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PRESENT STATUS OF THE PHENOMENOLOGICAL REGGE POLE THEORY. II

By

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ABSTRACT

In this paper we review most recent developments in various fields of phenomenological Regge pole theory of high energy scatterings.

Contents

§1 Introduction

This paper, second in a series devoted to Regge phenomenology, concerns mainly with theoretical topics which have been now calling our attention in relation to Regge pole phenomenology.

In the first issue of this work, entitled "Present Status of the Phenomenological Regge Pole Theory",[1] we have presented the close connection between the characteristic properties of Regge pole model and the observed features of high energy reactions. Further we have made in the last section in I several comments in concluding the systematic and critical review. Let us survey the recent developments in the phenomenological Regge pole model, referring to those comments.

(1) As for the spin dependence,[2] no prominent results have been obtained since then. Experimentally, by use of the polarized targets, the measurements of the polarization parameters of the recoil nucleons in πp processes are reported by Argonne group[3] in the momentum range 3.2~5.2 GeV/c, lower than CERN group (5.6 GeV/c and 11.2 GeV/c).

The former group analysed these data on the basis of the interference model,[4] which we have already presented in §5.3 of I (Regge pole exchanges + direct channel resonances).

The non-vanishing value of the density matrix element ρ_{00} of ω mesons produced by πN collisions seems to be obtained by adding other poles or absorption corrections.

On the whole the trouble of the spin dependence is avoided by introducing the additional non-Regge effects which come from such as s-channel resonances, absorption corrections or Regge cuts. It will be an interesting problem to see how the Regge pole model can cooperate with these non-Regge effects and how these non-Regge effects are interrelated each other.

(2) Much attention has been devoted to the processes where the conspirators of π or K Regge poles[5] are expected to play important roles. Detailed analyses are made in pn and $\bar{p}p$ charge exchange processes and photoproduction processes, and the sharp forward peaks observed in these processes seem to be rather beautifully reproduced by a conspiracy mechanism.

The successes in this direction bring us two important questions. The first is the question of evasions versus conspiracies. In the usual Regge exchange amplitude, we have treated a Regge pole as if it were independent of others, and then each Regge exchange amplitude should satisfy the conservation of angular momentum by itself. In this case we must always take the case of evasions. On the other hand, in the case of conspiracies, the kinematical constraint is satisfied in non-trivial ways by the coexistence of two or more Regge poles. Then it would be necessary to understand the individual Regge pole in a more unified way.

The second is the question about the existence of the conspirators. At present, we have not yet observed any particles that can be regarded as conspirators. Phillips, who first introduced the conspirators into the phenomenological stage, suggested that the conspirators might play the part for the absorptive effect. Then we are again met by "the absorptive corrections".

(3) It must also be noted that the experimental data are being accumulated day by day. Especially the data for rather high momentum transfer in addition

to the forward data might give us information on the properties of the trajectory and residue functions of the Regge poles.[6] We observed the second or third dips in some processes, which seem to indicate us the falling meson trajectories as far as we take the assumption that the Regge pole exchange mechanism dominates even at large momentum transfers. We had usually taken the trajectory function to be rather flat at large negative t as seen from the Pignotti form[7] for example. Further we have not yet found any reasonable foundations to assume the Regge pole dominance in the region of large negative J values. Nevertheless such unified understanding of the dip mechanism seems to us very fascinating on account of the fact that the dip-bump structure occurs regularly. Another possible explanation of the dip mechanism had been proposed earlier in connection with the direct channel resonances.

How can we discriminate these two mechanisms experimentally or theoretically?

Is there any interrelation between the s-channel resonances and the Regge pole exchanges?

(4) The possibility of rising meson trajectories[8] has been recently investigated in the framework of S-matrix theory, stimulated by the argument made by Van Hove.[9] Experimentally it appears very plausible that the observed resonances lie on Chew-Frautschi plot.[10] Furthermore meson and baryon trajectories have their own characteristic features: In the meson trajectories the exchange degeneracy[11] holds in a good approximation, whereas it does not for baryon trajectories. On the other hand, the baryon trajectories seem to satisfy the so-called "MacDowell symmetry".[12]

By use of these properties, several attempts are now in progress to obtain the "dynamical equation of the Regge parameters". At this stage, however, we have no complete theories to predict all the Regge parameters.

Thus whole situation seems to require us to understand the Regge poles in a deeper level, and some orientations[13] are now proposed by several authors to the problem: "Where Regge poles come from?"

In the part I, we announced that in this issue we would treat mainly the subject of conspiracy. Since then, the several compact reviews[14] are prepared by several authors. Now we feel it is necessary to systematize the complicated situation of Regge phenomenology from a more critical point of view and get a clue to the structure of hadrons.

The aim of this paper is, therefore, to present a review of some topics which seem important for our purpose. The recent progress of the phenomenological analyses are shortly reviewed only in §2 and the following sections (§3~§6) are devoted to the interesting topics mentioned above. In the last section (§7) we shall present some orientations recently proposed by several authors.

§2. Recent analyses with Regge pole model[15]

Up to the present numerous experimental data have been accumulated on the forward scatterings of elastic processes at high energies. The data on the quasi-two-body forward scatterings and the elastic backward scatterings, though they are not at so high energies, have been also accumulated to some extent. On the other hand, those at intermediate and large angles are less abundant, but they are increasing at present.

Regge pole analyses of elastic scatterings are rather complicated, as more poles can take part in this case than in inelastic processes. Several analyses with many poles have been carried out in the forward elastic scatterings, and it has been found that Pomeranchuk pole is necessary to have very flat trajectory in contrast with others[16]. Rarita et al.[17] have made fits to the $\pi^{\pm}p$, pp and $\bar{p}p$ data above $p_L=5.9$ and up to about $-t=1$ in a new model, in which only P, P' and $\rho(\omega)$ with linear trajectories are taken into account for $\pi^{\pm}p$ (pp and $\bar{p}p$) processes.

Now, in this section, we present a review whose main part consists of the following topics:

(1) Conspiracy (forward scatterings of inelastic two body reactions and photoproductions).

(2) Dip mechanism (the higher momentum transfer scatterings of elastic processes).

(3) Application of Regge pole models to large angle scatterings of pp elastic process.

(4) Fermion trajectories (backward scatterings of πp elastic processes).

2. 1 *Forward scatterings of inelastic two body reactions and photoproductions*

The most striking among the sucesses of Regge pole models in the last year is the explanation of the very sharp forward peaks in pn charge exchange scattering as well as charged π photoproduction in terms of the exchange of conspiring pion trajectory. Let us first consider the pn charge exchange scattering. As mentioned in §§3.2 of I, pn and $\bar{p}p$ charge exchange scattering data, especially the very sharp forward peak ($\sim\exp(-50|t|)$ for $|t|<0.02$) in the former, can not be reproduced in the Regge pole model with only ρ and R poles. This circumstance can not be improved as far as one insists on the conventional Regge pole model, as fully discussed by Phillips.[18] Once one considers a mechanism based on conspiracy of Regge poles, the situation changes drastically, however.

The story is roughly as follows. Since ρ and R poles are not sufficient to explain the experiment, one has to invoke another mechanism in which Regge poles of unnatural parity may play a prominent role. The most important among them is the pion pole because of its closeness to the physical region together with its known strength of coupling to nucleons. The pion is known to contribute to the following s-channel helicity amplitudes

$$\phi_2{}^s=f^s{}_{++;--},\qquad \phi_4{}^s=f^s{}_{+-;-+} \tag{2.1}$$

which are expressed at $t=0$ by the t-channel helicity amplitudes via the Trueman-Wick crossing relations:[19]

$$\left.\begin{aligned}\phi_2{}^s&=\frac{1}{2}(\phi_1{}^t-\phi_2{}^t+\phi_3{}^t-\phi_4{}^t)\\ \phi_4{}^s&=\frac{1}{2}(\phi_1{}^t-\phi_2{}^t-\phi_3{}^t+\phi_4{}^t)\end{aligned}\right\}\text{ at } t=0 \tag{2.2}$$

with standard notations.[20] The pion contributes only to $\phi_1{}^t-\phi_2{}^t$ in the form $\beta_\pi(t)s^{\alpha_\pi(t)}$, and so

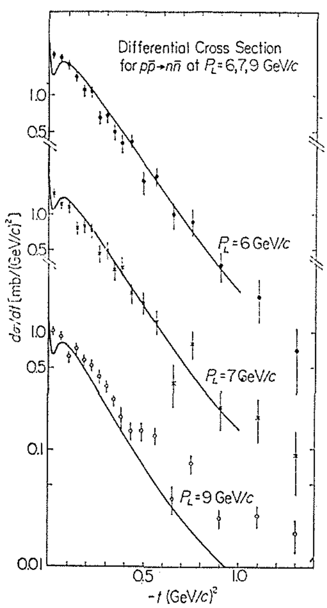
$$\phi_2{}^s(\pi)=\phi_4{}^s(\pi)=\mathrm{O}(t)\quad\text{as}\quad t\to 0, \tag{2.3}$$

the latter equality being a consequence of angular momentum conservation in the forward direction. Hence we conclude that the pion by itself contributes nothing at $t=0$. This is not consistent with the experimental data.

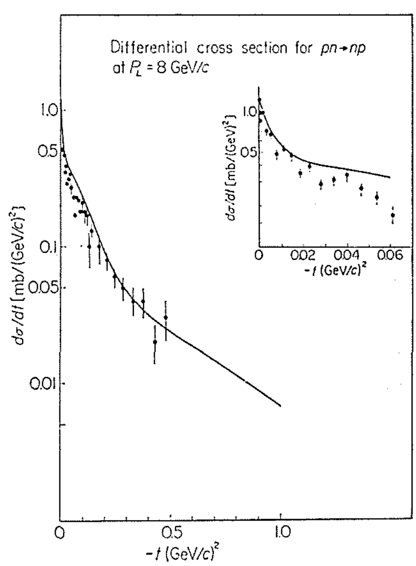
The constraint $\phi_4{}^s=O(t)$ is satisfied without requiring $\phi_2{}^s=O(t)$ as well, however, if in addition to the pion trajectory one considers[21] a natural parity Regge pole, say π_C, contributing to $\phi_3{}^t-\phi_4{}^t$ in the form $\beta_C(t)s^{\alpha_C(t)}$ in such a way that $\alpha_\pi(0)=\alpha_{\pi_C}(0)$ and $\beta_\pi(t)-\beta_{\pi_C}(t)=O(t)$, while $\beta_\pi(t)+\beta_{\pi_C}(t)=O(1)$. In this case the double helicity-flip amplitude $\phi_2{}^s$ contributes with normal size at $t=0$, thereby helping to circumvent the difficulty above. This ends up a rough description of the pion conspiracy.

Phillips[22] was the first to point out the important consequence of the pion conspiracy in the processes under consideration. More refined treatments were carried out by Arbab and Dash[23] and also by Fukui and Morita.[24] A typical fit to the data is shown in Fig. 1, quoted from Ref. 24.

In the high energy π^+ photoproduction from proton a sharp forward peak similar to the one in pn charge exchange scattering seems to exist.[25] As no single-particle exchanges contribute to photoproduction at $t=0$, the conspiracy mechanism has been suggested to explain the peak. Models with pion conspirator have been presented by Ball et al.[26] and Henyey;[27] these models predict that $s^2\dfrac{d\sigma}{dt}(t=0)$ is almost constant at high energies. The experimental data up to 16 GeV, however, show that it decreases considerably as s increases, with different rates at different $-t$ values. In order to explain these features and



(a)



(b)

Fig. 1. Differential cross sections for the forward scatterings of the processes $\bar{p}p\to\bar{n}n$ (a) and $pn\to np$ (b). Solid curves present the fit according to the $\pi+\pi_C+\rho+R+B$ model.[24] In this fit only the 7 GeV/c data of $\bar{p}p\to\bar{n}n$ and the 8 GeV/c data of $pn\to np$ are used, and therefore other curves are predictions according to the model. The curve for $pn\to np$ is the calculated one multiplied by 0.75 in view of normalization errors.

to avoid the strong t-dependence of residue functions several attempts have been presented with conspiracy mechanism. Dietz and Korth[28] have employed a pair of conspirator, C and C', with $G=-1$ and $I=1$, and obtained good fits with

$$\alpha_\pi(t) = -0.007+0.4t, \tag{2.4a}$$

$$\alpha_C(t) = \alpha_{C'}(t) = -0.12+0.78t\,; \tag{2.4b}$$

here, pion is assumed not to conspire. Borgese and Colocci[29] have given a model with B meson conspiracy in addition to pion conspiracy, whose trajectories are taken as

$$\alpha_\pi(t) = \alpha_{\pi_C}(t) = t-0.02, \tag{2.5a}$$

$$\alpha_B(t) = \alpha_{B_C}(t) = t-0.32. \tag{2.5b}$$

On the other hand a model without conspiracy reproducing the peak has been presented by Amati et al.;[30] the background, which can be originated from a Regge cut, or from absorption correction, or from fixed poles, has been taken into account. They have formulated the contribution of the background as fixed pole contribution and taken

$$\alpha_\pi(t) = t-0.02, \tag{2.6a}$$

$$\alpha_B(t) = t-0.3, \tag{2.6b}$$

$$\alpha_{A_2}(t) = 0.5t+0.4. \tag{2.6c}$$

The comparison of their results with experimental data is shown in Fig. 2.

Since the π and K mesons belong to the same octet of $SU(3)$, if pion has its conspirator, it is expected that the K meson also has its conspirator K_C. Experimental data on the strangeness exchange processes are scarce and Regge pole analyses in these processes are few at present. Several models with K meson conspiracy, however, have been presented to the K meson photoproduction; it seems that K meson conspires weakly or evades[26),27] for lack of the forward peak.

Furthermore the experimental data of charged pion photoproduction at the higher energies might turn out to show the forward dip, as maybe suggested by a slight fall in the forward direction of the differential cross section at 16 GeV/c. Ader et al.[31] have investigated the process $\gamma p\to\pi^+n$ as well as $\gamma p\to\pi^0 p$ and $\gamma p\to K^+\Lambda^0(\Sigma^0)$ in nonconspiracy model with $\pi+B$, $\pi+\omega$ and $K+K^*$ exchange, respectively.

On the other hand Frautschi and Jones[32] have investigated Reggeization of pion exchange without conspiracy, analyzing the differential cross section of the process $\pi N\to\rho\Delta$ in detail and other processes in which pion exchange is expected to dominate the forward peak as suggested by experimental density matrices, i.e., $\pi N\to\rho N$, $KN\to K^*\Delta$, $\pi N\to f^0\Delta$ and $\pi N\to f^0 N$. Taking the trajectory as

$$\alpha_\pi(t) = -0.02+t \tag{2.7}$$

and the reduced residues as linear functions of t, they have reproduced the experimental data fairly well.

As pion conspiracy mechanism has worked successfully for pn and $\bar{p}p$ charge exchange processes, from the stand-point of Lorentz-pole classification it

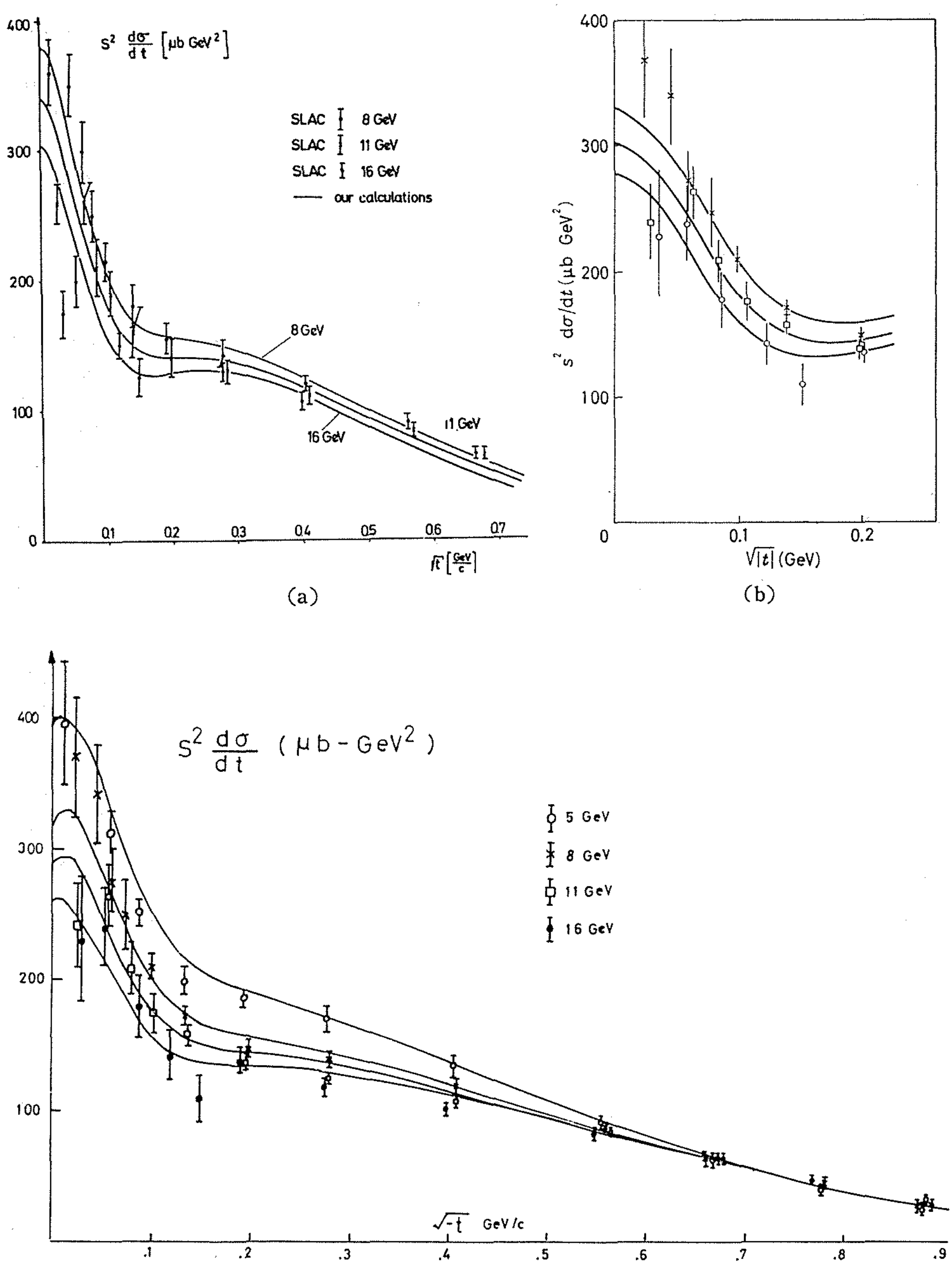


Fig. 2. Differential cross sections for $\gamma p \to \pi^+ n$. Solid curves show the fits according to the following models: (a) The model with conspiracy of a pair of trajectories with $G=-1$ and $I=1$.[28] (b) The model with double conspiracy (π and B).[29] $\times$: 8 GeV, □: 11 GeV, ○: 19 GeV (c) The model without conspiracy but with background.[30]

should work also in other processes to which pion can contribute. More detailed analyses as well as experimental data are now required for the definite conclusion on the conspiracy.

Several other attempts have been presented in the analyses of the forward scattering data of inelastic two body processes. Various processes have been analysed in various models without conspiracy mechanism. Here we enumerate only some of the processes; $\pi^+p\rightarrow\pi^0\Delta^{++}$,[33] $\pi^-p\rightarrow\pi^0 n$,[34] $\pi^-p\rightarrow\eta n$,[34] $\pi^+n\rightarrow\omega p$,[34),35] $\pi^+p\rightarrow K^+\Sigma^+$.[36]

2. 2 *Dips at the higher momentum-transfer scatterings of elastic processes*

As mentioned in *I* dips have been observed at $-t\approx0.7$ in $\pi^\pm p$ elastic scattering differential cross sections up to about $p_L=4$, accompanied with secondary bumps at $-t\approx1.2$. Chiu et al.[37] have investigated Frautschi's proposal[38] that the vanishing of helicity-flip amplitude of P' at $\alpha_{P'}(t)=0$ can be used to explain the secondary bump (see also §§3.3 in *I*), extensively by fitting the bumps together with the other high-energy data of pp and $\bar{p}p$ as well as $\pi^\pm p$ processes. They have found that their solutions with this mechanism do not have good x^2 values and that the dip-bump structure can be explained naturally by the vanishing of the helicity-nonflip amplitude at $\alpha_{P'}(t)=0$ (no-compensation mechansim).

Recent experimental data[39] show several additional structures in differential cross section of elastic scattering; for π^-p process a prominent dip is seen at $-t\approx3$, for $\bar{p}p$ there is an evidence for a secondary dip at $-t\approx1.8$ (the first dip at $-t\approx0.5$) and for K^-p there seems to be some structure at $-t\approx0.9$ (see Fig. 3). On the contrary the pp data are relatively smooth except for a slight break near $-t=1.6$ and the K^+p data also seem to be smooth.

Barger and Phillips[40] have suggested that the dip-bump structure can be explained in terms of the zeros of the Regge pole amplitudes associated with the exceptional points $\alpha(t)$=non-positive integer, and given a model which contains essentially energy independent Pomeranchuk amplitude and two Regge poles P' and ω with a single degenerate trajectory. In their model, as $\alpha(t)$ goes through zero or negative integers, the P' and ω amplitudes come alternatively in and out of phase with P because of the opposite signature of P' and ω, and the interference terms in the differential cross section tend to oscillate. In order to reproduce plausible structure for $\pi^\pm p$ and $\bar{p}p$ and smoothness for pp, the residue functions must have cyclic character which have double zeros at right signature points and are correlated between P' and ω. This model suggests the following predictions; K^+p and K^-p scattering should qualitatively resemble pp and $\bar{p}p$; the oscillations should disappear as $s^{\alpha-1}$ as $s\rightarrow\infty$; similar dips should occur at large $-t$ in inelastic two-body reactions associated with P' or ω exchanges.

There is another way to interpret the dip-bump structure, especially for the lower energies. For example Hoff[41] has proposed an interpretation in the interference model, i.e., the secondary maxima result from the presence of the resonance amplitude. This mechanism may be applied to the interpretation of the possible dip-bump series. According to this the ratio of the differential cross section at two points of the series should not have such a strong s-dependence as that predicted by Barger and Phillips.

There are many ambiguities in Regge pole analyses (not only at relatively

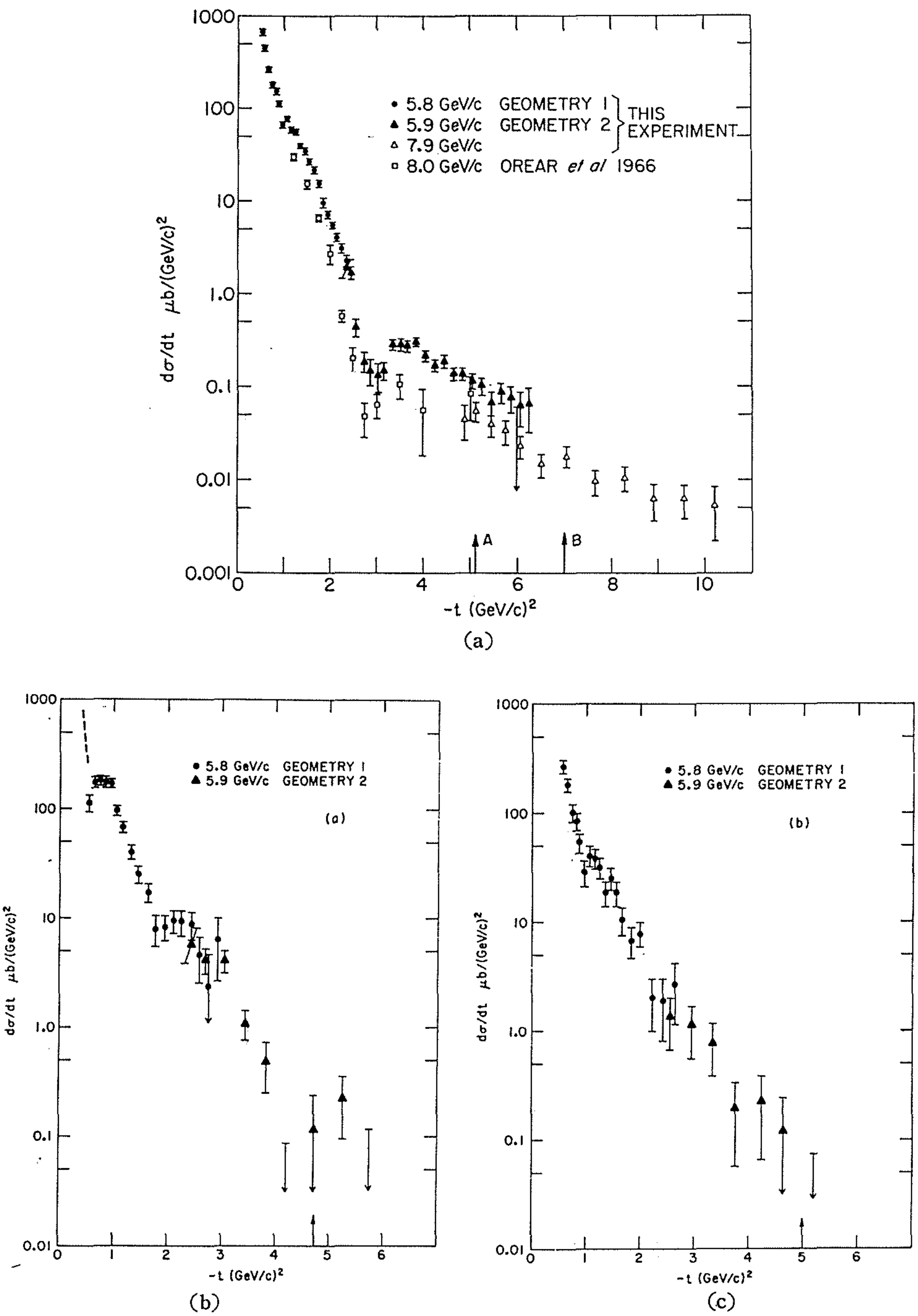


Fig. 3. Dips at the high momentum-transfer scatterings of $\pi^- p(a)$, $p\bar{p}(b)$ and $K^- p(c)$ elastic processes.[39] The arrows on the abscissa indicate 90° (*c.m.* system).

large $-t$ but also at forward scatterings). In ordinary Regge pole analyses it is assumed that several Regge poles dominate the scattering amplitude, but, as there is possibility that, in the region under consideration, a lot of poles with the same quantum numbers may contribute or other effects may exist, we should consider the obtained Regge parameters as the effective ones. Therefore, when we extend the Regge pole model to the higher momentum-transfers, we have little definite information on the behavior of Regge poles.

2. 3 *Large angle scatterings of pp elastic process*

Although experimental data of large angle scatterings at high energies are insufficient for detailed study of Regge pole theory in large angles, but rather accurate data in pp scatterings have been accumulated to some extent. The large angle pp elastic scattering data[42),43)] show that the angular distribution is nearly isotropic and the energy dependence is as follows ;[42)]

$$\frac{d\sigma}{dt}(\theta_{CM}=90°)=Ae^{-ap_{CM}^2}, \tag{2.8}$$

where

$$a=\begin{cases} 3.29 \\ 1.51 \end{cases} \quad \text{for} \quad p_L \lessgtr 8. \tag{2.9}$$

Several authors have attempted to explain the break of energy dependence within Regge pole model. Huang et al.[44)] have tried to reproduce the break assuming that Pomeranchon with linear trajectory is dominant in the smaller $-t$ region and its contribution vanishes at $-t\approx 7$ because of wrong-signature, and then the moving cut associated with P becomes dominant. They have assumed that the contribution of the moving cut with branchpoint $\alpha_C(t)$ is the same as that of Regge pole with the trajectory $\alpha^C(t)$, and found $\alpha_P(t)=1+0.29t$ and consequently $\alpha^C(t)=1+0.145t$. Subsequently Huang and Pinsky[45)] have analyzed the process in detail with two trajectories, which have been found consistent with $1+0.5t$ and $1+0.25t$, and suggested that they correspond to the Pomeranchuk pole and a Regge cut generated by it.

In these analyses the Regge pole terms have been assumed to behave as $s^{\alpha(t)}$, but it is not correct because $\cos\theta_t$ approaches to a finite value as s approaches to the infinity with *c.m.* angle fixed. Therefore explicit form must be used for Regge pole terms. Sakmar and Wojtaszek[46)] have employed $P_\alpha(\cos\theta_t)$ as the Regge pole term and attempted to reproduce the break with only Pomeranchon whose trajectory is made curved suitably and approaches to zero in the infinite limit. Though they have not discussed the angular distribution, it was pointed out[47)] that their model, as well as that of Huang et al., predicts wrong fixed-s angular dependence near the break region. There is another attempt[48)] in which trajectories are assumed to be linear in t and $Q_{-\alpha-1}(\cos\theta_t)$ is taken for the Regge pole term. This model, under the assumption of one pole dominance, can reproduce both the s- and t-dependence of the experimental data fairly well.

On the other hand there is a discussion on the total contribution of moving cuts arising from many Regge poles exchanges to relatively high momentum transfer scatterings[49)] and another based on dipole exchange.[50)]

2. 4 Backward scatterings of πp elastic processes

As for the $\pi^{\pm}p$ backward scatterings at low and intermediate energies ($p_L \lesssim 6$) a lot of papers have appeared in interference model with considerable success (see §3). At high energies simple Regge pole models with only baryon Regge poles have been presented (see also §4 in *I*).

Recent experimental data[51] show that π^-p differential cross section has a dip at $-u\approx 0.2$ with sharp backward peak and π^-p has a wide backward peak. Then Barger and Cline[52] have analyzed these data along with other high energy data above $p_L=4$ on backward $\pi^{\pm}p$ elastic scattering; they have found that both N_α and Δ_δ trajectories are in agreement with straight line extrapolations through the known resonance region, appearing to deviate at large $-u$, and that possible contribution from N_γ amplitude necessarily must be small. Figure 4 shows the fits to the new data with

$$\alpha_{N_\alpha}(\sqrt{u}) = -0.38+0.88u. \tag{2.10a}$$

$$\alpha_{\Delta_\delta}(\sqrt{u}) = 0.19+0.87u. \tag{2.10b}$$

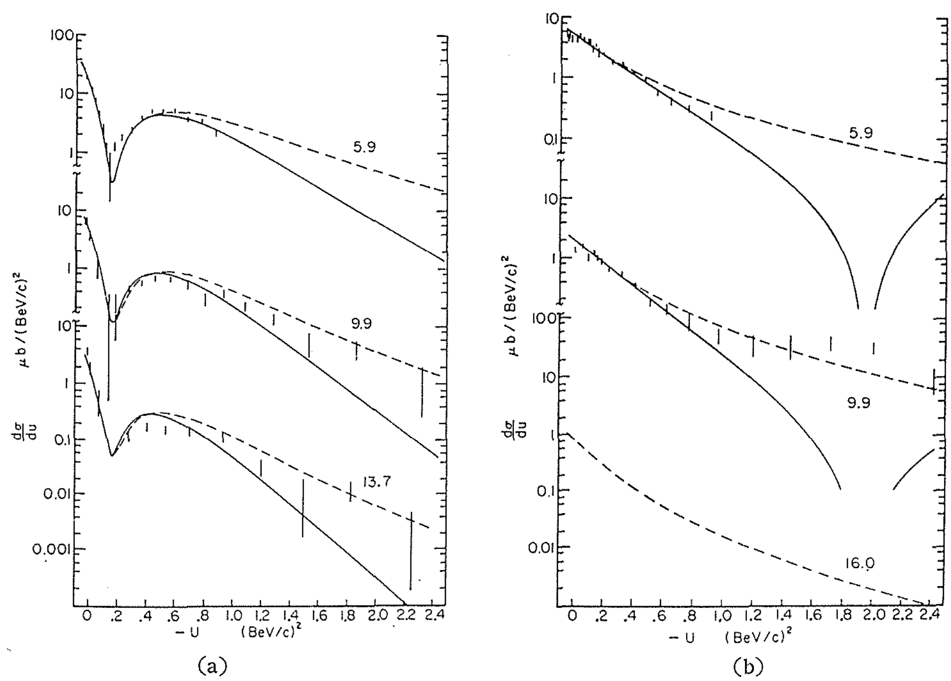


Fig. 4. Regge pole fits to $\pi^{\pm}p$ backward scattering data:[52] (a) π^+p with N_α, Δ_δ and (b) π^-p with Δ_δ. The solid curves show the case of liner trajectors and the dashed curves nonlinear trajectories.

The dip near $-u=0.2$ of π^+p is reproduced by the zero associated with wrong-signature nonsense value of N_α trajectory, and on the other hand that of Δ_δ produces dip near $-u=1.9$ in π^-p differential cross section, which has not been observed experimentally (as the contribution of Δ_δ in π^+p is much smaller than that of N_α, the possible dip due to Δ_δ is buried in the N_α amplitude). In order to avoid this trouble the case of nonlinear trajectories has been also investigated. The fit with

$$\alpha_{\Delta_\delta}(\sqrt{u}) = \frac{0.21+0.9u}{1-1.6u}, \tag{2.11a}$$

$$\alpha_{N_\alpha}(\sqrt{u}) = \frac{-0.37+0.9u}{1-0.5u}, \tag{2.11b}$$

is also shown in Fig. 4. Another analysis[53] based on the same mechanism has appeared with

$$\alpha_{\Delta_\delta}(\sqrt{u}) = 0.15+0.90u, \tag{2.12a}$$

$$\alpha_{N_\alpha}(\sqrt{u}) = -0.33+0.11\sqrt{u}+1.06u. \tag{2.12b}$$

On the other hand an alternative explanation of the dip of π^+p has been proposed by Contogouris et al.[54] In this model two fermion trajectories N_α and N_γ contribute with comparative real parts, and then an interference term oscillates with the difference between the imaginary parts of the trajectories. Neglecting the Δ_δ exchange as suggested by the smallness of its contribution to π^-p scattering (the differential cross section at 180° of π^-p, the amplitude of which is expected to be dominated by the Δ_δ trajectory, is smaller of one order than that of π^+p) and assuming

$$\alpha_{N_\gamma}(\sqrt{u}) = -2.087\sqrt{u}+2.03u, \tag{2.13a}$$

$$\alpha_{N_\alpha}(\sqrt{u}) = -0.5+\frac{0.22}{1.1-u}, \tag{2.13b}$$

they have had good fits to the differential cross section data. This model has the following features which are in contrast with the models mentioned above; the value of $-u$ at which the dip occurs decreases with s, and polarization of recoil nucleon has rather large value, oscillating with u.

§3. Interference model

As we saw in I and the previous section, Regge pole exchange model provides very satisfactory descriptions in many processes at high energies. On the other hand, the extrapolation of this model to lower energy region can not be justified neither from theoretical nor experimental point of view: The Regge pole term does no more dominate the scattering amplitude and other terms such as background integral can not be neglected in lower energy region. In fact the experimental data show a typical behaviour of resonance amplitude near the resonant energy and the scattering amplitude should have poles corresponding to the resonances in the unphysical sheet of the complex s plane, which are never included in the Regge pole exchange amplitude.

The scattering amplitudes are, therefore, generally represented by two terms, the resonance amplitudes, f^s, and the background amplitudes, f^B (usually represented by Regge exchange amplitude, f^R) for lower energies. This is the basic idea of the interference model.

In the following, we shall give a historical review of the interference model and its applications. Further some arguments recently made in connection with interference model are presented.

3. 1 Historical review of the interference model

The phenomenological analyses are made mostly in πN scattering data. In the following, the main results of the interference model are presented.

From the data of the total π^+p and π^-p cross section ($\sigma^\pm$), Höhler and Giesecke[55] analysed the charge exchange total cross section from 1 to 6 GeV/c in laboratory momentum. The experimental data show resonance-like structures as shown in Fig. 5, and at the energy of the maxima of $\sigma^\pm$ the curve in the Argand diagram describes an arc of a circle in the counterclockwise direction. The background amplitude is calculated and it was found that its energy dependence is the same as the one of ρ Regge exchange amplitude. This seems to indicate that the background amplitude is not very different from ρ Regge exchange amplitude. This fact is confirmed by more detailed analysis of πN charge exchange scattering data by Höhler et al.[56]

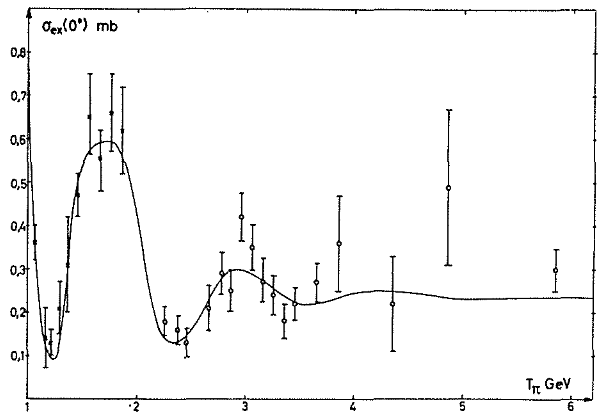


Fig. 5. The cross section of π^-p charge exchange scattering at 0° via the incident laboratory energy. The curve shows resonant-like structure.

From the detailed analyses in π^-p charge exchange data in forward direction about 2 GeV/c by the interference modell, Carroll et al.[57] determined the spins of $N^*_{I=1/2}$ (2190), $N^*_{I=3/2}$ (1920) and $N^*_{I=3/2}$ (2360), and further showed that the process can be well described by the interference model. Also dip and the secondary maximum of π^-p elastic scattering cross section in the region from 1.7 to 2.5 GeV/c are interpreted as being due to resonance effects.[41),58)] This may be confirmed by the analysis of Barut and Kleinert,[59] according to which only a few partial waves contribute to the cross section near the secondary peak whereas all partial waves together make up the diffraction peak.

In order to get information about the spin of a resonance from the behaviour of the differential cross section near the resonant energy, the magnitude of the background amplitude should not be much larger than that of the resonance amplitude. For higher resonances, the situation is favourable in the backward region. Heinz and Ross[60] analysed the backward $\pi^\pm p$ cross sections between 2.1 and 5.5 GeV/c, where they assumed the background amplitude is primarily due to N exchange.

Systematic analyses are made by Barger, Cline and Olsson[61] in differential cross section at 180° of π^-p elastic scattering, the difference of σ^- and σ^+, and the differential cross section at 0° of $\pi^- p$ charge exchange scattering. Some analyses are made by this model in connection with the finite polarization effect observed in $\pi^- p$ charge exchange scattering.[62]

In all cases, the resonance amplitude is assumed to be represented by the sum of Breit-Wigner resonance formulae, and the sum of the amplitudes, f^s and f^R, does not guarantee the unitarity.[63] We shall give an important comment on this point in §§3.3.

3. 2 *The validity of the interference model—double counting*

Objections are made against the interference model in regard to the possibility of double counting from two points of view. One is the formal suggestion made by Schmid,[64] and the other is the overall phenomenological analyses in the πN data made by Chiu and Stirling.[65]

(a) The Regge amplitude is usually expressed as

$$\left.\begin{aligned} &f=\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}\beta(t)\left(\frac{E}{E_0}\right)^{\alpha} \\ &\alpha(t)=\alpha_0+\alpha' t \end{aligned}\right\}, \tag{3.1}$$

where the residue function $\beta(t)$ vanishes at the point where $\alpha(t)=0,\ -1,\ -2,\cdots\cdots$, and the residual factor of $\beta(t)$ is usually taken to be constant unless more strongly ghost-killing factor is assumed. According to Schmid[64] the partial wave amplitudes f_l in the s channel describe circles in the Argand diagram as the energy increases, which are usually associated with resonances. For the $\pi^- p$ charge exchange, the energy at the top of the circles is plotted for each l and he concludes that these series correspond to N^* Regge trajectory in the direct channel. Thus he argued that the direct-channel resonances are already contained in the Regge amplitude of the crossed channel and the sum of the two terms leads to double counting.

In order to understand the situation qualitatively, let us take the residue function as

$$\beta(t)=\sin\pi\alpha\cdot\gamma. \tag{3.2}$$

Then the partial wave expansion is obtained,[66]

$$\begin{aligned} f_l&=\frac{\gamma}{2}\int_{-1}^{1}(1\pm e^{-i\pi\alpha})e^{\alpha\ln(E/E_0)}P_l(z)dz \\ &=\left(\frac{E}{E_0}\right)^{\alpha_0-2k^2\alpha'}j_l(-2k^2\alpha'\ln E/E_0) \\ &\pm\left(\frac{E}{E_0}\right)^{\alpha_0-2k^2\alpha'}e^{-i\pi(\alpha_0-2k^2\alpha'-ml)}j_l(2k^2\alpha'\sqrt{(\ln E/E_0)^2+\pi^2}) \\ &\left(\tan m\pi=\frac{-\pi}{\ln(E/E_0)}\right), \end{aligned} \tag{3.3}$$

where we used the following formulae,

$$\left.\begin{aligned} &e^{az}=\sum_l(2l+1)j_l(a)P_l(z) \\ &j_l(e^{im\pi}z)=e^{iml\pi}j_l(z) \end{aligned}\right\}. \tag{3.4}$$

From Eq. (3.3) we see the oscillating propery is due to the spherical Bessel function j_l.

(b) Chiu and Stirling[65] analyzed all the available data on πN scattering and divided the experimental data into three groups.

(i) The data with which the interference model is compatible:

π^+p total cross section, π^-p forward differential cross section, π^-p polarization, and π^-p charge exchange differential cross section.

(ii) The data of which the interference model does not give a unique quantitative description:

π^-p differential cross section at 180°.

(iii) The data for which the interference model does not work:

π^+p differential cross section at 180°, π^+p polarization, and the sum of σ^+ and σ^-.

In most cases of groups (i) and (ii), there are large cancellations between nearby resonances, or resonance contributions are very small. In the case of group (iii), the prominent resonances enter with the same sign in the scattering amplitude. Especially in the sum of the σ^+ and σ^-, the resonances contribute constructively to the background Regge amplitude and the total amplitude does not oscillate around it, contradictorily with the experiment.

3. 3 *The validity of the interference model—againt the possibility of double counting*

The unified understanding of the phenomena in the intermediate energy on the basis of the interference model has been disturbed not only on account of the ambiguity of the qualitative description, but also on account of the larger and constructive contributions of the resonances. How can we eliminate the double counting or overestimations?

(a) As was pointed out in §§3.1, unitarity is not guaranteed by the sum of amplitudes f^s and f^R. It is well known in nuclear physics[67] that the resonance amplitude is not in general described by the simple Breit-Wigner terms with constant widths and elasticities except when the incident energy is near the resonance energy and the background contribution is neglibible. The more general expression[68] for resonance amplitude can be expressed in elastic case as

$$f^s = \frac{1}{k}\sum_l \frac{x_l f_l(k) e^{2i\delta_l}}{\epsilon_l - i} P_l(z), \tag{3.5}$$

$$\epsilon_l = \frac{2(k_l - k)}{\Gamma(k)}, \tag{3.6}$$

where k is the *c.m.* momentum, k_l corresponds to the mass of the resonance, $\Gamma(k)$ is the total width and x_l is elasticity. The important factor δ_l is the phase of the background amplitude and $f_l(k)$ is the form factor, which satisfies the following relations by definition,

$$\left.\begin{array}{l} f_l(k_l) = 1, \\ f_l(k) \underset{k\to 0}{\sim} k^{2l+1} \end{array}\right\}, \tag{3.7}$$

and in general $f_l(k)$ is decreasing function of $(k_l - k)^2$. These two factors $f_l(k)$ and $e^{2i\delta_l}$, are omitted in the usual Breit-Wigner approximation.

In order to see the difference, we shall show in Fig. 6 the typical behavior of these total amplitudes in the Argand diagram.[69] If we use this general

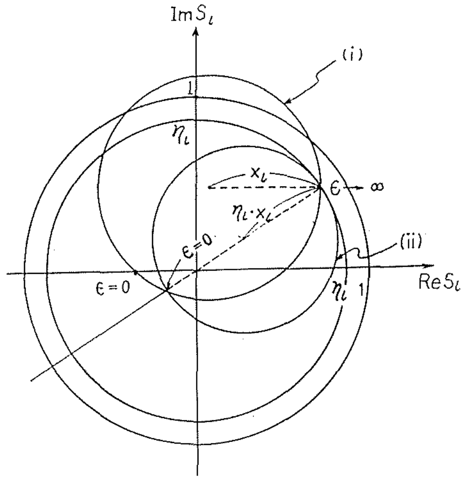


Fig. 6. The Argand diagram of $S_l=e^{2i\varphi_l}$.
(i) The Breit-Wigner type of resonance formula are used.
(ii) The phase factor is taken into account as is seen in Eq. (3.5).
Note that the curve in the case (ii) is always within the circle with its center in origin and radius η_l.

expression, the interference model reproduces the experimental data on the π^+p backward scattering and the sum of $\pi^\pm p$ total cross sections[70] (these belong to group (iii)). More overall calculation will be desired by the modified model. Another modification, due to Takagi and Miyamura[71] is to put

$$f=f^s+(1-R)f^R, \tag{3.8}$$

where R depends on the incident energy.

(b) We have already seen in the previous section that the circles in the Argand diagrams are direct consequence of the phase factor $\exp(-i\pi\alpha)$, and not the result of a particular form of a Regge amplitude. The phase factor is a general feature of the amplitude behaving asymptotically like $s^\alpha(\ln s)^\beta(\ln \ln s)^\gamma\cdots\cdots$ for fixed t and follows from the crossing symmetry.[72] Furthermore one should not associate the circles with resonances in the direct channel,[73] since there are not poles in the unphysical sheet in the partial wave amplitude of Regge exchange terms.

3. 4 Some comments in connection with the interference model

In concluding this section, we are to make some comments on the finite energy sum rules and propose some problems in connection with the interference model.

Finite energy sum rule was first proposed by Igi and Matsuda,[74] who assumed that the scattering amplitude is approximated by the sum of resonance amplitude (f^s) in low energy region and by the Regge pole amplitude (f^R) in high energy region: i.e.,

$$f=\begin{cases} f^s & \text{for } k\lesssim k_N \\ f^s+f^R & \text{for } k_N\lesssim k\lesssim k_0 \\ f^R & \text{for } k_0\lesssim k \end{cases} \tag{3.9}$$

and make use of the following superconvergence relation,

$$\int_0^\infty \mathrm{Im}(f-f^R)\,dk=0. \tag{3.10}$$

Recently it is proposed that finite energy sum rule might be used for bootstrap-like calculation.[75),76)] In order to determine the Regge parameters from the information of the crossed channel, the Regge amplitude should vanish in the low energy region. We have no reason, however, to make this assumption. In fact we have already seen in §§3.1 the background amplitude is well represented by ρ Regge exchange amplitude at least in $\pi^- p$ charge exchange process. Accordingly we must put

$$f=f^R+f^s \quad \text{for} \quad k\lesssim k_0, \tag{3.11}$$

and then Eq. (3.10) leads

$$\int_0^{k_0} \mathrm{Im}\, f^s dk=0, \tag{3.12}$$

where we have no Regge parameters.

Even if we do assume

$$\begin{cases} f\approx f^s, \\ f^R\approx 0, \end{cases} \quad (\text{for} \quad k\lesssim k_0). \tag{3.13}$$

then superconvergence relation reduces to the following relation,

$$\int_0^{k_0} \mathrm{Im}\, f^s dk=\int_0^{k_0} \mathrm{Im}\, f^R dk\approx 0. \tag{3.14}$$

Thus we can not get a bootstrap-like equation.

Dolen, Horn and Schmid[75)] proposed the modified representation of the following form,

$$f=f^R+f^s-\langle f^s\rangle. \tag{3.15}$$

In this case superconvergence relation leads to a trivial result as follows,

$$\frac{1}{k_0}\int_0^{k_0} \mathrm{Im}\, f^s dk=\mathrm{Im}\langle f^s\rangle. \tag{3.16}$$

In order to get a usual finite energy sum rule, we must assume the bootstrap-like equation itself, i.e.,

$$\int_0^{k_0} \mathrm{Im}\, f^s dk=\int_0^{k_0} \mathrm{Im}\, f^R dk. \tag{3.17}$$

From the above discussions, the question[75)] about the interference model from the view point of finite energy sum rules seems to have no clear cut meaning.

Next we must note that the separation of the amplitude into the resonance and background term is not unique as is well known in nuclear physics. The most general definition of the two terms are that the resonance term fluctuates very rapidly with energy and the background term is the sum of effects other than those contained in f^s. The far-away resonances can be included, therefore, either in f^s or f^R.[61)] In order to see more detailed mechanism of these two terms, we are obliged to study the origin of the resonances, and then we might be able to see the structure of the so-called "elementary particles".

The third comment is on the crossing symmetry. Overall representation of Regge pole amplitude valid both in the direct and crossed channels is not yet obtained. In particular, the full amplitude is not approximated by the t-channel Regge pole terms alone in low energy region in s-channel or in the case of direct t-channel, since background integral can not be neglected. Some attempts are now in progress, in relation to the modified Regge amplitude,[77] the interpretation of the Regge pole model discussed[77),78] by Van Hove[79] (see §7) or unusuall properties of Pomeranchuk trajectory[80] (see §2).

4. Contribution from others than moving poles

In the complex J plane, in addition to the moving poles, there may exist fixed singularities or moving cuts of kinematical or dynamical origin. Several theoretical arguments for the existence of these singularities, based on Mandelstam representation or in the field theory, have been presented. The estimation of their contribution, however, is difficult in Regge pole analyses because of little information about their behavior and of many ambiguities in the experimental data.

On the other hand the unusual behavior of Pomeranchon is widely known. This may have relation not only with the singularities montioned above but also with other effects like absorptive correction.

In the following we take up these problems.

4. 1 Fixed singularities[81]

It has been pointed out that partial wave amplitudes have fixed singularities in the complex J plane at nonsense points of wrong signature which come from singularities of second kind rotation function and the existence of the third double spectral function in the case of relativistic scattering. Gribov and Pomeranchuk[82] have argued that the unitarity relation, when such fixed singularities exist, is respected only by the appearance of essential singularities (Gribov-Pomeranchuk singularities). Mandelstam[83] has shown, however, that in the presence of cuts in the J plane the arguments for the essential singularities must be modified and that the essential singularities do not occur on the real axis of the J plane. Jones and Teplitz,[84] and subsequently Mandelstam and Wang,[85] have suggested that even in the presence of cuts fixed poles can exist where the Gribov-Pomeranchuk singularities would be expected in the absence of cuts.

Such poles make only small contributions to the asymptotic behavior but would modify in effect the form of the moving pole contribution. In the case that the contribution of the third double spectral function is not too small, they invalidate the old mechanism (see §3 in *I*) for the presence of dips at wrong signature points, which are expected to occur in the absence of the fixed poles; they bring about shifts of the dips away from the points where the trajectory actually crosses the real points, or fillings of the dips. Experimentally there is no clear-cut evidence for such shifts or fillings of dips due to fixed poles.

4. 2 Cuts contribution

There are some theoretical arguments for the existence of moving cuts in the frame of field theory.[83),86] We have no reliable tools in studying the behavior of the possible cuts, and so the behavior of cuts obtained from Feynman diagram calculation is usually consulted.

It is expected that simultaneous exchange of several Regge poles[87] gives rise to moving cuts. The branch point of the cut arising from the exchange of two Regge poles with trajectories $\alpha_1(t)$ and $\alpha_2(t)$ is given by the maximum of $\alpha_1(t')+\alpha_2(t'')-1$, under the constraint that t', $t''\leqslant 0$ and $(-t')^{1/2}+(-t'')^{1/2}\leqslant (-t)^{1/2}$. For linear trajectories it is given by

$$\alpha^C(t)=\alpha_1\Big[\Big(\frac{\alpha_2'}{\alpha_1'+\alpha_2'}\Big)^2 t\Big]+\alpha_2\Big[\Big(\frac{\alpha_1'}{\alpha_1'+\alpha_2'}\Big)^2 t\Big]-1, \tag{4.1}$$

where α_1' and α_2' are slopes of original trajectories. In the case of simultaneous exchange of n identical poles with trajectory $\alpha(t)$, the branch points of the induced cuts are given by

$$\alpha^C{}_{(n)}(t)=n\alpha\Big(\frac{t}{n^2}\Big)-n+1 \quad (n=2,\,3,\,\cdots\cdots). \tag{4.2}$$

The contribution of the cuts to the high energy scattering amplitudes takes the following form;

$$f^C(s,t)=\int^{\alpha^C(t)} dJ\; C(J,t)\Big(\frac{s}{s_0}\Big)^J \frac{1\pm e^{-i\pi J}}{2\sin\pi J}, \tag{4.3}$$

where $C(J,t)$ is the reduced discontinuity of the t-channel partial wave amplitude in the J-plane with the threshold dependence $(q_t^i\, q_t^f)^J$ factored out.

Two attempts are made to explain the experimental data, in which moving cuts play important role: One is the problem of the energy dependence of recoil neutron polarization in the process $\pi^- p\to\pi^0 n$. In addition to ρ-pole the leading cut associated with the ρ- and P-exchange is taken into consideration by Lany et al.[88] The model gives increasing energy dependence of the polarization, while alternative explanations predict decreasing polarization;[89] the experimental data are so scarce and ambiguous that we can not exclude either of them.

Another is the attempt to reproduce the differential cross section of pp elastic scattering at *c.m.* angle 90°, employing the cut arising from the exchange of two Pomeranchons in addition to the Pomeranchuk pole. Experimentally, in the vicinity of incident momentum 8 GeV/c, a remarkable break has been discovered in the differential cross section at 90°. Huang et al.[44] have explained the break by the scheme that P pole exchange contribution dominates the scattering amplitude in the smaller $-t$ region, vanishing at $-t=7$ (GeV/c)^2 because of the zero of the signature factor, and that then the moving cut dominates in the larger $-t$ region. In this model, however, it has been assumed that the cut is treated in a simple way as if there were a Regge pole at $J=\alpha^C(t)$.

As for the total contribution of all the cuts arising from multi-Pomeranchon exchange, Anselm and Dyatlov[49] have estimated it; the contribution to the amplitude is written as

$$A(s,t)=s\sum_{n=1}^{\infty}B_n(t,\xi)\Big(\frac{s}{4m^2}\Big)^{\alpha^C{}_{(n)}(t)-1}, \qquad \xi=\ln\frac{s}{4m^2}, \tag{4.4}$$

and the coefficients $B_n(t,\xi)$ are estimated as

$$B_n(t,\xi)=(-1)^n\exp[-n\cdot a(\ln\tau,\ln\xi)], \qquad \tau=-\alpha_P'(0)t, \tag{4.5}$$

with an unknown well-behaved function $a(\ln\tau,\ln\xi)$. Then they have obtained for the differential cross section at intermediate angles a formula like Orear's, i.e., it decreases exponentially with $(-t)^{1/2}$ and also exhibits additional oscillations.

On the other hand under the assumption that scattering amplitude behaves, for large s, as

$$f(s,t)\simeq g(t)s^{\alpha(t)}(\ln s)^{\beta(t)}(\ln\ln s)^{\gamma(t)}\cdots\cdots, \tag{4.6}$$

Gervais and Yudaráin[90] have shown

$$\alpha(t)\equiv 1 \quad \text{for} \quad t<0, \tag{4.7}$$

if all the cuts arising from Pomeranchuk pole are taken into account. This means that the diffraction peak hardly shrinks; this result is favorable for elastic scattering. In inelastic two body processes, however, shrinkings have been observed experimentally, for example, in πN charge exchange scattering, whereas the result of non-shrinking properties can be obtained also for inelastic processes in a similar way.

Although it can not be excluded at present the possibility that the slow shrinking due to logarithmic factors in Eq. (2.4) can reproduce the experimental data, a way out of the above situation is to assume that the Pomeranchuk intercept is

$$\alpha_P(0)=1-\epsilon, \qquad \epsilon>0. \tag{4.8}$$

Then, the branch points arising from exchange of a pole with trajectory $\alpha(t)$ and many Pomeranchons no longer accumulate at $\alpha_P(0)$, but rather give an effective twisted trajectory $\alpha_{eff}(t)$ which becomes flatter as $-t$ increases. In this case one obtains again a behavior similar to Regge pole type diffraction peak (not only for inelastic processes but also for elastic processes), but the total cross section should tend to zero as $s\to\infty$ (it can not be excluded experimentally at present). The twisted trajectories[91] may correspond to the bended trajectories which are sometimes employed in phenomenological Regge pole analyses; present experimental data do not give any definite answer to the question how the trajectories behave at larger $-t$ value, linear or bended.

Now, leaving aside the difficulty of the infinite accumulation of branch points associated with multi-Pomeranchon exchange, let us continue to present another aspects of moving cut contribution. As for the latter in the forward direction Freund and O'Donovan[92] have estimated the cut effect due to simultaneous two-pole exchange, by calculating simple AFS cut[86] contributions to the unphysical amplitude; they have suggested that it is smaller than 5% in inelastic $\pi\pi$ scattering at $t=0$, while it is seizable, i.e., up to 15~30% in elastic $\pi\pi$ channel. We note also that the presence of cuts introduces spin-dependent effects in the forward scattering, which are absent in pure Regge pole models. Dunne[93] has investigated the processes $pp\to pp$, $Kp\to K^*p$, $\gamma p\to\pi^0 p$ and $\pi p\to A_2 p$, and pointed out that these effects are actually present only if one considers cuts generated by three- or more-pole exchange; the spin dependent amplitude of pp scattering can behave as $s/(\ln s)^5$ at $t=0$ (pion conspiracy mechanism predicts that it behaves as $s^{\alpha_\pi(0)}$; see also Eq. (6.10)). Various discussions on the tests of moving cuts have been presented also in other processes; we refer the reader to Refs. 94 and 95 for NN scattering, Ref. 96 for photoproduction of π^0 from nucleon and Ref. 97 for the reactions $\pi^-(K^-)p\to K^+(\pi^+)\Sigma^-$.

4. 3 Unusuality of Pomeranchon and absorptive correction

In ordinary Regge pole analyses the trajectory of Pomeranchuk pole must be taken to have very small slope in negative t region, which is not even inconsistent with a fixed pole; this particular behavior comes from the nonshrinkage of the forward peaks in the elastic scattering differential cross sections. Extrapolating this trajectory function to the positive t region, we can not find any known resonances which correspond to the Regge recurrence of Pomeranchon. Other trajectories can have, in negative t region, slopes not too different from the almost universal slope of 1 $(\mathrm{GeV}/c)^{-2}$ and be associated with series of known resonances which correspond to their Regge recurrences. The assumption of the existence of Pomeranchon has been requested solely from the behavior of total cross section, that is to say, the latter seems to reach to constant value in the high energy limit. Therefore we should now consider that the Pomeranchuk trajectory does not belong to Regge trajectory but corresponds to other effect.

Harari[80] has suggested, using the resonance approximation to finite-energy sum rules, that the Pomeranchuk pole is mostly built from the nonresonating background in the low energy amplitudes, while the other poles can be usually described in terms of the resonance approximation for the low-energy region. The starting point of his argument is the observation that some processes such as K^+p, pp scatterings seem not to involve any important resonances in the low energy region whereas others such as K^-p, $\bar{p}p$ exhibit very rich resonance structure, and on the other hand that the Pomeranchuk pole dominates the high energy scattering of all these processes. According to this, in case of the scattering with no important resonances, cancellations should occur between the imaginary parts of the Regge pole contributions. Subsequently Gilman et al.[98] have given quantitative discussion along this scheme, considering the $C=+1$, $I=0$ t-channel amplitudes for πN and KN scattering to find Regge pole parameters of P'.

Oehme[99] has suggested the possibility that the Pomeranchuk pole is a fixed pole, but subsequently Finkelstein and Tan[100] have pointed out that Mandelstam's mechanism, which was suggested by Oehme, is not sufficient to allow the Pomeranchon to be fixed pole. There are some other arguments[101],[102] in relation to the possibility that Pomeranchon is not a usual Regge pole. In these arguments it has been intended to avoid the essential singularity at $t=0$ which comes from the series of branch points arising from simultaneous exchange of many Pomeranchuk poles in the case of $\alpha_P(0)=1$.

There is an attempt[103] to replace the contribution of the vacuum pole by shadow scattering in elastic scattering; in this attempt the absorption correction works also in inelastic processes (see §5 in *I*). On the other hand Squires[104] has derived a formula for cut contribuion to inelastic amplitude, which looks like absorption correction. Furthermore Phillips[105] has suggested the possibility that absorption correction plays, as the singularities in the complex J plane, some part in generating a conspiracy of poles.

§5. Dynamical determination of Regge parameters and rising trajectories

Regge parameters in potential scattering can be calculated either by solving directly Schrödinger equation or by using Cheng-Sharp equation[106),107)] for Regge parameters themselves. The latter scheme is based upon general principles such as analyticity and unitarity and so can be easily generalized[107)] to a relativistic case. There are, however, essential differences to be noticed between nonrelativistic and relativistic Cheng-Sharp schemes:

1) Only one channel in potential scattering but many in relativistic scattering.

2) Potential determines subtraction constants, while they should be determined by crossing in relativistic case.

3) Regge trajectory turns over above some finite energy in potential model, whereas all evidence points to infinitely rising trajectories in relativistic scattering.

Let us first discuss nonrelativistic Cheng-Sharp scheme, then extention of it to a relativistic case with a particular attention to the asymptotic behavior of Regge parameters, and finally mention a few applications.

5. 1 Analyticity properties of $\alpha(s)$ and $\beta(s)$ and unitarity

In potential scattering with a superposition of Yukawa potentials we know[108)] that $\alpha(s)$ and $\beta(s)/(4q^2)^{\alpha(s)}$, trajectory and reduced residue functions of a Regge pole, are real analytic in the cut s-plane with one cut extending from 0 to $+\infty$ unless the pole collides with another one (this case will not be considered here). Here, $s=q^2/2m$, q=relative momentum, and m=reduced mass=1/2 (put). Moreover, we know that for $s\to 0$

$$\left.\begin{aligned}&\mathrm{Im}\alpha(s)=\mathrm{O}(s^{\alpha(0)+1/2}),\\&\beta(s)=\mathrm{O}(s^{\alpha(0)}),\end{aligned}\right\}\tag{5.1}$$

and for $s\to\infty$

$$\left.\begin{aligned}&\alpha(s)\to -n-ig^2/2\sqrt{s}\,,\\&\beta(s)\to\frac{g^2}{2s}\,,\end{aligned}\right\}\tag{5.2}$$

for n-th Regge pole (denoted by α_n and β_n). g^2 is given by an integral of spectral function entering in a superposition of Yukawa potential. We then write dispersion relations for $\alpha_n(s)$ and $\beta_n(s)$ subtracted at infinity:

$$\alpha_n(s)=-n+\frac{1}{\pi}\int_0^\infty\frac{\mathrm{Im}\alpha_n(s')}{s'-s-i\varepsilon}ds',\tag{5.3}$$

$$\beta_n(s)=\frac{g^2}{2s}\prod_{i=1}^{n-1}\left(\frac{s-s_i}{s}\right)\exp\left[\frac{1}{\pi}\int_0^\infty\frac{\arg\beta_n(s')-\mathrm{Im}\,\alpha_n(s')\ln(s'/s)}{s'-s-i\varepsilon}ds'\right].\tag{5.4}$$

One may also write down dispersion relations subtracted at finite $s=s_0$; subtraction constants then depend on s_0. Note that $\beta_n(s)$ has $(n-1)$ zeros on the negative real axis which are explicit in (5.4).

Unitarity

$$a(J,s)-a^*(J^*,s)=2i\sqrt{s}\,a^*(J^*,s)a(J,s)\qquad(s>0)\tag{5.5}$$

relates $\mathrm{Im}\,\alpha(s)$ to $\beta(s)$. The relation depends on the representation of the partial-wave amplitude $a(J,s)$ and on how many poles are retained in unitarity relation. The simplest approximation is to take only one trajectory $\alpha_n(s)$ and write $a(J,s)=\beta_n(s)/(J-\alpha_n(s))$; in this one-pole approximation one obtains

$$\mathrm{Im}\,\alpha_n(s)=\sqrt{s}\,\beta_n(s), \qquad \beta_n(s): \text{ real.} \tag{5.6}$$

Combining this relation to (5.3) and (5.4) provides us a set of integral equations for Regge parameters (Cheng-Sharp equations). It is not hard to include more poles and improve the representation of $a(J, s)$ in order to get more reliable numerical results on $\alpha(s)$ and $\beta(s)$; we refer the reader to the literature[109),110)] on this point.

A set of infinitely many coupled Cheng-Sharp equations is, in principle, equivalent to Schrödinger equation. In practice, one- or two-pole approximations are made with appropriate representations of $a(J, s)$. Agreement of such crudely approximate solutions to the exact one[111)] is not bad and in some cases excellent.[106),110)]

The above Cheng-Sharp scheme can be easily extended[107)] to a relativistic case. If we assume the validity of Mandelstam representation, we can prove[107),112)] (1) one-cut real analyticity of $\alpha(s)$ and $\beta_{ij}(s)/(4q_iq_j)^{\alpha(s)}$, (2) holomorphy of the partial-wave amplitude $a_{ij}(J, s)$ for $\mathrm{Re}\,J>N\geqq 0$ and (3) the validity of once-subtracted dispersion relations for α and β_{ij} (the subscripts i, j, distinguish two-particle channels and q_i denotes relative momentum in the channel i). Here, and in the following, s is total *c.m.* energy squared. Two-particle unitarity reads

$$a_{ij}(J, s)-a_{ij}^*(J^*, s)=2i\sum_k \rho_k(s)a_{ki}^*(J^*, s)a_{kj}(J, s)\theta(s-s_k), \tag{5.7}$$

where $\rho_k(s)=2q_k/\sqrt{s}$ and s_k means threshold of the channel k. From (5.7) one can prove factorization of residues: $\beta_{ij}^2=\beta_{ii}\beta_{jj}$.

In exactly the same way as in a potential model, one writes down a set of coupled integral equations for α's and β's; the relation of the latter two now depends also on how many channels are taken into account. This set of equations, we stress, is valid only for trajectories which turn over above some finite energy. We also note that at this stage subtraction constants are free parameters to be determined from experiment.

There are, however, a number of experimental indications that Regge trajectories rise indefinitely with energy. We shall give a few in the next subsection. For such trajectories one needs two subtractions in the dispersion integral for $\alpha(s)$. Also the boundary condition on $\beta(s)$ may be quite different from what we would expect for Regge trajectories which turn over above some finite energy. On the other hand, there is a theoretical question as to whether infinitely rising and falling trajectories are allowed in the present framework of S-matrix theory. We shall come back to this question later.

5. 2 *Rising trajectories*

It is possible to determine $\alpha(s)$ and $\beta(s)$ at two or three points by measurements of the spin and width of the appropriate particles or for $s\leqq 0$ of the asymptotic behavior in the crossed channel.

Strictly speaking, it is only ρ trajectory that $\alpha(s)$ and $\beta(s)$ have been precisely determined for $-1\ (\mathrm{GeV}/c)^2\leqq s\leqq 0$. $\alpha(s)$ is clearly linear in s and $\beta(s)/(4q^2)^{\alpha(s)}$ has an exponential dependence for $s<0$. The trajectory may or may not be projected linearly to $s>0$. The former seems to be more likely; many resonances found by Focacci et al.[113)] seem to lie on the ρ trajectory if the lat-

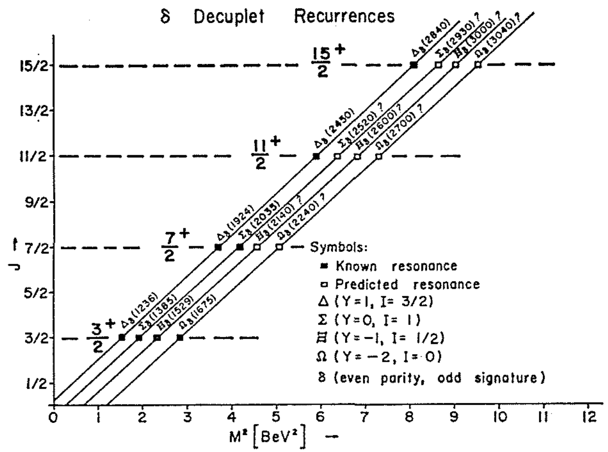


Fig. 7(a). Chew-Frautschi plot of baryon Regge recurrences of δ decuplet quoted from Ref. 61 (V. Barger, the fourth paper cited).

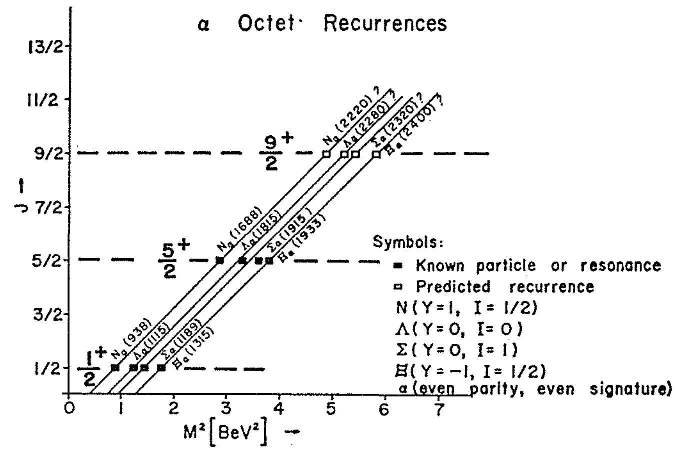


Fig. 7(b). The same for α octet recurrences as in Fig. 7(a).

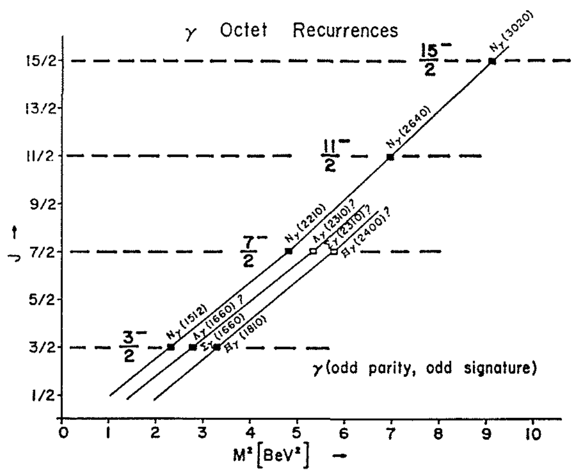


Fig. 7(c). The same for γ octet recurrences as in Fig. 7(a).

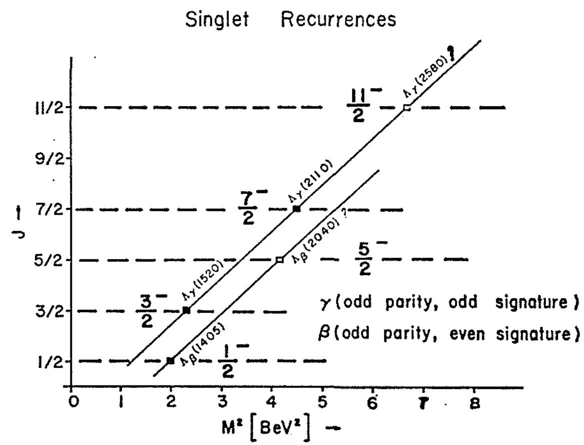


Fig. 7(d). The same for γ and β singlet recurrences as in Fig. 7(a).

ter increases linearly with s.

Regge dip mechanisms, applied to larger momentum transfer region as described in §2, if correct, wold constitute the first evidence for falling trajectories (also linearly in s) in the space-like region up to, say, $-s \approx 3$ $(\mathrm{GeV}/c)^2$.

As for baryon trajectories, the situation is less clear regarding the s-dependence of $\alpha(s)$ for $s<0$, while we have more information on $\alpha(s)$ for $s>0$ than for meson trajectories. Thus it appears that many πN resonances as well as $\bar{K}N$ ones lie on Regge trajectories which also increase like s with increasing s (up te about 3^2 $(\mathrm{GeV}/c)^2$). Fig. 7 depicts the situation for baryon resonances.

Now we are in a position to ask whether Regge trajectories which satisfy

$$\alpha(s) \to \pm\infty \quad \text{as} \quad s \to \pm\infty \tag{5.8}$$

coexist peacefully with the standard assumptions of the S-matrix theory, as first raised by Khuri.[114] The standard assumptions mean the following. Let $f(s) = \exp(|s|^{1/2-\varepsilon})$ with $\varepsilon>0$. For the simplest case of equal mass (m) spinless particles, (1) the scattering amplitude $f(s, t)$ is analytic in the cut s-plane and bounded by $f(s)$ in $|s|$ as $|s| \to \infty$ for fixed finite t; (2) $f(s, t)$ is bounded by $f(s)$ in $|s|$ as $|s| \to \infty$ for fixed $-1 \leqq \cos\theta_s \leqq 1$; (3) the partial-wave amplitude $a(J, s)$ satisfies necessary conditions for the Sommerfeld-Watson transform to be valid; (4) $a(J, s)$ is bounded by $f(s)$ as $|s| \to \infty$ for J not near a Regge pole. Moreover, (5) both $\alpha(s)$ and $\gamma(s) = \beta(s)/(s-4m^2)^{\alpha(s)}$ are real analytic with one cut from $s=4m^2$ to $+\infty$ and $\alpha(s)$ but not $\gamma(s)$ are bounded by $f(s)$ as $|s| \to \infty$ in all directions. We purposely did not include in (5) polynomial boundedness of $\beta(s)$ since it is clearly violated for trajectories (5.8).[115] Khuri[114] showed that the above assumptions imply, in a single pole approximation, that

$$\lim_{s\to\infty} |\beta(s) P_{\alpha(s)}(\cos\theta_s)| < f(s) \quad (\text{fixed} \quad \cos\theta_s). \tag{5.9}$$

Since $P_\alpha(z) < e^{\zeta\alpha}$ as $\alpha \to +\infty$ where $\zeta>0$, (5.8) is consistent with (5.9) if and only if $\beta(s)$ decreases exponentially or faster as $s \to +\infty$. Thus Regge trajectories satisfying (5.8) are not inconsistent with the standard assumptions if the residues $\beta(s)$ show an exponential or faster decrease with $\alpha(s)$ as $s \to +\infty$. Then how fast can Regge trajectories increase as $s \to \infty$? They cannot do more slowly than $|s|^{1/2}/\ln|s|$ as $|s| \to \infty$ as shown by Khuri. It is shown by Ida[116] that under physically plausible assumptions on $\alpha(s)$ (but not any on the residues $\beta(s)$) Regge trajectories cannot increase faster than linearly in s and more slowly than $|s|^{1/2+\varepsilon}$, with an arbitrary $\varepsilon>0$. For such trajectories one can write the dispersion relation

$$\alpha(s) = A(s-s_b) + B + \frac{s-s_b}{\pi} \int_{s_0}^{\infty} \frac{\operatorname{Im}\alpha(s')}{(s'-s)(s'-s_b)} ds', \tag{5.10}$$

where s_b is the subtraction point. This is consistent with experimental indications discussed above. We take (5.10) as being valid for Regge trajectories to be discussed below.

In passing we remark that an exponential decrease of $\beta(s)$ is consistent within the present experimental errors for πN resonances, as pointed out by Goldberg.[117]

5. 3 Dynamical determination of Regge parameters

Our next task is to find an integral representation for $\beta(s)$ in accord with the boundary condition (5.9) and the Mandelstam symmetry. The latter (see §§4.1 in I) is usually guaranteed by assuming

$$\frac{\beta(s)}{(4q^2)^{\alpha(s)}}=\frac{b_1(s)}{\Gamma(\alpha(s)+3/2)}. \tag{5.11}$$

If we put $b_1(s)=e^{-cs}b(s)$ with $c=d+A(1-\ln A)$ and $b(s)$ being a polynomial, then we have $\beta(s)\to e^{-ds}$ as $s\to+\infty$:

$$\frac{\beta(s)}{(4q^2)^{\alpha(s)}}=\frac{e^{-ds}e^{A(\ln A-1)s}\,b(s)}{\Gamma(\alpha(s)+3/2)} \quad (d>0). \tag{5.12}$$

(Note that $\alpha(s)\to As$ as $s\to+\infty$ and $\Gamma(z)\underset{z\to\infty}{\to}e^{z\ln z-z-(1/2)\ln z}$.) For this choice of $\beta(s)$, one has the following integral representation subtracted at $s=s_b$:

$$\beta(s)=b(s_b)e^{-ds}e^{-A(1-\ln A)}(4q^2)^{A(s-s_b)+B}\Gamma^{-1}(\alpha(s)+3/2)\times$$
$$\times\exp\Big[\frac{s-s_b}{\pi}\int_{s_0}^{\infty}\frac{ds'}{(s'-s-i\varepsilon)(s'-s_b)}\{\theta_\Gamma(s')+\theta_\beta(s')-\operatorname{Im}\alpha(s')\ln(q'^2/q^2)\}\Big], \tag{5.13}$$

where $\Gamma(\alpha(s)+3/2)=|\Gamma(\alpha(s)+3/2)|e^{i\theta_\Gamma(s)}$ and $\beta(s)=|\beta(s)|e^{i\theta_\beta(s)}$.

If we now use the unitarity (5.7) to the expression $a_{ij}(J,s)=\beta_{ij}(s)/(J-\alpha(s))$ and go to the pole position $J=\alpha(s)$, we obtain

$$\operatorname{Im}\alpha(s)\beta_{ij}(s)=\sum_k\theta(s-s_k)\rho_k(s)\beta_{ki}(s)\beta_{kj}(s) \qquad (\beta_{ij}:\text{ real}). \tag{5.14}$$

Integral representations of the type (5.13) for $\beta_{ij}(s)$ and (5.10) together with this relation (5.14) constitute a set of coupled integral equations for Regge parameters $\alpha(s)$, $\beta_{ij}(s)$.

The simplest case is to take elastic channel only $(i=j=k)$; (5.14) reads then

$$\operatorname{Im}\alpha(s)=\rho(s)\beta(s), \tag{5.15}$$

but this is not consistent with (5.12) and the fact that $\operatorname{Im}\alpha(s)\propto\Gamma_{tot}$ at a particular resonance position and Γ_{tot} has rather small s-dependence as implied by experiment. In order to avoid this inconsistency, we can invoke the explanation that in (5.14) sum over $k\neq i=j$ cannot be neglected as well as contributions from multiparticle intermediate states. One has then effectively instead of (5.15)[118]

$$\operatorname{Im}\alpha(s)=\rho(s)\beta(s)e^{ds} \tag{5.16}$$

where d is defined by (5.12). Then $\operatorname{Im}\alpha(s)\to s^{-1}$ as $s\to+\infty$.

The set of equations (5.10), (5.13) and (5.16) was used by Epstein and Kaus[118] for calculating the ρ trajectory in $\pi\pi$ scattering. Subtraction constants $b_1(s_b)$ and A (since $\operatorname{Re}\alpha_\rho(m_\rho^2)=1$, $B=1$ if $s_b=m_\rho^2$) are determined from $\Gamma_{\rho\to\pi\pi}$ and the intercept $\alpha_\rho(0)$, the latter being well determined from fit to $\pi^-p\to\pi^0n$ as seen in I. Their solution for a typical case is depicted in Fig. 8.

Mandelstam[119] has pointed out that in relativistic scattering the subtraction constants are not free parameters to be determined from experiment but are determined by crossing. From the latter point of view he proceeded by employing the narrow-resonance approximation ($\operatorname{Im}\alpha=0$ and the absence of exponential in (5.13) for β) and introducing the generalized superconvergence relation as crossing relation in a framework of bootstrap dynamics. His result

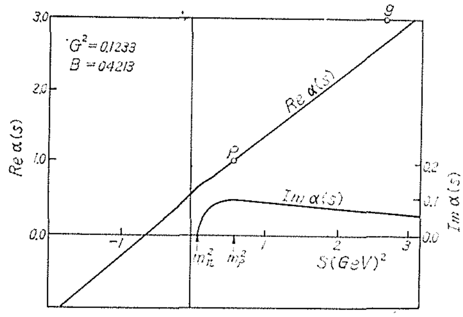


Fig. 8(a). The ρ-trajectory calculated by Epstein and Kaus (Ref. 118) in terms of the Cheng-Sharp equations described in the text (§§5.3).

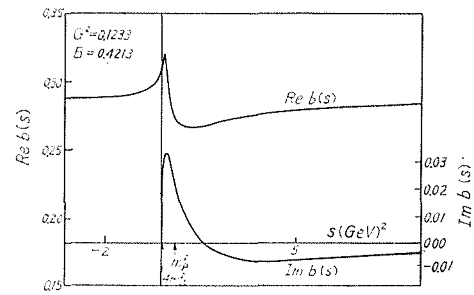


Fig. 8(b). The reduced residue function, $b(s)$, of the ρ-trajectory as calculated by Epstein and Kaus (Ref. 118) with the same parameters as in Fig. 8(a). $b(s)$ is defined by Eq. (5.12) in §§5.3.

is only preliminary and more realistic case should be carried out as to whether the above scheme is useful for calculating Regge parameters.

§6. Conspiracy of Regge poles and related problems

As briefly mentioned in *I*, it has been recognized[120),121)] that relativistic Regge pole theory has unsuspected conspiracy features. Roughly speaking, this amounts to saying that Regge pole does not exist as an isolated one at zero energy, that is to say, a set or sets of infinite Regge poles but not a single one do form an irreducible state at $t=0$. This will be made clear in the subsection 2.

It has been also realized that the Regge representation has a simple and natural group theoretical meaning. Joos' remark[122)] on this point has been followed by much work[123)~128)] on the so-called little group decomposition of the S-matrix elements which we shall summarize in the first part of this section.

It is to be noted that a possible conspiracy (in a narrow sense) was first realized by Volkov and Gribov[20)] who studied restrictions imposed on Regge pole model due to crossing and analyticity. The case they considered turned out to be a particular case of more general phenomenon which one encounters[129)] in the normal Regge theory. It will be treated in §§6.3.

Phenomenological applications of conspiracy of Regge poles to those reactions where normal Regge pole model fails to be applied have been rather successful as seen in §2. In the last part of this section some qualitative features of conspiracy will be revealed.

6. 1 *Little group decomposition of a two-body scattering amplitude*

The littel group decomposition of a two-particle scattering amplitude is the diagonalization of the S-matrix elements $S(p_3\lambda_3,\ p_4\lambda_4\,;\ p_1\lambda_1,\ p_2\lambda_2)$ for fixed energy $P=p_1+p_2=p_3+p_4$ or fixed momentum transfer $Q=p_1-p_3=p_4-p_2$. For fixed P_μ, we put together particles 1 and 2 into in-state and 3 and 4 into out-state. The in- and out-states are decomposed into partial waves each of which belongs to a unitary irreducible representation (IR) of the little group $SO(3)$ which leaves P_μ unchanged. Since S is an invariant operator which maps in-state onto out-state, Wigner-Eckart theorem states that S is diagonal in J, the Casimir value which specifies a unitary IR of $SO(3)$; [130] this corresponds to the usual partial-wave expansion of Jacob and Wick.[131]

Alternatively, one may make S diagonal according to a little group to which Q_μ is stable. This case corresponds to expansions of the S-matrix elements in terms of unphysical representations of the Poincaré group and Joos[122] was the first who recognized any possible relevance of this expansion to Regge theory. For $-Q_\mu^2=Q_0^2-\vec{Q}^2=t<0$, the little group is $SO(2,1)$; its IR[132] is specified by a parameter α which takes the significance of complex angular momentum in Regge theory; this IR is unitary and infinite-dimensional if $(\alpha+1/2)^2<0$ or $-1<\alpha<-1/2$ or α=integer or half integer. Grouping particles 1 and 3 to pseudo left-state and 4 and 2 to pseudo right-state and expanding each pseudo state into "crossed" partial waves which are characterized by α, one sees that S is made in a block form, each block being specified by α. Since pseudo states are not ordinary physical states, it is not necessary for α to be in the ranges of the unitary representations.[133] It is worth noticing, however, that the Regge background integral corresponds to the integral over one of unitary domains of α ($\alpha=-1/2+i\sigma$, $-\infty<\sigma<+\infty$; principal series).[123],[125],[126]

For $t=0$, the little group depends on the external masses since the apex of the light cone $Q_\mu^2=0$ is a physical point in the s-channel only for pair-wise equal mass cases. For the latter, therefore, it is the homogeneous Lorentz group $SO(3, 1)$; thus for this particular case S is diagonal in the Casimir values σ, M where σ is continuous and M discrete. A pair of parameters (σ, M) specifies an IR of $SO(3, 1)$.[134] Submatrix elements $S_{\sigma,M}$ may be assumed to be analytically continued in complex σ-plane and be meromorphic in σ; poles of $S_{\sigma,M}$ in σ-plane are called Lorentz poles. For unequal mass particles, on the other hand, the little group is $E(2)$, the Euclidean group of a plane. The expansion of S into unitary IR's of $E(2)$ is similar to the impact parameter representation.[135]

Since the scattering amplitude is analytic near $t=0$, the expansion according to $SO(2, 1)$ for $t<0$ should in some way go over into the one at $t=0$ (and eventually be continued to $t>0$). If we restrict ourselves to the unitary representations of the Poincaré group, this is the case in the general mass configuration, as shown by Feldman and Mathews.[127] It should be noted, however, that any Regge pole contribution does not have this property unless $m_1=m_3$ or $m_2=m_4$.[136] We shall come back to this problem later. In a particular case of pair-wise equal mass particles, this continuity leads to the conclusion, due to Toller and Sciarrino,[137] that Regge poles appear in families and one or more families should degenerate into a single Lorentz pole at $t=0$. This arrangement of Regge poles at $t=0$ (conspiracy) was implicit in Domokos and Suranyi[138] and encountered also by Freedman and Wang,[139] and should not depend on the external mass configuration since unitarity of the S-matrix requires Regge pole to couple with any pair of particles irrespective of their masses unless some selection rule forbids the coupling. This point of view has actually a deeper meaning as will be soon stated below.

6. 2 *Classification group of Regge bound states at zero energy*

As mentioned above a Regge-pole contribution to a scattering amplitude for unequal-mass particles is not analytic at $t=0$. In fact, it is proportional to the quantity

$$R_{\lambda\mu}=\beta(t)\frac{1}{2}\left\{\mathfrak{D}^{\alpha(t)}_{\lambda\mu}(0,\theta_t,0)\pm\eta\mathfrak{D}^{\alpha(t)}_{\lambda\mu}(0,\pi-\theta_t,0)\right\}, \tag{6.1}$$

where

$$\mathfrak{D}^{J}_{\lambda\mu}(\phi,\theta,-\phi)=-\frac{1}{\pi}\tan\pi(J-\lambda)\ E^{-J-1}_{-\mu-\lambda}(\phi,\theta,-\phi), \tag{6.2}$$

and $E^J{}_{\lambda\mu}$ is a rotation function of 2nd kind defined in Ref. 140. (η is a suitable phase factor.) Note that the $\mathfrak{D}^J{}_{\lambda\mu}$'s are identical to the functions entering in Mandelstam's Reggeization procedure. In particular, $\mathfrak{D}^J{}_{00}\,(\phi,\theta,-\phi)=-1/\pi\tan\pi J\times Q_{-J-1}(\cos\theta)$. Now the t-channel $c.m.$ scattering angle θ_t is related to the invariants, s, t, and external masses by

$$\cos\theta_t=\frac{t(\sum m_i^2-t-2s)+(m_1^2-m_3^2)(m_2^2-m_4^2)}{[\{t-(m_1+m_3)^2\}\{t-(m_1-m_3)^2\}\{t-(m_2+m_4)^2\}\{t-(m_2-m_4)^2\}]^{1/2}}\,. \tag{6.3}$$

Unless $m_1=m_3$ or $m_2=m_4$, $\cos\theta_t(t=0)=\varepsilon((m_1-m_3)(m_2-m_4))$ ($\varepsilon(x)=+1$ for $x>0$ and -1 for $x<0$). On the other hand, $d^J{}_{\lambda\mu}(\theta)=\mathfrak{D}^J{}_{\lambda\mu}(0,\theta,0)$ has a logarithmic singularity at $\cos\theta=-1$, as is easily seen by expressing it by a hypergeometric function. Therefore, if $m_1\neq m_3$ and $m_2\neq m_4$, $R_{\lambda\mu}$ is singular at $t=0$ since $\beta(t)$, the residue, cannot have zeros of infinite order.

Of course, the total amplitude should be analytic near $t=0$ and hence the above singularities have to be cancelled in some way.

The origin of the above trouble may be phrased in a group theoretical language as follows. According to Domokos and Tindle,[136] in order to investigate analytic property of a scattering amplitude near the trouble point, it is necessary to define the four-momentum Q_μ in such a way that 1) it is analytic near $t=0$ and 2) it goes into the light-like momentum $(p,0,0,p)$ for $t\to0$ which is the case for the general mass configuration. They proposed the choice $Q_\mu=(Q_0,\vec{Q})=(\sqrt{t+p^2},\ 0,0,p)$ and investigated the structure of the algebra of the little group which leaves this Q_μ invariant as function of t. It is found that the little group changes its structure for $t\to0$, in other words, it contracts. This contraction of the little group is the origin of the appearance of the singularity above.

In order to avoid the occurrence of the unwanted singularities in the transition amplitude, they proposed that a correct spin algebra of Regge bound states at $t=0$ should be such that 1) it is subalgebra of $SL(2,C)$ and 2) it is preserved at a contraction point ($t=0$). They have shown that only the algebra which has all the required properties is the algebra $SL(2,C)$ itself.[141]

In consequence we have the following results:

1) Regge poles at $t=0$ should be classified according to $SL(2,C)$ whatever the invariance group of an amplitude is at $t=0$. It is thus necessary to distinguish the latter from the classification group of Regge bound states at $t=0$.

2) Since an IR (σ,M)[142] of $SL(2,C)$ contains an infinite number of angular momentum states, $J=\sigma-\kappa$, $\kappa=0,1,\cdots\cdots$, a set of Regge poles at $J=\sigma-\kappa$, $\kappa=0$, $1,\cdots\cdots$ (a family) forms an irreducible state at $t=0$, called a Lorentz pole as the last subsection.

3) A Lorentz pole contribution to a scattering amplitude is regular at $t=0$,

while a Regge pole one is not.

4) If parity and PCT operation are included as symmetries, then each irreducible state is characterized by a set of parameters (σ, τ_l, η) if $M=0$ or (σ, τ_l, M) if $M>0$. Here τ_l is the Lorentz signature, η the intrinsic parity[143] of a Lorentz pole. In each irreducible state (σ, τ_l, M) there are at least two series of Regge poles with different η (parity doubling).

6. 3 *Invariant and helicity amplitudes and their relationship*

Let us now try to understand what relation exists between the above $SL(2, C)$ classification of Regge poles and kinematic constraints on Regge amplitudes at $t=0$. The latter are derived in various ways.[144] Here we want to discuss them on the basis of the relationship between invariant and helicity amplitudes. It may be written as

$$\tilde{f}_i^s = (K_i^s)^{-1} f_i^s = \sum_j \tilde{S}_{ij} F_j, \tag{6.4a}$$

$$\tilde{f}_i^t = (K_i^t)^{-1} f_i^t = \sum_j \tilde{T}_{ij} F_j, \tag{6.4b}$$

where $F_i(s, t)$ are invariant amplitudes free from kinematical singularities and zeros, while f_i^s and f_i^t are parity symmetry conserving helicity amplitudes[145] of the s-channel $1+2\to3+4$ and t-channel $\bar{4}+2\to3+\bar{1}$ scatterings, respectively. Each f_i^s is of the form

$$\bar{f}^s{}_{\lambda_3\lambda_4;\,\lambda_1\lambda_2} + \eta \bar{f}^s{}_{-\lambda_3-\lambda_4;\,\lambda_1\lambda_2}, \tag{6.5}$$

with a suitable phase factor η, where

$$\bar{f}^s{}_{\lambda_3\lambda_4;\,\lambda_1\lambda_2} = (1-\cos\theta_s)^{-|\lambda-\mu|}(1+\cos\theta_s)^{-|\lambda+\mu|} f^s{}_{\lambda_3\lambda_4;\,\lambda_1\lambda_2}, \tag{6.6}$$

and similarly for f_i^t. $S_{ij}=K_i^s\tilde{S}_{ij}$ and $T_{ij}=K_i^t\tilde{T}_{ij}$ are transformation coefficients where $\tilde{S}_{ij}$ and $\tilde{T}_{ij}$ are polynomials in s and t. Equations (6.4) state that $\tilde{f}_i^s$ and $\tilde{f}_i^t$ are free from kinematical singularities and zeros. It should be emphasized here that additional zeros will occur in certain linear combinations of $\tilde{f}_i^s$ or $\tilde{f}_i^t$ at a point where $\det\tilde{S}$ or $\det\tilde{T}$ vanish.[144],[146] In such a way we have kinematical constraints among otherwise independent amplitudes f_i.

Let us confine ourselves to the t-channel where Regge poles are assumed to appear. Kinematical constraints are written as

$$\sum_i [\tilde{T}]_{ij} \tilde{f}_i^t = O(t-t_0) \qquad \text{for} \qquad t\to t_0, \tag{6.7}$$

where $[\tilde{T}]_{ij}$ is a cofactor of $\tilde{T}$ belonging to $\tilde{T}_{ij}$ and we have assumed that $\det T = O(t-t_0)$ for $t\to t_0$. In the general mass case t_0 is equal to either $(m_1\pm m_3)^2$ or $(m_2\pm m_4)^2$ or 0. A conspiracy relation corresponds to the last case since only in this case there are involved helicity amplitudes which receive contributions from *different Regge poles*.

Let us now suppose that each $\tilde{f}_i^t$ in (6.7) is replaced by its Reggeized form

$$\tilde{f}_i^t \to (K_i^t)^{-1} \frac{1\pm e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)} R_{\lambda\mu}, \tag{6.8}$$

with appropriate λ, μ, where $R_{\lambda\mu}$ is given by (6.1). Before applying (6.7) (with $t_0=0$), we invoke the daughter mechanism (the $SL(2, C)$ conspiracy) in order to have Reggeized $\tilde{f}_i^t$ regular at $t=0$ since otherwise (6.7) loses its meaning. Then one sees that a conspiracy relation enforces a definite relationship between *different Regge families* unless it is satisfied in a trivial way.[147] (The equal mass case is not an exceptional case since Regge poles must be treated in the framework of $SL(2, C)$ at $t=0$.)

Such a relationship will correspond to the one which exists between *different Regge families* within an irreducible state (Lorentz pole) of $SL(2, C)$. By the same token, Cosenza et al.[148] have shown that any kinematical constraint of the type discussed above is automatically satisfied if one considers a Lorentz pole contribution in a generalized sense. Roughly speaking, the latter means that we calculate it as if the scattering amplitude at $t=0$ were invariant under $SL(2, C)$.

Various authors have worked out some special cases; we refer the reader to Ref. 20 for $N\bar{N}\to N\bar{N}$, Ref. 149 for $N\bar{N}\to Y\bar{Y}$, Ref. 146 for $\pi N\to VN$ and Ref. 150 for $\gamma N\to\pi N$.

Extension of the Lorentz pole concept to $t\neq 0$ is being tried by Salam et al.,[151] Cosenza et al.,[148] and Domokos et al.[152]

6. 4 *Physical meaning of the Lorentz quantum number M*

The quantity M of a Lorentz pole has a simple physical meaning other than its group theoretical one. First of all we note that

$$M\leqq \text{min. of } \{S_1+S_3,\ S_2+S_4\}, \tag{6.9}$$

where S_i is the spin of the i-th particle. Hence $M=0$, 1 for $NN\to NN$. There are thus three types of Lorentz poles which couple to $N\bar{N}$ system. Class I: $M=0$, $\eta=+1$. Class II: $M=0$, $\eta=-1$. Class III: $M=1$ (Lorentz signature is dropped out).

The asymptotic forward s-channel (not vanishing) helicity amplitudes due to a Lorentz pole exchange is given by[148]

$$f^s{}_{\lambda_3\lambda_4;\,\lambda_1\lambda_2}(s,\theta_s=0^0)\underset{s\to\infty}{\longrightarrow}\delta_{\lambda_3-\lambda_1;\,\lambda_4-\lambda_2}s^{\sigma-|M-|\lambda_3-\lambda_1||}, \tag{6.10}$$

and the same for all the mass configurations. (Note that $\lambda_1-\lambda_2=\lambda_3-\lambda_4$ implies angular momentum conservation.) Thus in the forward direction a single Lorentz pole with a given value of M contributes asymptotically only to the amplitudes with helicity flip equal to $\pm M$. The normal Regge theory, on the contrary, predicts no spin dependence in the asymptotic forward amplitudes.

The behaviour for $t\to 0$ of the helicity amplitudes due to an exchange of a Lorentz pole (σ, M) is given by[153]

$$f^s{}_{\lambda_3\lambda_4;\,\lambda_1\lambda_2}\propto t^{(1/2)|\lambda-\mu|+(\lambda_{min}-M)} \qquad \text{for} \qquad \lambda_{mim}\geqq M,$$

$$f^s{}_{\lambda_3\lambda_4;\,\lambda_1\lambda_2}\propto t^{(1/2)|\lambda-\mu|+(1/2)\lambda_{min}} \qquad \text{for} \qquad \lambda_{max}\geqq M>\lambda_{min},$$

$$f^s{}_{\lambda_3\lambda_4;\,\lambda_1\lambda_2}\propto t^{(1/2)|\lambda-\mu|+M-(1/2)(\lambda_{max}-\lambda_{min})} \qquad \text{for} \qquad \lambda_{max}<M, \tag{6.11}$$

where $\lambda_{max}=\text{max. of }(|\lambda|, |\mu|)$ and $\lambda_{min}=\text{min. of }(|\lambda|, |\mu|)$ with $\lambda=\lambda_1-\lambda_2$ and $u=\lambda_3-\lambda_4$. It should be noted that the helicity-non-flip amplitude is suppressed by a factor t^M, while the amplitudes with $\lambda_{min}=M$ or 0 and $\lambda_{max}\geqq M$ are allowed to contribute with the normal strength. It is due to this property and (6.10) that conspiracy works so well for the reactions $pn \to np$ and $\gamma p \to n\pi^+$ at high energies (*cf.* §2).

§7. Models for Regge poles and further outlook

So far we have presented the review of recent progress in the phenomenological Regge pole theory and some related topics which would become important in the course of future developments in the Regge pole theory.

The Regge pole hypothesis, combined with unitarity condition, leads to many interesting results. The recent development of phenomenological analyses in the high energy reactions of hadrons and the discovery of various kinds of higher resonances seem to make clear of the validity of the Regge pole hypothesis.

It seems that we are now in a stage to investigate the Regge pole model from a more fundamental standpoint.[154] As is well known, the Regge pole theory is originated from the bound state problem in the non-relativistic potential theory.[155] We may understand, therefore, that the Regge pole behaviour has its origin in the composite structure of hadrons. Much efforts are being made to connect the possible inner structure with the Regge behaviour.

In the following, we shall give a brief review of several suggestions in connection with the interpretation of Regge poles.

7. 1 The early stage of the Regge pole model

A few attempts were made to interpret the Regge poles in the early stage of Regge pole theory. Tanaka and Inoue[156] pointed out that the propagation of a particle with a finite extension might be closely related to the Regge pole behaviour. They found that both a particle with finite extension[157] and a "Regge field"[158] are equivallent to an assembly of the infinite local fields with different spins. A similar idea was also proposed independently by Licht.[159] He discussed several non-relativistic models in field theories, such as the rigid rotator models, the spherical harmonic oscillator model or the hydrogen-like atom model. He showed that the propagators of the second quantized fields, which depends on additional quantum numbers, exhibit Regge pole behaviour. These attempts are interesting in the sense that the Regge pole behaviour is understood to be due to the propagation of particle with internal structure.

7. 2 Models for the Regge poles

(a) Non-local models for Regge poles

Tanaka et al.[160] further investigated the bi-local field proposed by Yukawa in 1949[161] as the simplest model of an extended particle, where the bi-local field is quantized so as to represent the Regge pole behavior and compared with the one quantized due to Fierz's[157] decompositions. The interesting fact was pointed out that the bi-local field thus quantized takes the duplicate roles. First, when it arises with a time-like four momentum, it can be decomposed into local fields with different spins and masses. Second, when it is exchanged with a space-like four momentum between two colliding particles, it gives to the Regge pole behavior and the decomposition loses its meaning. In the former case, among the decomposed local fields only several ones belonging to low-lying masses and spins may be effective in the low energy region.

It becomes now important to examine the propagator of particles with internal structure especially for the case when the particles propagates with a space-like four momentum. We refer to the reader for the paper of Shirafuji[162] where the Green's functions of bi-local fields are investigated.

Tanaka and Hamamoto, and Salam et al. further proposed the idea of the Reggeization of unitary spin.[163]

(b) Feynman-diagram models for Regge poles

Another attempt to interpret the Regge pole behaviour is made in terms of an infinite tower of elementary particles.[72),164)~166)]

Van Hove[72] noticed the considerable success of single particle exchange models with absorptive corrections in the two body reactions in lower energy region, and tried to seek for conditions in which two models can become equivalent. He considered a series of particles with a mass spectrum $M(J)$ which satisfies the following property,

$$M(J) \to \infty \quad \text{for} \quad J \to \infty. \tag{7.1}$$

Under the single particle exchange approximation, the scattering amplitude is given as follows,

$$f(s,t) = \sum_J \frac{b(J,t)}{M^2(J)-t} P_J(\cos\theta_t). \tag{7.2}$$

With suitable analyticity properties of $b(J,t)$ and $M(J)$ in the complex J plane, the scattering amplitude for $s \to \infty$ is obtained by applying the well-known Sommerfeld-Watson transformation in agreement with the Regge behaviour. Also the Regg-pole amplitude is obtained by summing the most divergent terms resulting from an infinite set of single-particle exchanges.[164]

It is suggested by several authors[165),166)] that this model might be useful as a guide to understand such aspects of Regge pole model as daughters or conspirators.

On the other hand, some interesting comments are given by Halpern[167] to which we only refer the reader.

(c) Infinite component (local) models for Regge pole

The above argument leads us to further conjecture, which may be phrased as follows: Each elementary particle in the above infinite series can never be independent of each other, but strongly be related as if they are by nature some kind of unified entity. Therefore it seems natural to consider that such a series of particles can be described by the infinite-component field theory.

The first attempt was made by Cocho et al.,[168] who showed that the Regge behaviour can be obtained by calculating the amplitudes related to the exchange of the infinite multiplet which belongs to the IR of $SL(2,C)$. Later more detailed and systematic investigation have been made by Fronsdal,[169] considering the infinite multiplet of the groups $SO(3,1)$ and $SO(4,1)$. We can thus obtain the vertex function and compared it with the experimental data. We note that in these theories, the calculation was made in the time-like case and then analytically continued to the space-like region. The relationship of the locality of the infinite component theory and the non-local field is under discussion by several authors.[170),171)]

(d) Composite models for Regge poles

On the other hand, we should notice the recent development of composite model of hadrons.

In addition to the considerable success in the classification of hadrons based on the composite model, eager attempts[172] were made successfully to understand the high energy reactions in terms of the composite model. The "additivity assumption", that the scattering amplitudes are given by the sum of the scattering amplitudes of individual constituent particles, has given many successful explanations, especially for the forward or backward scatterings.

Further this model gives some remarkable results which are not predicted by Regge pole model[173] such as the spin dependence of the scattering amplitude or the ratio of the total cross section of baryon-baryon scattering to that of meson-baryon scattering.

Machida and Yoshida[174] investigated the relation between the exchange of a composite system and that of a Regge pole. For simplicity, they use a quark model and investigate the behaviour of the amplitude in the case when ρ trajectory is exchanged. The quark-antiquark scattering amplitude are Reggeized and πp charge exchange amplitudes are calculated by use of the additivity assumption. They concluded that the residue function must have an exponential dependence in t in order to fit the experimental data.

Another attempt was made by Squires[175] to calculate the ρ trajectory as a series of bound states of quark-antiquark system. Here the $\pi-\pi$ channel is treated as to give a small correction to the $q-\bar{q}$ system.

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REFERENCES

1) M. Bando and K. Morita, "*Present Status of the Phenomenological Regge Pole Theory. I*", the preceding paper refered to as *I* hereafter.
2) The references to this topic are seen in §6 in *I*.
3) D. D. Drobnis, J. Lales, R. C. Lamb, R. A. Lundy, A. Moretti, R. C. Niemann, T. B. Novery, J. Simanton, A. Yokosawa and D. D. Yovanovitch, Phys. Rev. Letters **20** (1968), 274.
4) R. C. Lamb, R. A. Lunty, T. B. Novey, A. Yokosawa and D. D. Yovanovitch, Phys. Rev. Letters **20** (1968), 353.
5) The references on this topic are seen in §§2.1.
6) The references on this topic are seen in §§2.2.
7) A. Pignotti, Phys. Rev. Letters **10** (1963), 416.
8) The references on this topic are seen in §5.
9) L. Van Hove, Phys. Letters **24B** (1967), 183.
10) C. F. Chew and S. C. Frautschi, Phys. Rev. Letters **8** (1962), 191.
11) R. C. Arnold, Phys. Rev. Letters **14** (1965), 657; Phys. Rev. **153** (1967), 1506.
12) V. Barger and D. Cline, Phys. Rev. Letters **20** (1968), 298; Phys. Rev. **153** (1967), 1575; D. H. Lyth, Phys. Rev. Letters **20** (1968), 641.
13) The references are seen in §7.
14) See for instance, L. Bertocchi, "*Theoretical Aspects of High Energy Phenomenology*",

Proceedings of the Heidelberg International Conference on Elementary Particles, 197 (1967) ; V. Barger, "*Total Cross sections and Elastic Scattering near the Forward Direction*", Review paper in the Topical Conference on High-Energy Collisions of Hadrons held at CERN in 1968.
15) In this section we omit the units of $t(u)$ and p_L ; unless mentioned otherwise, they are $(\mathrm{GeV}/c)^2$ and GeV/c, respectively.
16) See also §§3.3 in I.
17) W. Rarita, R. J. Riddell, Jr., C. B. Chiu and R. J. N. Phillips, Phys. Rev. **165** (1968), 1615.
18) R. J. N. Phillips, p. 268 in *Strong and Weak Interactions —Present Problems—*, Academic Press, New York and London, 1966.
19) T. L. Trueman and G. C. Wick, Ann. Phys. 26 (1964), 322.
20) M. L. Goldberger, M. T. Grisaru, S. W. MacDowell and D. Y. Wong, Phys. Rev. **120** (1960), 2250.
21) D. V. Volkov and V. N. Gribov, Soviet Phys. JETP 17 (1963), 720.
22) R. J. N. Phillips, Nucl. Phys. **B2** (1967), 394.
23) F. Arbab and J. M. Dash, Phys. Rev. **163** (1967), 1603.
24) I. Fukui and K. Morita, Prog. Theor. Phys. 36 (1968), 538.
25) A. M. Boyarski, F. Bulos, W. Busza, R. Diebold, S. D. Ecklund, G. E. Fischer, J. R. Rees and B. Richter, Phys. Rev. Letters **20** (1968), 300.
26) J. S. Ball, W. R. Frazer and M. Jacob, Phys. Rev. Letters **20** (1968), 518.
27) F. S. Henyey, Phys. Rev. **170** (1968), 1619.
28) K. Dietz and W. Korth, Phys. Letters **26B** (1968), 394.
29) A. Borgese and M. Colcocci, Nuovo Cim. **56A** (1968), 245.
30) D. Amati, G. Cohen-Tannoudji, R. Jengo and Ph. Salin, Phys. Letters **26B** (1968), 510.
31) J. P. Ader, M. Capdeville and Ph. Salin, Nucl. Phys. **B3** (1967), 407.
32) S. Frautschi and L. Jones, Phys. Rev. **164** (1967), 1918.
33) T. Yoshida, Prog. Theor. Phys. **39** (1968), 1603.
34) M. Barmawi, Phys. Rev. **166** (1968), 1857.
35) M. Nobuyama, to be published in Prog. Theor. Phys. Vol. 41, No. 3.
36) K. Koike, to be published in Prog. Theor. Phys. Vol. 41. No. 2.
37) C. B. Chiu, S.-Y. Chu and L.-L. Wang, Phys. Rev. **161** (1967), 1563.
38) S. Frautschi, Phys. Rev. Letters **17** (1966), 722.
39) A. Ashmore, C. J. S. Damerell, W. R. Frisken, R. Rubinstein, J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan and D. H. White, Phys. Rev. Letters **21** (1968), 387.
40) V. Barger and R. J. N. Phillips, Phys. Rev. Letters **20** (1968), 564.
41) G. T. Hoff, Phys. Rev. Letters **18** (1967), 816.
42) C. W. Akerlof, R. H. Hieber, A. D. Krish, K. W. Edwards, L. G. Ratner and K. Ruddick, Phys. Rev. Letters **17** (1966), 1106.
43) J. V. Allaby, G. Cocconi, A. N. Diddens, A. Klovning, G. Matthiae, E. J. Sacharidis and A. M. Wetherell, Phys. Letters **25B** (1967), 156.
44) K. Huang, C. E. Jones and V. L. Teplitz, Phys. Rev. Letters **18** (1967), 146.
45) K. Huang and S. Pinsky, preprint (Massachusetts Institute of Technology), CTP 19.
46) I. A. Sakar and J. H. Wojtaszek, Nuovo Cim. **56A** (1968), 92.
47) C. B. Chiu, J. Harte and C.-I. Tan, Nuovo Cim. **53A** (1968), 174.
48) T. Yoshida, to be published in Prog. Theor. Phys. Vol. 41, No. 2.
49) A. A. Anselm and T. T. Dyatlov, Phys. Letters **24B** (1967), 479.
50) T. Sawada, Phys. Rev. **165** (1968), 1848.
51) J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken and R. Rubinstein, Phys. Rev. Letters **21** (1968), 389.

52) V. Barger and D. Cline, Phys. Rev. Letters **21** (1968), 392.
53) Y. Noirot, M. Rimpault and Y. Saillard, Phys. Letters **26B** (1968), 454.
54) A. P. Contogouris, J. Tran Thank Van and M. Le Bellac, Nucl. Phys. **B5** (1968), 683.
55) G. Höhler and J. Giesecke, Phys. Letters **12** (1964), 149.
56) G. Höhler, G. Ebel and J. Giesecke, Physik **180** (1964), 430.
57) A. S. Carroll, J. F. Corbett, G. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg and W. S. C. Williams, Phys. Rev. Letters **16** (1966), 288.
58) The another interpretation of the dip mechanism is that it corresponds to the exceptional point of the trajectory function in a Regge pole exchange model (see §2).
59) A. O. Barut and H. Kleinert Phys. Rev. Letters **16** (1966), 950.
60) R. M. Heinz and M. H. Ross Phys. Rev. Letters **14** (1965), 1091.
61) V. Barger and D. Cline, Phys. Rev. Letters **16** (1966), 913; Phys. Rev. **155** (1966), 1792; V. Barger and M. Olesson, Phys. Rev. 151 (1966), 1123; V. Barger, Rev. Mod. Phys. 40 (1968), 129.
62) See §6 in I.
63) This was already pointed out by Barger and Olesson.
64) C. Schmid, Phys. Rev. Letters **20** (1968), 689.
65) C. B. Chiu and A. V. Stirling, Phys. Letters **26B** (1968), 236.
66) C. B. Chiu and A. Kotanski, CERN preprint.
67) See, for example, H. Feshbach "*The Compound Nucleus*" in *Nuclear Spectroscopy, part B*, edited by Fay Ajzenberg-Selove.
68) M. Bando and S. Machida, Prog. Theor. Phys. **40** (1968), 317.
69) The authors thank to Dr. Sawada for suggesting this graph.
70) M. Bando and T. Hattori, preprint KUNS 139 (Kyoto).
71) F. Takagi and Miyamura, private communication.
72) L. Van Hove, "*Theoretical Problems in Strong Interaction at High Energies*". Lectures at CERN in 1964.
73) P. D. B. Collins, R. C. Johnson and E. J. Squires, Phys. Letters **27B** (1968), 23.
74) K. Igi and S. Matsuda, Phys. Rev. Letters 18 (1967), 625; Phys. Rev. **163**, (1967), 1622.
75) R. Dolen, D. Horn and C. Schmid Phys. Rev. **166** (1968), 1768.
76) P. G. O. Freund, Phys. Rev. Letters **20** (1968), 235.
77) L. Durand, III, Phys. Rev. **166** (1968), 1680.
78) V. A. Alessandrini, D. Amati and E. J. Squiires, CERN preprint.
79) L. Van Hove, Phys. Letters **24B** (1967), 183.
80) H. Harari, Phys. Rev. Letters **20** (1968), 1396.
81) One can see detailed discussions in the following papers: W. Drechsler, CERN preprint, T. H. 841; R. Jengo, ,CERN preprint, T. H. 869.
82) V. N. Gribov and I. Pomeranchuk, Phys. Letters **2** (1962), 232.
83) S. Mandelstam, Nuovo Cim. **30** (1963), 1148.
84) C. E. Jones and V. L. Teplitz, Phys. Rev. 159 (1967), 1271.
85) S. Mandelstam and L. Wang, Phys. Rev. **160** (1967), 1490.
86) D. Amati, S. Fubini and A. Stanghellini, Phys. Letters **1** (1962), 29.
87) Such calculations for cuts are based on the assumption that Regge poles behave like as if they appear independent particles, (see §7) and not as poles in the total scattering matrix.
88) V. M. de Lany, D. J. Gross, I. J. Muzinich and V. L. Teplitz, Phys. Rev. Letters 18 (1967), 149.
89) See also §5 in *I*.
90) J.-L. Gervais and F. J. Yudurain, Phys. Rev. Letters **20** (1968), 27.
91) Y. Srivastava, Phys. Rev. Letters **19** (1967), 47.
92) P. G. O. Freund and P. J. O'Donovan, Phys. Rev. Letters **20** (1968), 1329.
93) S. A. Dunne, Phys. Rev. Letters **19** (1967), 1299.

94) D. Branson, S. Nussinov, S. B. Treiman and W. I. Weisberger, Phys. Letters **25B** (1967), 141.
95) R. J. N. Phillips, Phys. Letters **25B** (1967), 517.
96) F. Cooper, Phys. Rev. **167** (1968), 1314.
97) R. E. Mickens, Phys. Rev. **166** (1967), 1844.
98) R. J. Gilman, H. Harari and Y. Zarmi, Phys. Rev. Letters **21** (1968), 323.
99) R. Oehme, Phys. Rev. Letters **18** (1967), 1222.
100) J. Finkelstein and C.-I. Tan. Phys. Rev. Letters **19** (1967), 1061.
101) T. Sawada, Phys. Rev. **165** (1968), 1848.
102) J. H. Schwarz, Phys. Rev. **167** (1968), 1342.
103) G. Cohen-Tannoudji, A. Morel and H. Navelet, Nuovo Cim. **48A** (1967), 1075.
104) E. J. Squires, Phys. Letters **26B** (1968), 461.
105) R. J. N. Phillips, Nucl. Phys. **B2** (1967), 394.
106) H. Cheng and D. Sharp, Phys. Rev. **132** (1963), 1854.
107) H. Cheng and D. Sharp, Ann. Phys. **22** (1963), 481.
108) See, for instance, J. C. Taylor, Phys. Rev. **127** (1962), 2257.
109) S. C. Frautschi, P. E. Kaus and F. Zachariasen, Phys. Rev. **133** (1964), 1607.
110) D. Hankins, P. Kaus and C. J. Pearson, Phys. Rev. **137** (1965), B1034.
111) C. Lovelace and D. Masson, Nuovo Cim. **26** (1962), 472. A. Ahmadzadeh, P. Nurke and C. Tate, Phys. Rev. **131** (1963), 1315.
112) A. O. Barut and D. E. Zwanziger, Phys. Rev. **127** (1962), 974.
113) M. N. Focacci et al., Phys. Rev. Letters **17** (1966), 890.
114) N. N. Khuri, Phys. Rev. Letters **18** (1967), 1098.
115) C. E. Jones and V. L. Teplitz, Phys. Rev. Letters **19** (1967), 135.
116) M. Ida, Prog. Theor. Phys. **40** (1968), 901, 904.
117) H. Goldberg, Phys. Rev. Letters **19** (1967), 139.
118) G. Epstein and P. Kaus, Phys. Rev. **166** (1968), 1633.
119) S. Mandelstam, Phys. Rev. **166** (1968), 1539, 1391.
120) M. Toller, Nuovo Cim. **53A** (1968), 671, **54A** (1968), 296.
121) D. Z. Freedman and J.-M. Wang, Phys. Rev. **160** (1967), 1560.
122) H. Joos, *Lectures in Theoretical Physics* (University of Colorado Press, Boulder, Colo., 1964), Vol. 7A, p. 132.
123) L. Sertorio and M. Toller, Nuovo Cim. **33** (1964), 413.
124) M. Toller, Nuovo Cim. **37** (1965), 631; and Refs. 120.
125) F. T. Hadjiannou, Nuovo Cim. **44A** (1966), 185.
126) J. F. Boyce, J. Math. Phys. **8** (1967), 675.
127) G. Feldman and P. T. Mathews, Phys. Rev. **168** (1968), 1587.
128) As to potential scattering see G. Cocho, C. Fronsdal, I. T. Grodsky, and V. G. Kadyshevsky, Phys. Rev. **168** (1968), 1766.
129) See, for instance, E. Leader, Phys. Rev. **166** (1968), 1599; L. Bertocchi, Ref. 14.
130) In order to form a complete set in terms of which the S-matrix elements are expanded, the representations of the corresponding spinor group $SU(2)$ are required.
131) M. Jacob and G. C. Wick, Ann. Phys. **26** (1964), 322.
132) All the unitary IR's of the corresponding spinor group $SU(1,1)$ were found by V. Bargmann, Ann. Math. **48** (1947), 568. See also, A. O. Barut and C. Fronsdal, Proc. Ray. Soc. (London) **A287** (1965), 532.
133) For nonunitary representations neither completeness nor orthogonality conditions exist. So the reader may hesitate to use non-unitary representations in order to expand the S-matrix elements which sould be developed using a complete set of states. However, we may expect that all Regge poles are to the left of Re $J=-1/2$ for sufficiently large negative t. In such circumstances, we may represent the scattering amplitude by the $O(2,1)$ expansion of a square integrable function using a complete set of unitary IR's. When t increases to positive values, singularities may

penetrate the contour. Non-unitary representations may appear in this way and be identified with Regge pole terms.

134) All the unitary *IR*'s of the corresponding spinor group $SL(2, C)$ are found by I. M. Gel'fand and M. A. Naimark; see M. A. Naimark, *Linear Representations of the Lorentz Group*, London, 1964. See also, A. Sciarrino and M. Toller, J. Math, Phys. 8 (1967), 1252.

135) T. Miyazaki, Prog. Theor. Phys. **39** (1968), 1319.

136) G. Domokos and G. L. Tindle, Phys. Rev. (1968), 1906.

137) A. Sciarrino and M. Toller, Ref. 134.

138) G. Domokos and P. Suranyi, Nucl. Phys. **54** (1964), 529.

139) D. Z. Freedman and J.-M. Wang, Phys. Rev. **153** (1967), 1596.

140) M. Andrews and J. Gunson, J. Math. Phys. **5** (1964), 1391.

141) See also, G. Domokos, *Proceedings of the Novel Symposium* VIII, Göteborg, 1968.

142) The representations we are considering are generally not unitary since σ is related by $\sigma=\alpha(0)$ to the intercept $\alpha(0)$ of the leading trajectories in a Lorentz family and experimentally $\alpha(0)$ seems to differ from integer or half-integer except for possibly the Pomeranchuk pole.

143) The intrinsic parity η is defined by $\eta=P\cdot P_J$ for a given Regge pole with parity P and signature P_J. Natural parity corresponds to $\eta=+1$ and unnatural parity to $\eta=-1$.

144) G. Cohen-Tannoudji, A. Morel and H. Navelet, Ann. Phys. **46** (1968), 239. A complete references to this subject can be found in Ref. 14.

145) M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx and F. Zachariasen, Phys. Rev. **133** (1964), B145.

146) S. Frautschi and L. Jones, Phys. Rev. **167** (1968), 519.

147) Trivial solutions correspond to the so-called evasive ones.

148) G. Cosenza, A. Sciarrino, and M. Toller, Roma university preprint No. 158 (1968).

149) K. Morita, Prog. Theor. Phys. **39** (1968), 531; **40** (1968), 192 (Erratum) and **40** (1968), 1087.

150) S. Drell and J. Sullivan, Phys. Rev. Letters **19** (1967), 268; J. S. Ball, W. R. Frazer and M. Jacob, Ref. 26.

151) R. Delbourgo, A. Salam and J. Strathdee, Phys. Rev. **164** (1967), 1981; A Salam and J. Strathdee, *Proceeding of the Novel Symposium* VIII, Göteborg, 1968.

152) G. Domokos and P. Suranyi, *Proceedings of the Topical Conference on High Energy Hadron Collisions*, CERN, 1968.

153) E. Leader, Ref. 129.

154) S. Tanaka, Prog. Theor. Phys. Suppl. No. 41, 194 (1968).

155) T. Regge, Nuovo Cim. **14** (1959), 961; **18** (1960), 947.

156) S. Tanaka and T. Inoue, report at the annual conference of the Physical Society of Japan, Sept. 1962.

157) M. Fierz, Phys. Rev. **78** (1950), 184; Helv, Phys. Acta **23** (1950), 412.

158) The decomposition of Regge field is made by using the following formula,
$$\frac{P_\alpha(-z)}{\sin\pi\alpha}=\frac{1}{\pi}\sum_{l=0}^{\infty}\left(\frac{1}{\alpha-1}-\frac{1}{\alpha+l+1}\right)P_l(z),\qquad \text{for}\qquad |z|<1.$$

159) A. L. Licht, Nuovo Cim. **30** (1963), 117.

160) M. Bando, T. Inoue, Y. Takada and S. Tanaka, Prog. Theor. Phys. **38** (1967), 715.

161) H. Yukawa, Phys. Rev. **77** (1950), 219; **80** (1950), 1047.

162) T. Shirafuji, Prog. Theor. Phys. **39** (1968), 1047.

163) S. Tanaka and S. Hamamoto, Prog. Theor. Phys. **38** (1967), 1411; A. Salam and J. Strathdee, Phys. Rev. Letters **19** (1967), 339; R. Delbourgo, A. Salam, M. A. Rashid and J. Strathdee.

164) L. Durand, III, Phys. Rev. **161** (1967), 1610.

165) R. Blankebecler and R. L. Sugar, Phys. Rev. **168** (1968), 1597.

166) Y. Iwasaki, preprint.
167) M. B. Halpern, Phys. Rev. **159** (1967), 1328.
168) G. Cocho, C. Fronsdal, I. T. Grodsky and V. G. Kadyshevsky, Phys. Rev. **168** (1968), 1766.
169) C. Fronsdal, Phys. Rev. **168** (1968), 1845.
170) C. Fronsdal, Phys. Rev. **156** (1967), 1653.
171) H. D. I. Abarbanel and Y. Frishman, preprint (SLAC).
172) S. Machida and M. Namiki, Prog. Theor. **33** (1965), 125. Y. Fujimoto, S. Machida and M. Namiki, Prog. Phys. Suppl. Extra Number (1965), **304**: M. Imachi, T. Matsuoka, K. Ninomiya and S. Sawada, Prog. Theor. Phys. **40** (1968), 353. Complete references can be seen in the paper, "*Phenomenological Analyses of High Energy Scattering*". (Prog. Theor. Phys. Suppl. Extra Number (1967), Part II).
173) K. Ninomiya, S. Sawada, and M. Imachi, private commnication.
174) S. Machida and T. Yoshida, Prog. Theor. Phys. **39** (1968), 1304.
175) E. J. Squire and P. J. S. Watson, Ann. Phys. **41** (1967), 409; E. J. Squires, preprint (Trieste).