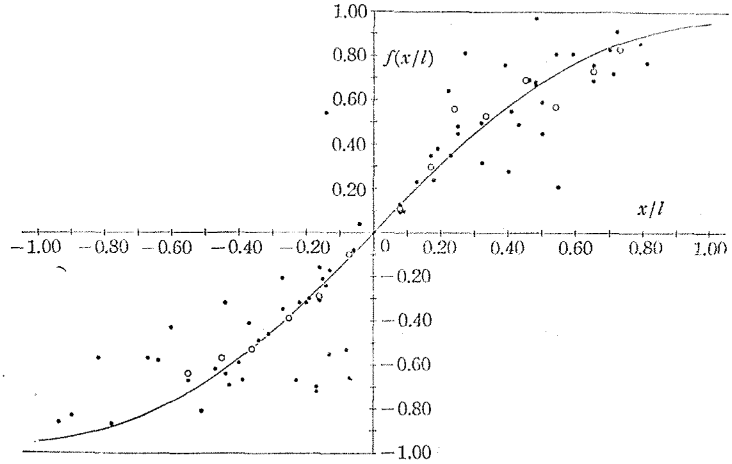


Fig. 15. Longitudinal distribution of chlorinity converted from the time change of chlorinity with reference to the concept of mixing length theory.

$$\operatorname{erf}\left(\frac{x/l}{2\sqrt{\eta t/l}}\right) = 0.95 \qquad \text{for} \quad \frac{x}{l} = 1, \tag{2.5}$$

Hence by a table of error function, we get

$$\frac{l}{2\sqrt{\eta t}} = 1.4 \qquad \text{or} \qquad \eta t = \frac{1}{7.8} l^2 . \tag{2.6}$$

In our case t is equal to T, so that

$$\eta = \frac{1}{7.8} ul = 0.13\, ul , \tag{2.7}$$

which is the same with the formula (2.1) and we get

$$\alpha = 0.13 ; \tag{2.8}$$

in other words, about 10% of the water mixes every half-period of a tidal current (S. Hayami *et al.*, (7)).

The water of Seto Inland Sea communicates with open sea through narrow straits such as Akashi, Naruto, Hayasui and Shimonoseki. From the evidence mentioned above, it may be inferred that the exchange between the water of Seto Inland Sea and open sea water is mainly operated by tidal mixing.

10. Exchange of water in Tanabe Bay

Tanabe Bay is located on the southwest coast of Kii Peninsula. The bay opens to west and connects with Pacific Ocean (Fig. 7). The width of the bay is about 5 km. In November, 1956, an investigation was carried out on a survey line through the mouth of the bay. Sampling was taken hourly during 12 hours a day, and every day from November 2nd to 8th. The result of the investigation is shown in Fig. 16. From this figure, it is found that (i) from 2nd to 5th, the daily variation of chlorinity follows the change of sea level, and after 6th it nearly disappears; (ii) the daily mean value of chlorinity increases gradually; and (iii) in the variation of $\Delta\sigma_0$, tidal change is conspicuously seen, and its tidal range fluctuates according to the sea level caused by a long-period tide.

The gradual increase of the daily mean value of chlorinity is due to the effect of rainfall. In Table 4, a record of rainfall observed at Seto Marine Biological Laboratory is given. Due to the heavy rainfall on October 31, the chlorinity of surface water suddenly decreased and gradually recovered probably by tidal mixing. From the evidence that the daily variation of chlorinity is not practically on

Table 4.

Date (1956)	October			November								
	29	30	31	1	2	3	4	5	6	7	8	9
Rainfall (mm)	3.9	0	130.5	0	0	0	22.5	0	0	0.53	0.84	0

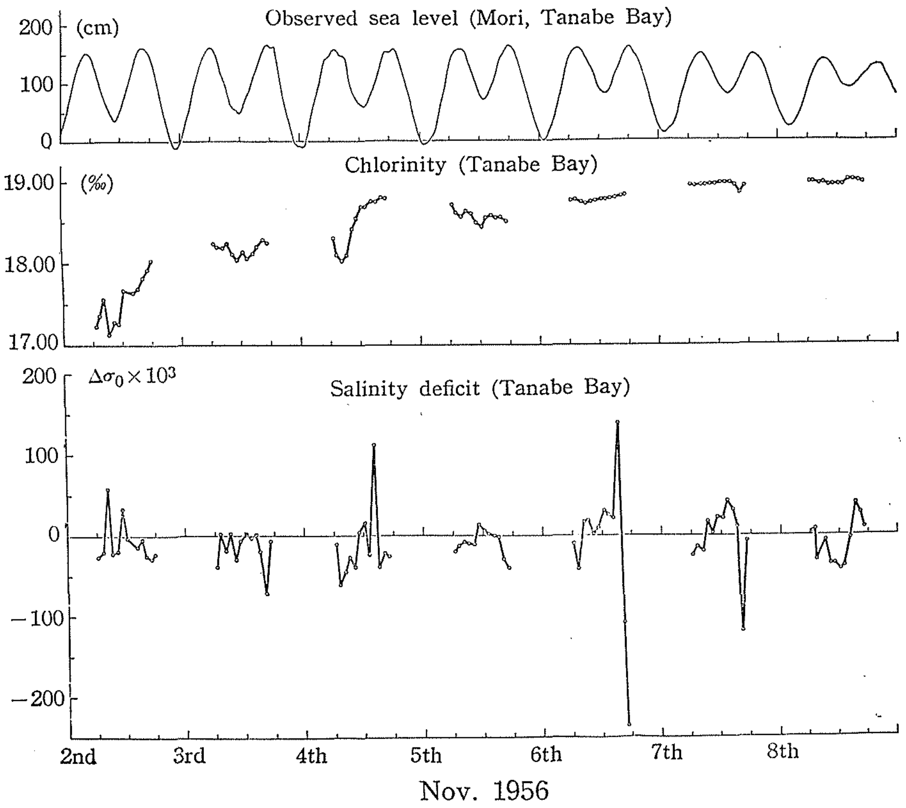


Fig. 16. Tidal variations of sea level, chlorinity and salinity deficit on the bay mouth line of Tanabe Bay.

November 7 and 8, it is supposed that the chlorinity in the bay had risen to the value of those outside the bay.

The correlation coefficients between sea level and salinity deficit are shown in Table 5, and they are all negative. This indicates that the salinity deficit is higher in the bay than outside the bay, and it is suggested that the salt of sea water is consumed in the bay. (This point will be discussed fully in Part III.)

Table 5. Correlation coefficient between sea level ζ and salinity deficit $\Delta\sigma_0$. (Tanabe Bay, November, 1956)

Date	2	3	4	5	6	7	8
$R(\zeta\sim\Delta\sigma_0)$	−0.71	−0.71	−0.50	−0.97	−0.38	−0.50	−0.71

Using the recovering process of the chlorinity, the value of the coefficient of tidal mixing will be estimated as follows. As a working model, we take a rectangular bay with uniform depth of mixing layer and take coordinate axes as

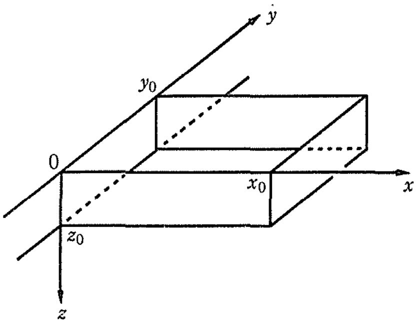


Fig. 17. Bay model and coordinate system.

shown in Fig. 17. The diminution of chlorinity from the value before the rainfall will be denoted as q. Then, q is diffused by tidal mixing.

The diffusion equation for q is given by

$$\frac{\partial q}{\partial t}=\eta_1\frac{\partial^2 q}{\partial x^2}+\eta_2\frac{\partial^2 q}{\partial y^2}+\eta_3\frac{\partial^2 q}{\partial z^2}, \qquad (2.9)$$

where η_1, η_2 and η_3 are the mixing coefficients in the x-, y- and z-directions, respectively.

Boundary conditions for q are

$$\left.\begin{aligned}&\eta_2\frac{\partial q}{\partial y}=0,\quad \text{at}\quad y=0\quad\text{and}\quad y_0\quad\text{(both sides)},\\&\eta_3\frac{\partial q}{\partial z}=0,\quad \text{at}\quad z=0\ \text{(surface)}\quad\text{and}\quad z_0\ \text{(depth of mixing layer)}.\end{aligned}\right\} \qquad (2.10)$$

Integrating (2.9) from the surface to the depth of mixing layer and from one side to the other, we have

$$\frac{\partial Q}{\partial t}=\eta_1\frac{\partial^2 Q}{\partial x^2}, \qquad (2.11)$$

where Q is the mean value of q in a cross section of the bay. The initial and boundary conditions for Q are assumed as:

$$\left.\begin{aligned}&Q=Q_0\quad\text{at}\quad t=0,\qquad 0<x\leqq x_0,\\&Q=Q_1(t)\quad\text{at}\quad t>0,\quad x=0\qquad\text{(bay mouth)},\\&\frac{\partial Q}{\partial x}=0\qquad\text{at}\qquad x=x_0\qquad\text{(bay head)},\end{aligned}\right\} \qquad (2.12)$$

where Q_0 is a certain numerical constant and $Q_1(0)=0$. If $Q_1(t)$ varies slowly with time, the solution satisfying these conditions is expressed with sufficient accuracy by

$$Q\simeq Q_1+\frac{4Q_0}{\pi}\sin\frac{\pi x}{2x_0}\exp\left(-\frac{\eta_1\pi^2}{4x_0^2}t\right),$$

or

$$Q-Q_1\simeq K\exp(-kt), \qquad (2.13)$$

where

$$K\equiv\frac{4Q_0}{\pi}\sin\frac{\pi x}{2x_0},\quad k\equiv\frac{\eta_1\pi^2}{4x_0^2}. \qquad (2.14)$$

If we denote the mean chlorinity in the bay and at the bay mouth by $Cl(t)$ and

$Cl_1(t)$ respectively, and assume that the bay water, which had originally the same chlorinity with the outer water, was suddenly diluted at $t=0$ by the inflow of land water due to the rainfall, and that the direct effect of the rainfall over the sea surface is alike everywhere, we may put

$$Q(t) = Cl_1(0) - Cl(t)\,,$$
$$Q_0 = Cl_1(0) - Cl(0)\,,$$
$$Q_1(t) = Cl_1(0) - Cl_1(t)\,.$$

Hence,

$$Q - Q_1 = Cl_1(t) - Cl(t)\,.$$

Assuming that the time change of $Cl_1(t)$ is small as compared with that of $Cl(t)$, and then differentiating (2.13) with respect to t, we get

$$\frac{\partial Cl}{\partial t} = Kk \exp(-kt)\,, \tag{2.15}$$

which gives

$$\log \frac{\partial Cl}{\partial t} = \log(Kk) - kt\,. \tag{2.16}$$

Using observed daily mean values of $Cl(t)$, we get from (2.16)

$$k = 0.6 \qquad \text{(per day)}\,,$$

and

$$K = 3.33 \qquad (\%)\,.$$

Putting $x_0=5$ km and $k=0.6$/day in (2.14), we obtain

$$\eta = 7.05 \times 10^5 \qquad (\text{cm}^2/\text{sec})\,. \tag{2.17}$$

Assuming the relation (2.1) and putting $\alpha=0.1$ and $T=6$ hrs., we have

$$u = 18 \qquad (\text{cm/sec}) \tag{2.18}$$

which seems to be of right order of magnitude in Tanabe Bay (S. Hayami *et al.*, (**9**)).

On August 10, 1957, the chlorinity and salinity deficit of various depths were investigated at twenty-six stations distributed uniformly in Tanabe Bay, sea water being sampled at 0, 5, 10, 15, 20 and 25 m layers.

On 11th, mean water at the same layers was sampled along the same line through the bay mouth as in the previous investigation (November 1956). The results of the investigation are given in Figs. 18~20.

The change of salinity deficit at the bay mouth has a clear component of 3-hour-period at all layers. In Akashi Straits and probably in Hayasui Straits, it was already seen that the salinity deficit had the component of 6-hour-period. It may be due to eddies which are generated every 6 hours at or around the time of slack water of semi-diurnal tide. Then the component of 3-hour-period in this

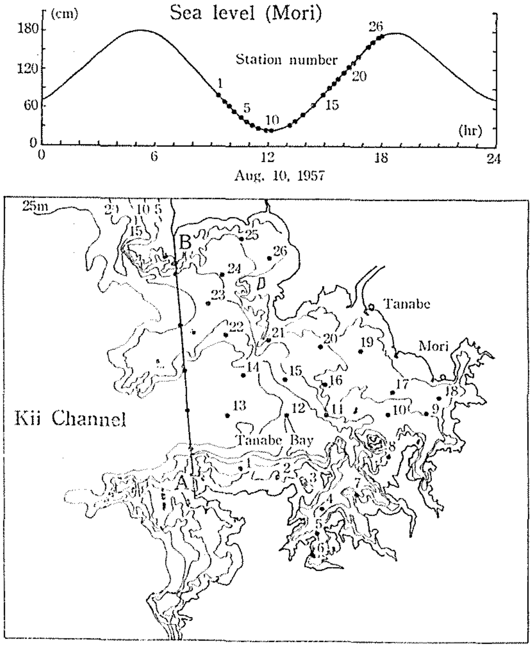


Fig. 18. Location of sampling stations where the figure at each station indicates the number of station in the sequence of observations (lower). Time of observation and phase of tide at each station (upper).

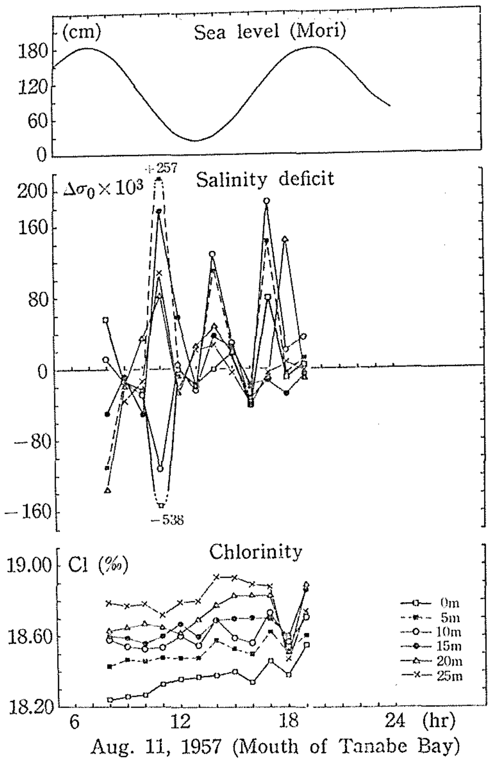


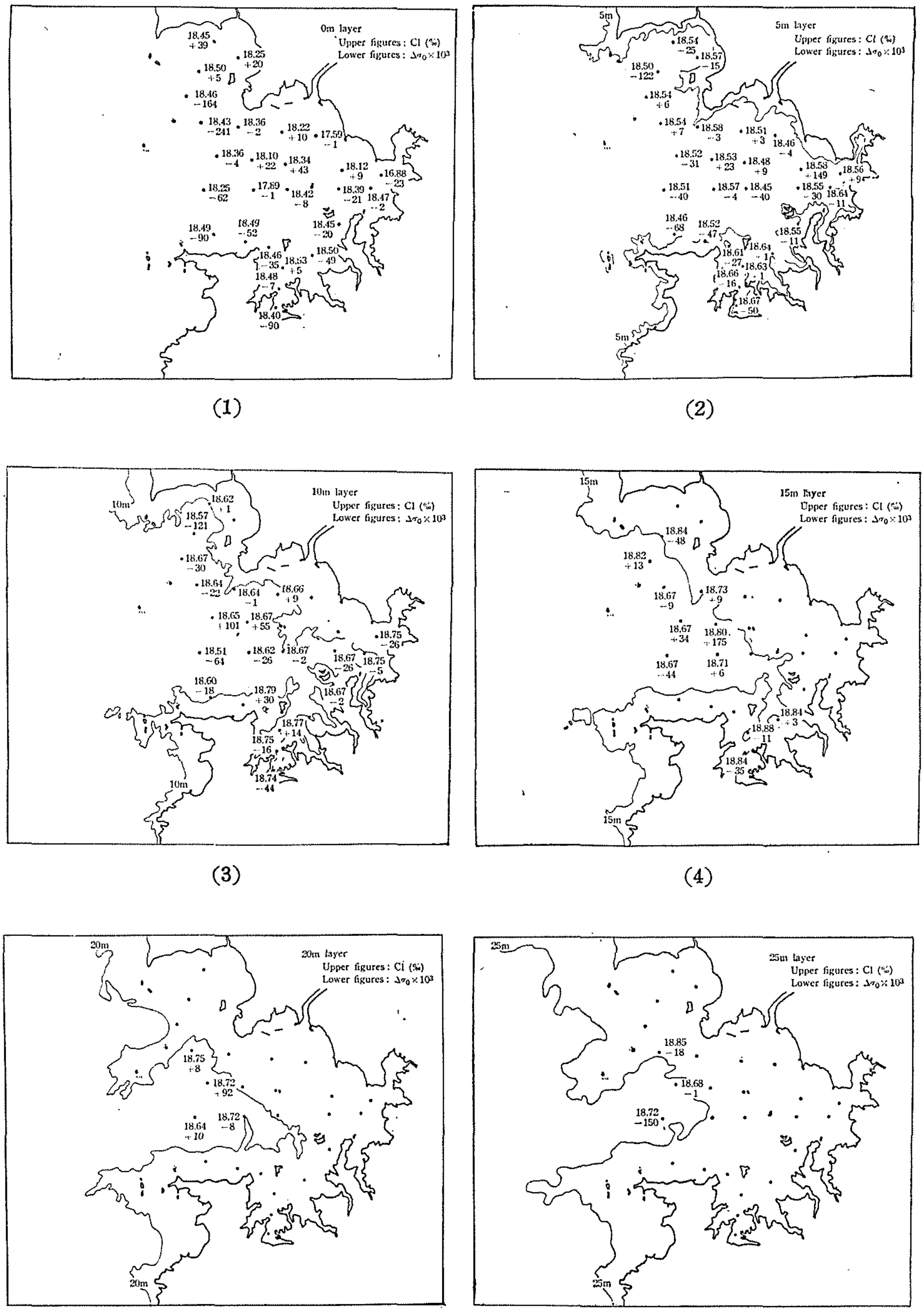
Fig. 19. Tidal variations in the salinity deficit and chlorinity in Tanade Bay, Aug. 11, 1957.

bay may be due to eddies associated with the slack water of quatile diurnal tide. The tidal range of $\Delta\sigma_0$ does not vary much with the depth. This is interesting as compared with the distribution of $\Delta\sigma_0$ in the bay. Table 6 shows the mean chlorinity and the mean salinity deficit at 26 stations in the bay at every layer. Except the value at 25 m layer, $\Delta\sigma_0$ increases with the depth.

Table 6.

Depth (m)	0	5	10	15	20	25
Chlorinity (‰)	18.28	18.55	18.67	18.77	18.71	18.75
Salinity deficit $\Delta\sigma_0\times10^3$	−27.7	−12.9	−10.6	+18.5	+25.5	−56.3

The salinity deficit is diffused by tidal mixing in the same way as the chlorinity, temperature and so on. At the time of investigation carried out in November 1956, the salinity deficit of the bay water was higher than that of the outer water. In



(1) (2)

(3) (4)

(5) (6)

Fig. 20. Distribution of chlorinity and salinity deficit in Tanabe Bay, Aug. 10, 1957.

such a case, the salinity deficit flows out of the bay through the bay mouth by tidal mixing. This flux of salinity deficit is given by $\eta\frac{\delta\varDelta\sigma_0}{\delta x}$, where η is mixing coefficient. The range of tidal change in $\varDelta\sigma_0$ at the bay mouth is a change of $\varDelta\sigma_0$ for $\delta x = uT$, where u is the mean current velocity at the specified depth during the half-period T of tide. Since the mixing coefficient η is proportional to u^2T, the flux varies directly with $u\delta\varDelta\sigma_0$. On the other hand, the tidal range of $\varDelta\sigma_0$ does not sensibly change with the depth, while current velocity decreases with the depth. Hence, the flux of salinity deficit decreases with the depth. The abrupt increase below the depth of 15 m seems to have a connection with the corresponding contraction of the bay below this level which will cause a rapid decrease of current velocity (S. Hayami *et al.*, (**10**)).

Such a phenomenon is seen again in the horizontal distribution. As is seen in Fig. 20, the relatively high values of $\varDelta\sigma_0$ occur in the central area and the small values of $\varDelta\sigma_0$ surround them. It is remarked that the higher values of $\varDelta\sigma_0$ are found at every layer in the same region. This suggests the existence of a large eddy in this area, in accordance with tidal current observations (A. Nakamura (**11**)). The high value of salinity deficit is probably due to the stagnation of water. The distribution of the salinity deficit seems to be closely related with the degree of mixing.

Part III

SALINITY DEFICIT AND PRODUCTIVITY OF THE SEA

11. Rate of consumption of dissolved matter in sea water

It was found in the preceding part that the salinity deficit was a valuable factor to represent the character of sea water. The water with a relatively high salinity deficit was found in the central part of Tanabe Bay. As mentioned in the introduction, the decrease of salinity deficit means the replenishment of the salt into sea water. In the case of distribution as shown in Fig. 20, the bay water may be supplied with a certain amount of salts from the outer water of lower salinity deficit through the tidal mixing. If we presume that the above distribution is stationary, the same amount of salts should be consumed in the bay. This relation will be given quantitatively by the diffusion equation of salinity deficit.

Since the increment of 10^{-6} in specific gravity corresponds to the addition of about 1.2 g of salts into one cubic meter of sea water, the salinity deficit is converted into the mass of salt q contained in a unit volume of sea water:

$$q = 1.2\times10^3\varDelta\sigma_0 \qquad (\mathrm{g/m^3})\,. \tag{3.1}$$

Tanabe Bay is modelled again as given in Fig. 17. The diffusion equation of the salinity deficit under no mean current is given by

$$\frac{\partial q}{\partial t}=\eta_1\frac{\partial^2 q}{\partial x^2}+\eta_2\frac{\partial^2 q}{\partial y^2}+\eta_3\frac{\partial^2 q}{\partial z^2}+c(x,y,z,t)\,, \tag{3.2}$$

where η_1, η_2 are the coefficients of horizontal mixing, while η_3 is that for the vertical mixing and c the rate of consumption.

The depth of mixing layer is assumed to be uniform, and the boundary conditions are taken as follows:

$$\left.\begin{aligned}&\eta_2\frac{\partial q}{\partial y}=0 \quad \text{at} \quad y=0 \quad \text{and} \quad y_0 \text{ (both sides)}\,,\\ &\eta_3\frac{\partial q}{\partial z}=0 \quad \text{at} \quad z=0 \text{ (surface)} \quad \text{and} \quad z_0 \text{ (depth of mixing layer)}\,,\\ &\eta_1\frac{\partial q}{\partial x}=0 \quad \text{at} \quad x=x_0 \qquad \text{(bay head)}\,.\end{aligned}\right\} \tag{3.3}$$

Integrating (3.2) with respect to y and z, we have

$$\frac{\partial Q}{\partial t}=\eta_1\frac{\partial^2 Q}{\partial x^2}+C\,, \tag{3.4}$$

where Q and C are the mean value of q and c respectively over a cross section of the bay.

In the steady state, we have

$$0=\eta_1\frac{\partial^2 Q}{\partial x^2}+C\,. \tag{3.5}$$

Assuming C to be independent of x and using the condition at bay head, we get

$$\eta_1\frac{\partial Q}{\partial x}=C(x_0-x)\,, \tag{3.6}$$

while at the bay mouth, we get

$$\eta_1\left(\frac{\partial Q}{\partial x}\right)_0=Cx_0\,. \tag{3.7}$$

This equation indicates that the consumption of salts in the bay balances with the supply from the bay mouth. The solution of Eq. (3.6) is given by

$$Q=Q_m+\frac{C}{2\eta_1}[2x_0x-x^2]\,, \tag{3.8}$$

where Q_m is the salinity deficit at the bay mouth.

From 6th to 7th of August 1957, I. Yamazi (**12**, **13**) investigated the distribution of zooplankton in the same bay. The numbers in Fig. 21 represented the settling volume of zooplankton. The plankton net was hauled in the upper 5 m layer, and 50 litres of sea water were passed through the net at every haul.

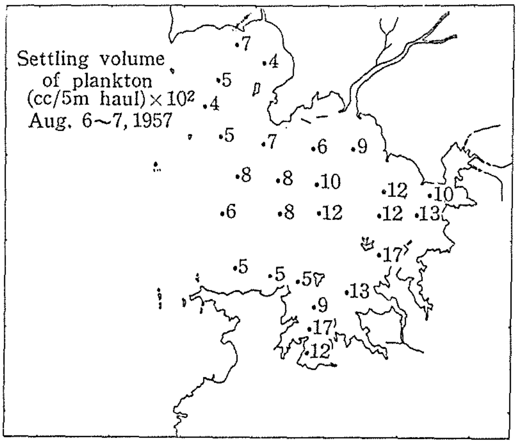


Fig. 21. Distribution of the settling volume of zooplankton in the upper 5 m layer.

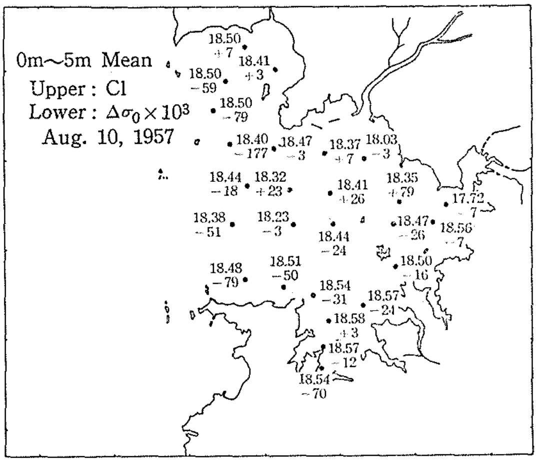


Fig. 22. Distribution of the mean values of chlorinity and salinity deficit at the surface and 5 m layer.

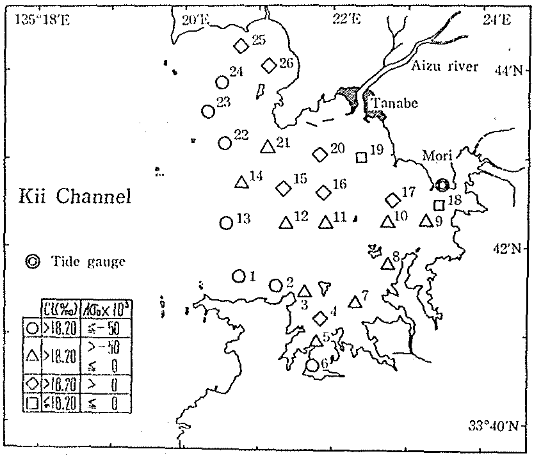


Fig. 23. Chlorinity and salinity deficit distribution represented by the symbols in table.

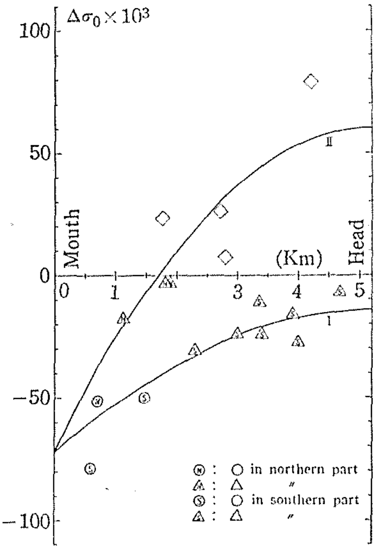


Fig. 24. Relation between the salinity deficit and the distance from the bay mouth line,

Since abundance of phytoplankton causes the growth of zooplankton, zooplankton population may be regarded as a measure of the consumption of salts and other materials in the water. In order to compare $\Delta\sigma_0$ distribution with zooplankton distribution, the mean values at 0 m and 5 m depths of Cl and $\Delta\sigma_0$ were calculated as shown in Fig. 22. Using symbols, Fig. 22 was redrawn as shown in Fig. 23 and the longitudinal distribution of the salinity deficit in Fig. 22 was shown in Fig. 24. In this figure, it was seen that the points in the southern part were arranged along a curve denoted by I, while the points in the northern part were arranged along a curve denoted by II.

Assuming that the distribution shown in Fig. 24 is for the steady state, Eq. (3.8) will give the forms of the curves I and II in Fig. 24. Taking the mean value of seven ○ symbols as Q_m, we get

$$Q_m = -72\times 1.2 \qquad (\text{g/m}^3)\,.$$

Most plausible values of $C/(2\eta_1)$ in Eq. (3.8) are found by the method of least squares for the curves I and II respectively. The result is given in Table 7. The curves I and II in Fig. 24 are drawn by using these values.

Table 7.

	$C/(2\eta_1)$(g/m^5)
Curve I (southern part)	0.26×10^{-5}
Curve II (northern part)	0.59×10^{-5}

The ratio of $C/(2\eta_1)$ in the southern part to that in the northern part is

$$\frac{(C/(2\eta_1))_{\text{I}}}{(C/(2\eta_1))_{\text{II}}} = \frac{1}{2.28}\,.$$

Since the rate C is not known definitely, it is assumed tentatively that the rate C depends on the settling volume of zooplankton, then

$$\frac{C_{\text{I}}}{C_{\text{II}}} = \frac{P_{\text{I}}}{P_{\text{II}}} = 1.13\,, \quad \frac{\eta_{1\cdot\text{I}}}{\eta_{1\cdot\text{II}}} = 2.58\,,$$

where P_{I} and P_{II} are the mean values of the settling volume for the curves I and II respectively. The rate of consumption in the southern part is roughly equal to that in the northern part, and $\eta_{1\cdot\text{I}}$ is about two and half times as large as $\eta_{1\cdot\text{II}}$, that is, the replenishment of salt through the bay mouth is about two and half times larger in the southern part than in the northern part.

Taking the mean of $C/(2\eta_1)$ in the southern and northern parts and putting η_1 as 7.05×10^5 (c.g.s.), we get

$$C_{\text{mean}} = 0.85\times10^{-5}\,\eta_{\text{mean}} \quad (\text{g/m}^3\,\text{sec})\,, \tag{3.9}$$
$$= 52 \quad (\text{g/m}^3\,\text{day}) \quad (\text{Y. Fukuo } (\mathbf{14}))\,. \tag{3.10}$$

The value of consumption rate is also obtained by Eq. (3.7) in the case where the gradient of salinity deficit is known at the bay mouth. It was seen in Akashi Straits that the space distribution was derived from the tidal change by the concept of mixing length. From the result of investigation on the bay mouth line in November 1956, the gradient of $\Delta\sigma_0$ will be calculated. Taking the mean tidal range of $\Delta\sigma_0$ to be 80 and the mean velocity of tidal current to be the value of u as given by (2.18), we get

$$\left(\frac{\partial Q}{\partial x}\right)_0 = 2.5\times10^{-2} \qquad (\text{g/m}^4)\,.$$

Taking $x_0=5$ km, $\eta_1=7.05\times10^5$ (c.g.s.)

$$C = 0.5\times10^{-5}\,\eta_1\,, \quad (\text{g/m}^3\,\text{sec}) \tag{3.11}$$
$$= 31 \quad (\text{g/m}^3\,\text{day}) \quad (\text{S. Hayami } et\ al., (\mathbf{9}))\,. \tag{3.12}$$

E. S. Nielsen (**15**) measured the productivity in various regions by using C^{14} isotope. He reported that the 3.8 (g Carbon/m^2 day) of productivity was found in Walvis Bay (South Africa 14°W 23°S) and the photosynthetic layer was 0.8 m deep at that time. Calculating the ratio of the total amount to Carbon according to the table of relative composition of plankton organisms (see "*The Oceans*" p. 234), we get

$$\text{Total}: \ \text{Carbon} = 1.64 : 1$$

The value of this ratio makes it possible for us to estimate roughly the gross production from the carbon production reported by him. We get

$$C = 7.8 \qquad (\text{g/m}^3\,\text{day})\,. \tag{3.13}$$

This value is considerably different from the values given in (3.10) and (3.12).

This discrepancy may be due to over-estimation of the mixing coefficient η_1. The value of the mixing coefficient given by Eq. (2.17) was estimated by the observed value at surface layer. The mean value of the mixing coefficient in the mixing layer should be taken as η_1 in Eq. (3.9) or (3.11). If we assume as rough estimation that the tidal current decreases linearly downward and vanishes at the mixing layer depth, then the mean tidal current becomes one half of the surface current. The mean value of mixing coefficient $\bar{\eta}$ is given by

$$\bar{\eta} = \alpha\overline{ul} = \alpha\bar{u}^2T = \frac{1}{4}\alpha u^2T = \frac{1}{4}\eta_{\text{surface}}$$

Using the value $\bar{\eta}$ as the mixing coefficient, we will get 13 (g/m^3 day) and 7.8 (g/m^3 day) in place of the values given by (3.10) and (3.12) respectively.

12. Variations of plankton population and salinity deficit

In the preceding section, the consumption and replenishment of salts in sea water have been discussed with reference to the salinity deficit, and the rate of consumption has been roughly estimated. Also, it has been already mentioned that plants utilize the dissolved materials to grow and contribute to the increase of salinity deficit. The salt may also be consumed by many other organisms beside plankton. But, as the growth of these organisms depends fundamentally on the amount of plankton with respect to food relations, the amount of plankton may be considered to represent a convenient index for the consumers of salt and materials contributing to the salinity deficit.

From such a point of view, a long continued investigation from July, 1958 to July, 1959 was carried out for various quantities which seem to influence the salinity deficit. The survey line was stretched from Tsunashirazu to Tanabe, as seen in Fig. 25. The water was taken at the depth of about 50 cm below the sea surface. Sampling was made at each of 10 stations set at equal intervals on the survey line. As soon as the water was taken at each station, it was poured into one aged-glass bottle in equal quantities, and one sample of mean water was made up. Such a mean water sample was taken once a day at or near the time of mid-water during about 400 days.

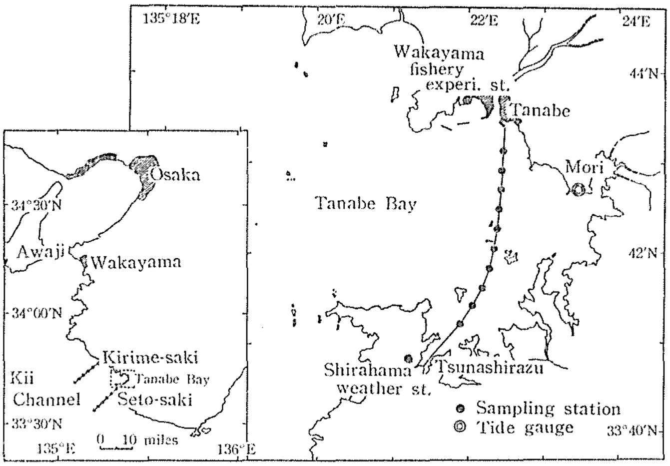


Fig. 25. Locations of survey line in the bay and of station outside the bay.

On the same survey line the surface temperature and the plankton population were examined at the same time by I. Yamazi, S. Fuse, E. Harada and others. The chlorinity, salinity deficit, phosphate-P and silicate-Si of mean water sample were measured. The settling volume of plankton was measured by I. Yamazi. The determination of phosphate-P and silicate-Si were made by S. Fuse and E. Harada.

The result of the investigation is given in Fig. 26. In this figure, the daily mean value of tidal range was calculated from the records of tide-gauge set at Mori (situated at bay head) and the rainfall is the mean of daily values observed at Tsunashirazu and Tanabe.

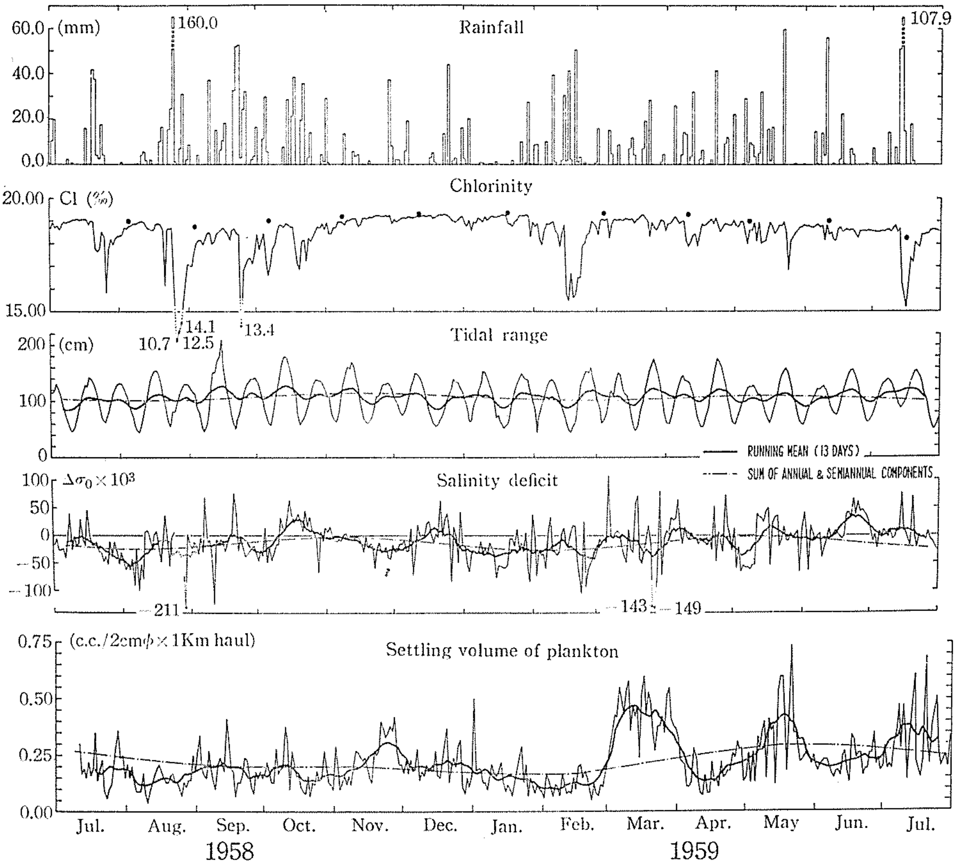


Fig. 26. Variations over the year in rainfall, chlorinity, tidal range, salinity deficit and plankton population on the survey line stretched from Tsunashirazu to Tanabe.

The chlorinity of surface water outside the bay was measured at 10 stations shown by inset map in Fig. 25 by Wakayama Fisheries Experiment Station, and the mean chlorinity of 10 stations was plotted by symbol ● above the variation in the chlorinity of mean water on the survey line. The chlorinity diminution of mean water from the smooth curve through the points ●'s is fairly similar to the variation of daily mean rainfall. The salinity deficit and plankton population have a short-period component and change complicatedly, but both variations seem to have regularities in the mean trend.

At first, the annual and semi-annual components of the tidal range H, salinity deficit $\Delta\sigma_0$ and plankton population P were calculated by harmonic analysis. The results are given in Table 8. All semi-annual components took their maximum values in November and May. The same phase between salinity deficit and plankton population may support the assumption that the plankton's growth contributes to the increase of salinity deficit. The same phase between the tidal range and salinity deficit means that the salinity deficit is lower in the bay than outside the bay, since the increase of tidal range tends to accelerate the velocity of tidal current and then intensify the tidal mixing. This is contrary to the result of bay mouth investigation carried out in November 1956 (Fig. 16). It may be due to the excess of replenishment by land water influx over the consumption of the salt on the average of a long term.

Table 8.

		Mean	Amplitude	Phase lag
Tidal range H (cm)	Annual	107.1	2.1	143°
	Semi-annual		5.0	258°
Salinity deficit ($\Delta\sigma_0 \times 10^3$)	Annual	−13.8	0.8	12°
	Semi-annual		11.3	243°
Settling volume of plankton (cc/0.314 m^3)	Annual	0.225	0.052	327°
	Semi-annual		0.025	252°

Epoch : 0 hr. 1st. July, 1958

In order to rule out the short-period fluctuation, a running mean of $\Delta\sigma_0$ was calculated at 13-day-intervals, as the peaks of short fluctuation of $\Delta\sigma_0$ appeared at about 13-day-intervals. On account of the mutual comparisons between the variations of different quantities, running means of tidal range, chlorinity and plankton population were also calculated with the same time-interval. These running means were drawn by thick lines in Fig. 26. The variation of about 30-day-period seems to exist except in the chlorinity. So, the correlation coefficients among H, $\Delta\sigma_0$ and P were examined every month. Table 9 gives the coefficients obtained.

The correlation coefficients $R(H\sim\Delta\sigma_0)$ between the tidal range and the salinity deficit have positive signs except in winter and early spring as seen in the semi-annual component. It is inferred that the salinity deficit varies also outside the bay. Fig. 27 shows the result of investigation which was carried out on two

Table 9. Correlations between H, $\Delta\sigma_0$ and P.
H: Running mean value of tidal range; $\Delta\sigma_0$: Running mean value of salinity deficit; P: Running mean value of plankton population.

Year	Month	$R(H\sim\Delta\sigma_0)$	$R(P\sim\Delta\sigma_0)$
1958	7	−0.73	+0.85
	8	+0.61	+0.11
	9	+0.05	−0.16
	10	+0.35	−0.44
	11	+0.92	−0.99
	12	−0.99	+0.40
1959	1	+0.61	−0.16
	2	−0.67	+0.06
	3	−0.58	+0.61
	4	−0.05	−0.05
	5	+0.37	+0.44
	6	+0.78	+0.95
	7	+0.54	+0.59

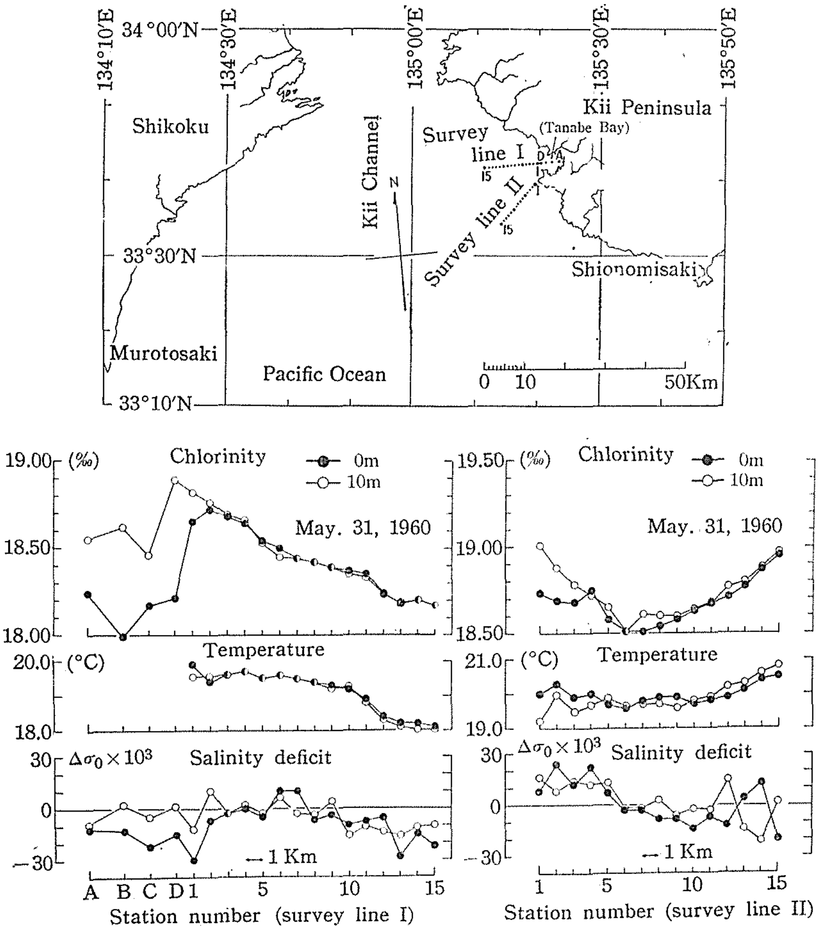


Fig. 27. Distribution of the chlorinity, temperature and salinity deficit outside the bay.

survey lines stretched about 20 km offshore. It is interesting to note that the distributions of $\Delta\sigma_0$ undulate along both lines, and especially along the line II the value of the salinity deficit is likely to converge to zero offshore.

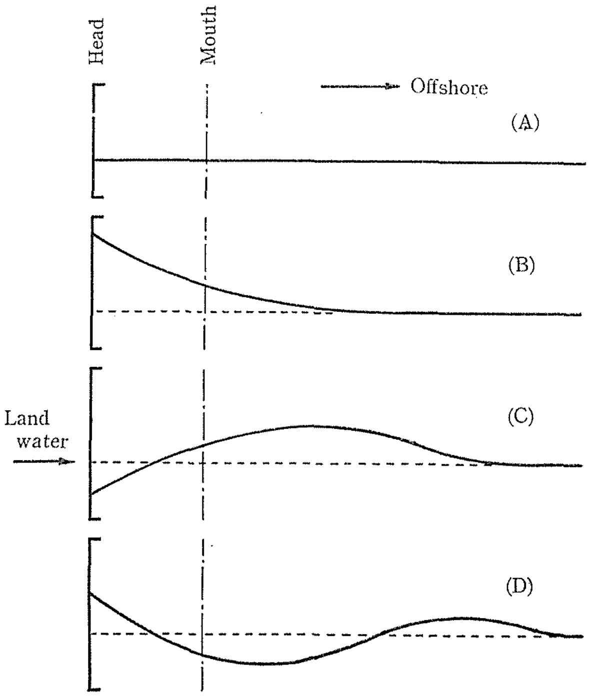


Fig. 28. Schematic distribution of salinity deficit from the bay head to offshore.

Unless the inflow of land water and the consumption of salts take place in the bay, the salinity deficit would be uniform everywhere as shown by (A) in Fig. 28. In fact, by the influx and the consumption, the distribution (A) would be deformed. If there is a continuous consumption of salts in the bay, the distribution (A) would be transformed into the distribution (B) and furthermore the distribution (B) changes into (C) by the influx of land water. Being deformed by the combination of such successive processes, the distribution would be carried away and diffused by the current offshore and ultimately the salinity deficit would vanish. Along the survey line I, the chlorinity becomes suddenly lower in the bay and, probably owing to its lowering, the distribution of salinity deficit is similar to (C). Along the survey line II, the distribution seems to be in the state of (D).

In the cases of (B) and (D), the correlation between H and $\Delta\sigma_0$ becomes negative while in the case of (C) the correlation becomes positive. When the correlation is positive, it must be noted that the salinity deficit is increased, both by the outflow of salts through the tidal mixing and by the consumption of salts. The correlation between the salinity deficit and plankton population $R(P\sim\Delta\sigma_0)$ is negative from autumn to winter. It may be considered that during this season plankton population decays under poor environment and is decomposed by some agents such as bacteria.

Fig. 29 shows the variations of the monthly mean value of various quantities

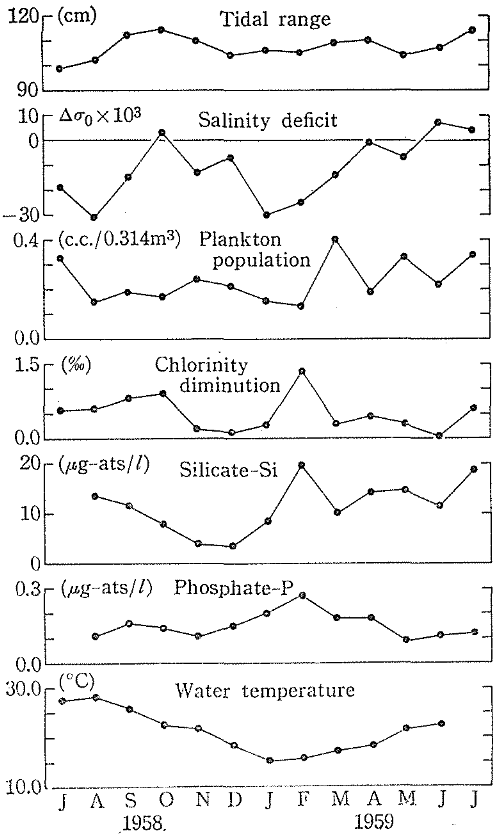


Fig. 29. Variations in monthly mean values of various quantities in Tanabe Bay.

observed. Most of quantities have the extrema in February, indicating a peculiar circumstance. Such a peculiarity probably causes the spring blooming of plankton. The change of chlorinity diminution is very similar to that of silicate-Si which is mainly brought about by the inflow of land water. The relation of salinity deficit to phosphate-P is contrary to each other in rough trends, i.e. phosphate-P decreases with the increase of salinity deficit which reflects the limiting character of the nutrient salt.

13. Differential equations for salinity deficit and plankton population

The relations between various quantities were investigated in the above section. They are very intricate and we cannot grasp them by such a simple treatment. In the following, an effort is made to unravel their inter-relationships as far as possible.

According to H. W. Harvey, the factors influencing plants' growth are as follows: (i) The incident light; (ii) The rate of replenishment of nutrients; (iii) The rate at which the zooplankton are eating the plants; (iv) The rate at which the plants are being carried down by turbulence below the photosynthetic zone; (v) The effect of increased temperature.

Although little is known about the nature of consumption of materials in sea water by plankton, it may safely be said that there is a limit in the amount of consumption, hence there must exist an upper bound of salinity deficit. This upper bound is denoted as q_0 (in the following, salinity deficit will be expressed by q as given in Eq. (3.1)). The quantity q_0-q will represent the total amount of salt to be prepared for the consumption by plankton. So, it is probable that the larger q_0-q of sea water is, the more easily plankton grows. Now, the population will be expressed by the mass of plankton contained in a unit volume of sea water. The population is increased either by the growth of individuals or by the increase of the number of individuals. Hence, the increase of population may depend on q_0-q. For the sake of simplicity, it is assumed that the increasing rate of population is proportional to q_0-q, i.e.

$$\frac{dp}{dt}=\gamma_1(q_0-q)\,,$$

where p is the population (mass/unit volume).

In a population, some species grow and others die. If the growth exceeds the death, the population will increase. It is assumed again that the increasing rate of population is proportional to itself. Then

$$\frac{dp}{dt}=\beta_1 p\,,$$

where if $\beta_1>0$, the population increases, and if $\beta_1<0$, the population decreases. Furthermore, it must be taken into account that the population is dependent on the light quantities, grazing intensity, water temperature and others. For the moment, the effect of these factors upon the increasing rate is put by a constant k_1.

Then, the diffusion equation for plankton population p is given by

$$\frac{\partial p}{\partial t}=\eta_1\frac{\partial^2 p}{\partial x^2}+\eta_2\frac{\partial^2 p}{\partial y^2}+\eta_3\frac{\partial^2 p}{\partial z^2}+\beta_1 p+\gamma_1(q_0-q)+k_1\,. \tag{3.14}$$

The working model of Tanabe Bay and the coordinates are taken as given by Fig. 17. Boundary conditions are

$$\left.\begin{array}{lll}\eta_2\dfrac{\partial p}{\partial y}=0 & \text{at}\ \ y=0\ \ \text{and}\ \ y=y_0 & \text{(both sides)}\,,\\ \eta_3\dfrac{\partial p}{\partial z}=0 & \left\{\begin{array}{ll}\text{at}\ \ z=0 & \text{(surface)}\,,\\ \text{at}\ \ z=z_0 & \text{(mixing layer depth)}\,,\end{array}\right. & \\ \eta_1\dfrac{\partial p}{\partial x}=0 & \text{at}\ \ x=x_0\qquad \text{(bay head)}\,. & \end{array}\right\} \tag{3.15}$$

Integrating (3.14) with respect to x, y and z, and using the boundary conditions (3.15), we get

$$\frac{\partial\bar{p}}{\partial t}=-\frac{\eta_1}{x_0}\left(\frac{\partial\bar{p}_s}{\partial x}\right)_0+\beta_1\bar{p}-\gamma_1\bar{q}+(\gamma_1 q_0+k_1)\,, \tag{3.16}$$

where

$$\left.\begin{aligned}\bar{p}&=\frac{1}{V}\int_0^{x_0}\int_0^{y_0}\int_0^{z_0}p\,dx\,dy\,dz\,,\\ \bar{q}&=\frac{1}{V}\int_0^{x_0}\int_0^{y_0}\int_0^{z_0}q\,dx\,dy\,dz\,,\\ \bar{p}_s&=\frac{1}{S}\int_0^{y_0}\int_0^{z_0}p\,dy\,dz\,,\end{aligned}\right\} \tag{3.17}$$

and V is the entire volume of the bay above the depth of mixing layer and S is the cross section area of the bay above the same depth.

Representing the daily mean values of $\bar{p}$, $\bar{q}$ and $\bar{p}_s$ by P, Q and P_s respectively, we get from Eq. (3.16)

$$\frac{dP}{dt} = -\frac{\eta_1}{x_0}\left(\frac{\partial P_s}{\partial x}\right)_0 + \beta_1 P - \gamma_1 Q + (\gamma_1 q_0 + k_1)\,. \tag{3.18}$$

The running means of the plankton population and salinity deficit observed are regarded as the daily mean values P and Q respectively. The first term on the right-hand side represents the outflow of population from the bay. For the lack of data, the value of $\left(\frac{\partial P_s}{\partial x}\right)_0$ is boldly assumed to be constant K_p during some time intervals T, and it is supposed that η_1 is the sum of the mean value $\bar{\eta}_1$ over the same intervals T and the deviation $\delta\eta_1$ which is proportional to tidal range H. Then

$$\eta_1 = \bar{\eta}_1 + \delta\eta_1 \fallingdotseq \bar{\eta}_1 + aH \qquad (a:\text{ constant})\,,$$

$$\frac{\eta_1}{x_0}\left(\frac{\partial P_s}{\partial x}\right)_0 \fallingdotseq \frac{\bar{\eta}_1 + aH}{x_0}K_p = \frac{\bar{\eta}_1}{x_0}K_p + \frac{aK_p}{x_0}H\,.$$

Consequently, Eq. (3.18) is simplified to the following form:

$$\frac{dP}{dt} = \alpha_1 H + \beta_1 P - \gamma_1 Q + \kappa_1\,, \tag{3.19}$$

where

$$\alpha_1 \equiv -\frac{a}{x_0}K_p\,, \quad \kappa_1 \equiv \gamma_1 q_0 + k_1 - \frac{\bar{\eta}_1}{x_0}K_p\,. \tag{3.20}$$

The diffusion equation for salinity deficit was treated in § 11, where the influx of land water was not taken into account. We must determine how to estimate the amount of influx. It was previously seen that the change of chlorinity diminution corresponds clearly to the one of silicate-Si (Fig. 29). The influx of land water may be estimated from the chlorinity diminution. When δM kg of land water mixes with 1 kg of sea water, the chlorinity diminution δCl is given by $\delta Cl = Cl - (Cl/(1+\delta M)) \fallingdotseq Cl\,\delta M$, where Cl is the chlorinity of sea water, and the chlorinity of land water is not considered. In the case where the mean chlorinity diminution in the volume V is calculated, the chlorinity outside the bay should be taken as Cl. The chlorinity of the outer water may be regarded as a constant during the short term. Hence, δCl is nearly proportional to δM. The chlorinity diminution is diffused by tidal mixing and the diffusion equation of δCl was already treated in § 10. The diffusion equation for the chlorinity diminution δCl is

$$\frac{\partial(\delta Cl)}{\partial t} = \eta_1\frac{\partial^2(\delta Cl)}{\partial x^2} + \eta_2\frac{\partial^2(\delta Cl)}{\partial y^2} + \eta_3\frac{\partial^2(\delta Cl)}{\partial z^2}\,. \tag{3.21}$$

Boundary conditions are taken as

$$\left.\begin{aligned} \eta_2\frac{\partial(\delta Cl)}{\partial y} &= 0 \quad \text{at} \quad y=0 \quad \text{and} \quad y=y_0\,, \\ \eta_3\frac{\partial(\delta Cl)}{\partial z} &= 0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=z_0\,. \end{aligned}\right\} \tag{3.22}$$

In the same manner as in the plankton population p, representing the mean values of δCl over the volume V and the cross section area S by $\overline{\delta Cl}$ and $\overline{\delta Cl_s}$ respectively, and further, denoting the daily mean values of $\overline{\delta Cl}$ and $\overline{\delta Cl_s}$ by L and L_s respectively, we obtain from Eqs. (3.21) and (3.22)

$$\frac{\partial L}{\partial t} = \frac{\eta_1}{x_0}\left(\frac{\partial L_s}{\partial x}\right)_{x_0} - \frac{\eta_1}{x_0}\left(\frac{\partial L_s}{\partial x}\right)_0\,,$$

or

$$\frac{\eta_1}{x_0}\left(\frac{\partial L_s}{\partial x}\right)_{x_0} = \frac{\partial L}{\partial t} + \frac{\eta_1}{x_0}\left(\frac{\partial L_s}{\partial x}\right)_0\,. \tag{3.23}$$

Eq. (3.23) represents the influx of the chlorinity diminution due to land water at bay head. Assuming that $\left(\frac{\partial L_s}{\partial x}\right)_0$ is the sum of a constant K_l which is mean value of $\left(\frac{\partial L_s}{\partial x}\right)_0$ over the interval T and the deviation $\delta\left(\frac{\partial L_s}{\partial x}\right)_0$ which is proportional to L, we have

$$\begin{aligned} \frac{\eta_1}{x_0}\left(\frac{\partial L_s}{\partial x}\right)_{x_0} &\fallingdotseq \frac{dL}{dt} + \frac{\bar{\eta}_1 + aH}{x_0}(K_l + bL) \\ &= \frac{dL}{dt} + \frac{\bar{\eta}_1 K_l}{x_0} + \frac{aK_l}{x_0}H + \frac{\bar{\eta}_1 b}{x_0}L\,, \end{aligned} \tag{3.24}$$

where b is a constant and a term $abHL/x_0$ has been neglected in the second row.

The differential equation for salinity deficit corresponding to (3.19) will be obtained in the following manner. As the consumption of dissolved matters in sea water may depend on other factors beside plankton population, the term $c(x, y, z, t)$ on the right-hand side of Eq. (3.2) is divided into a term proportional to plankton population, $\beta_2 p$ and a term due to other factors which is assumed as a constant k_2. Then,

$$\frac{\partial q}{\partial t} = \eta_1\frac{\partial^2 q}{\partial x^2} + \eta_2\frac{\partial^2 q}{\partial y^2} + \eta_3\frac{\partial^2 q}{\partial z^2} + \beta_2 p + k_2\,. \tag{3.25}$$

Treating this equation in the same manner as those for p and δCl, and taking the boundary conditions in the form:

$$\left.\begin{aligned} \eta_2\frac{\partial q}{\partial y} &= 0 \quad \text{at} \quad y=0 \quad \text{and} \quad y=y_0\,, \\ \eta_3\frac{\partial q}{\partial z} &= 0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=z_0\,, \end{aligned}\right\} \tag{3.26}$$

we get

$$\frac{dQ}{dt}=\frac{\eta_1}{x_0}\left(\frac{\partial Q_s}{\partial x}\right)_{x_0}-\frac{\eta_1}{x_0}\left(\frac{\partial Q_s}{\partial x}\right)_0+\beta_2 P+k_2. \qquad (3.27)$$

In this equation, $\frac{\eta_1}{x_0}\left(\frac{\partial Q_s}{\partial x}\right)_{x_0}$ represents the influx of salinity deficit due to land water at bay head. If δM kg of land water which has a salt content s g/kg inflows into one kilogram of the sea water which has the chlorinity Cl, the salinity deficit produced is given by

$$\frac{rCl}{1+\delta M}-\frac{rCl+s\delta M}{1+\delta M}=-\frac{s\delta M}{1+\delta M}\doteqdot -s\delta M,$$

where r is the Cl-ratio of normal sea water and the chlorinity of land water has been neglected. Hence the salinity deficit due to land water is approximately proportional to the influx of land water, and consequently to the chlorinity diminution due to land water. Then

$$\begin{aligned}\frac{\eta_1}{x_0}\left(\frac{\partial Q_s}{\partial x}\right)_{x_0} &\doteqdot m\frac{\eta_1}{x_0}\left(\frac{\partial L_s}{\partial x}\right)_{x_0} \quad (m \text{ being a constant})\\ &= m\frac{dL}{dt}+\frac{m\bar{\eta}_1}{x_0}K_l+\frac{amK_l}{x_0}H+\frac{bm\bar{\eta}_1}{x_0}L. \qquad (3.28)\end{aligned}$$

$\left(\frac{\partial Q_s}{\partial x}\right)_0$ is boldly assumed to be constant K_q during the interval T by the same reason as in $\left(\frac{\partial P_s}{\partial x}\right)_0$. Then

$$\frac{\eta_1}{x_0}\left(\frac{\partial Q_s}{\partial x}\right)_0\doteqdot\frac{\bar{\eta}_1+aH}{x_0}K_q=\frac{\bar{\eta}_1}{x_0}K_q+\frac{aK_q}{x_0}H, \qquad (3.29)$$

and substituting from (3.28) and (3.29) into (3.27), we get

$$\frac{dQ}{dt}=\alpha_2 H+\beta_2 P+\lambda_2 L+\mu_2\frac{dL}{dt}+\kappa_2, \qquad (3.30)$$

where

$$\left.\begin{aligned}\alpha_2 &\equiv \frac{am}{x_0}K_l-\frac{a}{x_0}K_q,\\ \lambda_2 &\equiv \frac{bm}{x_0}\bar{\eta}_1,\\ \mu_2 &\equiv m,\\ \kappa_2 &\equiv k_2+\frac{m}{x_0}\bar{\eta}_1 K_l-\frac{1}{x_0}\bar{\eta}K_q.\end{aligned}\right\} \qquad (3.31)$$

Eqs. (3.19) and (3.30), which have been obtained under various assumptions, give the relation between the plankton population and salinity deficit. In these equations, dP/dt represents the plankton production and is decided by the terms on the right-hand side, and so, the terms on the right-hand side mean the productivity, of which $\gamma_1 Q$ is the most important factor that is concerned with the growth of living bodies. The change of Q is governed by Eq. (3.30). dP/dt,

dQ/dt and dL/dt are put as the daily differences of P, Q, H and L respectively, and then the values of the coefficients (α_1, β_1, γ_1, κ_1) and (α_2, β_2, λ_2, μ_2, κ_2) were determined in every month, which was regarded as some time intervals T, by applying the method of least squares to Eqs. (3.19) and (3.30). The values of coefficients obtained are shown in Table 10 and the variations of coefficients are shown in Fig. 30. If H, L and dL/dt are given as known functions, Eqs. (3.19) and (3.30) express simultaneous differential equations of P and Q. They were solved numerically by Runge-Kutta's method at 4-day-intervals. The result of this calculation fits fairly well with the observed values as shown in Fig. 31.

Table 10. Coefficients in Eqs. (3.19) and (3.30).
Unit of P=Settling volume 1 cc/0.314 m^3;
Unit of $Q=10^3\times\Delta\sigma_0$.

Year	Month	$\alpha_1(P/\text{day}\cdot\text{cm})$	$\beta_1(1/\text{day})$	$\gamma_1(P/\text{day}\cdot Q)$	$\kappa_1(P/\text{day})$
1958	Aug.	-0.623×10^{-3}	0.112	-0.497×10^{-3}	0.063
	Sep.	0.118 〃	0.018	0.360 〃	−0.023
	Oct.	−0.676 〃	0.069	0.059 〃	0.065
	Nov.	−0.154 〃	0.300	−1.934 〃	−0.024
	Dec.	0.365 〃	−0.135	−0.561 〃	−0.008
1959	Jan.	−0.139 〃	0.042	0.116 〃	0.003
	Feb.	0.390 〃	0.190	−0.044 〃	−0.060
	Mar.	0.323 〃	−0.129	−1.022 〃	0.033
	Apr.	1.299 〃	−0.191	1.506 〃	−0.116
	May	−1.537 〃	0.226	1.049 〃	0.071
	Jun.	0.067 〃	0.236	0.185 〃	−0.056
	Jul.	−1.394 〃	0.082	−0.225 〃	0.133

Year	Month	$\alpha_2(Q/\text{day}\cdot\text{cm})$	$\beta_2(Q/\text{day}\cdot P)$	$\lambda_2(Q/\text{day}\cdot Cl)$	$\mu_2(Q/Cl)$	$\kappa_2(Q/\text{day})$
1958	Aug.	0.14	−160.2	2.4	−16.6	8.3
	Sep.	0.02	107.6	1.5	− 6.0	−24.1
	Oct.	0.23	88.1	4.6	− 0.3	−45.1
	Nov.	−0.05	− 10.5	4.9	50.9	6.6
	Dec.	0.10	− 71.0	−36.0	−14.3	8.7
1959	Jan.	−0.04	− 6.4	− 7.2	6.0	6.8
	Feb.	0.16	58.4	− 1.8	12.6	−21.9
	Mar.	0.17	− 56.5	−22.5	−21.6	11.6
	Apr.	0.20	96.1	14.5	−41.6	−46.8
	May	−0.46	17.8	27.8	12.1	36.9
	Jun.	0.27	− 77.3	10.3	−19.4	−11.5
	Jul.	0.04	47.8	0.8	8.3	−22.0

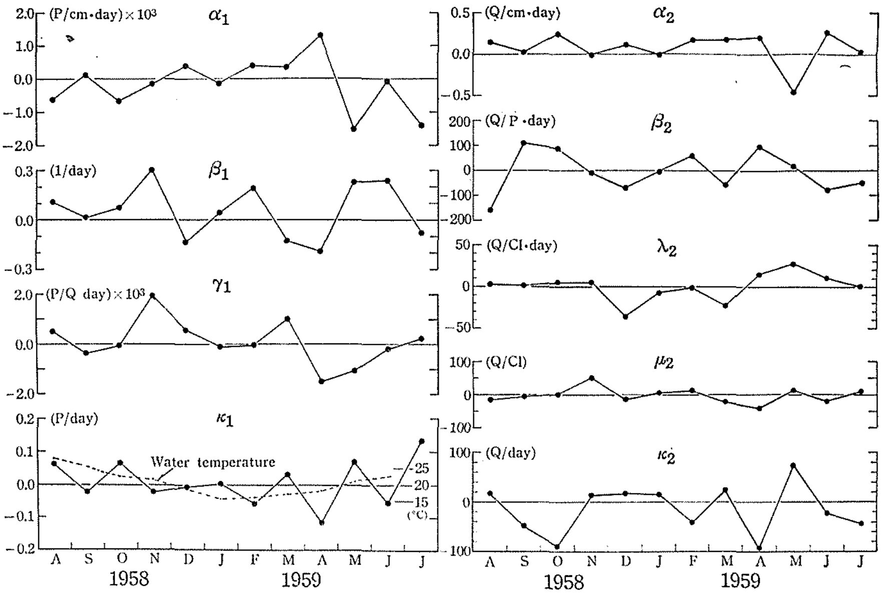


Fig. 30. The changes of coefficients in differential equations for plankton population and salinity deficit.

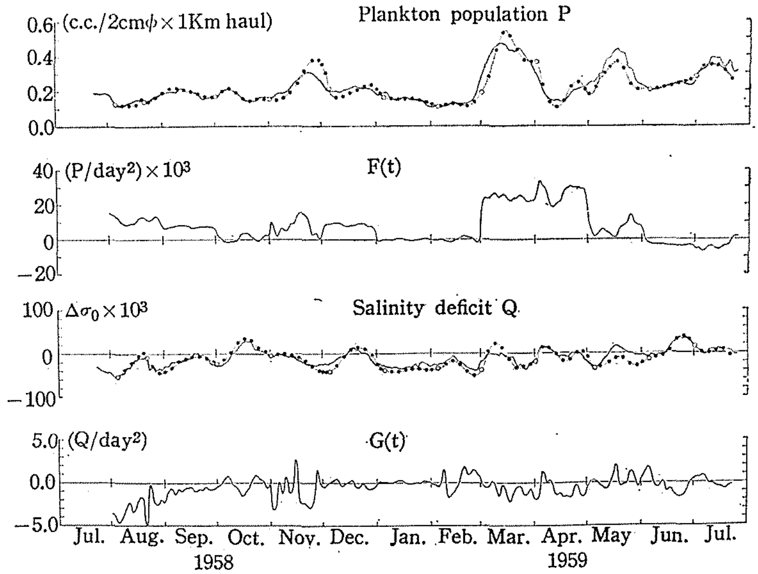


Fig. 31. Comparison between the observed variation and numerical calculation by Runge-Kutta's method for the plankton population P and salinity deficit Q. (Solid line shows the observed variation and dotted line shows the result of calculation.)

14. Discussions

The coefficients in Eqs. (3.19) and (3.30) are monthly mean values assumed as constants. Using monthly mean values of coefficients ($\alpha_1, \beta_1, \cdots, \mu_2, \kappa_2$) of the Nth month, the values of P and Q on the 1st day of the ($N+1$)th month were calculated. These P and Q were taken as the initial values of the ($N+1$)th month and the coefficients of the Nth month were discontinuously replaced by the coefficients of the ($N+1$)th month. Using above procedure as a rule, P and Q of every month were calculated likewise. In the case, however, when the result of calculation started from the values of P and Q on the 1st day did not suitably correspond to the observed values, the calculation on the previous month was continued and the day replacing the coefficient was delayed.

Table 11.

Date of replacement	
22,	Aug.
1,	Oct.
1,	Nov.
5,	Dec.
5,	Jan.
4.5,	Feb.
1,	Mar.
1,	Apr.
8,	May
5,	Jun.
1,	Jul.

In Fig. 31, the point ○ on curves of P and Q shows the day of replacement and Table 11 gives the date when new values of the coefficients were used. As shown in Fig. 30, the coefficients ($\alpha_1, \beta_1, \cdots, \kappa_2$) fluctuate over the year, indicating their dependency upon the time. If the value of the coefficient changes considerably within one month, the coefficient should be decided by the method of least squares for a shorter interval. The shift of day of replacement may be a convenient means to avoid such trouble.

However, judging from the satisfactory correspondence of the calculation to the observation, the coefficients ($\alpha_1, \beta_1, \cdots, \kappa_2$) may be regarded, in the first approximation, as constants between the replacements. If so, P or Q is eliminated from Eqs. (3.19) and (3.30), and the differential equation for P or Q is obtained, that is,

$$\frac{d^2P}{dt^2}-\beta_1\frac{dP}{dt}+\gamma_1\beta_2P=\alpha_1\frac{dH}{dt}-\gamma_1\alpha_2H-\gamma_1\lambda_2L-\gamma_1\mu_2\frac{dL}{dt}-\gamma_1\kappa_2\equiv\mathbf{F}(t)\,, \tag{3.32}$$

$$\begin{aligned}\frac{d^2Q}{dt^2}-\beta_1\frac{dQ}{dt}+\gamma_1\beta_2Q&=\alpha_2\frac{dH}{dt}+(\beta_2\alpha_1-\beta_1\alpha_2)H-\beta_1\lambda_2L\\&\quad+(\lambda_2-\beta_1\mu_2)\frac{dL}{dt}+\mu_2\frac{d^2L}{dt^2}+(\beta_2\kappa_1-\beta_1\kappa_2)\equiv\mathbf{G}(t)\,.\end{aligned} \tag{3.33}$$

Esq. (3.32) and (3.33) are the ones that are fully examined in the theory of vibration. According as β_1 is either positive or negative, the solution increases or decreases exponentially. If $\beta_1^2-4\gamma_1\beta_2<0$, the solutions of both equations exhibit the oscillations which have the same proper frequency $4\pi/\sqrt{4\gamma_1\beta_2-\beta_1^2}$. Those values are shown in Table 12.

From this table, it is seen that the plankton population increases exponentially in November and oscillatingly decreases in December. Likewise, the plankton

Table 12.

Year	Month	β_1	$\beta_1^2-4\gamma_1\beta_2$	$4\pi/\sqrt{3\gamma_1\beta_2-\beta_1^2}$
1958	Aug.	0.112	-0.3058	22.7 days
	Sep.	0.018	-0.1546	32.0 〃
	Oct.	0.069	-0.0160	99.3 〃
	Nov.	0.300	0.0085	—
	Dec.	-0.135	-0.1411	33.5 〃
1959	Jan.	0.042	0.0048	—
	Feb.	0.190	0.0463	—
	Mar.	-0.129	-0.2144	27.1 〃
	Apr.	-0.191	-0.5424	17.1 〃
	May	0.226	-0.0236	81.9 〃
	Jun.	0.236	0.1129	—
	Jul.	0.082	0.0498	—

population increases in February and oscillatingly decreases from March to May. When a disturbance is given to a system, the system is decided as stable or unstable according as the given disturbance will decay oscillatingly or grow to infinite. It is interesting to note that the environment in November, 1958 and February, 1959, where most of quantities take the extrema, are unstable. This instability may cause the blooming of population.

Plankton grows by absorbing the salt in sea water. When the sea water ages and cannot support the population, the death of plankton begins and subsequently the accumulation of salts starts. γ_1 is the increasing factor of population due to the salts, that is, it governs the growth of plankton. β_2 is the interesting parameter deciding the productivity, playing an important role to the ageing of sea water. β_1 represents the vitality of the population itself. The possibility of the oscillation of population depends on the sign of their combination given by $\beta_1^2-4\gamma_1\beta_2$.

The influx of land water and tidal mixing influence the quantity Q and consequently the growth of population. This situation is shown by the value on the right-hand side of Eq. (3.32), which represents the external forces that modify the population as shown in Fig. 31. The mode of oscillation of this external force is roughly similar to the change of population, so it may suggest that the long-period variation of population is forced by external conditions.

It is worth noticing that the annual trend of the change of coefficient κ_1 corresponds to one of the water temperature (see Fig. 30). It indicates that the water temperature is the dominant factor of κ_1. In Tanabe Bay, which is located about 33°42′N and 135°18′E, light quantity is sufficient for plant's growth. In such an area, it is well known that the growth of phytoplankton mainly depends on the water temperature rather than incident light (H. W. Harvey, 1955).

The term $\beta_2 P$ represents the consumption of salts proportional to plankton population and gives the time rate of productivity. When $\beta_2 > 0$, the growth of plankton exceeds the death of plankton and the organic matter is produced by photosynthesis. When $\beta_2 < 0$, the death of plankton exceeds the growth and the plankton decomposes. From August 1958 to July 1959, the annual average of $\beta_2 P$ is only 36 g/m³ year. This value may indicate that the materials absorbed by living bodies are restored almost into sea water, as is usually seen in open sea. The mean value of $\beta_2 P$ for the months in which β_2 is positive gives the mean rate of increase of salinity deficit proportional to plankton population. Its mean value is

$$\overline{\beta_2 P} = 16 \qquad (\mathrm{g/m^3\ day}),$$

which is comparable with the value given by (3.13).

Acknowledgements

The writer wishes to express his sincere appreciation to Prof. Shōitirō Hayami for his cordial guidance and encouragement in this work.

His thanks are also due to the staff of the Seto Marine Biological Laboratory, Kyoto University, especially to Dr. I. Yamazi, Dr. S. Fuse and Dr. E. Harada for their hearty supports.

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