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PUMPING EFFECTS IN MULTIPLE-FLOWING-WELLS SYSTEM

BY

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ABSTRACT

The pumping tests were at procedured various places in the old city of Beppu Spa, and showed that the pumping effects in multiple-flowing-wells system have close relations with the density of the well number arround the pumped wells, that is, we can treat the discharges of the flowing wells, as uniform leakage from the aquifer. By this treatment we have derived some relations between the leakage factor and the density of well number surrounding the pumped well. The results are as follows: (1) Linear relation is found between the leakage factor and the density of well number surrounding the pumped well. (2) The influence upon the surrounding wells caused by pumping varies with the density of well number and with the variation of the leakage through the semi-confining beds from the aquifer. (3) In cases when the distribution of wells is not uniform, a marked influence occurs upon the area of less density of well number.

1. Introduction

When a newly bored well discharges or is pumped in a place where there exist numerous flowing wells, the discharges of the surrounding flowing bored wells decrease. It is a very important and interesting geophysical subject to study how the influences happen upon the multiple flowing wells basin and by what physical mechanism this phenomenon is caused in an aquifer, so that we have obtained the drawdown distribution of pressure head or the decreased discharges of the surrounding flowing wells caused by pumped well.

Recently the developments of spa resources have been carried out in many places and it appears that the increased amount of the discharge causes the decrease or stoppage of some flowing hot springs in the neighbourhood as well as the fall of spring temperature. Such being the case, the above-mentioned research has an important role for the protection or the development of the hot springs resources.

The spa sources in the old city of Beppu run about 3.2 km from south to north and about 1.4 km from east to west, and there are more than 700 wells bored in the region, most of which are flowing wells. Indeed, there are 30 wells bored per hectare in the densest part.

At the Geophysical Research Station of Kyoto University at Beppu numerous results have been obtained by various investigators and they are reported in the bulletin of "Tikyūbuturi" (Geophysics), but, concerning the effect of the pumping, we have only the data obtained by Dr. K. Habu [1]*.

One of the metods of investiating the mutual interaction of flowing well discharge is the pumping test due originally to Prof. K. Seno. The writer has carried out pumping tests since 1949 at various places in the city of Beppu and reported the results several times [2,3]. In the present paper, the writer gives the explanation and interpretation of some interesting facts obtained by these pumping tests, such as local characteristics in the effect of pumping and the peculiar correlation between the decrease of well discharge and the density of surrounding well number, verifying the results of pumping tests with theoretical consideration.

2. Outline of spa sources in the old city of Beppu

The observations of the existing states of hot springs, namely sites, spring temperatures, well discharges, bored depths and sizes of well pipes etc., were made several times in the past [4,5,6]. The geographic distributions of the observed hot springs are shown in Fig. 1, and Table 1 shows the summarized results.

By a glance at the distribution of hot springs as shown in Fig. 1, it will be seen that there are numerous hot springs extending from the coast toward the center of the city. In the past there were numerous natural flowing hot springs everywhere in this zone, from where the development of hot spring resources started. With the increase of the

Fig. 1. Distribution of spa sources in the old city of Beppu.

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* Square brackets show the references listed at the end of this paper.

Table 1. The statistical results of spa sources in the old city of Beppu

Odservation year & month	Total number of spa sources	Mean bored depth	Mean temp.	Mean discharge	Total discharge
1924 8-9	741	47 m	53.74°C	12.88 lit/min	9.54 m^3/min
1933 7-8	702	69	54.50	16.09	11.33
1949 7-8	653	82	52.35	18.09	11.81
1959 2-3	713	99	52.30	14.09	10.04

number of wells, however, a number of the natural hot springs ceased flowing, and only a few of them still remain nowadays.

The depths of well boring are shallowest in the western Bluff, and also shallow in the dense part of well number in the center of the city, while they become deeper with the approach to the coast apart from this zone. They are condiderably deep down to 300 m in the nortern part. The bored depths were increased year by year, and in 1959 the mean bored depth becomes twice as large as that in 1924. The number of wells of depths less than 30 m was 18.3% of the whole number in 1924, but in dropped to 1.2% in 1959.

The total discharge was least in 1924 probably due to insufficient development. The discharge increases so much when it rains heavily. According to the statistics of observation data for many years [7], the discharge is most abundant in September, the rainy season, and is least in April, this range being about 20% of the mean. Even in consideration of this range, the total discharge has scarcely change since 1933, whereas the number of wells has increased and decreased at times and the mean bored depth has increased by 30 m due to the development of deeper aquifer. Such being the case, it is a noteworthy fact that the total discharge has not so varied in the intervals observed, suggesting that the supply is equal to the flowing-out of the thermal ground water in these aquifers.

3. Methods of test

It is difficult to prospect what effect the start of discharge of a newly bored well will give the discharge of the adjacently bored wells, we can know it by pumping test on a bored well, with the approximate condition of a new boring. On a test of pumping, it is best convenient to research, in the conditions where pumped and obervation wells are bored in the same aquifer. This condition is seldom satisfied in the old city of Beppu, there were so dense number of bored wells that the variation of the discharge rate and of spring temperature have been measured by the two follwoing methods.

1. To overpump on a bored well.
2. To shutdown the discharge of a bored well.

Generally speaking, the minor variation of the discharge is difficult to measure, compared with that of the pressure head. It frequently happens that the protective equipments of the well object the observation, so, it is impossible to observe the pressure heads all the wells surrounding a pumped well. In consequence, we selected observation wells of which discharges are measured accurately and easily.

The more the amount of water is pumped, the lower becomes the pressure head, consequently the decrease of the discharge of surrounding wells becomes so effective that the effects can be measured easy, but the discharge rate of overpumping is restrained by the bore diameters of less than 5 cm of wells, the water temperature, the character of the aquifer, kinds of casing pipes and strength of casing pipes, etc.. On one occasion, a casing pipe was broken down by overpumping, the discharge finally stopped. The discharge by overpimping was actually measured in the range of 20 to 120 lit/min.

A longer period pumping will give more effective result. But the influences of rainfall or barometric pressure etc. superpose on a pumping effect, which makes the analysis much difficult, and moreover because of circumstances of the well owners' utilities or some other reasons, the overpumping intervals were restrained in from 2 to 6 hours. The result showed that the steady state condition is satisfied approximately in above time intervals, as the thermal water is the confined water. (See Fig. 3)

As the influential distance from the pumped well was not yet known, the measurement was carried out on the wells within 100 m distant from the pumped well. Indeed, many tests are ascertained approximately to be adquate.

4. Analysis of pumping tests

The pumping tests were carried out at 21 places as shown in Fig. 1, and some of them show neither effect nor sufficient data to analyse, the causes of which are small discharge of wells or no observation wells near the pumped well, or may be the character itself of the thermal water aquifer.

In the old city of Beppu, there is no well not influenced more or less by the ocean tide, therefore the tidal effect of the respective observation well was observed prior to the pumping test, with which the tidal influence under the pumping test is corrected. These detailed data and analyses were reported in other reports [8,9]. Figs. 2, 3 and 4 show some of the results of the pumping tests. Some discharges of the surrounding wells do not indicate the same amount of the decrease on the same distance from the pumped well, but indicate the distinct characteristic of direction. This is inferred mainly by the irregular distribution of the wells and by the property of the aquifer.

We have numerous data [10] in regarding to the geological logs concerned to boring of wells, but not yet detected the definite geological structures in this basin.

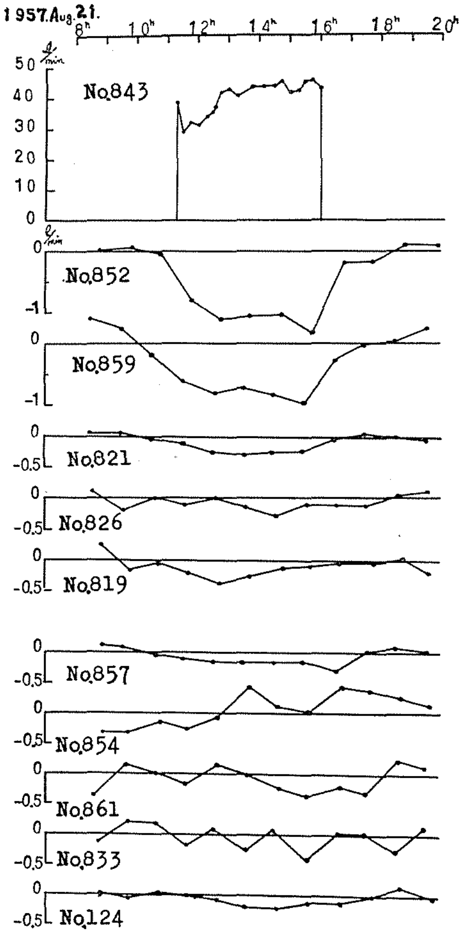

Fig. 2, A. Variation of discharge of wells by Well No. 843 pumping test.

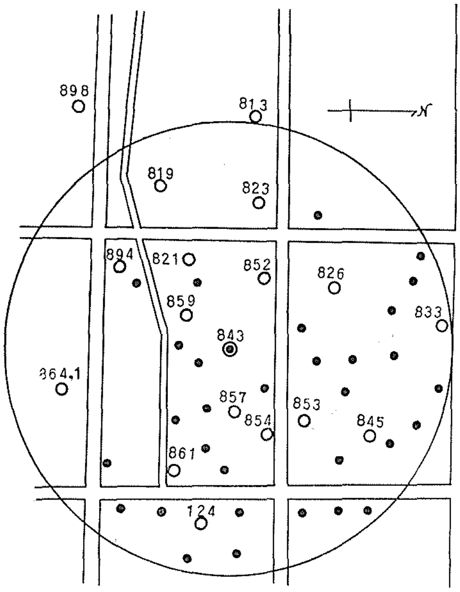

Fig. 2, B. Distribution of wells around the pumped well No. 843.
Numbers are shown as hot spring No.
Radius of cycle is 100 m.

On the whole, the deeper the boring, the higher becomes the pressure head and the temperature of the thermal water. The thermal water aquifers seem not to be completely confined individually by the definite aquifuges but communicated each other.

To be compared with the local characteristics of the pumping effect, or with the effects caused by the different aquifers, the decreases of the discharge are converted into the ones under the standard discharge 50 lit/min of the pumped well and at the standard distance from the pumped well. All observed wells on each pumping test are not always bored at the same distance. The stanadard distance from the pumped well was set as 40 m where is the most determinable distance for the dectease interpolated from the data observed in each test, on a basis of least errors.

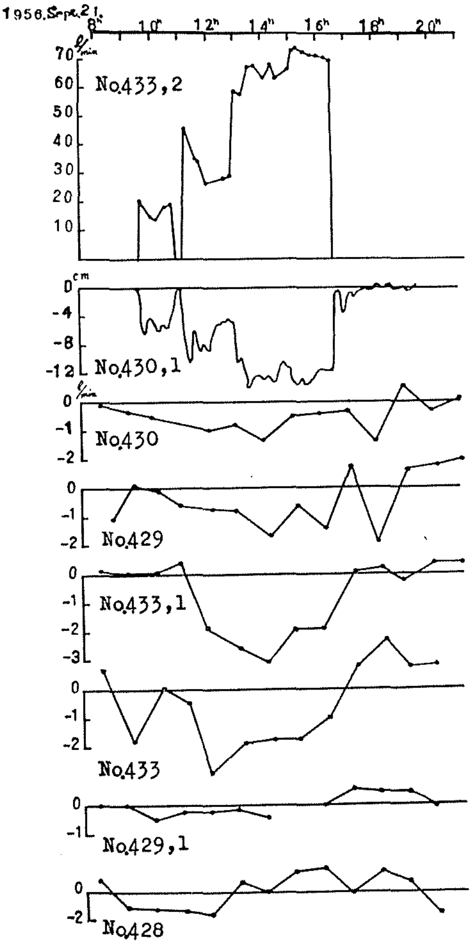

Fig. 3, A. Variation of discharge of wells by Well No. 433-2 pumping test.

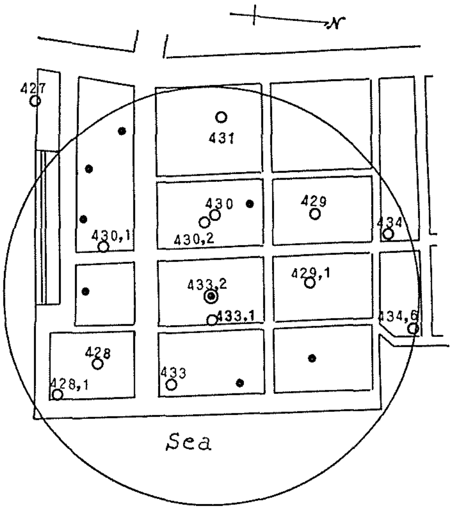

Fig. 3, B. Distribution of wells around the pumped well No. 433-2. Radius of cycle is 100 m.

In camparison with the decrease effected by the standard pumped discharge 50 lit/min of the pumped well, the decreases are much on the regions of low density of well number in the surroundings of a pumped well and, little on the ones of high density of well number. As the distribution of wells is not uniform, the different value will be obtained owing to the area scale, but the result of pumping rests shows that the area of influence for the discharge about 50 lit/min covers nearly 100 m or so from the pumped well. Thus, the well number density per unit area is calculated by the number of wells within 100 m from the pumped well.

The relation between the well number density and the decrease of the discharge of the virtual well at the point 40 m apart from the pumped well, and at the rate of the

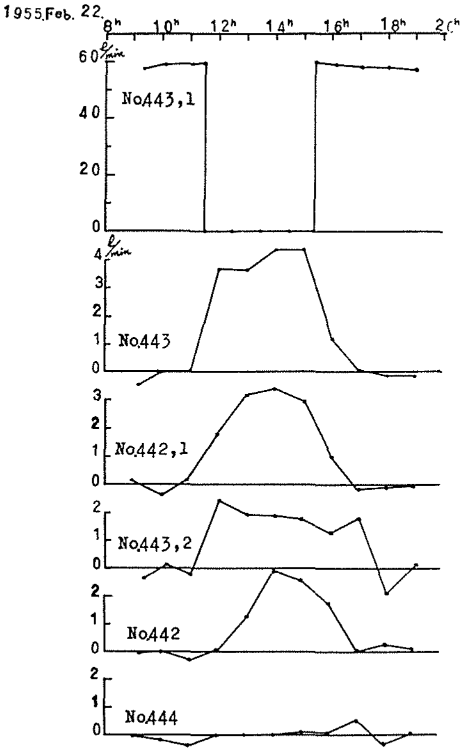

Fig. 4, A. Variation of discharge of wells by Well No. 443-1 shut down test.

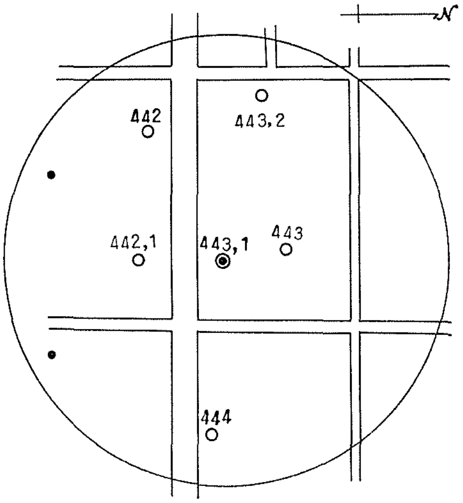

Fig. 4, B. Distribution of wells around the pumed well No. 443-1. Rrdius of cycle is 100 m.

pumped discharge as 50 lit/min is shown in Fig. 5. In the wide basin with multiple flowing wells, the depths of the wells tapped the aquifer are different locally, that is, there is discrepancy of the structure or characteristic of aquifers. Whereas, the result shows that the decrease becomes great at a small density of well number and the effects are reduced with increasing of the density of well number. In this case, it is evident that a close relation should lie between the decrease and the well number density around the pumped well.

5. A theory of pumping test in an aquifer with uniform leakage

In a case where the ground water flows out from an aquifer through numerous wells and leaks out from or leaks into the aquifer through the semi-confining beds, if a new well is bored into this aquifer and pumping begins, the discharges of the bored flowing wells and the leakage around this new well will change owing to the drawdown of the pressure head in this aquifer. The treatment of this problem becomes very com-

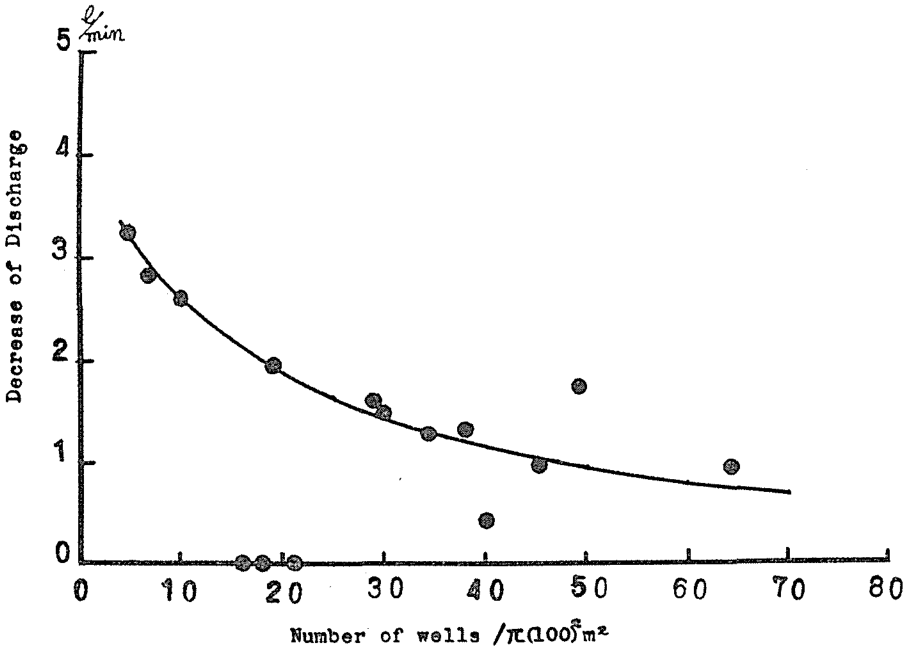

Fig. 5. Relation between the decrerse of discharge and number of wells around the pumped well (for discharge at rete 50 l/min. and 40 m distant from the pumped well).

plicated in analysis to apply the earlier formulas of the pumping test for practical purposes.

Generally, the theoretical consideration of pumping tests is based on Thiem's, Theis' and Nomitsu's theories. It is, however, not adequate to apply them in such a case as in Beppu where there are numerous flowing wells and leakage from an aquifer would be supposed.

Theoretical studies of the pumping tests in the leaky aquifer were developed by Hayami, Kunishi [11] and Hantush, Jacob [12]. The solution of non-steady distribution of drawdown was obtained in a case where the leakage is assumed with a simple relation, as the leakage from any point of an aquifer takes place in proportion to the pressure drawdown.

It must be discussed enough whether or not the discharge of flowing wells can be treated as uniform leakage from any point of an aquifer. But the method that the dotted flowing wells distribution are converted into the continuous leaky one has newly been introduced and the result of the tests mentioned above seems to give a suggestion of this possibility.

Now, it is assumed that many same flowing wells are bored and uniformly distributed in an infinitely wide aquifer with uniform transmissibility. When a well is pumped at a constant discharge from this aquifer and a steady state condition has attained after a long time, the drawdown distribution of the pressure head in the vicinity of this well is expressed as the following differential equation.

$$T\left(\frac{d^2P}{dr^2}+\frac{1}{r}\frac{dP}{dr}\right)-f(N)W(P)=0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where T is the coeficient of transmissibility; P is the drawdown of pressure head; r is the distance from the pumped well; f(N)W(P) is the quantity of leakage from this aquifer through the flowing wells per unit area. f(N)W(P), "quantity of leakage" may chiefly vary with N,N being the number of wells in unit area, and with W(P) the function of P.

T. Nomitsu; K. Seno and Ka. Yamashita [13] found a following relation from the measurments of the discharge and the height of a well mouth in the old city of Beppu.

$$\Delta q = c(P-P_g) \qquad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where Δq is the variation of discharge; P is the variation of the statical pressure head at which the discharge is zero; Pg is the pressure variation accompanied with the variation of the height of well mouth; c is a certain constant, relating to the diameter of the well bore and the formation.
As in Beppu well mouths are fixed at the constant heights, we take Pg=0 in Eq. (2),

then
$$\Delta q = cP \qquad \cdots\cdots\cdots\cdots\cdots\cdots(2')$$

which represents the variation of the discharge of a well taking place in proportion to that of the statical pressure head in an aquifer at the wall of the well. If the pressure drawdown caused by pumping is expressed as s, W(P) in Eq. (1) is represented as follows at first approximation.

$$W(P) = -c\rho gs \qquad \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

where ρ is the density of water; g is the acceleration of gravity.
As the quantity of the leakage f (N) is a parameter, putting cf(N)=b, substitute the above notation in Eq. (1),

$$\frac{d^2s}{dr^2}+\frac{1}{r}\frac{ds}{dr}-\frac{b}{T}s=0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(4)$$

the boundary conditions are

$$r\to\infty \qquad s=0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(5, a)$$

$$r\to 0 \qquad -2\pi rT\frac{ds}{dr}=Q \qquad \cdots\cdots\cdots\cdots\cdots\cdots(5, b)$$

where Q is the constant discharge of the pumped well. Eq. (4) is the modified Bessel's differential equation and its general solution [14] is expressed as follows,

$$s = AI_0\left(\sqrt{\frac{b}{T}}\, r\right) + BK_0\left(\sqrt{\frac{b}{T}}\, r\right) \qquad \cdots\cdots\cdots\cdots(6)$$

where A and B are arbitrary constant; $I_0(x)$ and $K_0(x)$ are respectively the modified zero-order Bessel functions of the first and the second kinds. The boundary condition (5,a) demands A=0, and B is expressed as follows by (5,b).

$$\lim_{r\to 0}\left[-2\pi r T\frac{ds}{dr}\right] = 2\pi TB = Q \qquad B = \frac{Q}{2\pi T}$$

Then the solution of Eq. (4) is given by

$$s = \frac{Q}{2\pi T}K_0\left(\sqrt{\frac{b}{T}}\, r\right) \qquad \cdots\cdots\cdots\cdots(7)$$

On the other hand, Eq. (2) may be understood as expressing the relation between the discharge rate and the pressure drawdown at the wall of a well when the discharge rate of a well changes. With regard to the pressure drawdown at the wall of a well caused by pumping, if $\sqrt{\frac{b}{T}}r_w$ is very small, r_w being the radius of a well, $-2\pi r T\frac{ds}{dr}\Big|_{r=r_w} \fallingdotseq Q$. In the old city of Beppu, $2r_w$ is 0.038~0.050 m and its mean value is 0.045 m, and $\sqrt{\frac{b}{T}}<0.1$ as obtained later. Hence for the practical purposes, substituting the above conditions into Eq. (7),

$$s_w = \frac{Q}{2\pi T}K_0\left(\sqrt{\frac{b}{T}}r_w\right) \qquad \cdots\cdots\cdots\cdots(8)$$

then from Eq. (2) Δq means to equal to Q

$$-\Delta q = cs_w \qquad c = -2\pi T/K_0\left(\sqrt{\frac{b}{T}}r_w\right) \qquad \cdots\cdots\cdots\cdots(9)$$

If Δq_r is the increase of the well discharge at the point of r from the pumped well, from Eqs. (7) and (9) it is given by

$$\Delta q_r = -cs_r = QK_0\left(\sqrt{\frac{b}{T}}\, r\right)\Big/K_0\left(\sqrt{\frac{b}{T}}\, r_w\right) \qquad \cdots\cdots\cdots\cdots(10)$$

If $\sqrt{\frac{b}{T}}$ is known, the decrease of the well discharge around the pumped well at the steady state condition is estimated by Eq. (10).

As $\frac{b}{T}$ is the ratio of b, being called "leakance" (HANTUSH, 1949) to the transmissibility T of the aquifer, the values of $\frac{b}{T}$ are estimated from the decrease of the discharge of a well by Eq. (10) at the distance of 40 m from the pumped well and the standard pumped discharge 50 lit/min. These values are shown in Table 2.

Hantush (1956) depeloped a graphical method by which the field values of $\frac{b}{T}$ can be determined from data obtained from tests in such a type of flow [15]. During the

Table 2. The pumping effect and the leakage factor (Pumping discharge 50 lit/min)

Pumped well	Number of wells within 100 m from the pumped well	The decrease of the discharge at the distance of 40 m from the pumped well (lit/min)	$\frac{b}{T}$ (m^{-2})
433-2	19	1.96	1.01×10^{-3}
434-18	10	2.60	0.69 "
398	18	–	–
1186	16	–	–
19	34	1.30	1.65 "
443-1	7	2.85	0.59 "
536	21	–	–
448-2	5	3.25	0.48 "
331-2	40	0.43	4.10 "
44	38	1.35	1.63 "
52	29	1.63	1.27 "
843	45	1.00	2.12 "
694	30	1.53	1.36 "
769	64	1.00	2.12 "
221-1	49	1.80	1.13 "

period of pumping test of Well No. 433-2 in the old city of Beppu, successive drawdown curve was obtaind at Well No. 430-2, being 37 m distant from the pumped well (Fig. 3). Time and recovery were measured after the pumping shutdown. Fig. 6 is the graph of the recovery against time *t* on the semi-logarithmic paper. If the recovery of the pressure head and the slope of this curve at the inflective point are s_i, m_i respectively, these are given theoretically as follows.

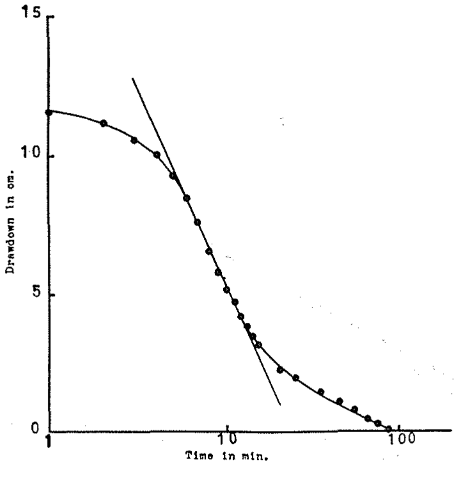

Fig. 6. Time-drawdown curve by No. 433-2 pumping test.

$$s_i = \frac{Q}{4\pi T}K_0\left(\sqrt{\frac{b}{T}}\,r\right) = \frac{1}{2}s_m \qquad \cdots\cdots\cdots\cdots(11)$$

where s_m is the drawdown of pressure head at the steady state caused by pumping.

$$m_i = \frac{2.3Q}{4\pi T}\exp\left(-\sqrt{\frac{b}{T}}\,r\right) \qquad \cdots\cdots\cdots\cdots(12)$$

From Eqs. (11), (12)

$$f\left(\sqrt{\frac{b}{T}}\,r\right) = K_0\left(\sqrt{\frac{b}{T}}\,r\right)\exp\left(\sqrt{\frac{b}{T}}\,r\right) = 2.3\,s_i/m_i \qquad \cdots\cdots\cdots(13)$$

In the case of the pumping test at Well No. 433-2 from Fig. 6, s_i is estimated as 0.059 m. The slope at the inflective point of the curve is measured as 0.135. Then from Eq. (13), $f\left(\sqrt{\frac{b}{T}}r\right)=1.005$. From this value, $\sqrt{\frac{b}{T}}r=1.3$. Well No. 430-2 is 37 m distant from the pumped well. Hence

$$\frac{b}{T} = 1.23\times10^{-3} \quad (m^{-2})$$

the Table 2 shows $\frac{b}{T} = 1.01\times10^{-3} \quad (m^{-2})$

The values obtained from the both methods are approximately equal each other.

The relation between the values of $\frac{b}{T}$ and the density of well number around respective pumped well is shown in Fig. 7. It is evident that the points are on a straight line, therefore, $\frac{b}{T}$ is represented by the linear function of parameter N.

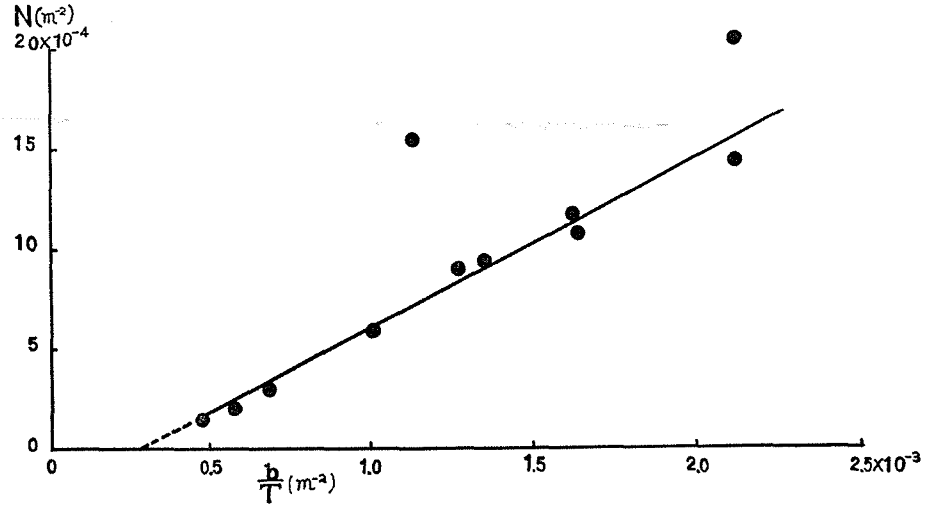

Fig. 7. Relation between leakage factor and number of wells around the pumped well.

The pumped aquifers may have respectively different values of T in locality, the distribution of wells is not uniform and it is not clear whether or not the discharges of all the surrounding wells are flowing out from the same aquifer, but the results show that there is a linear relation between the leakage factor and the density of of well number. This fact indicates that the hypothesis of assuming the dotted well flowing as uniform leakage is appropriate. With some exceptions, the relationship is obtained as follows.

$$\frac{b}{T} = 1.20N + 2.84\times10^{-4} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(14) \qquad \begin{matrix} N(m^{-2}) \\ \frac{b}{T}(m^{-2}) \end{matrix}$$

In the above Eq., $\frac{b}{T}$ is not zero at N=O. This shows that the leakage is due not only to flowing out from the wells, but also to the leakage from the pumped aquifer to or from the adjacent aquifers. The value of $\frac{b}{T}$ at N=0 in Eq. (14) is the characteristic of the leakage among the aquifers, this is called as "the inner leakage factor", and that caused by flowing well as "the outer leakage factor", and both combined as "the leakage factor". Such a decrease of the leakage from the thermal water aquifer to a shallow one or an increase of the leakage from the deeper aquifer caused by the drawdown of the pressure head, may be shown as an example in the case of pumping of Well No. 44 (150 m bored, 55.8 °C of flowing water temperature) in the old city of Beppu. The pumping effect have been represented distinctly both on Well No. 42 (91 m bored, 47.0 °C of flowing water temperature), and Well No. 43 (108 m bored, 46.5 °C of flowing water temperature), these being different from the pumped aquifer. (See Fig. 8)

The increase of well discharge of Well No. 442-1 (39 m distant from the pumping well) caused by the shutdown in Well No. 443-1 (60.25 lit/min of the discharge) was measured as $\Delta q=3.60$ lit/min. It is given by $\Delta q=5.48$ lit/min calculated by Eqs. (10) and (14) from the density of well number. The calculated value is larger than the measured one. If the value of $\frac{b}{T}$ corresponds to the measured one, the inner leakage factor must be given as $3.19\times10^{-4}(\mathrm{m}^{-2})$. Thus the value of the inner leakage factor seems to vary with the localities. The distribution of the inner leakage factor will play an important role to the research of thermal water aquifer.

Thus we derived Eq. (14), an expression between the leakage factor and the density of well number surrounding the pumped well. As Eq. (10) is an expression of the deceases of the flowing well discharge caused by pumping in such a type of flow, combining both, we have

$$\Delta q_r = Q\,\frac{K_0(\sqrt{1.20N+2.84\times10^{-4}}\,r)}{K_0(\sqrt{1.20N+2.84\times10^{-4}}\,r_w)} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(15)$$

where the value of $K_0(x)$ is given by the table of the function.

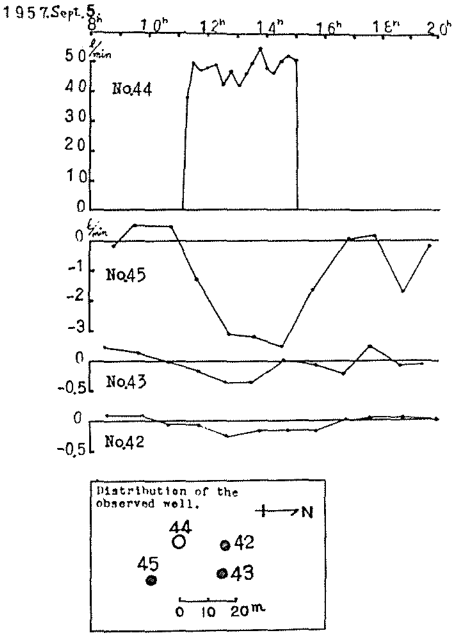

Fig. 8. Variation of discherge of wells by Well No. 44 pumping test.

6. Local characteristics of pumping effect

As mentioned in the previous section, the decrease of the discharge of the surrounding flowing wells caused by pumping can be estimated by Eq. (15) with the density of well number around the pumped well, and concerning to the detailed data of the above, there are some cases where N can not be regarded as the mean density of well number. Moreover, the inner leakage factor is not uniform owing to the local difference of the formation and the random distribution of wells in the surroundings of the pumped well. Therefore, the pumping effects did not indicate the same decrease on the same distance from the pumped well. There are some cases classified as follows where characteristics locally appeared in the results of pumping test.

Table 3. Classification of local characteristics

Type	Characteristics of the pumping effect	Well No. of the pumped well
1	Pumping effects are uniform in any direction from pumped well	443-1, 769, 221-1, 331-1 19, 44, 448
2	The case where the pumping effects have the characteristics by radial direction from pumped well	694, 843, 433-2, 434-18 52

The first type

This is a normal type. In the center region of Beppu, wells are distributed nearly uniform, there is no variation of decrease in any direction. Consequently the characteristics of direction do not appear on the pumping effect. Some irregular dispersion which appeared on the pumping effect may be caused partly by non uniformity of the inner leakage factor and by the facts that the pumped aquifer and the aquifer of the observation well are different each other, etc..

The second type

Case 1. (See Fig. 2)....The pumped wells No. 694 and No. 843 are both nearly situated at the place of the western edge of hot springs group. Those pumping effects remarkably appear in the west side, where there are a few wells, compared with the east side where there is denser well number. The cause of these different pumping effects may be inferred chiefly from the fact of non uniform distribution of flowing wells. Indeed, the leakage factors are different in the east and the west sides. Let the leakage factor be represented by $\frac{b_1}{T}$ in the west side ($x<0$) and that of $x>0$ by $\frac{b_2}{T}$, then the drawdown of the pressure head caused by pumping is considered as follows.

In the case where $\frac{b}{T}$ varies with θ, s is the function of the variables r and θ. Eq. (4) must be as follows.

$$\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}+\frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2}-\frac{b}{T}(\theta)s=0 \qquad \cdots\cdots\cdots\cdots(16)$$

As the above mentioned, $\frac{b}{T}(\theta)$ largely varies near the boundary line ($x=0$), but on the remnant area, any variation is seldom. Therefore, the variation s for θ is considered to be minute except in the vicinity of the boundary line also. Regarding the drawdown of the pressure head s, in both domains as $\frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2}\fallingdotseq 0$ of Eq. (16), it becomes the same form as Eq. (4) and s is approximately represented as follows.

$$s\cong\frac{Q}{2\pi T}K_0\left(\sqrt{\frac{b}{T}(\theta)}r\right)$$

Accordingly, the drawdown of the pressure head on the east and the west is given by

$$x<0 \qquad s_1\cong\frac{Q}{2\pi T}K_0\left(\sqrt{\frac{b_1}{T}}\,r\right) \qquad \cdots\cdots\cdots\cdots(17)$$

$$x>0 \qquad s_2=\frac{Q}{2\pi T}K_0\left(\sqrt{\frac{b_2}{T}}\,r\right) \qquad \cdots\cdots\cdots\cdots(18)$$

Applying Eqs. (17) and (18) to two cases of the pumping test mentioned above and calculating the pumping effects, we obtain as shown in Tables 4, 5 and in Figs. 9 and 10.

Of these two cases, the observed values in the western part ($x<0$) approximately equal to the calulated ones, but the decreased amounts of the observation wells in the eastern ($x>0$) are measured less than the calculated ones. This disagreement, as

Table 4. Well No. 694 Pumping test (Discharge 50 lit/min)

Western part from the pumped well ($x<0$)

$$\frac{b_1}{T}=1.0\times10^{-3}\ (\mathrm{m}^{-2})$$

Well No. of observation wells	Distance from the pumped well (m)	Δq_r (lit/min)	
		Observed value	Calculated value
635-1	49	1.10	1.53
659	60	0.97	0.87
690	84	0.33	0.36
689	72	0	0.56
692	102	0.46	0.19
685	102	0	0.19

Eastern part from the pumped well ($x>0$)

$$\frac{b_2}{T}=1.88\times10^{-3}\ (\mathrm{m}^{-2})$$

Well No. of observation well	Distance from the pumped well (m)	Δq_r (lit/min)	
		Observed value	Calculated value
717	30	1.93	2.00
709	40	0.49	0.98
791	60	0.16	0.40
740	100	0.33	0.06
715	95	0	0.07
744	82	0	0.13

Table 5. Well No. 843 Pumping test (Discharge 50 lit/min)

Western part from the pumped well ($x<0$)

$$\frac{b_1}{T}=2.30\times10^{-3}\ (\mathrm{m}^{-2})$$

Well No. of observation wells	Distance from the pumped well (m)	Δq_r (lit/min)	
		Observed value	Calculated value
859	25	1.45	2.28
852	35	1.31	1.20
821	45	0.28	0.67
826	55	0.19	0.38
823	66	0	0.21
819	82	0.25	0.11
813	105	0	0.03

Eastern part from the pumped well ($x>0$)

$$\frac{b_2}{T}=3.60\times10^{-3}\ (\mathrm{m}^{-2})$$

Well No. of observation wells	Distance from the pumped well (m)	Δq_r (lit/min)	
		observed value	Calculated value
857	27	0.17	1.40
854	40	0	0.48
853	46	0	0.33
861	60	0	0.13
124	85	0.14	0.02
845	74	0	0.05

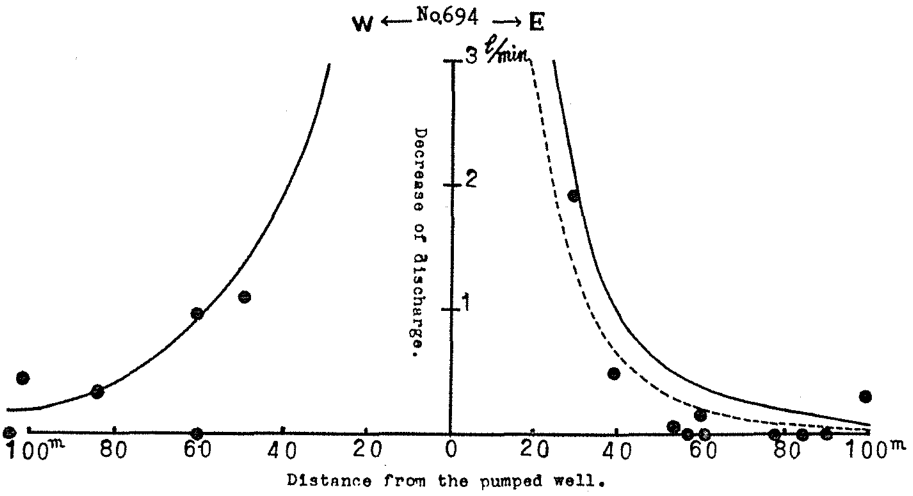

Fig. 9. Relation between the decrease of discharge and distance from the pumped well.

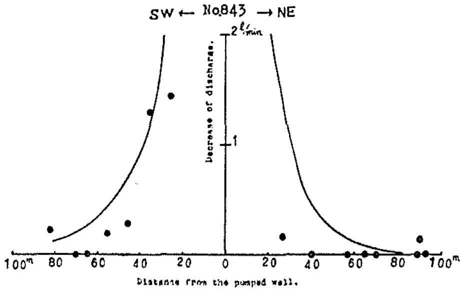

Fig. 10. Relation between the decrease of discharge and distance from the pumped well.

mentioned in the previous section, is caused by the differences of the inner leakage factor in the eastern part. For trial, in the case of $x>0$ pumping on Well No. 694, assumed the inner leakage factor as 1.4×10^{-3} (m^{-2}), the values are shown the dotted line in Fig. 9 and the calculated values come closer to the observed ones.

Case 2. (See Fig. 3)....On the case of the pumping tests of well No. 433-2 and No. 434-18, the effects remarkably appeared to the direction of the northeastern part. Both pumped wells are in the region adjacent to the sea coast of this basin. This region is the reclaimed ground and there are a few wells. In the western part of this region, the thermal water temperature is high and the individual well discharge is aboundant. The eastern part from the coast is the sea and no well. As can be seen in table 6 and Fig. 3, it is evident that both the thermal water temperature become lower and the discharge smaller with the approach to the northeastern part, which show the pressure head being at the low level [16].

Table 6. Discharge and temperature of the wells surrounding pumped well No. 433-2

Southwestern part from the pumped well			Northeastern part from the pumped well		
Well No.	Temp.	Discharge	Well No.	Temp.	Discharge
427	61.5°C	20.8 lit/min	433-1	59.5°C	11.6 lit/min
430-1	61.0	48.0	433	56.5	15.7
430	61.0	51.2	429-1	54.5	11.0
429	61.5	49.7	428-1	60.0	24.0
431	57.5	26.2	428	53.0	4.0
			432	55.0	24.0
			434-13	42.0	–

Accordingly, the characters of pumping effect in this case also are explained by the same influences as the case 1. However, the fact that the distribution of the pressure heads, or the distribution of the discharges has the character of direction, suggests the inner leakage factor being not uniform in this region, with the scanty data up to the present it is difficult to duscuss the character of the pumping effect.

7. Conclusion

The pumping tests at various places in the old city of Beppu Spa have shown that the pumping effects in multiple-flowing-wells system have close relation with the density of the well number around the pumped well, and this correlation can be treated as if the discharge of the flowing wells leaking uniformly through semi-permeable beds from an aquifer. By means of this treatment we could derive the relation between the leakage factor and the density of well number surrounding the pumped well.

1. The decrease of the flowing well discharge surrounding the pumped well is ex-

pressed by the modified zero-order Bessel function of the second kind in which the variable is $\sqrt{\frac{b}{T}}r$, $\frac{b}{T}$ being the leakage factor and r being the distance from the pumped well.

2. The relation between the leakage factor and the density of well number surrounding the pumped well is expressed as follows.

$$\frac{b}{T} = 1.20N + 2.84 \times 10^{-4} \qquad \text{N: The density of wells surrounding the pumped well } (m^{-2})$$

The first term in the right side of the above equation is the outer leakage factor, and the second is the inner leakage factor.

3. The effect caused by pumping upon the surrounding wells varies owing to the outer and the inner leakage factor, but in the case of the irregular distribution of wells with direction, a marked influence appears upon the area of less density of well number.

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