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KINEMATICAL CHARACTER OF THE NEAR-BY STARS

BY

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ABSTRACT

In this paper, the orbital parameters of the near-by stars within 20 pc of the sun were statistically investigated. The main results are: (i) All of these parameters show clearly the age-effect as seen in Figs. 3, 4 and 5. (ii) The frequency distribution function of each orbital parameter is well represented by the respective theoretical distribution function derived on the assumption of the velocity-ellipsoid. (iii) From the kinematical view-point all the near-by stars are classified in large into two categories of (A, F) and (G, K, M).

The possible causes of the age-effect of the orbital parameters were discussed in connection with the evolutional development of our Galaxy. The variation of the force-field is likely to have played an important role for the age-effect, though the velocity dispersion of the stars at the birth time as well as the encounters with massive clouds afterward might also have taken some part.

1. Introduction

Hitherto, the relations between the kinematical parameters and physical quantities of the stars have been investigated from various view-points. In order to get some statistical clues to the problem concerning the physical state and the evolution of the Galaxy, however, it is convenient to choose some parameters of the stellar orbit as the indication of the kinematical stellar character. Such investigations have been carried out basing on the parameters of the Keplerian orbit or alike orbit (**1, 2, 3**), the epicycle one (**4**), the orbit like as given by Contenpoulos (**5**). The present study with reference to the formulae for the two-dimensional orbit by one of us (S-1) (**6**) was also intended to examine statistically the kinematical characters of the near-by stars in details for the purpose of discussing the kinematical evolution of the Galaxy.

The contents were arranged as follows: The five sections from §2 to §6 were devoted to the statistical study on the kinematical character of the near-by stars. Beginning with the description of the original statistical data as well as the ways for obtaining the various orbital parameters in §2, we examined the H-R diagrams for the groups of the stars due to their values of the respective orbital parameters in §3. It was shown there that the H-R diagram changed quite systematically depending on each of all the orbital parameters. In §4 the frequency distribution of every orbital parameter was compared with the corresponding theoretical one derived by us and it was found that the agreement was respectively satisfactory. The differences of the orbital parameters among the spectral groups were examined in §5 according to the significance tests. And in §6 the dependence of each orbital parameter on the stellar age or its age-effect was investigated rather closely.

In §7 the possible causes for producing the age-effect, such as the encounters with the massive clouds, the variation of the force-field and the initial velocity-dispersion, were discussed.

2. The statistical data and the ways for getting the orbital parameters

All the original data were taken from Gliese's Catalogue of the near-by stars within 20 pc of the sun (7), in which 463 single stars with space velocities were chosen. As for the physical quantities of the stars such as their absolute magnitudes and spectral types, the catalogued values were used as they were.

The fundamental orbital parameters in our case are the peri- and apo-galacticon distance α and β (measured inward and outward respectively from the local standard of rest) and the velocity-component perpendicular to the Galactic plane at the time of its passage $\dot{\zeta}_0$. It is remarked that since we made use of our two-dimensional velocity-diagram for deriving α and β*, the actual peri- and apo-galacticon distances may differ more or less unless $\dot{\zeta}_0$ is small, but we sometimes use the terms in stead of these parameters.

The first thing to do was the conversion from the listed three rectangular velocity-components, $u(l=148°, b=0°)$, $v(l=58°, b=0°)$ and $w(b=90°)$ to $\dot{\xi}_0$, $\dot{\eta}_0$ and $\dot{\zeta}_0$*. But, because the stars in question were within 20 pc of the sun, it held with a good approximation that

$$\dot{\xi}_0 = u-u_{\odot},\ \dot{\eta}_0 = v-v_{\odot},\ \dot{\zeta}_0 = w-w_{\odot},$$

where $u_{\odot}$, $v_{\odot}$ and $w_{\odot}$ are the components of the solar motion for which the following standard value was adopted:

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* Our velocity-diagram has been made with the following assumptions (see S-1). (i) The local standard of rest moves along a circular locus around the Galactic center, keeping the radius $\varpi_0=8.2$ kpc and the angular velocity $\omega_0=26.4$ km/sec/kpc. (ii) The force is axially symmetric about the axis passing the Galactic ceter perpendicularly to the plane. Letting $\xi(l=148°, b=0°)$, $\eta(l=58°, b=0°)$ and $\zeta(b=90°)$ be the rotating cylindrical coordinates with the local standard of rest as the origin, the force is expressed by

$$\frac{\partial V}{\partial \varpi}=\frac{\partial V}{\partial \xi}=5716-1064.9\xi+134.61\xi^2\ [\mathrm{km^2/sec^2/kpc}]\ \ (\xi:\ \text{in kpc})$$

corresponded to the case of $c=d=\cdots=0$. (iii) Each star is now passing the circle $\xi=0$ with $\dot{\xi}=\dot{\xi}_0$, $\dot{\eta}=\dot{\eta}_0$.

$V_{\odot} = 19.5\,\text{km/sec},\ l_{\odot} = 23°,\ b_{\odot} = +22°\,;$

or $u_{\odot} = -10\,\text{km/sec},\ v_{\odot} = +15\,\text{km/sec},\ w_{\odot} = +7\,\text{km/sec}.$

After getting $\dot{\xi}_0, \dot{\eta}_0, \dot{\zeta}_0$ we read both α and β on the velocity-diagram given in S-1* as Fig. 6 for all the available data, but the seven stars such as Nos. 191, 239, 451, 764, 827 and 861 were omitted because of their too large values of α or/and β to be allowed for our approximation.

As the auxiliary orbital parameters the mean distance from the Galactic center $\boldsymbol{a}$, the eccentricity of the radial motion $\boldsymbol{e}$ defined respectively by

$$\boldsymbol{a} = \frac{2\varpi_0 - \alpha + \beta}{2} \quad \text{and} \quad \boldsymbol{e} = \frac{\alpha + \beta}{2\varpi_0 - \alpha + \beta},$$

and the amplitude of the radial motion $\alpha+\beta$ were also evaluated.

3. The relations between the kinematical parameters and the HR-diagrams

In their work Wooley and Eggen (8), by grouping the near-by stars taken from Gliese's Catalogue according to their peri-galacticon distances and by plotting the CM-diagram for each group, found an interesting fact that the upper-left end of the main sequence for each group appeared to displace quite systematically toward the lower-right as the group's peri-galacticon distance increased. This prompted us to examine the fact more in detail not only for the groupings due to the peri-galacticon distance but also to all the other orbital parameters.

Fig. 1 represents the changes of the HR-diagrams according to the parameter-values of the successive groupings such as (a) by the peri-galacticon distance α, (b) by the apo-galaction distance β, (c) by the mean distance $\boldsymbol{a}$, (d) by the eccentricity $\boldsymbol{e}$ and (e) by the amplitude $(\alpha+\beta)$. The full curve in each diagram indicates the so-called initial or zero-aged main sequence given by Johnson and Hiltner (9).

It is readily seen in Fig. 1(a) that the steady variation of the main sequence with the peri-galacticon distance looks just the same as in the diagram of Wooley and Eggen. But with respects to the other parameters too, the similar systematic change can be traceable as seen in Figs. 1(b)～1(e).

In Table 1 are given the numerical quantities for every group, namely both the spectral type $\mathrm{Sp}_{\mathrm{UL}}$ and the absolute magnitude $\mathrm{Mv}_{\mathrm{UL}}$ of the upper end of the main sequencce, the number of stars n, the mean value of the parameter and $\sigma_{\dot{\zeta}}$ or the dispersion of $\dot{\zeta}_0$. As for $\mathrm{Sp}_{\mathrm{UL}}$ and $\mathrm{Mv}_{\mathrm{UL}}$ are given those of the first upper-left end star as well as the means from the first three ones ($\overline{\mathrm{Sp}}_{\mathrm{UL3}}$, $\overline{\mathrm{Mv}}_{\mathrm{UL3}}$) within the strip of 1 mag. below the zero-aged main sequence, by letting

Table 1. Numerical data for the groupings due to the values of the respective parameters.

(a)

α (kpc)	0.0~1.0	1.0~2.0	2.0~3.0	3.0~4.0	4.0~5.0	5.0~
n	221	97	67	39	23	9
$\bar{\alpha}$ (kpc)	0.38	1.39	2.46	3.40	4.32	5.50
Sp_{UL}	A3	F3	F6	G1	G0	G6
Mv_{UL} (mag.)	1.8	2.9	3.5	4.6	4.1	5.7
Quality	b~c	c	d	c	d	c
Sp_{UL3}	A3	F5	F9	G2	G1	K5
Mv_{UL3} (mag.)	1.8	3.7	4.3	4.8	4.4	7.7
$\sigma_{\dot{\zeta}}$ (km/s)	15.6	19.8	23.5	26.8	26.2	30.6

(b)

β (kpc)	0.0~1.0	1.0~2.0	2.0~3.0	3.0~4.0	4.0~5.0	5.0~
n	211	116	58	29	12	30
$\bar{\beta}$ (kpc)	0.35	1.32	2.39	3.28	4.40	6.59
Sp_{UL}	A3	A7	A3	A5	F2	F7
Mv_{UL} (mag.)	1.7	2.2	1.9	2.6	3.5	3.9
Quality	b~c	a	c	e	d	b
Sp_{UL3}	A4	F2	F0	F2	G2	F8
Mv_{UL3} (mag.)	2.0	2.8	3.0	3.4	5.1	3.9
$\alpha_{\dot{\zeta}}$ (km/s)	17.5	20.8	16.8	23.7	21.3	30.6

(c)

$\boldsymbol{a}$ (kpc)	~6.7	6.7~7.7	7.7~8.7	8.7~9.7	9.7~
n	39	87	182	93	55
$\bar{\boldsymbol{a}}$ (kpc)	6.28	7.13	8.13	9.10	10.58
Sp_{UL}	G2	F6	A3	A3	A5
Mv_{UL} (mag.)	4.8	4.0	1.7	1.9	2.6
Quality	b~d		b~c	c	e
Sp_{UL3}	G3	F9	A4	F0	A9
Mv_{UL3} (mag.)	4.9	4.5	2.0	3.0	2.9
$\sigma_{\dot{\zeta}}$ (km/s)	23.6	22.8	16.5	17.5	25.4

(d)

$\boldsymbol{e}$	0.00~0.10	0.10~0.20	0.20~0.30	0.30~0.40	0.40~
n	101	176	106	43	30
$\bar{\boldsymbol{e}}$	0.057	0.141	0.241	0.335	0.486
Sp_{UL}	A3	A3	F2	F7	G0
Mv_{UL} (mag.)	1.7	1.9	3.3	4.0	4.1
Quality	b~c	c	d	b	d
Sp_{UL3}	A4	A6	F5	G0	G2
Mv_{UL3} (mag.)	2.0	2.4	3.8	4.5	4.6
$\sigma_{\dot{\zeta}}$ (km/s)	11.8	17.4	22.4	24.8	33.8

(e)

$\alpha+\beta$ (kpc)	0.0~1.0	1.0~2.0	2.0~3.0	3.0~4.0	4.0~5.0	5.0~6.0	6.0~
n	47	100	112	81	55	23	38
$\overline{\alpha+\beta}$ (kpc)	0.63	1.45	2.44	3.35	4.41	5.49	7.62
Sp_{UL}	A5	A3	A3	A5	F2	F8	F7
Mv_{UL} (mag.)	2.2	1.7	1.9	2.6	3.3	3.8	4.0
Quality	c~d	b~c	c	e	d	d	b
Sp_{UL3}	A5	A4	F0	F0	F6	G6	G2
Mv_{UL3} (mag.)	2.1	1.8	3.0	3.0	3.9	5.4	4.6
$\sigma_{\dot{\zeta}}$ (km/s)	10.7	14.5	18.0	19.2	24.7	26.4	30.8

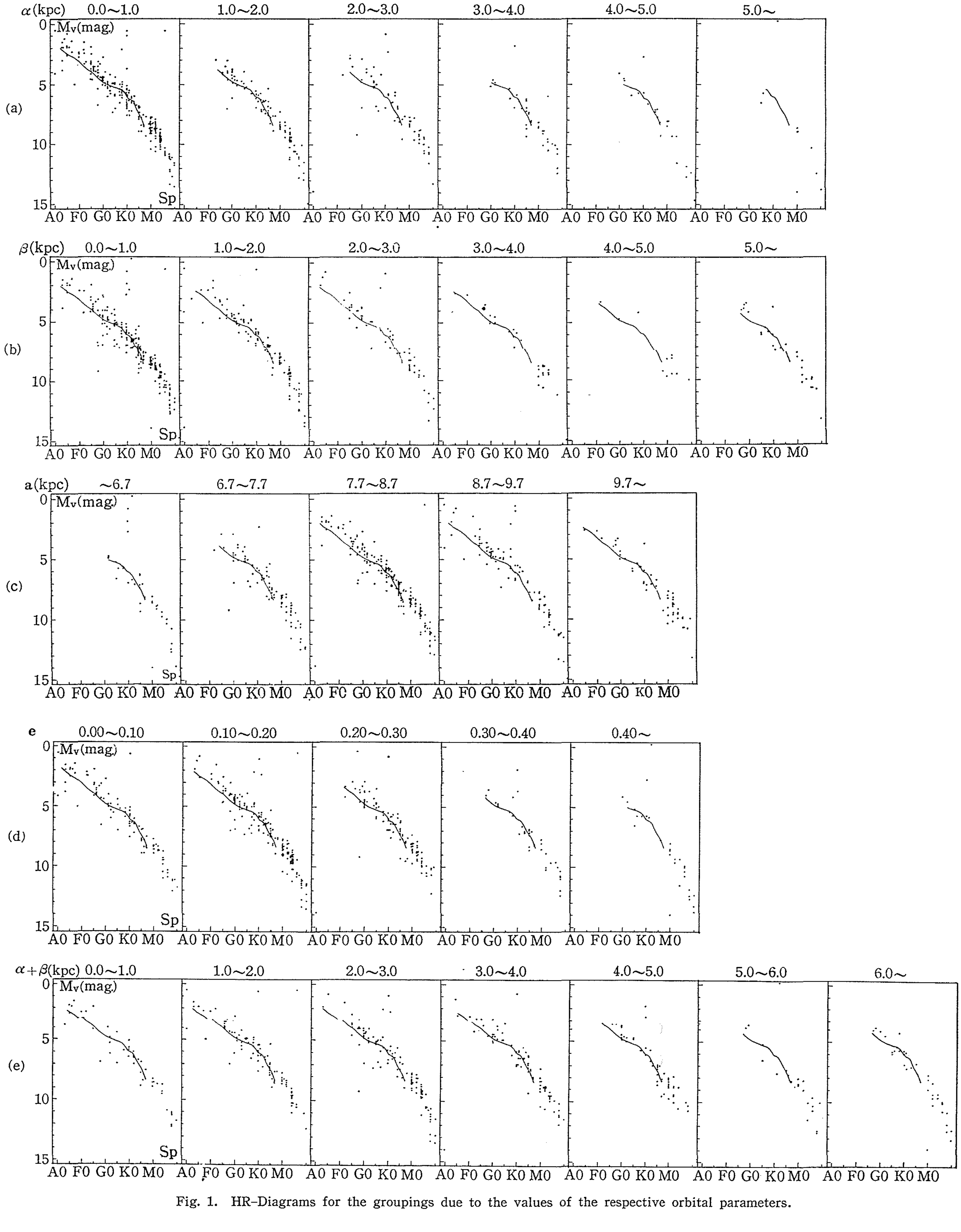

Fig. 1. HR-Diagrams for the groupings due to the values of the respective orbital parameters.

Table 2. Numerical data on the less luminous stars in each group corresponded to Table 1.

(a)

α (kpc)	0.0~1.0	1.0~2.0	2.0~3.0	3.0~4.0	4.0~5.0	5.0~
n	221	97	67	39	23	9
$\bar{\alpha}$ (kpc)	0.38	1.39	2.46	3.40	4,32	5.50
n_s	14	4	5	2	1	3
Quality	(a1, c1, d3, e7, f2)	(b2, e2)	(a1, c1, e2, f1)	(d1, f1)	(a1)	(b1, c1, f1)
Percentage	6.3	4.1	7.5	5,1	4.4	33.3

(b)

β (kpc)	0.0~1.0	1.0~2.0	2.0~3.0	3.0~4.0	4.0~5.0	5.0~
n	211	116	58	29	12	30
$\bar{\beta}$ (kpc)	0.35	1.32	2.39	3.28	4.40	6.59
n_s	18	6	2	2	0	1
Quality	(a2, b1, c1, d3, e7, f4)	(a1, b1, c1, e3)	(c1, f1)	(b1, d1)		(e1)
Percentage	8.5	5.2	3.4	6.9	0.0	3.3

(c)

$\boldsymbol{a}$ (kpc)	~6.7	6.7~7.7	7.7~8.7	8.7~9.7	9.7~
n	39	87	182	93	55
$\bar{\boldsymbol{a}}$ (kpc)	6.28	7.13	8.13	9.10	10.58
n_s	7	4	10	7	1
Quality	(a1, b1, c1, d1, e1, f2)	(a1, e2, f1)	(b1, c1, d2, e5, f1)	(a1, b1, c1, e3, f1)	(d1)
Percentage	17.9	4.6	5.5	7.5	1.8

(d)

$\boldsymbol{e}$	0.00~0.10	0.10~0.20	0.20~0.30	0.30~0.40	0.40~
n	101	176	106	43	30
$\bar{\boldsymbol{e}}$	0.057	0.141	0.241	0.335	0.486
n_s	8	12	4	2	3
Quality	(d2, e5, f1)	(a2, b1, c1, d1, e5, f2)	(b1, c1, e1, f1)	(a1, d1)	(b1, c1, f1)
Percentage	7.9	6.8	3.8	4.7	10.0

(e)

$\alpha+\beta$ (kpc)	0.0~1.0	1.0~2.0	2.0~3.0	3.0~4.0	4.0~5.0	5.0~6.0	6.0~
n	47	100	112	81	55	23	38
$\overline{\alpha+\beta}$ (kpc)	0.63	1.45	2.44	3.35	4.41	5.49	7.62
n_s	4	7	8	2	4	2	2
Quality	(d1, e3)	(d1, e5, f1)	(a2, b1, c1, e2, f2)	(d1, f1)	(a1, b1, c1, d1)	(c1, e1)	(b1, f1)
Percentage	8.5	7.0	7.1	2.5	7.3	8.7	5.3

into account the intrinsic scatter seemed to be of this order (**10, 11**).

The distinct change of Sp_{UL} or M_{UL} is found in the grouping of α, but some sort of a systematic trend is also clearly recognized for each of the other groupings. Hence, it is likely true that the α-grouping may serve as measure of the stellar age as Wooley and Eggen's suggestion, but as for the kinematical age-parameter of the stellar grouping either of groupings due to ζ_0 and $\sigma_{\dot{\zeta}}$ seems to be preferable as will be shown later.

By the way let us pay attention to the stars being fainter, say by 1 mag. or more than the initial main sequence in Figs. 1 (a)~1 (e). A general tendency is noticed there that their percentage number increases as the mean distance $\boldsymbol{a}$ decreases and their apo-galacticons are likely concentrated towards $\beta=0.0\sim1.0$ kpc (Table 2). But the existence of another group of the subdwarfs moving nearly in common with the local standard of rest is also suspected, judging from that the percentage number in $\boldsymbol{a}=7.7\sim8.7$ kpc or $\boldsymbol{e}=0.0\sim0.1$ does not necessarily correspond to the respective minimum value and yet the percentage number of $(\alpha+\beta)=0.0\sim1.0$ kpc appears as maximum.

4. The frequency distribution of the orbital parameters for the whole stars

For all the available data the frequency distributions of (a) the peri-galacticon distance α, (b) the apo-galacticon distance β, (c) the mean distance $\boldsymbol{a}$, (d) the eccentricity $\boldsymbol{e}$ and (e) the amplitude of the radial motion $(\alpha+\beta)$ were derived as shown by the block diagrams in Fig. 2. As for the mean distance $\boldsymbol{a}$ its observed frequency distribution appears nearly as Gaussian but those for the other parameters turn out not to be normal but take particular forms respectively. Then, it was next considered what type of the frequency distribution is expected for each orbital parameter.

As regards the whole near-by stars the frequency distribution of the velocity components parallel to the Galactic plane may be in the first approximation represented by

$$F(\dot{\xi}_0, \dot{\eta}_0)d\dot{\xi}_0 d\dot{\eta}_0 = \frac{1}{2\pi\sigma^2}\left(-\frac{B_0}{\omega_0}\right) \exp\left\{-\frac{1}{2\sigma^2}\left(-\frac{B_0}{\omega_0}\dot{\xi}_0^{\,2} + \dot{\eta}_0^{\,2}\right)\right\}d\dot{\xi}_0\, d\dot{\eta}_0\,, \qquad (1)$$

where ω_0 and B_0 denote the angular velocity and the Oort's constant of the Galactic rotation at the local standard of rest ($\omega_0=8.2$ kpc). If (1) is assumed, we are able to derive the analytical expression of the frequency distribution of each orbital parameters.

The velocity-components $\dot{\xi}_0$ and $\dot{\eta}_0$ defined at the reference circle $\varpi=\varpi_0$ are written by means of our formulae [S-1] in terms of $\boldsymbol{a}$ and $\boldsymbol{e}$ as follows :

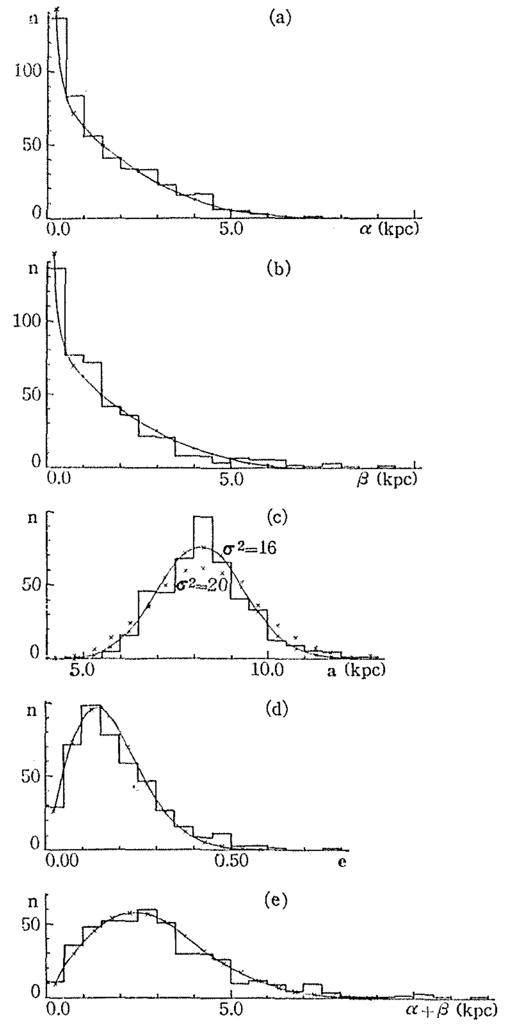

Fig. 2. The frequency distribution of each orbital parameter for all the stars.
The histrogram : the observed frequency distribution
$\times$: the theoretical frequency integrated in a range of 0.5 kpc
$\cdot$: the theoretical value at the midpoint of a range (multiplied by the necessary factor)

$$\dot{\xi}_0^2 = -\frac{4B_0}{\omega_0}\Theta_0^2\frac{a}{\varpi_0}\left\{e^2-\left(\frac{\varpi_0}{a}-1\right)^2\right\},\ C_0^2=(\Theta_0+\dot{\eta}_0)^2 = -\frac{4B_0}{\omega_0}\Theta_0^2\left\{\frac{a}{\varpi_0}(1-e_0^2)-\left(1+\frac{\omega_0}{4B_0}\right)\right\}. \quad (2)$$

Putting $\frac{\omega_0}{a}=1+x(-1\leqq x\leqq 1)$* we get

$$\dot{\xi}_0^2 \doteqdot -\frac{4B_0}{\omega_0}\Theta_0^2(e^2-x^2),\quad \dot{\eta}_0 \doteqdot \frac{2B_0}{\omega_0}\left\{x-\left(1+\frac{B_0}{\omega_0}\right)x^2+e^2\right\},\quad (e\geqq x). \quad (3)$$

With the aid of (3) the formula in (1) is transformed into

$$\left.\begin{aligned} G(x,e)dxde &= \frac{4}{\pi}\left(\frac{B_0}{\omega_0}\frac{\Theta_0}{\sigma}\right)^2\frac{e}{\sqrt{e^2-x^2}}\exp\left\{-\left(\frac{B_0}{\omega_0}\frac{\Theta_0}{\sigma}\right)^2e^2\right\}dxde \\ &= \frac{2h^2}{\pi}\frac{e}{\sqrt{e^2-x^2}}\exp(-h^2e^2)dxde,\quad h\equiv\sqrt{2}\left(-\frac{B_0}{\omega_0}\frac{\Theta_0}{\sigma}\right), \end{aligned}\right\} \quad (4)$$

where $\Theta_0=\varpi_0\omega_0$ or the linear velocity of the Galactic rotation at ϖ_0.

Integration of (4) with respect to e or x brings out the required distribution function of $x=1+\varpi/a$ or e respectively. Neglecting higher order terms we obtain

$$f_a(a)da = \frac{2h}{\sqrt{\pi}}\left(\frac{\varpi_0}{a}\right)^2\exp\left\{-h^2\left(\frac{\varpi_0}{a}-1\right)^2\right\}\frac{da}{\varpi_0} = \frac{2h}{\sqrt{\pi}}\exp\left\{-h^2\left(\frac{\varpi_0}{a}-1\right)^2\right\}d\left(\frac{\varpi_0}{a}\right), \quad (5)$$

$$f_e(e)de = 2h^2e\exp(-h^2e^2)de/[1-\exp(-h^2)]^{**}. \quad (6)$$

It is noticed that so far as the present approximation concerns the frequency distribution of $\frac{\omega_0}{a}$ is of the normal type or $N(1, 1/\sqrt{2}h)$, while that of e appears practically to be of χ-distribution with its degree of freedom $\nu=2$, or saying alternatively, the distribtion of e^2 is of the normal type like as $N(0,1/\sqrt{2}h)$.

In order to get the other required formulae, $\dot{\xi}_0$ and $\dot{\eta}_0$ must be expressed by $\alpha=\varpi_0-a(1-e)$ and $\beta=a(1+e)-\varpi_0$. After some reductions from (2) we have

$$\begin{aligned} \dot{\xi}_0^2 &= -4B_0\omega_0\alpha\beta\left(1+\frac{\beta-\alpha}{\varpi_0}\right)^{-1}, \\ \dot{\eta}_0 &= -B_0\left(\beta-\alpha-\frac{2\alpha\beta}{\varpi_0}\right)\left(1-\frac{\beta-\alpha}{2\varpi_0}\right)^{-1}\left\{1+\frac{B_0}{\omega_0}\frac{\beta-\alpha-\frac{2\alpha\beta}{\varpi_0}}{2\varpi_0+\beta-\alpha}+\cdots\right\}. \end{aligned} \quad (7)$$

By taking only the principal order into account, namely

$$\dot{\xi}_0^2 \doteqdot -4B_0\omega_0\alpha\beta,\quad \dot{\eta}_0 \doteqdot -B_0(\beta-\alpha), \quad (7')$$

and by changing the variables from $(\dot{\xi}_0, \dot{\eta}_0)$ to (α, β) we get

$$H(\alpha,\beta)d\alpha d\beta = \frac{2k^2}{\pi}\left(\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{\alpha}{\beta}}\right)\exp\{-k^2(\beta+\alpha)^2\}d\alpha d\beta,\quad k\equiv\frac{1}{\sqrt{2}}\left(-\frac{B_0}{\sigma}\right). \quad (8)$$

It is seen there that the bivariate frequency distribution turns out in the

* None of stars with $a<\frac{\varpi_0}{2}$ can reach to $\varpi=\varpi_0$.

** $1/[1-\exp(-h^2)]$ is a normalizing factor resulting from its integration range of $0\leqq e\leqq 1$, but the term $\exp(-h^2)$ contributes little in the present case of $h^2=25$.

first approximation to be symmetric. The frequency distribution of $u=\alpha+\beta$ is readily obtained by adopting u, α as the independent variables and integrating with respect to α.

$$f_{\alpha+\beta}(\alpha+\beta)d(\alpha+\beta) = 2k^2(\alpha+\beta)\exp\{-k^2(\alpha+\beta)^2\}\,d(\alpha+\beta) \tag{9}$$

This is the same type as (6).

Whereas the integration with respect to α is given in terms of Weber's function such as $D_{-3/2}$ and $D_{-5/2}$, accordingly it is expressed by a power series. If we put $u/\alpha-1=\sqrt{s}$ and expand $\exp(-2k^2\alpha s^{1/2})$ in a power series, we get

$$H_1(s,\alpha)dsd\alpha = \frac{k^2}{\pi}\alpha\exp(-k^2\alpha^2)d\alpha(-s^{1/4}+s^{-3/4})\exp\{-k^2\alpha^2(2s^{1/2}+s)\}\,ds\,, \tag{10}$$

Hence
$$\left.\begin{aligned} f_\alpha(\alpha)d\alpha &= \frac{k^2}{\pi}\exp(-k^2\alpha^2)\frac{1}{\sqrt{k\alpha}}\Big[\Gamma\left(\frac{3}{4}\right)\Big\{1-\frac{1}{2!}(k\alpha)^2+\frac{3}{4!}(k\alpha)^4-\cdots \\ &\quad -\frac{(4n-5)!!}{(2n)!}(k\alpha)^{2n}-\cdots\Big\}+\frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Big\{(k\alpha)+\frac{1}{3!}(k\alpha)^3 \\ &\quad +\frac{1\cdot5}{5!}(k\alpha)^5+\cdots+\frac{(4n-3)!!}{(2n+1)!}(k\alpha)^{2n+1}+\cdots\Big\}\Big]d\alpha\,, \\ \text{or}\quad f_\alpha(\alpha)d\alpha &= \frac{k}{\pi}\exp(-k^2\alpha^2)\Big[\frac{1.225}{\sqrt{k\alpha}}\Big\{1-\frac{(k\alpha)^2}{2}-\frac{(k\alpha)^4}{8}-\frac{7}{270}(k\alpha)^6-\cdots\Big\} \\ &\quad +1.479\sqrt{k\alpha}\Big\{1+\frac{(k\alpha)^2}{6}+\frac{(k\alpha)^4}{24}+\cdots\Big\}\Big]d\alpha\,. \end{aligned}\right\} \tag{11}$$

On the other hand, if we put $u/\alpha-1=t$ and expand $\exp(-k^2\alpha^2t^2)$ into a power series we have alternatively

$$f_\alpha(\alpha)d\alpha = \frac{4k}{\sqrt{2\pi}}\exp(-k^2\alpha^2)\Big[1+\frac{1}{16(k\alpha)^2}-\frac{15}{512(k\alpha)^4}+\cdots+\frac{(-1)^{n+1}}{n!}\frac{(4n-3)!!}{(4k\alpha)^{2n}}+\cdots\Big] \tag{11'}$$

The former is useful for $k\alpha(k\beta)<1$, while the latter is for $k\alpha(k\beta)\geq1$.

Thus the analytical formulae of all the orbital parameters have been obtained. By means of these formulae (11) or (11'), (5), (6) and (9), and by adopting as trial, the following round values:

$h=5.0$, $k=0.30$ (corresponding to $-B_0/\sigma=0.43_1$ ($\varpi_0=8.2$ kpc), $-B_0/\sigma=0.42_4$ respectively, then $\sigma=22$ km/sec ($B_0=-9.5$ km/sec/kpc)) the theoretical frequencies for the five orbital parameters were calculated in every 0.5 kpc interval, and these were indicated by crosses in Fig. 2. It is found there that agreement between the observed and the calculated distribution is quite satisfactory.

5. The dependences of the orbital parameters on the spectral type

Here is taken up the dependence of the spectral type on the orbital parameters. In Table 3 the means and the dispersions of the respective orbital parameters are given for every spectral group in comparison with those for all the stars combined together. The frequency curves of each orbital parameter for successive spectral types were proved not to differ fundamentally in type, so we eliminate showing the diagrams of the frequency curves for every spectral subgroup.

Table 3. The means and the dispersions of the orbital parameters in each spectral group.

Sp		A0~A8	F0~F9	G0~G8	K0~K3	M0~M6
n		20	48	94	123	171
(a)	$\bar{\alpha}$ (kpc)	0.70	0.78	1.47	1.64	1.60
	σ_{α} (kpc)	0.96	0.76	1.37	1.33	1.48
(b)	$\bar{\beta}$ (kpc)	1.21	1.81	1.57	1.27	1.75
	σ_{β} (kpc)	0.95	2.06	1.64	1.53	1.87
(c)	$\bar{\boldsymbol{a}}$ (kpc)	8.47	8.75	8.28	8.08	8.29
	σ_{a} (kpc)	0.63	1.21	1.21	1.13	1.31
(d)	$\bar{e}$	0.114	0.139	0.187	0.185	0.210
	σ_{e}	0.092	0.083	0.114	0.115	0.135
(e)	$\overline{\alpha+\beta}$ (kpc)	1.91	2.53	3.04	2.91	3.37
	$\sigma_{\alpha+\beta}$ (kpc)	1.43	1.92	1.82	1.67	2.15

But it is worthy of test whether the apparent differences of both mean and variance among various spectral type groups may be ascribed to the sampling error or not.

In carrying out these tests, for the sake of simplicity, the t- and the F-test for the normal distribution were made in common use, but as for $\dot{\zeta}_0$ and $\boldsymbol{a}$ directly by regarding their observed distribution as normal, while with regards α, β, $\boldsymbol{e}$ and $(\alpha+\beta)$ indirectly by transforming their observed distributions into the almost normal ones with the aid of the variable conversion from x's to log x's. That the $\boldsymbol{a}$'s distribution was regarded as normal is merely because its apparent distribution looks as nearly normal. After such transformations, the uniformity of the variance at first and then that of the mean among the spectral type groups were examined with the aids of the F- and the t-test respectively.

Table 4. Significance test.

$$\alpha : (\underline{\mathrm{A}, \ \mathrm{F}}) - (\underline{\mathrm{G}, \ \mathrm{K}, \ \mathrm{M}})$$
$$\beta : (\underline{\mathrm{A}, \ \mathrm{F}, \ \mathrm{G}, \ \mathrm{K}} \times \mathrm{M})$$
$$\boldsymbol{a} : \mathrm{A} - (\mathrm{F} \times \underline{\mathrm{G}, \ \mathrm{K}, \ \mathrm{M}})$$
$$\boldsymbol{e} : (\underline{\mathrm{A}, \ \mathrm{F}} \times \underline{\mathrm{G}, \ \mathrm{K}, \ \mathrm{M}})$$
$$\alpha+\beta : (\underline{\mathrm{A}, \ \mathrm{F}} \times \underline{\mathrm{G}, \ \mathrm{K}, \ \mathrm{M}})$$
$$\zeta : \mathrm{A} - \mathrm{F} - \mathrm{G} - \mathrm{K} - \mathrm{M}$$

The results of these tests are schematically summarized in Table 4. The symbols found there signify as follows: the spectral groups in a pair of parenthese or those underlined together have no significant differences among their variances or the means respectively, and both sides of the bar or the cross are significantly different in the variance or the mean respectively with its significance level of 1% if the marks are written as thick, or of 5% if thin.

A general view of the above diagram might be reduced to such a conclusion that the near-by stars are divided into two large categories of (A, F) and (G, K, M) just essentially the same as Parenago's result (**12**) or Shimizu's one (**13**), though the more detailed classifications such as (A), (F), (G, K, M); (A), (F), (G, K), (M) and finally each of the spectral type would make of course a smaller dispersion in each class.

6. Some relations between the stellar age and the orbital parameters

Even though the star members in each of our HR-diagram have been selected merely by the values of some orbital parameters among the near-by field stars, the age corresponding to the turning point in each HR-diagram may serve as a measure of the mean age of the group members. On this view-point we tried to inquire into the age-effect on the orbital parameters with the aid of the data in Table 1.

In order to estimate the age t(yr) for the turning point we made use of Schwarzschild's table on the life time of the main sequence stars (**14**) and the mean $\log t$ interpolated independently from both arguments Sp and Mv was adopted as the representative age of the group in question. The similar mean $\log t$ corresponding to $\overline{\mathrm{Sp}}_{\mathrm{UL3}}$ and $\overline{\mathrm{Mv}}_{\mathrm{UL3}}$ was also evaluated for checking how our assigned $\mathrm{Sp}_{\mathrm{UL}}$ and $\mathrm{Mv}_{\mathrm{UL}}$ were reliable. Against these $\log t$'s the means in successive subgroups for every orbital parameter given in Table 1 were plotted as in Fig. 3. The upper limit of t may be $(1.5\pm0.5)\times10^{10}$yr, so that only for

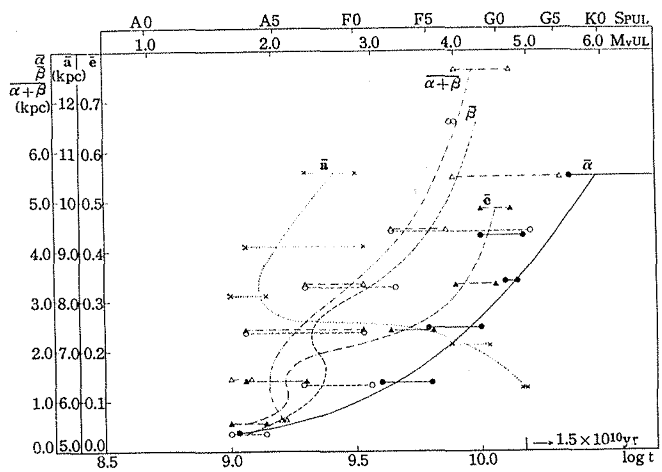

Fig. 3. Age-effect of the corbital parameters.

With respect to each pair of marks, the left and the right correspond to $\mathrm{Sp}_{\mathrm{UL}}(\mathrm{Mv}_{\mathrm{UL}})$ and $\overline{\mathrm{Sp}}_{\mathrm{UL}}(\overline{\mathrm{Mv}}_{3\mathrm{UL}3})$ respectively. The upper limit of t may be $\sim1.5\times10^{10}$yr, then for $t\simeq1.5\times10^{10}$yr only the dependence on $\mathrm{Sp}_{\mathrm{UL}}$ may be meaningful.

$t<(1.5\pm0.5)\times10^{10}$yr the dependence on Sp_{UL} may be meaningful.

Some interesting features are noticed here: (i) The stars moving nearly in a circular orbit or $a\simeq\varpi_0(=8.2\,\text{kpc})$ are likely the youngest of all, and for those with a being the larger or the smaller than 8.2 kpc their $\log t$'s turn out always the larger though in a respective way rapidly or slowly. It must be taken account, however, that the stars passing through some bounded region are limited according to the value of a. (ii) As regards the other parameters, the parameter value increases in general as $\log t$ increases. (iii) There is an evidence, however, that the curve associated with β such as those of β, $\alpha+\beta$ and e undulate in an interval of $9<\log t<9.5$. This may be, unless due to sampling errors, corresponding to a discontinuity of the kinematical character between both spectral groups (A, F) and (G, K, M) mentioned in §4.

It is certain that the correlations found in Fig. 3 may be represented exaggeratedly somewhat, since the parameter means have been taken from the groupings due to the very parameter values. Nevertheless, as a general conclusion from Fig. 3, it may well be said that the orbital parameters depend not only on the age but also possibly on the kinematical condition at the time of the stellar birth, namely how the arm of gas clouds from which the stars were born had moved deviatedly from the circular orbit.

Next let us examine the dependences of $\sigma_{\dot\zeta}$ or the dispersion of the vertical velocity component $\dot\zeta_0$ on the orbital parameters. These are represented as Fig. 4 which is also another way of plotting of Table 1. It is seen that there exists some clear correlation between $\sigma_{\dot\zeta}$ and each of the orbital parameters. The similar clear correlation as for the mean of $\dot\zeta_0$ was proved too, even though its

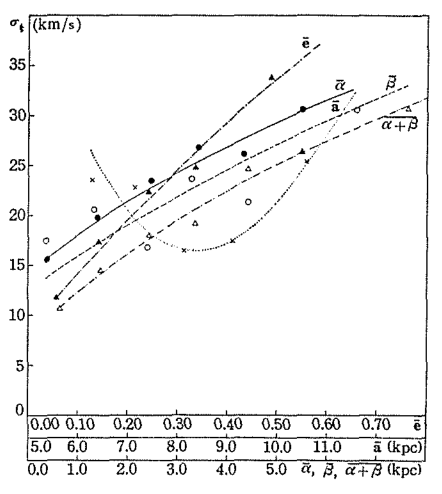

Fig. 4. The dependence of $\sigma_{\dot\zeta}$ on the orbital parameters.

illustration is omitted here. Reflecting upon the age effect on the orbital parameters, the fact may mean that $\sigma_{\dot{\zeta}}$ as well as $\overline{\dot{\zeta}_0}$ serve as the kinematical age-indicators, even though their use is limited. Before going to this problem, however, it must be remarked that our orbital parameters had referred to the two-dimensional orbit and consequently their strong correlations with $\dot{\zeta}_0$ or $\sigma_{\dot{\zeta}}$ ought to be expected regardless of the age effect.

For the purpose of seeing the age-effect on $\sigma_{\dot{\zeta}}$, the values of $\sigma_{\dot{\zeta}}$ were plotted

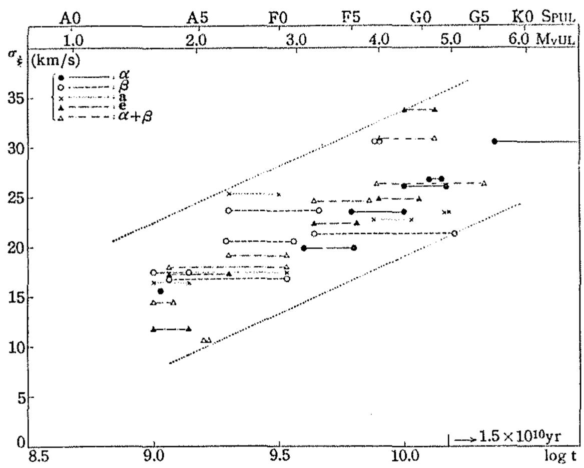

Fig. 5. The age-effect of $\sigma_{\dot{\zeta}}$.

against $\log t$ as shown in Fig. 5 with the aid of Table 1 over again. We can readily ascertain there that the age-effect on $\sigma_{\dot{\zeta}}$ is almost straightforward against $\log t$, even if some fluctuations from the linearity might be pointed out. In connection with this it may be advisory to mention that Fig. 5 is almost unbiased in a sense remarked as for Fig. 3.

7. Possible causes for the change of the orbital parameters

Up to now we have seen how each of the orbital parameters depends on the stellar age, then our next concerns are what has brought about such state of affairs. The differences in orbital parameters may be caused by $(\Delta\dot{\xi}_0, \Delta\dot{\eta}_0)$ or the velocity change of each star and by $(\Delta\omega_0, \Delta A_0)$ or the change of the force. The circumstance of how each factor affects the respective orbital parameters is approximately estimated by deriving the differential formulae from the expressions of the parameters. If we make use of the formulae given in S-1 and put $\frac{a}{\varpi_0}=1$ by adopting the reference radius to be equal to the star's mean radius, we arrived at, after some calculations,

$$
\left.\begin{array}{r}
\dfrac{\varDelta \boldsymbol{a}}{\boldsymbol{a}}=-\dfrac{\omega_0}{4B_0} \\
\dfrac{\varDelta \boldsymbol{e}}{\boldsymbol{e}}=\left[\dfrac{\omega_0}{8B_0}\right] \\
\dfrac{\varDelta(\alpha+\beta)}{(\alpha+\beta)}=-\dfrac{\omega_0}{8B_0} \\
\dfrac{\varDelta \alpha}{\alpha}=\left[\dfrac{\omega_0}{4B_0}\left(1+\dfrac{e}{2}\right)\right] \\
\dfrac{\varDelta \beta}{\beta}=-\dfrac{\omega_0}{4B_0}\left(1-\dfrac{e}{2}\right)
\end{array}\right\} \times \frac{\varDelta \dot{\xi}_0^2+\varDelta \dot{\eta}_0^2}{\Theta_0^2}+\left\{\begin{array}{c}
0 \\
-\dfrac{\omega_0}{4B_0}\dfrac{1}{e^2} \\
-\dfrac{\omega_0}{4B_0}\dfrac{1}{e^2} \\
-\dfrac{\omega_0}{4B_0}\dfrac{1}{e(1-e)} \\
-\dfrac{\omega_0}{4B_0}\dfrac{1}{e(1+e)}
\end{array}\right\} \times \frac{\varDelta \dot{\xi}_0^2}{\Theta_0^2}
$$

$$
+\left\{\begin{array}{l}
\left[\dfrac{\omega_0}{2B_0}\right] \\
\left[\left(\dfrac{3}{2}-\dfrac{A_0}{\omega_0}\right)\dfrac{\omega_0}{2B_0}\right] \\
\left[\left(\dfrac{5}{2}-\dfrac{A_0}{\omega_0}\right)\dfrac{\omega_0}{2B_0}\right] \\
-\dfrac{\omega_0}{2B_0}\left\{1-\left(\dfrac{3}{2}-\dfrac{A_0}{\omega_0}\right)e\right\}^{*} \\
\left[\dfrac{\omega_0}{2B_0}\left\{1+\left(\dfrac{3}{2}-\dfrac{A_0}{\omega_0}\right)e\right\}\right]
\end{array}\right\} \times \frac{\varDelta \omega_0}{\omega_0}+\left\{\begin{array}{c}
0 \\
-\dfrac{\omega_0}{2B_0} \\
-\dfrac{\omega_0}{2B_0} \\
-\dfrac{\omega_0}{2B_0}e \\
-\dfrac{\omega_0}{2B_0}e
\end{array}\right\} \times \frac{\varDelta A_0}{\omega_0}. \tag{12}
$$

Remark: [ ]: negative always. * : nagative if $A_0<\left(\frac{3}{2}-\frac{1}{e}\right)\omega_0$, but positive if $A_0>\left(\frac{3}{2}-\frac{1}{e}\right)\omega_0$. The other terms: positive always.

We have already the evidence that the age-effect on the orbital parameters, letting $\boldsymbol{a}$ whose observed distribution must be influenced strongly by our present position in the Galaxy be out of consideration now, has worked so as to increase the parameter values, so consequently $\varDelta\dot{\xi}_0^2>0$, $\varDelta\omega_0<0$ and/or $\varDelta A_0>0$ for $\varDelta t>0$ should be satisfied. In order to see the effect numerically, the change of $(\varDelta\dot{\xi}_0^2+\varDelta\dot{\eta}_0^2)$, $\varDelta\dot{\xi}_0^2$, $\frac{\varDelta\omega_0}{\omega_0}$ or $\frac{\varDelta A_0}{\omega_0}$ necessary for an increment of 1/10 in $\frac{\varDelta \boldsymbol{a}}{\boldsymbol{a}}$, $\frac{\varDelta \boldsymbol{e}}{\boldsymbol{e}}$, $\frac{\varDelta(\alpha+\beta)}{(\alpha+\beta)}$, $\frac{\varDelta\alpha}{\alpha}$ or $\frac{\varDelta\beta}{\beta}$ respectively was estimated in Table 5 ($\omega_0=26.4$ km/sec/kpc, $A_0=16.9$ km/sec/kpc).

This table may give a general idea concerning the relative importance of the respective effect, though the values tabulated could hardly be accepted as they are on account of our approximate foundation. The effect of 0.1 in $\frac{\varDelta\omega_0}{\omega_0}$ or $\frac{\varDelta A_0}{A_0}$ is comparable with that of $10^3\sim10^4$(km/sec)^2 in $\varDelta(\dot{\xi}_0^2+\dot{\eta}_0^2)$ but with that of the far less value of $\varDelta\dot{\xi}_0^2$ if $\boldsymbol{e}$ approaches to zero (or to 1 too for $\varDelta\alpha$). In the present case where the orbital parameters except for $\boldsymbol{a}$ tend to zero for the circular orbit, it may be more convenient to see the increment itself rather than

Table 5. Changes of the constants correspond to 10% change of each orbital parameter.

		$\frac{\omega e}{\Delta}=0.1$	$\frac{\Delta e}{e}=0.1$	$\frac{\Delta(\alpha+\beta)}{\alpha+\beta}=0.1$	$\frac{\Delta\alpha}{\alpha}=0.1$	$\frac{\Delta\beta}{\beta}=0.1$	unit
$\Delta(\dot{\xi}_0^2+\dot{\eta}_0^2)$		$+6.7\times10^3$	-1.3×10^4	$+1.3\times10^4$	$-(6.7\sim4.5)+10^3$ $\downarrow$ $\downarrow$ $e=0$ $e=1$	$+(6.7\sim13)\times10^3$ $\downarrow$ $\downarrow$ $e=0$ $e=1$	km²/sec²
$\Delta\dot{\zeta}_0^2$	$e=0.01$	0	$+0.67$	$+0.67$	$+6.6\times10$	$+6.8\times10$	km²/sec²
	0.05	0	$+1.7\times10$	$+1.7\times10$	$+3.2\times10^2$	$+3.5\times10^2$	
	0.1	0	$+6.7\times10$	$+6.7+10$	$+6.0\times10^2$	$+7.4\times10^2$	
	0.2	0	$+2.7+10^2$	$+2.7+10^2$	$[+1.1\times10^3]$	$[+1.6\times10^3]$	
	0.5	0	$[+1.7\times10^3]$	$[+1.7\times10^3]$	$[+1.7\times10^3]$	$[+5.0\times10^3]$	
	0.8	0	$[+4.3\times10^3]$	$[+4.3\times10^3]$	$[+1.1\times10^3]$	$[+9.7\times10^3]$	
	0.9	0	$[+5.4\times10^3]$	$[+5.4\times10^3]$	$+6.0\times10^2$	$[+1.1\times10^4]$	
	0.95	0	$[+6.1\times10^3]$	$[+6.1\times10^3]$	$+3.2\times10^2$	$[+1.2\times10^4]$	
	$e=0.99$	0	$[+6.6\times10^3]$	$[+6.6\times10^3]$	$+6.5\times10$	$[+1.3\times10^4]$	
$\frac{\Delta\omega_0}{\omega_0}$		-0.072	-0.084	-0.039	$0.072\sim0.51$ $\downarrow$ $\downarrow$ $e=0$ $e=1$	$-(0.072\sim0.039)$ $\downarrow$ $\downarrow$ $e=0$ $e=1$	
$\frac{\Delta A_0}{\omega_0}$		0	$+0.072$	$+0.072$	$0\sim0.072$ $\downarrow$ $\downarrow$ $e=0$ $e=1$	$0\sim0.072$ $\downarrow$ $\downarrow$ $e=0$ $e=1$	

the percentage one. In the next table the changes of $\Delta(\dot{\xi}_0^2+\dot{\eta}_0^2)$ etc. necessary for $\Delta e=0.01$ for the successive values of e are shown as an example.

We can notice in both tables that the encounter effect is mainly due to the coefficient of $\Delta\dot{\xi}_0^2$ which turns out appreciable only when the eccentricity is small (or near to 1 too for $\Delta\alpha$).

Now we consider the effect due to the change of the squared velocity components which are supposedly caused by the encounters with the gas clouds. According to Spitzer and Schwarzschild's view that the increase of the velocity dispersion with advancing spectral type along the main sequence might be explained as the result of the random encounters with the cloud complex having the mass of an order of 10^6 solar mass, the root-mean-square velocity (referred

Table 6. Changes of the constants correspond to $\Delta e=0.01$.

e	0.01	0.05	0.1	0.2	0.5	0.8	0.9	
$\Delta(\dot{\xi}_0^2+\dot{\eta}_0^2)$	-1.3×10^4	-2.7×10^3	-1.3×10^3	-6.7×10^2	-2.7×10^2	-8.4×10	-7.4×10	(km/sec)²
$\Delta\dot{\xi}_0^2$	6.7×10^{-1}	3.4	6.7	1.3×10	3.4×10	5.4×10	6.1×10	(km/sec)²
$\frac{\Delta\omega_0}{\omega_0}$	-8.4×10^{-2}	-1.7×10^{-2}	-8.4×10^{-8}	-4.2×10^{-3}	-1.7×10^{-3}	-1.0×10^{-3}	-9.3×10^{-4}	
$\frac{\Delta A_0}{\omega_0}$	-7.2×10^{-2}	-3.6×10^{-2}	-7.2×10^{-3}	-3.6×10^{-3}	-1.4×10^{-3}	-9.0×10^{-4}	-8.0×10^{-4}	

to the local standard of rest) of a star at t yr after its birth is expressed by

$$V(t) = V(0)(1+t/t_E)^{1/3}, \qquad t_E = \frac{4\,V^3(0)}{3\pi^{3/2}G^2 n_c m_c^2\cdot\ln\alpha\cdot F\left(\frac{B}{\omega}\right)}, \tag{13}$$

where $V(0)$ is the initial r.m.s. velocity and the parameter t_E is estimated as 2×10^8 yr with $V(t=1.5\times10^9\,\text{yr})=20$ km/sec and $V(0)=10$ km/sec (**15**). Von Hoerner has supported this view by applying (13) to the near-by stars (**16**). In Spitzer and Schwartzschild's treatment, however, the term relating the dynamical friction (**17**) has not been taken account, which is nearly proportional to $\eta_0=\left(\sqrt{1+\frac{4B_0}{\omega_0}e^2}-1\right)\Theta_0<0$ ($\varpi_0=a$) on the average in a round of the epicyclic orbit, and accordingly as e increases the dynamical friction works so as to reduce the velocity.

We treat the matter simply without regard to the Galactic differential rotation but including the dynamical friction implicitly. The increment of the squared velocity $\Delta V^2=V^2-V_0^2$ due to the encounters with the clouds is written by

$$\Delta V^2 = \sum\Delta V_{||}^2+\sum(\Delta V_{||})^2+\sum(\Delta V_\perp)^2 .$$

The expressions for the last two terms are given by Chandrasekhar (**18**) and the first one can be derived in a similar way to the others (in the three-dimensions).

$$\left.\begin{aligned}
\sum \Delta V_{||}^2 &= 8\pi G^2 n_c m_c^2\left(1+\frac{m}{m_c}\right)\frac{\ln qV^2}{V}[x_0\Phi'(x_0)-\Phi(x_0)]\Delta t,\\
\sum(\Delta V_{||})^2 &= 8\pi G^2 n_c m_c^2\frac{\ln qV^2}{V}\frac{1}{2x_0}[\Phi(x_0)-x_0\Phi'(x_0)]\Delta t,\\
\sum(\Delta V_\perp)^2 &= 8\pi G^2 n_c m_c^2\frac{\ln qV^2}{V}\frac{1}{2x_0}[x_0\Phi'(x_0)+(2x_0^2-1)\Phi(x_0)]\Delta t,
\end{aligned}\right\}$$

where $x_0=\frac{V}{\sqrt{2\,\overline{V_c^2}}}$, $q=\frac{D_0}{Gm_c}\left(1+\frac{m}{m_c}\right)^{-1}$, $\Phi(x)$: the error integral. Then, assuming that $m_c\gg m$ and $\ln qV^2\simeq3$ and $n_c m_c=3\times10^{-24}\,\text{gr/cm}^3$, we have

$$\Delta V^2 = 24\pi G^2 n_c m_c^2\frac{\Phi'(x_0)}{\sqrt{2\,\overline{V_c^2}}}\Delta t = \frac{4.3\times10^{-4}}{(\sqrt{\overline{V_c^2}}/10)\,\text{km/sec}}\left(\frac{m_c}{\Theta}\right)\Phi'(x)\left(\frac{\Delta t}{10^8\,\text{yr}}\right)\left(\frac{\text{km}}{\text{sec}}\right)^2. \tag{14}$$

The numerical values of ΔV^2 for $m_c=10^5$ solar mass and $\sqrt{\overline{V_c^2}}=10$ km/sec are shown in Table 7. This table gives a favourable evidence for our anticipation that the encounter effect is sensitive only when e remains small. With reference to the formula (14) the table also informs us what order of the encounter effect may be expected from the assumed cloud's mass.

The numerical values of ΔV^2 for $m_c=10^5$solar mass, $\sqrt{\overline{V^2}}=10$ km/sec are shown in Table 7.

Table 7. Increment of the squared velocity of a star due to the encounters with the clouds. ($\Delta t=10^8$yr)

x_0	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
$\sqrt{\overline{V^2}}$ (km/sec)	0	2.8	5.7	8.5	11.3	14.1	17.0	19.8	22.6	25.5	28.3	35.4	42.4	49.5	56.6
ΔV^2 (km/sec)^2	17	17	17	14	12.5	10	8	6.5	5	3	2	0.8	0.2	4×10^{-2}	6×10^{-3}

The table gives a favorable evidence for our anticipation that the encounter effect is sensitive only when e remains small. With reference to the formula (14) the table also informs us what order of the encounter effect may be expected from the assumed cloud's mass.

Thus, we are inclined to think that if the star had formed from the cloud moving almost in a circular orbit, the encounters with the massive clouds of an order of 10^5 solar mass would have changed its orbital parameters appreciably, but with increasing the eccentricity the action might have grown weak. In our view, therefore, even though the increase of the velocity dispersion with the stellar age up to 5×10^9 yr is represented well empirically by the first formula of (13) with $t_E=2\times10^8$ yr, this may not be necessarily because of the encounter effect only.

On the other hand, there is a possibility that the pair of the factors $\frac{\Delta\omega_0}{\omega_0}$ and $\frac{\Delta A_0}{\omega_0}$ describing the change of the force field may affect appreciably the orbital parameters as seen in Table 6. Then, we are likely to imagine that the age-effect on a may be overwhelmingly due to the secular decrease of ω_0, while that of the other parameters owes to the encounters in a certain period of the early stage when e is kept as small, but not before long the role to enlarge these parameter-values is replaced by influence of the variation of the force field. This variation is something like that ω_0 decreases but A_0 increases.

In the above discussion with respect to the initial velocity dispersion or the turbulent motion of the clouds from which the stars were born was not especially mentioned. The difference in the initial velocity is included in either $\dot{\xi}_0$, $\dot{\eta}_0$, $\dot{\zeta}_0$ or $\Delta\dot{\xi}_0$, $\Delta\dot{\eta}_0$, $\Delta\dot{\zeta}_0$ in our formulation. Of course this must be also taken into account.

For a detailed discussion based on our anticipation, the matters connected with the variation of the force-field should be examined, but it will be done in a future paper.

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