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NONLINEAR PLASMA OSCILLATIONS NEAR ELECTRON CYCLOTRON HARMONICS

BY

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ABSTRACT

The second harmonic emission from plasma is studied when the plasma is irradiated by an exciting microwave signal. We have observed three kinds of resonant structures near electron cyclotron harmonics, which are concluded to result from inherent nonlinear properties of the plasma.

A series of resonant peaks is observed at magnetic fields higher than those of the cyclotron harmonics for the exciting microwave signal and is interpreted as the second harmonics of the radial standing plasma waves confined to a region near the axis of the plasma column in the magnetic field.

The other two kinds of resonant structures are observed at magnetic fields lower than those of the cyclotron harmonics. We propose one possible explanation of the origin of these structures.

§ 1. Introduction

In the absence of static magnetic field, nonlinear properties of plasma oscillations were theoretically studied to explain the abnormally intensive emission of the second harmonic component of solar radio outburst[1,2]. It was shown that plasma oscillations are very rich in harmonics and that the ratio of the intensity of the harmonic component to that of the fundamental increases with the amplitude of the fundamental oscillation. In a laboratory plasma, it has been reported recently[3] that second harmonics of the electron plasma oscillations in a positive column which were found as the Tonks-Dattner resonances were observed and explained as a phenomenon due to inherent nonlinear properties of plasma oscillations.

In the presence of static magnetic field, it has been shown theoretically[4] that longitudinal plasma waves can propagate without damping exactly at the right angle to the magnetic field within narrow pass-bands near electron cyclotron harmonics in an infinite collisionless plasma (Bernstein-mode). It has been reported[5-8] that resonant peaks near cyclotron harmonics are observed in a spectrum of microwave emission and absorption by a plasma column in the presence of axial magnetic field and they are explained as a manifestation of such plasma waves.

In a plasma column of finite radius, whose electron density decreases from the axis towards the walls, such plasma waves are confined to a region near the axis of the plasma column, since the existence of these waves is prohibited in the region of $f^2 \geq f_p^2(r) + f_b^2$, where f and f_b are the frequency of the wave and the electron cyclotron frequency respectively and $f_p(r)$ is the local plasma frequency at the radial position of r in the plasma column. Then, the longitudinal plasma waves form standing waves and appear as a series of resonant peaks near electron cyclotron harmonics in the emission and the absorption spectra[9-12]. It was also shown[13] that such oscillations can be excited strongly when the plasma column is irradiated by a transverse microwave field.

Up to the presnt, however, there are no reports on the nonlinear effects of the plasma connected with the longitudinal plasma waves in magnetic field. We have studied one of the nonlinear properties in the presence of static magnetic field, that is, the generation of second harmonics of the standing plasma waves excited strongly by an exciting microwave. A preliminary work was reported[14] previously and the detailed studies will be given in this paper.

§ 2. Experimental Apparatus and Procedures

The plasma under study is produced by a D. C. discharge of neon gas. The discharge tube is made of quartz tube (12.6 mm in inner diameter) and has a conical hollow anode (10 mm in diameter) and an indirectly heated oxide cathode (10 mm in diameter). The distance between the anode and the cathode is 75 mm. The discharge tube is inserted across a standard S-band waveguide (WRJ-4) with the tube axis normal both to the direction of propagation of the wave and to the electric field vector in the waveguide. A static magnetic field is generated by an air-core coil magnet and applied in parallel with the tube axis. The inhomogeneity of the field on the axis is within 5 percents over the plasma column in the waveguide. These arrangements are shown in Fig. 1.

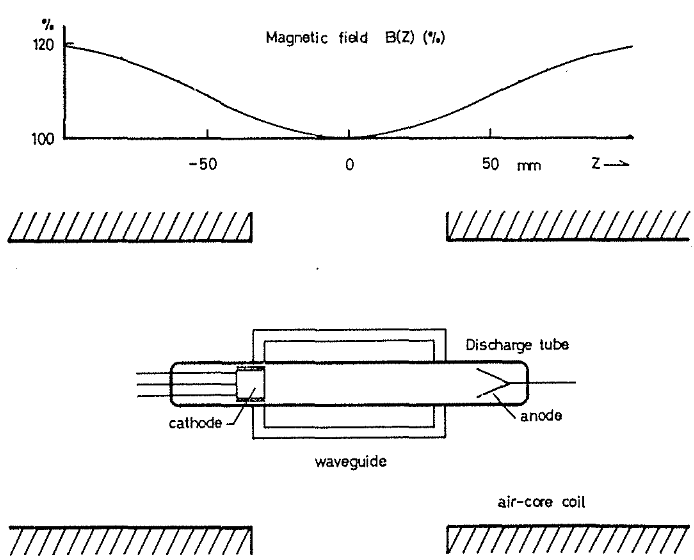

Fig. 1. Discharge tube and its arrangement.

Measurements on the spectra of microwave emission and absorption of the plasma column are made by keeping the measuring frequency at a fixed value and varying the strength of static magnetic field with the discharge current I_a as a parameter. These spectra are plotted by an XY-recorder, where the X- and the Y-axis are proportional to the strength of magnetic field and the intensity of the received microwave power respectively. The block diagram of our microwave circuit is shown in Fig. 2.

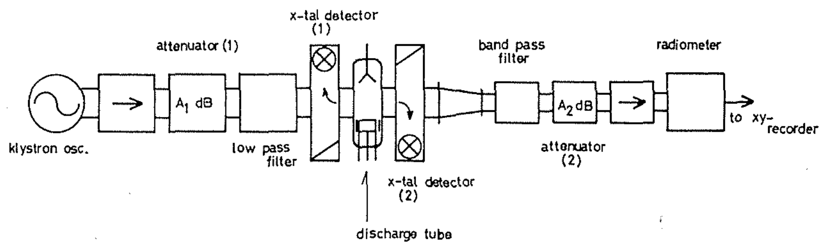

Fig. 2. Microwave circuit.

The exciting microwave is generated by a klystron oscillator (2K54A), and is fed to the plasma, its frequency being kept at $f_1=4{,}100$ MHz. The higher harmonics contained in this exciting microwave itself are removed by a low pass filter with the cutoff frequency of $f_c=4{,}100$ MHz. The rejection characteristics of the filter is greater than 50 dB attenuation from $1.25 f_c$ to $3 f_c$ and the insertion loss at 4,100 MHz is about 2 dB. The maximum power available to be fed to the plasma is about 250 mW. The power P_1 of the exciting microwave applied to the plasma is adjusted by the microwave attenuator (1) whose attenuation is denoted by A_1 dB. The powers of the exciting microwave reflected by and transmitted through the plasma are detected by the crystal detectors (1) and (2) respectively.

Microwave emission from the plasma is received by an X-band radiometer operated at a fixed frequency of $f_2=8{,}200$ MHz with a 5 MHz bandwidth. Here, a band pass filter with the center frequency of 8,200 MHz is used to remove the image frequency of heterodyne detection. The received power is adjusted by the microwave attenuator (2), whose attenuation is denoted by A_2 dB.

When measurements on the second harmonics of the plasma oscillations are made, the crystal detectors (1) and (2) are replaced by the matched loads, since they generate higher harmonics of the exciting signal and the required signals from the plasma are masked. These spurious harmonics contained in the exciting signal itself and generated in the microwave circuit are checked up in the absence of the plasma, and it is confirmed that the intensities of these spurious harmonics are negligibly small compared with the intensities of the harmonics of the plasma oscillations under study.

Measurements on the frequency dependence of the second harmonics of the plasma oscillations on the exciting microwave signal are made by keeping the frequency of the exciting signal at $f_1=4{,}100$ MHz and varying the receiving frequency f_2.

The power dependence of the second harmonic emissions on the exciting signal is measured as follows: The power P_1 of the exciting signal is decreased by the microwave attenuator (1), then, the microwave attenuator (2) is so adjusted that the detected power level, P_2, of the second harmonic emission is recovered. When the insertion losses of the attenuators (1) and (2) are A_1 and

A_2 dB respectively, it is clear that if $A_1=-A_2$, then P_2 is proportional to P_1, and if $2A_1=-A_2$, then P_2 is proportional to P_1^2.

§ 3. Experimental Results

In Fig. 3, curves (1) and (2) show the reflection and the transmissiom spectra for the exciting microwave signal of $f_1=4{,}100$ MHz as functions of magnetic field respectively. Here, the abscissa represents the strength of magnetic field normalized to f_1, i. e., f_b/f_1, as shown on the top. The ordinate represents the intensity of the reflected signal for the curve (1) and the magnitude of attenuation of the transmitted signal for the curve (2). The reflection and the transmission spectra show the resonant structures, which consist of many peaks, a, b, ... and A, B, ..., near $f_b/f_1=1/2$, 1/3, ... as functions of the magnetic field.

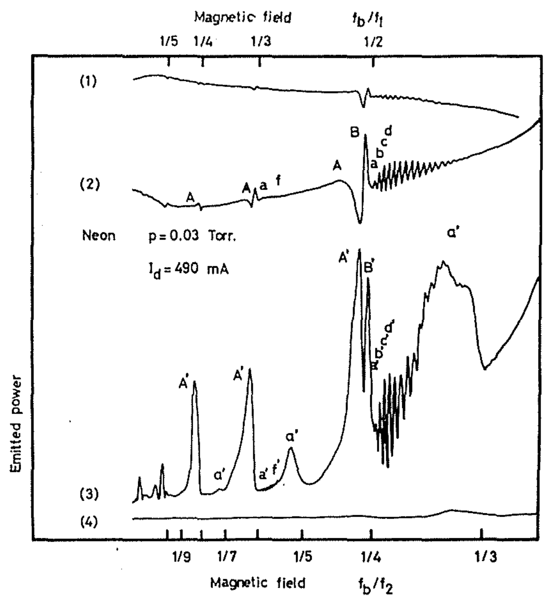

Fig. 3. Spectra as a function of a magnetic field.
(1) Reflection spectrum, $f_1=4{,}100$ MHz, $A_1=0$ dB.
(2) Transmission spectrum, $f_1=4{,}100$ MHz, $A_1=0$ dB.
(3) Second harmonic spectrum, $f_2=8{,}200$ MHz, $A_2=0$ dB. For the exciting signal, $f_1=4{,}100$ MHz, $A_1=0$ dB.
(4) Emission spectrum, $f_2=8{,}200$ MHz, $A_2=0$ dB.

The curve (3) shows the emission spectrum of the plasma for the receiving frequency of $f_2=8{,}200$ MHz in the presence of the exciting microwave of the frequency of $f_1=4{,}100$ MHz, where f_1 is set as $2f_1=f_2$. Hereafter, such a spectrum is called the second harmonic spectrum. The curve (4) shows the emission spectrum for the receiving frrquency of f_2 in the absence of the exciting microwave. Here, the abscissa represents the strength of magnetic field normalized to f_2, i. e., f_b/f_2, which is shown on the bottom of the figure and the ordinate the power emitted from the plasma. Many resonant peaks, denoted by a′, b′, ..., A′, B′, . . . and α', are observed near electron cyclotron harmonics, $f_b/f_2=1/3$, 1/4, . . . , in the curve (3), while a very faint peak is distinguishable only near $f_b/f_2=1/3$ in the curve (4).

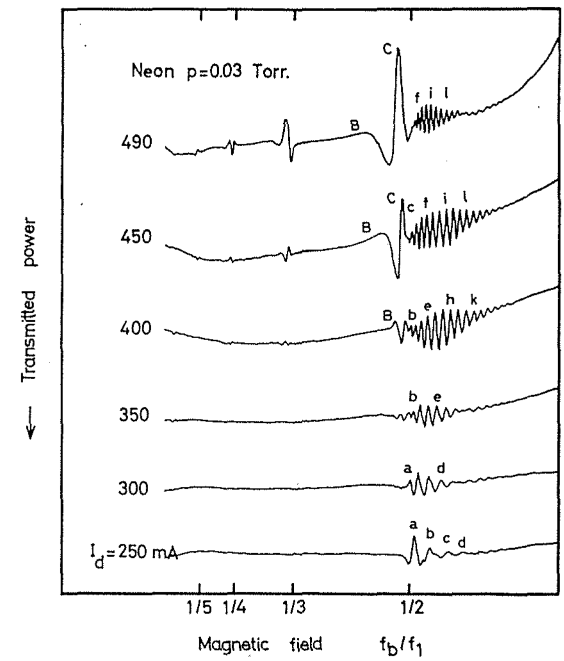

Fig. 4. Absorption spectra for the exciting microwave, $f_1=4{,}100$ MHz, $A_1=0$ dB.

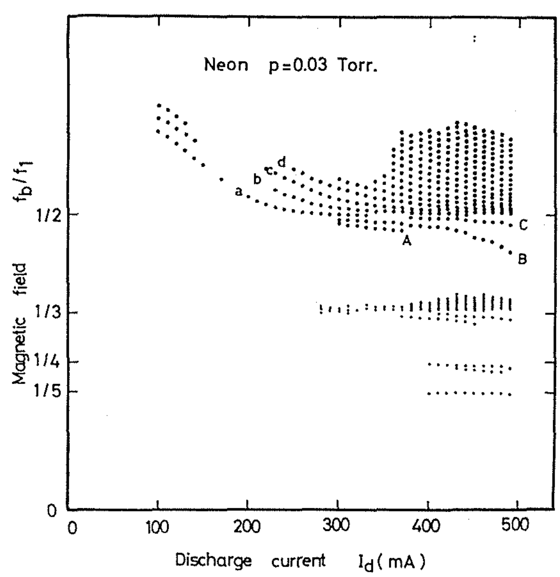

Fig. 5. f_b/f_1 vs. I_d diagram, which is plotted from the transmission spectra shown in Fig. 4.

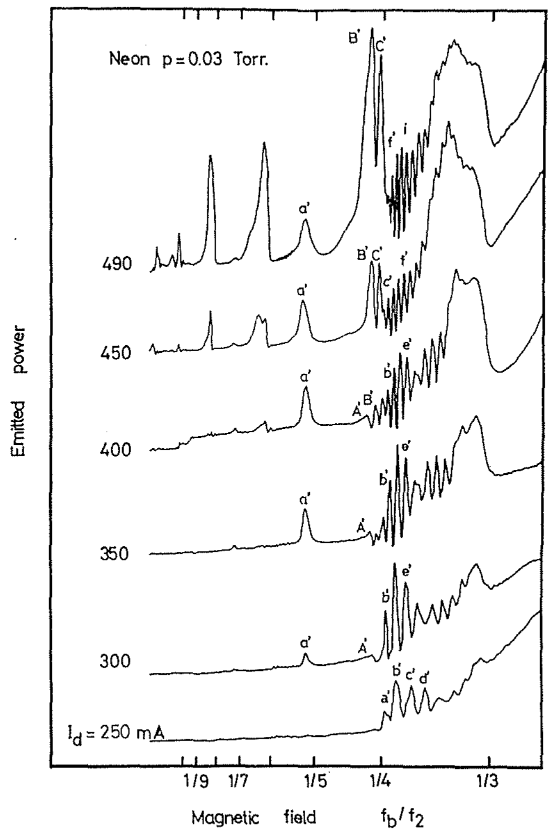

Fig. 6. Second harmonic spectra, f_2=8,200 MHz, A_2=0 dB. Here, f_1=4,100 MHz, A_1=0 dB for the exciting signal.

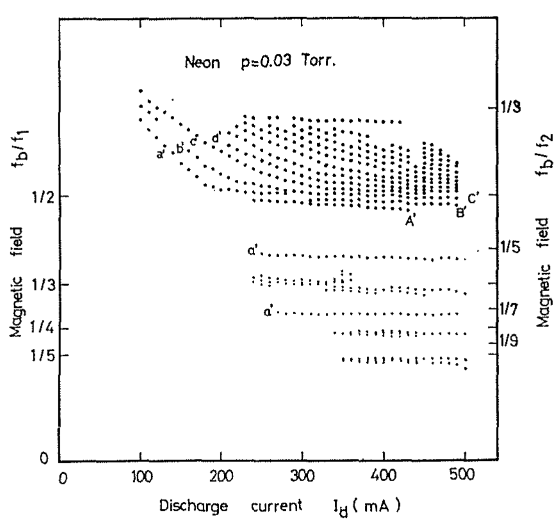

Fig. 7. f_b/f vs. I_d diagram, which is plotted from the second harmonic spectra shown in Fig. 6.

In order to find the dispersion relations for these peaks, measurements on the transmission and the second harmonic spectra are made with the discharge current I_d as a parameter, and the strengths of the magnetic field, at which these peaks are observed, are plotted versus I_d. In Fig. 4 are shown the transmission spectra with the discharge current I_d as a parameter, and in Fig. 5 are plotted the dispersion relations for the peaks, a, b, . . . and A, B, . . . , on the f_b/f_1 vs. I_d diagram. In Fig. 6 are shown the second harmonic spectra, and the dispersion relations for the peaks, a′, b′, . . . , A′, B′, . . . and α', are plotted on the f_b/f vs. I_d diagram as shown in Fig. 7.

Comparing the curve (3) with the curve (2) in Fig. 3 and the dispersion curves in Fig. 7 with those in Fig. 5, it is found that the peaks in the second harmonic spectrum are divided into three groups.

(i) Series of peaks a′, b′, . . .

It is seen in Fig. 3 that the series of resonant peaks, a′, b′, . . . , in the curve (3), are observed at the magnetic fields higher than $f_b/f_2=1/4$, $1/6, \ldots$, but not observed at the magnetic fields higher than $f_b/f_2=1/3$, $1/5, \ldots$. It is clear that the resonant peaks, a′, b′, . . . , are observed only at the magnetic fields where the resonant peaks, a, b, . . . , are observed in the curve (2). The dispersion curves for the peaks, a′, b′, . . . , in Fig. 7, are the same as those for the peaks, a, b, . . . in Fig. 5.

(ii) Series of peaks, A′, B′, . . .

The resonant peaks, A′, B′, . . . , appear at the magnetic fields lower than $f_b/f_2=1/4$, $1/6, \ldots$ as shown in the curve (3) of Fig. 3. They seem to appear near the magnetic fields at which the resonant peaks, A, B, . . . are observed in the transmission spectrum (2), but their correspondence is not clear. It is seen in Figs. 4 and 6 that the intensities of the peaks, A, B, . . . in the transmission spectra, increase with I_d and those of the peaks, A′, B′, . . . , also increase with I_d. The dispersion curves for the peaks, A′, B′, . . . , seem to be similar to those for the peaks, A, B, . . .

(iii) Peak, α'

The resonant peak, α', appears at the magnetic field lower than $f_b/f_2=1/3$, $1/5, \ldots$ as shown in the curve (3) of Fig. 3. It is noted that in the transmis-

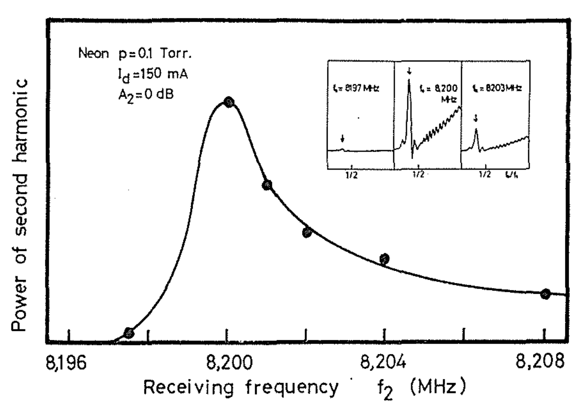

Fig. 8. Intensity distribution of second harmonic emission at $f_b/f_1=0.48$. For the exciting signal, $f_1=4{,}100$ MHz, $A_1=0$ dB.

sion spectrum (the curve (2)) there are no peaks corresponding to the peak, α', in the second harmonic spectrum. It is seen in Fig. 6 that the intensity of this peak does not change so much as those of the peaks, A′, B′, . . . , with the increasing I_a. From the dispersion curve shown in Fig. 7, it is found that the peak, α', shifts more slowly to the small value of f_b/f_2 than the peaks, A′, B′, . . . , with the increasing I_a and that the peak, α', seem to be observed when the electron density becomes so large that the condition, $f_p^2+f_b^2>f_1^2$, is fulfilled.

(iv) Frequency dependence

In Fig. 8 is shown the intensity distribution of the peak marked by the arrow as a function of receiving frequency f_2. Here, the half width of this curve is smaller than 5 MHz which is the bandwidth of our radiometer. It is found from this figure that all peaks in the second harmonic spectrum disappear if the receivig frequency is apart more than 10 MHz from two times the frequency of the exciting microwave, that is, $|f_2-2f_1|\geq 10$ MHz.

(v) Power dependence

In Fig. 9 (a) are shown the second harmonic spectra as a function of magnetic field. Here, the intensities of the second harmonics are the same in both curves, where $A_1=A_2=0$ dB for the curve (1) and $A_1=3$, $A_2=-6$ dB for the curve (2). In Fig. 9 (b) are plotted the relation of $-A_1$ vs. A_2 graphically for the peak at $f_b/f_2=0.29$, which is marked by the arrow in the curves (1) and (2). The vertical and the horizontal bars represent the magnitudes of errors in measuring the attenuation loss. The experimental plotts show that $-A_2=(1.8\sim1.9)A_1$ which means that the power P_2 of the second harmonic is nearly proportional to the square of the power of the exciting signal, P_1^2. Compaaring the intensity of the resonant peaks in the curve (2) with that in the curve (1) of Fig. 9 (a), it is found that the powers of all resonant peaks, a′, b′, . . . , A′, B′, . . . and α', are proportional to the square of the power of the exciting signal.

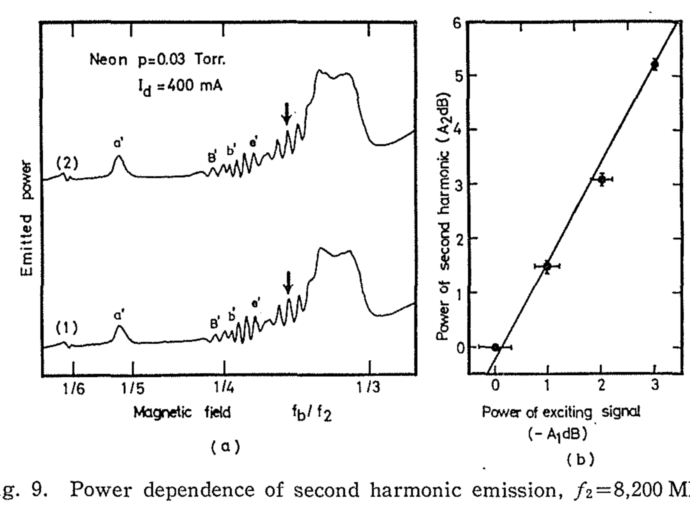

Fig. 9. Power dependence of second harmonic emission, $f_2=8{,}200$ MHz. For the exciting signal, $f_1=4{,}100$ MHz.

(a) Second harmonic spectra
Curve (1): $A_1=A_2=0$ dB, Curve (2): $A_1=3$ dB, $A_2=-6$ dB.

(b) $-A_1$ vs. A_2 diagram at $f_b/f_2=0.29$

The power of the peak marked by the arrow in the curve (1) is about two times the power radiated from a noise standard of temperature of about 1.1×10^4 °K ($\sim 10^{-12}$ W/5 MHz). On the other hand, the power absorbed by the plasma is estimated to be about 16 percent (the maximum absorbed power$\sim$40 mW) of the exciting microwave power, at the magnetic field $f_b/f_2=0.29$.

§ 4. Discussions

The experimental results described in § 3 are summarized as follows:

(I) When the plasma is irradiated by the microwave, many resonant peaks are observed to be excited in the second harmonic spectrum, though there is a very faint peak only near $f_b/f_2=1/3$ in the absence of the exciting microwave.

(II) All resonant peaks, a′, b′, . . . , A′, B′, . . . and α' in the second harmonic spectrum, disappear if $|f_2-2f_1|\geq 10$ MHz.

(III) The power of the second harmonic, P_2 is nearly proportional to the square of the power of the exciting signal, P_1^2.

(IV) The resonant peaks, a′, b′, . . . in the second harmonic spectrum, are observed respectively only at the magnetic fields where the resonant peaks, a, b, . . . , are observed in the transmission spectrum. The dispersion relations for the peaks, a′, b′, . . . , are the same as those for the peaks, a, b, ...

(V) The resonant peaks, A′, B′, . . . in the second harmonic spectrum appear respectively near the magnetic fields at which the resonant peaks, A, B, . . . , are observed in the transmission spectrum, but their correspondence is not clear. The dispersion relations for the peaks, A′, B′, . . . , seem to be similar to those for the peaks, A, B, ... The intensities of the peaks, A, B, . . . and A′, B′, . . . increase with I_a.

(VI) The resonant peak, α' in the second harmonic spectrum, appears at the magnetic fields lower than $f_b/f_2=1/3$, 1/5, . . . and there is no corresponding peak in the transmission spectrum.

It is considered that there are a number of nonlinear processes in plasmas, leading to the harmonic generation. The experimental results summarized in (I), (II) and (III) suggest that the resonant peaks in the second harmonic spectrum result from a nonlinear effect associated with the waves excited in the plasma by the microwave.

We first discuss the resonant peaks, a′, b′, ... The corresponding peaks, a, b, . . . in the absorption spectrum, have been studied in detail by several authors[8–13]. It has been shown[10] that the origin of the peaks, a, b, . . . is due to the radial standing plasma oscillations excited by a transverse *rf* electric field which is externally applied to the plasma column. Moreover, it has been reported[13] that the power of the microwawe emission ascribed to such a plasma oscillation is almost linearly proportional to the power absorbed by the plasma up to about 50 mW and that the maximum power of the emission is about 30 dB greater than the power emitted from the plasma which is not irradiated by the microwave.

The experimental results obtained in § 3 show that the peaks, a′, b′, . . . , are observed only at the magnetic fields higher than $f_b/f_2=1/4$, 1/6, . . . where the peaks, a, b, . . . are observed in the absorption spectrum as shown in Fig. 3. The peaks, a′, b′, . . . , correspond to the peaks a, b, . . . , respectively. These results suggest that the peaks, a′, b′, . . . , result from the second harmonics of the radial standing plasma oscillations excited by the microwave signal of f_1. The power dependence of $P_2 \propto P_1^2$ is interpreted as follows: the field intensity of the second harmonic of such a plasma oscillation is proportionl to the square of the field intensity of this oscillation, since its field intensity is linearly proportional to that of the exciting microwave.[13] Therefore, we can deduce that the peaks, a′, b′, . . . in the second harmonic spectrum, result from the inherent nonlinear property of the plasma oscillations.

Next, we discuss the peaks, A′, B′, ... It is known experimentally[10,13] that the peaks, A, B, . . . , are excited strongly by the microwave, though the origin of those peaks is not yet understood. From the experimental results summarized as (II), (III), and (V), the peaks, A′, B′, . . . , seem to be ascribed to the second harmonics due to nonlinear properties of the waves associated with the peaks, A, B, . . . , excited by the exciting microwave.

Lastly, we will consider the origin of the peak, α'. In the propagation of electromagnetic waves in plasmas, nonlinear effects must be taken into account, if the field intensity E_o is so large that the condition, $E_o \ll E_p$, is not fulfilled. Here, E_p is a so-called "plasma field" defined as the field under which an increment of the electron velocity amounts to the thermal velocity[15]. In our experimental conditions, where the electron temperature $T_e \sim 3\times10^4$ °K, the frequency $f_1=4,100$ MHz and the gas used is neon, the plasma field is estimated to be $E_p \sim$ 10 V/cm, while the maximum microwave field is about 5 V/cm. Therefore, it can be expected that the second harmonic is generated by a nonresonant process in a propagation of the exciting microwave in the plasma.

The peak, α', has no corresponding one in the absorption spectrum as seen in Fig. 3. However, in the emission spectrum (curve (4)) of the plasma which is not irradiated by the microwave, a very faint peak is distinguishable near the magnetic field where the peak, α', is observed in the curve (3). On the other hand, it can be expected that the resonant peaks, α, β, . . . , similar to A, B, . . . appear in the absorption spectrum of f_2 at the magnetic fields lower than $f_b/f_2=1/n$ (where $n=2, 3, \ldots$), if the exciting microwave of f_2 is applied to the plasma. Thus, we think that the peak, α', in the curve (3) corresponds to the peak, α, which may be observed in the absorption spectrum of f_2. The experimental results (II) and (III) suggest that the peak, α', results from the second harmonic due to nonlinear process in a propagation of the exciting microwave. So, we think that a resonant interaction of such a second harmonic wave with the plasma excites the waves in the plasma, which result in the appearance of the peak, α'.

§ 5. Conclusion

Measurements on the second harmonic emission from the plasma are made, when the plasma is irradiated by the microwave. The experimental results show that there are three kinds of the resonant structures near the electron cyclotron harmonics, $f_b/f_2=1/n$ ($n=2, 3, \ldots$). The resonant peaks, a′, b′, . . . , are observed only at the magnetic fields higher than $f_b/f_1=1/n$, and explained as the second harmonics resulting from an inherent nonlinear property of the radial standing plasma oscillations in a magnetic field.

For the origin of the other two series of peaks, A′, B′, . . . and α', one possible explanation is given. The peaks, A′, B′, . . . , are ascribed to the nonlinearity of large amplitnde oscillations excited in the plasma. The peak, α', is due to the nonresonant process in which the exciting microwave produces the second harmonic which is a resonant wave within the plasma. Further studies are required to fully understand these two series of peak A′, B′, . . . and α'.

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