

NUMERICAL TABLES ON THE RANDOM FORCE

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ABSTRACT

Numerical tables are given for evaluating characteristics of the random force or/and its time derivative.

Introduction

The idea of the random force originated from Holtsmark (1919) was first applied to the star field by Chandrasekhar and von Neumann (1942, 1943). According to the results of these authors, however, all the moments of the random force except for its mean do not converge. Such inaptness may be avoided by considering a test-star to have a non-zero radius, instead of a point mass, as shown by Camm (1963). Nevertheless, even with Camm's formulae the computed values of the mean and the variance turn out unreasonably larger than expected. Hence, the senior author tried to reformulate the random force under the following assumptions: (i) the stars with the same mass and radius are scattered randomly but uniformly in the large, (ii) the frequency distribution of the field-star's velocity is Maxwellian, and (iii) only the field-stars with the escape-velocities referred to the test-star can contribute to the random force on the test-star. Actual treatment to the required theoretical formulae is given elsewhere (Shimizu, 1969). Because of the last assumption, on which the binary encounter theory has also been based, the new result comes out reasonable and seems to be applicable to the real stellar system. But, the newly obtained theoretical formulae are not expressed, in general, by the elementary functions. Hence, we provided here the numerical tables together with the theoretical formulae for computing characteristics of the random force.

The following notations are used throughout.

G : gravitational constant,

m : mass of the star,

V_0 : velocity of the test-star,

δ : effective radius of the test-star,

n : average star density in space,

σ : dispersion in one velocity-component for field-stars, so the dispersion in the space-velocity for field-stars should be $\sqrt{3}\sigma$.

$\mu = 4Gm$, $x = \mu/2\sigma^2\delta$, $y = 2V_0^2/\sigma^2$, $y_1 = V_0/\sqrt{2}\sigma$.

$\mathbf{F} = (X, Y, Z)$: force acted on the unit mass of the test-star,

$F = |\mathbf{F}|$, $\dot{\mathbf{F}} = (\dot{X}, \dot{Y}, \dot{Z}) = d\mathbf{F}/dt$, $\dot{F} = dF/dt = |\dot{\mathbf{F}}|$,

$\Gamma\left(\nu + \frac{1}{2}\right)$: gamma function,

$\gamma\left(\nu + \frac{1}{2}, x\right)$: incomplete gamma function of the first kind,

$$I\left(\nu + \frac{1}{2}, x\right) = \gamma\left(\nu + \frac{1}{2}, x\right) / \Gamma\left(\nu + \frac{1}{2}\right),$$

$$\Pi(0, 3/2) = e^{-y/4} \sum_{\nu=0}^{\infty} \left\{ \frac{(y/4)^{\nu}}{\nu!} I\left(\nu + \frac{3}{2}, x\right) \right\},$$

$$\begin{aligned} \Pi\left(\frac{2k+1}{2}, \frac{2k+2l+1}{2}\right) &= e^{-y/4} \sum_{\nu=0}^{\infty} \left\{ \frac{(y/4)^{\nu}}{\nu!} \left(\nu + \frac{2k+1}{2}\right) \left(\nu + \frac{2k+3}{2}\right) \dots \right. \\ &\quad \left. \times \left(\nu + \frac{2k+2l-1}{2}\right) I\left(\nu + \frac{2k+2l+1}{2}, x\right) \right\}. \end{aligned}$$

(A) The moments of the random force acted on the unit mass of the moving test-star with its velocity V_0 in the z -direction due to the escaping field-stars are given by

$$\overline{F^2} = n_0 \sigma^2 \mu \text{CO2},$$

$$\overline{F^4} = n_0 \sigma^{10} \mu^{-1} \text{CO4} [1 + n_0 (5\mu^3 \text{CO2}^2 / 3\sigma^6 \text{CO4})],$$

$$\overline{F^6} = n_0 \sigma^{18} \mu^{-3} \text{CO6} [1 + n_0 (7\mu^3 \text{CO2} \cdot \text{CO4} / \sigma^6 \text{CO6}) + n_0^2 (35\mu^6 \text{CO2}^3 / 9\sigma^{12} \text{CO6})],$$

$$\begin{aligned} \overline{F^8} &= n_0 \sigma^{26} \mu^{-5} \text{CO8} [1 + n_0 (63\mu^3 \text{CO4}^2 / 5\sigma^6 \text{CO8}) + n_0 (12\mu^3 \text{CO2} \cdot \text{CO6} / \sigma^6 \text{CO8}) \\ &\quad + n_0^2 (42\mu^6 \text{CO2}^2 \cdot \text{CO4} / \sigma^{12} \text{CO8}) + n_0^3 (35\mu^3 \text{CO4}^4 / 4\sigma^{18} \text{CO8})], \end{aligned}$$

where CO2, CO4, CO6, CO8 are defined as followings.

$$\text{CO2} = (\pi/2)x[1 + (1/x)\Pi(3/2, 5/2) - \Pi(0, 3/2)],$$

$$\text{CO4} = (\pi/10)x^5[1 + (1/x^5)\Pi(3/2, 13/2) - \Pi(0, 3/2)],$$

$$\text{CO6} = (\pi/18)x^9[1 + (1/x^9)\Pi(3/2, 21/2) - \Pi(0, 3/2)],$$

$$\text{CO8} = (\pi/26)x^{13}[1 + (1/x^{13})\Pi(3/2, 29/2) - \Pi(0, 3/2)].$$

Throughout in this paper, the numerical values are expressed in the units of solar mass, parsec, and 10^6 years. As the unit of the velocity, km/s is adopted but this is nearly equal to pc/ 10^6 yr.

Table 1 gives the numerical values of CO2, CO4, CO6, CO8 for assigned values of x and y .

Table 2 gives $\log \text{CO2}$, $\log \text{CO4}$, $\log \text{CO6}$, $\log \text{CO8}$.

Table 3 gives $E1 = 5\text{CO2}/3\text{CO4}$, $E2 = 7\text{CO2} \cdot \text{CO4}/\text{CO6}$, $E3 = 35\text{CO2}/9\text{CO6}$, $E4 = 63\text{CO2}/5\text{CO8}$, $E5 = 12\text{CO2} \cdot \text{CO6}/\text{CO8}$, $E6 = 42\text{CO2} \cdot \text{CO4}/\text{CO8}$, $E7 = 35\text{CO2}/4\text{CO8}$.

Table 4 gives $\log E1$, $\log E2$, $\log E3$, $\log E4$, $\log E5$, $\log E6$, $\log E7$.

(B) The moments of the time derivative of the random force are given by

$$\overline{\dot{F}^2} = n_0 (\sigma^8 / \mu) (3D21 + 10D22^*),$$

$$\overline{\dot{Z}^3} = n_0 (\sigma^{15} / \mu^3) (D31^* - (2/3)D32^*),$$

$$\begin{aligned} \overline{\dot{F}^4} &= n_0 (\sigma^{22} / \mu^5) (D41 - (1/3)D42^* + 8D43^*) \\ &\quad + 2n_0^2 (\sigma^{16} / \mu^2) (D21 + 3D22^*) (5D21 + 16D22^*), \end{aligned}$$

where \dot{Z} is the component of $\dot{\mathbf{F}}$ along the motion of the test-star or along the z -axis. Terms marked with * vanish when the test-star is at rest.

Table 5 gives D21, D22, D31, D32, D41, D42, D43.

Table 6 gives $\log D21$, $\log D22$, $\log D31$, $\log D32$, $\log D41$, $\log D42$, $\log D43$.

(C) The cross moments of the random force and its time derivative when the test star is moving in the z -direction are given by

$$\overline{X^2 \dot{Z}} = \overline{Y^2 \dot{Z}} = -n_0 (\sigma^9 / \mu) E12^*,$$

$$\overline{Z^2 \dot{Z}} = 2n_0 (\sigma^9 / \mu) E12^*,$$

$$\overline{F^2 \dot{F}^2} = n (\sigma^{16} / \mu^3) (45E221 + 70E222^*) \text{ etc.}$$

X and Z are respectively the x- and the z- component of the random force. Terms marked by * vanish when the test-star is at rest.

Table 7 gives E12, log E12, E221, log E221, E222, log E222.

(D) Table 8 gives $I\left(\nu + \frac{1}{2}, x\right) = \gamma\left(\nu + \frac{1}{2}, x\right) / \Gamma\left(\nu + \frac{1}{2}\right)$.

Table 9 gives $G(N) \equiv \Gamma(\nu)$, $\log G(N) \equiv \log \Gamma(\nu)$, $G\left(N + \frac{1}{2}\right) \equiv \Gamma\left(\nu + \frac{1}{2}\right)$, $\log G\left(N + \frac{1}{2}\right) \equiv \log \Gamma\left(\nu + \frac{1}{2}\right)$.

(E) The probability $\tau(x, y_1)$ that an escaping star occurs at a distance x from the test-star is given by

$$\tau(x, y_1) = 1 - \frac{2}{\sqrt{\pi}} e^{-y_1^2} \sum_{\nu=0}^{\infty} \left\{ \frac{y_1^{2\nu}}{\nu!} I\left(\nu + \frac{3}{2}, x\right) \right\}.$$

Table 10 gives $T(x, y) \equiv \tau(x, y_1)$, $dT/dx \equiv \frac{\partial \tau}{\partial x}$, $ddT/ddx \equiv \frac{\partial^2 \tau}{\partial x^2}$,

$$I(x, y) \equiv - \int \frac{\partial \tau}{\partial x} \log x dx, J(x, y) \equiv \int \frac{1}{x} \frac{\partial \tau}{\partial x} dx.$$

Table 11 gives INT. $T(x, y) \equiv \int \tau(x, y_1) dx$, INT. $T/x \equiv \int \frac{\tau(x, y_1)}{x} dx$,

$$\text{INT. } T/x*2 \equiv \int \frac{\tau(x, y_1)}{x^2} dx, \text{ INT. } T/x*3 \equiv \int \frac{\tau(x, y_1)}{x^3} dx,$$

$$\text{INT. } T/x*4 \equiv \int \frac{\tau(x, y_1)}{x^4} dx, \text{ INT. } T \times x \equiv \int \tau(x, y_1) x dx.$$

In the above tables except for Table 8 and Table 9, each value for $x=50.0$ can hold still for $x>50.0$. And for the conveniency we expressed log 0 as 0.

(F) Table 12 gives the values of x and y for various sets of σ , δ and μ .

(G) Examples for calculations of the moments of $F = \sqrt{X^2 + Y^2 + Z^2}$ and its time derivative \dot{F} .

We shall show four examples in the case of $\delta = 4.51 \times 10^{-8}$ parsec ($= 2R_{\odot}$). As regards the frequency distribution function of F we adopted an approximate formula as follows,

$$w(F) = \frac{\alpha^{\frac{\gamma+1}{\beta}}}{2\Gamma\left(\frac{\gamma+1}{\beta}\right)} e^{-\alpha F^{\beta}} F^{\gamma},$$

where α, β, γ are adjustable constants. Then the k-th moment of F is given by

$$M_k = \frac{\Gamma\left(\frac{k+\gamma+1}{\beta}\right)}{\alpha^{\frac{k}{\beta}} \Gamma\left(\frac{\gamma+1}{\beta}\right)},$$

or

$$\log M_k = \log \Gamma\left(\frac{\gamma+1+k}{\beta}\right) - \log \Gamma\left(\frac{\gamma+1}{\beta}\right) - \frac{k}{\beta} \log \alpha.$$

Fitting with $\frac{\gamma+1}{\beta} = 1/2$ or 1 seems to be most convenient and probable.

The probability that a force less than an assigned value of F_0 occurs, or $\text{Pr}(F \leq F_0)$, is then given by the formula

$$\text{Pr}(F \leq F_0) = \frac{\gamma\left(\frac{\gamma+1}{\beta}, \alpha F_0^{\beta}\right)}{\Gamma\left(\frac{\gamma+1}{\beta}\right)} \equiv I\left(\frac{\gamma+1}{\beta}, \alpha F_0^{\beta}\right).$$

Case (a) $n_0=0.1, \sigma=10, \mu=1.8 \times 10^{-2}, y=2, x=2 \times 10^4$.

| from table | constants | fitted value |
|--|-------------------|----------------------------|
| $\log \bar{F}^2 = -0.2475$ | | $\log \bar{F}^2 = 0.8851$ |
| $\log \bar{F}^4 = 13.2894$ | $\alpha = 7.1$ | $\log \bar{F}^4 = 12.3185$ |
| $\log \bar{F}^6 = 28.4489$ | $\beta = 2/19$ | $\log \bar{F}^6 = 28.0077$ |
| $\log \bar{F}^8 = 44.3725$ | $\gamma = -17/19$ | $\log \bar{F}^8 = 46.4754$ |
| $\bar{F} = 9.0 \times 10^{-3}, \Pr(F \leq \bar{F}) = 0.9869$ | | |
| $\sqrt{\bar{F}^2} = 7.5 \times 10^{-1}, \Pr(F \leq \sqrt{\bar{F}^2}) = 0.9991$ | | |

As for \bar{F} too, the same approximate formulae as above are applicable.

| from table | |
|--------------------------------|--------------------------------------|
| $D21 = 2.78555 \times 10^2$ | |
| $D22 = 5.36165 \times 10$ | $\bar{F}^2 = 8.0 \times 10^{11}$ |
| $D41 = 1.70579 \times 10^{10}$ | $\sqrt{\bar{F}^2} = 8.9 \times 10^5$ |
| $D42 = 1.88187 \times 10^9$ | |
| $D43 = 6.37439 \times 10^6$ | $\bar{F}^4 = 8.7 \times 10^{39}$ |

Case (b) $n_0=3, \sigma=1, \mu=7.2 \times 10^{-2}, y=2, x=8 \times 10^5$

| from table | constants | fitted value |
|---|-----------------|----------------------------|
| $\log \bar{F}^2 = 0.8376$ | | $\log \bar{F}^2 = 0.4024$ |
| $\log \bar{F}^4 = 4.1644$ | $\alpha = 3.0$ | $\log \bar{F}^4 = 4.3404$ |
| $\log \bar{F}^6 = 10.1199$ | $\beta = 2/7$ | $\log \bar{F}^6 = 9.8083$ |
| $\log \bar{F}^8 = 16.8383$ | $\gamma = -5/7$ | $\log \bar{F}^8 = 16.2841$ |
| $\bar{F} = 2.6 \times 10^{-1}, \Pr(F \leq \bar{F}) = 0.8672$ | | |
| $\sqrt{\bar{F}^2} = 2.6, \Pr(F \leq \sqrt{\bar{F}^2}) = 0.9795$ | | |

| from table | |
|--------------------------------|------------------------------------|
| $D21 = 2.78555 \times 10^2$ | |
| $D22 = 5.36165 \times 10$ | $\bar{F}^2 = 6.0 \times 10^2$ |
| $D41 = 1.70579 \times 10^{10}$ | $\sqrt{\bar{F}^2} = 2.5 \times 10$ |
| $D42 = 1.88187 \times 10^9$ | |
| $D43 = 6.37438 \times 10^6$ | $\bar{F}^4 = 2.7 \times 10^{16}$ |

Case (c) $n_0=50, \sigma=5, \mu=1.8 \times 10^{-2}, y=5, x=8 \times 10^3$

| from table | constants | fitted value |
|---|-------------------|----------------------------|
| $\log \bar{F}^2 = 1.9876$ | | $\log \bar{F}^2 = 2.9943$ |
| $\log \bar{F}^4 = 13.7148$ | $\alpha = 3.3$ | $\log \bar{F}^4 = 13.0056$ |
| $\log \bar{F}^6 = 26.4723$ | $\beta = 2/13$ | $\log \bar{F}^6 = 25.9096$ |
| $\log \bar{F}^8 = 40.2152$ | $\gamma = -11/13$ | $\log \bar{F}^8 = 40.7066$ |
| $\bar{F} = 7.5 \times 10^{-1}, \Pr(F \leq \bar{F}) = 0.9588$ | | |
| $\sqrt{\bar{F}^2} = 9.9, \Pr(F \leq \sqrt{\bar{F}^2}) = 0.9912$ | | |

| from table | |
|--------------------------------|--------------------------------------|
| $D21 = 5.08873 \times 10^2$ | |
| $D22 = 2.16950 \times 10^2$ | $\bar{F}^2 = 4.0 \times 10^{11}$ |
| $D41 = 1.26771 \times 10^{11}$ | $\sqrt{\bar{F}^2} = 6.3 \times 10^5$ |
| $D42 = 1.81042 \times 10^{10}$ | |
| $D43 = 1.01546 \times 10^8$ | $\bar{F}^4 = 7.9 \times 10^{32}$ |

Case (d) $n_0=50, \sigma=1, \mu=1.8 \times 10^{-2}, y=0.0, x=2 \times 10^5$

| from table | constants | fitted value |
|----------------------------|-----------------|----------------------------|
| $\log \bar{F}^2 = 0.3264$ | | $\log \bar{F}^2 = -0.4402$ |
| $\log \bar{F}^4 = 5.4525$ | $\alpha = 4.8$ | $\log \bar{F}^4 = 4.7861$ |
| $\log \bar{F}^6 = 12.2818$ | $\beta = 1/5$ | $\log \bar{F}^6 = 12.0237$ |
| $\log \bar{F}^8 = 19.9207$ | $\gamma = -4/5$ | $\log \bar{F}^8 = 20.7116$ |

$$\bar{F} = 4.8 \times 10^{-2}, \quad \Pr(F \leq \bar{F}) = 0.9261$$

$$\sqrt{\bar{F}^2} = 1.5, \quad \Pr(F \leq \sqrt{\bar{F}^2}) = 0.9942$$

from table

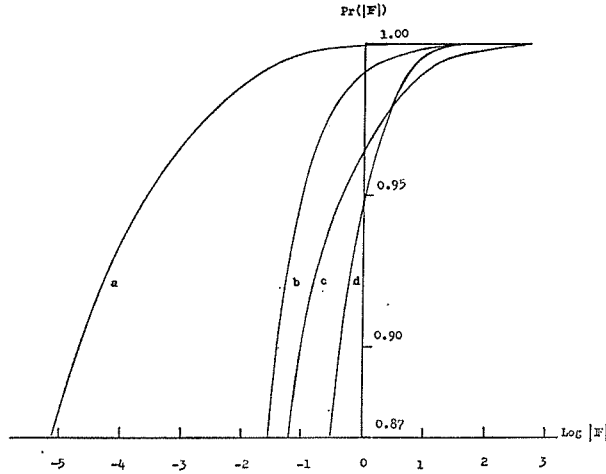
$$D21 = 1.64934 \times 10^2, \quad \bar{F}^2 = 1.4 \times 10^6$$

$$D41 = 1.65593 \times 10^9, \quad \sqrt{\bar{F}^2} = 1.2 \times 10^3$$

$$\bar{F}^4 = 4.4 \times 10^{10}$$

The part of curve of these approximated distribution function $\Pr(|F|) \equiv \Pr(F \leq F_0)$ is shown in Figure I.

Figure I



Caption of Tables

- Table 1. CO2, ..., CO8.
- Table 2. logCO2, ..., logCO8.
- Table 3. E1 = 5CO2/3CO4, ..., E7 = 35CO2/4CO8.
- Table 4. logE1, ..., logE7.
- Table 5. D21, ..., D43.
- Table 6. logD21, ..., logD43.
- Table 7. E12, ..., logE222.
- Table 8. $I\left(\nu + \frac{1}{2}, x\right) = \gamma\left(\nu + \frac{1}{2}, x\right) / \Gamma\left(\nu + \frac{1}{2}\right)$.
- Table 9. $G(N) \equiv \Gamma(\nu), \dots, \log G\left(N + \frac{1}{2}\right) \equiv \log \Gamma\left(\nu + \frac{1}{2}\right)$.
- Table 10. $T(x, y) \equiv \tau(x, y_1), \dots, J(x, y) \equiv \int \frac{1}{x} \frac{\partial \tau}{\partial x} dx$.

Table 11. $\text{INT. } T(x, y) \equiv \int \tau(x, y_1) dx, \dots, \text{INT. } T \times x \equiv \int \tau(x, y_1) x dx.$

Table 12. x and y .

REFERENCES

- Holtmark, J., *Ann. Phys.*, **58**, 577 (1919).
Chandrasekhar, S. and von Neumann, J., *Ap. J.*, **95**, 489 (1942); **97**, 1 (1943).
Camm, G., *M.N.*, **126**, 283 (1963).
Shimizu, T., to be published in *Publ. Ast. Soc. Japan*.

Table 3. (continued)

| y = 10.0 | | | | | | | | | | y = 0.0 | | | | | | | | | | | | | |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| x | E1 | E2 | E3 | E4 | E5 | E6 | E7 | x | E1 | E2 | E3 | E4 | E5 | E6 | E7 | x | E1 | E2 | E3 | E4 | E5 | E6 | E7 |
| 1.00E-04 | 1.39300E-13 | 1.99200E-13 | 1.59200E-13 | 8.65900E-25 | 0.00000E-00 | 0.00000E-00 | 0.00000E-00 | 1.00E-04 | 1.31160E-01 | 1.29260E-01 | 1.29260E-01 | 4.93450E-01 | 0.00000E-00 | 0.00000E-00 | 0.00000E-00 | 1.00E-04 | 1.31160E-01 | 1.29260E-01 | 1.29260E-01 | 4.93450E-01 | 0.00000E-00 | 0.00000E-00 | 0.00000E-00 |
| 5.00E-04 | 1.04720E-11 | 1.59380E-11 | 1.59380E-11 | 5.26890E-21 | 2.17817E-11 | 0.00000E-00 | 0.00000E-00 | 5.00E-04 | 1.18200E-01 | 1.11990E-01 | 2.17817E-01 | 1.09156E-01 | 0.00000E-00 | 0.00000E-00 | 0.00000E-00 | 5.00E-04 | 1.18200E-01 | 1.11990E-01 | 2.17817E-01 | 1.09156E-01 | 0.00000E-00 | 0.00000E-00 | 0.00000E-00 |
| 1.00E-03 | 1.39300E-10 | 1.99200E-10 | 1.59200E-10 | 8.65900E-19 | 1.08918E-10 | 2.72721E-10 | 2.69400E-20 | 1.00E-03 | 1.01160E-01 | 1.02960E-01 | 1.99200E-01 | 1.00125E-01 | 1.04350E-01 | 2.04900E-01 | 2.96440E-01 | 1.00E-03 | 1.01160E-01 | 1.02960E-01 | 1.99200E-01 | 1.00125E-01 | 1.04350E-01 | 2.04900E-01 | 2.96440E-01 |
| 5.00E-03 | 1.04720E-08 | 1.59380E-08 | 1.59380E-08 | 5.26890E-15 | 8.23435E-07 | 2.17815E-08 | 1.72440E-16 | 5.00E-03 | 8.02000E-01 | 8.19970E-01 | 1.57820E-01 | 7.83190E-01 | 1.83190E-01 | 4.69860E-01 | 5.48390E-01 | 5.00E-03 | 8.02000E-01 | 8.19970E-01 | 1.57820E-01 | 7.83190E-01 | 1.83190E-01 | 4.69860E-01 | 5.48390E-01 |
| 1.00E-02 | 1.39300E-07 | 1.99200E-07 | 1.59200E-07 | 8.65900E-13 | 1.08918E-07 | 2.72850E-07 | 2.69400E-20 | 1.00E-02 | 7.11690E-01 | 7.29640E-01 | 1.29260E-01 | 7.02220E-01 | 7.43400E-01 | 1.44300E-01 | 2.06440E-01 | 1.00E-02 | 7.11690E-01 | 7.29640E-01 | 1.29260E-01 | 7.02220E-01 | 7.43400E-01 | 1.44300E-01 | 2.06440E-01 |
| 5.00E-02 | 1.04720E-05 | 1.59380E-05 | 1.59380E-05 | 5.26890E-09 | 8.23090E-04 | 2.17735E-05 | 2.29520E-14 | 5.00E-02 | 5.01990E-01 | 5.19900E-01 | 9.74030E-01 | 4.91380E-01 | 5.33630E-01 | 1.00218E-01 | 1.43910E-01 | 5.00E-02 | 5.01990E-01 | 5.19900E-01 | 9.74030E-01 | 4.91380E-01 | 5.33630E-01 | 1.00218E-01 | 1.43910E-01 |
| 1.00E-01 | 1.39300E-04 | 1.99200E-04 | 1.59200E-04 | 8.65900E-07 | 1.08918E-04 | 2.72127E-04 | 2.69130E-08 | 1.00E-01 | 4.11650E-01 | 4.29270E-01 | 7.92770E-01 | 4.00710E-01 | 4.43290E-01 | 8.48470E-01 | 1.16280E-01 | 1.00E-01 | 4.11650E-01 | 4.29270E-01 | 7.92770E-01 | 4.00710E-01 | 4.43290E-01 | 8.48470E-01 | 1.16280E-01 |
| 5.00E-01 | 1.04720E-02 | 1.59380E-02 | 1.59380E-02 | 5.26890E-05 | 8.23435E-02 | 2.17735E-02 | 4.40250E-11 | 5.00E-01 | 2.01660E-01 | 3.09080E-01 | 5.70800E-01 | 2.79830E-01 | 3.23200E-01 | 6.02100E-01 | 8.03970E-01 | 5.00E-01 | 2.01660E-01 | 3.09080E-01 | 5.70800E-01 | 2.79830E-01 | 3.23200E-01 | 6.02100E-01 | 8.03970E-01 |
| 1.00E-01 | 1.39300E-01 | 1.99200E-01 | 1.59200E-01 | 8.65900E-04 | 1.08918E-01 | 2.72850E-01 | 2.69400E-08 | 1.00E-01 | 1.46860E-01 | 1.61800E-01 | 2.67720E-01 | 1.27770E-01 | 1.74860E-01 | 3.06590E-01 | 3.64920E-01 | 1.00E-01 | 1.46860E-01 | 1.61800E-01 | 2.67720E-01 | 1.27770E-01 | 1.74860E-01 | 3.06590E-01 | 3.64920E-01 |
| 5.00E-01 | 1.04720E-01 | 1.59380E-01 | 1.59380E-01 | 5.26890E-01 | 8.23435E-01 | 2.17735E-01 | 4.12640E-02 | 5.00E-01 | 1.11150E-01 | 1.21830E-01 | 1.85000E-01 | 8.25200E-01 | 1.34640E-01 | 2.26500E-01 | 2.47300E-01 | 5.00E-01 | 1.11150E-01 | 1.21830E-01 | 1.85000E-01 | 8.25200E-01 | 1.34640E-01 | 2.26500E-01 | 2.47300E-01 |
| 1.00E-01 | 1.39300E-01 | 1.99200E-01 | 1.59200E-01 | 8.65900E-01 | 1.08918E-01 | 2.72850E-01 | 2.69400E-08 | 1.00E-01 | 4.64630E-02 | 6.10200E-01 | 6.23060E-01 | 6.25980E-01 | 1.12710E-02 | 4.21020E-01 | 4.19450E-01 | 1.00E-01 | 4.64630E-02 | 6.10200E-01 | 6.23060E-01 | 6.25980E-01 | 1.12710E-02 | 4.21020E-01 | 4.19450E-01 |
| 5.00E-01 | 1.04720E-01 | 1.59380E-01 | 1.59380E-01 | 5.26890E-01 | 8.23435E-01 | 2.17735E-01 | 4.12640E-02 | 5.00E-01 | 9.60300E-01 | 1.16910E-01 | 1.59310E-01 | 3.09870E-01 | 2.10470E-01 | 3.70950E-01 | 5.16840E-01 | 5.00E-01 | 9.60300E-01 | 1.16910E-01 | 1.59310E-01 | 3.09870E-01 | 2.10470E-01 | 3.70950E-01 | 5.16840E-01 |
| 1.00E-01 | 1.39300E-01 | 1.99200E-01 | 1.59200E-01 | 8.65900E-01 | 1.08918E-01 | 2.72850E-01 | 2.69400E-08 | 1.00E-01 | 1.02030E-01 | 1.28800E-01 | 3.40280E-01 | 1.65480E-01 | 2.14770E-01 | 4.37680E-01 | 6.30330E-01 | 1.00E-01 | 1.02030E-01 | 1.28800E-01 | 3.40280E-01 | 1.65480E-01 | 2.14770E-01 | 4.37680E-01 | 6.30330E-01 |
| 5.00E-01 | 1.04720E-01 | 1.59380E-01 | 1.59380E-01 | 5.26890E-01 | 8.23435E-01 | 2.17735E-01 | 4.12640E-02 | 5.00E-01 | 1.04070E-01 | 2.07970E-01 | 3.59870E-01 | 4.11700E-01 | 2.49650E-01 | 4.87610E-01 | 6.80140E-01 | 5.00E-01 | 1.04070E-01 | 2.07970E-01 | 3.59870E-01 | 4.11700E-01 | 2.49650E-01 | 4.87610E-01 | 6.80140E-01 |
| 1.00E-01 | 1.39300E-01 | 1.99200E-01 | 1.59200E-01 | 8.65900E-01 | 1.08918E-01 | 2.72850E-01 | 2.69400E-08 | 1.00E-01 | 1.04020E-01 | 2.12260E-01 | 3.64230E-01 | 4.37680E-01 | 2.69790E-01 | 5.12160E-01 | 7.06860E-01 | 1.00E-01 | 1.04020E-01 | 2.12260E-01 | 3.64230E-01 | 4.37680E-01 | 2.69790E-01 | 5.12160E-01 | 7.06860E-01 |
| 5.00E-01 | 1.04720E-01 | 1.59380E-01 | 1.59380E-01 | 5.26890E-01 | 8.23435E-01 | 2.17735E-01 | 4.12640E-02 | 5.00E-01 | 1.04020E-01 | 2.12260E-01 | 3.64230E-01 | 4.37680E-01 | 2.69790E-01 | 5.12160E-01 | 7.06860E-01 | 5.00E-01 | 1.04020E-01 | 2.12260E-01 | 3.64230E-01 | 4.37680E-01 | 2.69790E-01 | 5.12160E-01 | 7.06860E-01 |

y = 0.5

y = 20.0

Table 4. (continued)

| y = 10.0 | | | | | | | | | | | | |
|----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| x | log E1 | log E2 | log E3 | log E4 | log E5 | log E6 | log E7 | log E1 | log E2 | log E3 | log E4 | log E5 |
| 1.00E-04 | 1.111695E-01 | 1.359635E-01 | 2.593635E-01 | 4.914025E-01 | 5.337225E-01 | 1.023945E-01 | 1.45323E-01 | 1.311695E-01 | 1.329655E-01 | 2.593635E-01 | 4.914025E-01 | 5.337225E-01 |
| 5.00E-04 | 1.102005E-01 | 1.119965E-01 | 1.174295E-01 | 1.091545E-01 | 1.091545E-01 | 0.000000E+00 | 0.000000E+00 | 1.102005E-01 | 1.119965E-01 | 1.174295E-01 | 1.091545E-01 | 1.091545E-01 |
| 1.00E-03 | 1.011695E-01 | 1.029655E-01 | 1.093935E-01 | 1.001285E-01 | 1.001285E-01 | 2.04905E-01 | 2.96443E-01 | 1.011695E-01 | 1.029655E-01 | 1.093935E-01 | 1.001285E-01 | 1.001285E-01 |
| 5.00E-03 | 8.02003E-02 | 8.19958E-02 | 8.37913E-02 | 8.39808E-02 | 8.39808E-02 | 8.39808E-02 | 8.39808E-02 | 8.02003E-02 | 8.19958E-02 | 8.37913E-02 | 8.39808E-02 | 8.39808E-02 |
| 1.00E-02 | 7.11695E-02 | 7.29648E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 | 7.11695E-02 | 7.29648E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 |
| 5.00E-02 | 5.02002E-02 | 5.19957E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 | 5.02002E-02 | 5.19957E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 |
| 1.00E-01 | 4.11695E-02 | 4.29648E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 | 4.11695E-02 | 4.29648E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 |
| 5.00E-01 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 |
| 7.50E-01 | 1.49168E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.49168E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 |
| 1.00E-02 | 1.11703E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.11703E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 |
| 5.00E-02 | -7.15633E-02 | 5.97641E-02 | -8.8925E-02 | -2.81173E-01 | 1.89154E-01 | -2.21112E-02 | -1.08777E-02 | -7.15633E-02 | 5.97641E-02 | -8.8925E-02 | -2.81173E-01 | 1.89154E-01 |
| 7.50E-02 | -9.37028E-01 | -9.16485E-01 | -2.33561E-01 | -1.39093E-01 | -1.39093E-01 | -8.09402E-01 | -2.02698E-01 | -9.37028E-01 | -9.16485E-01 | -2.33561E-01 | -1.39093E-01 | -1.39093E-01 |
| 7.50E-02 | -1.37432E-01 | -1.50459E-01 | -2.16481E-01 | -1.43297E-01 | -1.43297E-01 | -3.23859E-01 | -5.21650E-01 | -1.37432E-01 | -1.50459E-01 | -2.16481E-01 | -1.43297E-01 | -1.43297E-01 |
| 1.00E-01 | -2.16979E-01 | -1.90325E-01 | -2.77881E-01 | -3.98312E-01 | -2.77881E-01 | -1.83146E-01 | -4.08573E-01 | -2.16979E-01 | -1.90325E-01 | -2.77881E-01 | -3.98312E-01 | -2.77881E-01 |
| 1.35E-01 | -1.75232E-02 | -2.25675E-02 | -4.48498E-02 | -3.14703E-02 | -3.14703E-02 | -2.39705E-02 | -4.88730E-02 | -1.75232E-02 | -2.25675E-02 | -4.48498E-02 | -3.14703E-02 | -3.14703E-02 |
| 2.50E-01 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 |
| 5.00E-01 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 |

| y = 20.0 | | | | | | | | | | | | |
|----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| x | log E1 | log E2 | log E3 | log E4 | log E5 | log E6 | log E7 | log E1 | log E2 | log E3 | log E4 | log E5 |
| 1.00E-04 | 1.111695E-01 | 1.359635E-01 | 2.593635E-01 | 4.914025E-01 | 5.337225E-01 | 1.023945E-01 | 1.45323E-01 | 1.311695E-01 | 1.329655E-01 | 2.593635E-01 | 4.914025E-01 | 5.337225E-01 |
| 5.00E-04 | 1.102005E-01 | 1.119965E-01 | 1.174295E-01 | 1.091545E-01 | 1.091545E-01 | 0.000000E+00 | 0.000000E+00 | 1.102005E-01 | 1.119965E-01 | 1.174295E-01 | 1.091545E-01 | 1.091545E-01 |
| 1.00E-03 | 1.011695E-01 | 1.029655E-01 | 1.093935E-01 | 1.001285E-01 | 1.001285E-01 | 2.04905E-01 | 2.96443E-01 | 1.011695E-01 | 1.029655E-01 | 1.093935E-01 | 1.001285E-01 | 1.001285E-01 |
| 5.00E-03 | 8.02003E-02 | 8.19958E-02 | 8.37913E-02 | 8.39808E-02 | 8.39808E-02 | 8.39808E-02 | 8.39808E-02 | 8.02003E-02 | 8.19958E-02 | 8.37913E-02 | 8.39808E-02 | 8.39808E-02 |
| 1.00E-02 | 7.11695E-02 | 7.29648E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 | 7.11695E-02 | 7.29648E-02 | 7.47603E-02 | 7.47603E-02 | 7.47603E-02 |
| 5.00E-02 | 5.02002E-02 | 5.19957E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 | 5.02002E-02 | 5.19957E-02 | 5.37912E-02 | 5.37912E-02 | 5.37912E-02 |
| 1.00E-01 | 4.11695E-02 | 4.29648E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 | 4.11695E-02 | 4.29648E-02 | 4.47603E-02 | 4.47603E-02 | 4.47603E-02 |
| 5.00E-01 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 | 2.01991E-02 |
| 7.50E-01 | 1.49168E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.49168E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 | 1.65113E-02 |
| 1.00E-02 | 1.11703E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.11703E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 | 1.28658E-02 |
| 5.00E-02 | -7.15633E-02 | 5.97641E-02 | -8.8925E-02 | -2.81173E-01 | 1.89154E-01 | -2.21112E-02 | -1.08777E-02 | -7.15633E-02 | 5.97641E-02 | -8.8925E-02 | -2.81173E-01 | 1.89154E-01 |
| 7.50E-02 | -9.37028E-01 | -9.16485E-01 | -2.33561E-01 | -1.39093E-01 | -1.39093E-01 | -8.09402E-01 | -2.02698E-01 | -9.37028E-01 | -9.16485E-01 | -2.33561E-01 | -1.39093E-01 | -1.39093E-01 |
| 7.50E-02 | -1.37432E-01 | -1.50459E-01 | -2.16481E-01 | -1.43297E-01 | -1.43297E-01 | -3.23859E-01 | -5.21650E-01 | -1.37432E-01 | -1.50459E-01 | -2.16481E-01 | -1.43297E-01 | -1.43297E-01 |
| 1.00E-01 | -2.16979E-01 | -1.90325E-01 | -2.77881E-01 | -3.98312E-01 | -2.77881E-01 | -1.83146E-01 | -4.08573E-01 | -2.16979E-01 | -1.90325E-01 | -2.77881E-01 | -3.98312E-01 | -2.77881E-01 |
| 1.35E-01 | -1.75232E-02 | -2.25675E-02 | -4.48498E-02 | -3.14703E-02 | -3.14703E-02 | -2.39705E-02 | -4.88730E-02 | -1.75232E-02 | -2.25675E-02 | -4.48498E-02 | -3.14703E-02 | -3.14703E-02 |
| 2.50E-01 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 |
| 5.00E-01 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 | -1.80893E-02 | -2.39771E-02 | -4.85314E-02 | -4.85314E-02 | -4.85314E-02 |

Table 5.

| y = 2.0 | | | | | | | | | | | | |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| x | DEL | DE2 | D21 | D22 | D23 | D24 | D25 | D26 | D27 | D28 | D29 | D30 |
| 1.00E-04 | 4.18879E-12 | 4.18879E-11 | 6.01368E-26 | 7.18078E-26 | 6.01368E-24 | 9.44377E-08 | 1.15200E-09 | 1.36136E-10 | 8.54859E-12 | 1.33971E-14 | 4.37686E-26 | 6.83981E-39 |
| 5.00E-04 | 5.23599E-10 | 5.23599E-11 | 1.12800E-21 | 1.12800E-21 | 1.12800E-21 | 1.36136E-28 | 1.36136E-28 | 1.36136E-28 | 1.36136E-28 | 1.36136E-28 | 8.54859E-30 | 1.33971E-32 |
| 1.00E-03 | 4.18879E-09 | 4.18879E-10 | 6.01368E-18 | 7.18078E-20 | 6.01368E-18 | 9.44377E-26 | 1.15200E-26 | 1.36136E-26 | 1.36136E-26 | 1.36136E-26 | 4.37686E-27 | 6.83981E-30 |
| 5.00E-03 | 5.23599E-07 | 5.23599E-08 | 1.12800E-15 | 1.12800E-15 | 1.12800E-15 | 1.36136E-19 | 1.36136E-19 | 1.36136E-19 | 1.36136E-19 | 1.36136E-19 | 8.54859E-21 | 1.33971E-23 |
| 1.00E-02 | 4.18879E-06 | 4.18879E-07 | 6.01368E-12 | 7.18078E-14 | 6.01368E-12 | 9.44377E-17 | 1.15200E-17 | 1.36136E-17 | 1.36136E-17 | 1.36136E-17 | 4.37686E-18 | 6.83981E-21 |
| 5.00E-02 | 5.23599E-04 | 5.23599E-05 | 1.12800E-09 | 1.12800E-09 | 1.12800E-09 | 1.36136E-10 | 1.36136E-10 | 1.36136E-10 | 1.36136E-10 | 1.36136E-10 | 8.54859E-12 | 1.33971E-14 |
| 1.00E-01 | 4.18879E-03 | 4.18879E-04 | 6.01368E-06 | 7.18078E-08 | 6.01368E-06 | 9.44377E-08 | 1.15200E-08 | 1.36136E-08 | 1.36136E-08 | 1.36136E-08 | 4.37686E-09 | 6.83981E-12 |
| 5.00E-01 | 5.23599E-02 | 5.23599E-03 | 1.12800E-05 | 1.12800E-05 | 1.12800E-05 | 1.36136E-06 | 1.36136E-06 | 1.36136E-06 | 1.36136E-06 | 1.36136E-06 | 8.54859E-08 | 1.33971E-10 |
| 7.50E-01 | 1.70713E-01 | 1.70713E-01 | 1.05648E-01 | 1.27165E-02 | 1.05648E-01 | 5.20469E-01 | 5.20469E-01 | 5.20469E-01 | 5.20469E-01 | 5.20469E-01 | 3.28280E-01 | 5.13411E-04 |
| 1.00E-02 | 3.92869E-00 | 4.13159E-01 | 5.93688E-02 | 6.87708E-02 | 5.93688E-01 | 6.87708E-01 | 6.87708E-01 | 6.87708E-01 | 6.87708E-01 | 6.87708E-01 | 3.02464E-09 | 3.02464E-05 |
| 5.00E-02 | 4.68208E-01 | 5.57111E-01 | 1.21518E-03 | 1.60840E-01 | 1.21518E-03 | 2.25428E-05 | 2.25428E-05 | 2.25428E-05 | 2.25428E-05 | 2.25428E-05 | 1.52637E-04 | 2.31731E-01 |
| 1.00E-01 | 6.52831E-02 | 6.25492E-02 | 4.78259E-05 | 6.33830E-02 | 6.52831E-02 | 5.85975E-07 | 5.85975E-07 | 5.85975E-07 | 5.85975E-07 | 5.85975E-07 | 4.67490E-06 | 9.20902E-03 |
| 5.00E-01 | 2.40262E-02 | 4.25928E-01 | 1.74077E-05 | 3.24957E-03 | 2.40262E-02 | 8.20446E-08 | 8.20446E-08 | 8.20446E-08 | 8.20446E-08 | 8.20446E-08 | 7.54182E-07 | 1.76002E-05 |
| 1.00E-01 | 2.68441E-02 | 5.03555E-01 | 3.46209E-05 | 7.10031E-03 | 2.68441E-02 | 3.56651E-09 | 3.56651E-09 | 3.56651E-09 | 3.56651E-09 | 3.56651E-09 | 3.45718E-08 | 9.18355E-05 |
| 1.35E-01 | 2.77427E-02 | 5.31739E-01 | 5.00158E-05 | 1.10095E-04 | 2.77427E-02 | 9.68457E-09 | 9.68457E-09 | 9.68457E-09 | 9.68457E-09 | 9.68457E-09 | 1.00624E-09 | 3.02464E-05 |
| 2.50E-01 | 2.78595E-02 | 5.16144E-01 | 6.53101E-05 | 1.28397E-04 | 2.78595E-02 | 1.65647E-10 | 1.65647E-10 | 1.65647E-10 | 1.65647E-10 | 1.65647E-10 | 1.86316E-09 | 6.31466E-06 |
| 5.00E-01 | 2.79329E-02 | 5.16144E-01 | 6.52493E-05 | 1.28397E-04 | 2.79329E-02 | 1.88187E-09 | 1.88187E-09 | 1.88187E-09 | 1.88187E-09 | 1.88187E-09 | 6.31466E-06 | 6.31466E-06 |
| y = 5.0 | | | | | | | | | | | | |
| x | DEL | DE2 | D21 | D22 | D23 | D24 | D25 | D26 | D27 | D28 | D29 | D30 |
| 1.00E-04 | 4.18879E-12 | 1.04720E-12 | 1.19215E-23 | 2.83849E-25 | 1.19215E-23 | 1.29988E-34 | 1.29988E-34 | 1.29988E-34 | 1.29988E-34 | 1.29988E-34 | 4.27438E-35 | 4.27438E-35 |
| 5.00E-04 | 5.23599E-10 | 1.20900E-10 | 1.86274E-19 | 4.53508E-21 | 1.86274E-19 | 2.42164E-28 | 2.42164E-28 | 2.42164E-28 | 2.42164E-28 | 2.42164E-28 | 8.34919E-32 | 8.34919E-32 |
| 1.00E-03 | 4.18879E-09 | 1.04720E-09 | 1.19215E-17 | 2.83849E-19 | 1.19215E-17 | 1.29988E-25 | 1.29988E-25 | 1.29988E-25 | 1.29988E-25 | 1.29988E-25 | 4.27438E-26 | 4.27438E-26 |
| 5.00E-03 | 5.23599E-07 | 1.20900E-07 | 1.86274E-13 | 4.53508E-15 | 1.86274E-13 | 2.42164E-19 | 2.42164E-19 | 2.42164E-19 | 2.42164E-19 | 2.42164E-19 | 8.34919E-23 | 8.34919E-23 |
| 1.00E-02 | 4.18879E-06 | 1.04720E-06 | 1.19215E-11 | 2.83849E-13 | 1.19215E-11 | 1.29988E-16 | 1.29988E-16 | 1.29988E-16 | 1.29988E-16 | 1.29988E-16 | 4.27438E-17 | 4.27438E-17 |
| 5.00E-02 | 5.23599E-04 | 1.20900E-04 | 1.86274E-07 | 4.53508E-09 | 1.86274E-07 | 2.42164E-10 | 2.42164E-10 | 2.42164E-10 | 2.42164E-10 | 2.42164E-10 | 8.34919E-14 | 8.34919E-14 |
| 1.00E-01 | 4.18879E-03 | 1.04720E-03 | 1.19215E-05 | 2.83849E-07 | 1.19215E-05 | 1.29988E-08 | 1.29988E-08 | 1.29988E-08 | 1.29988E-08 | 1.29988E-08 | 4.27438E-08 | 4.27438E-08 |
| 5.00E-01 | 5.23599E-02 | 1.20900E-02 | 1.86274E-01 | 4.53508E-03 | 1.86274E-01 | 2.42164E-04 | 2.42164E-04 | 2.42164E-04 | 2.42164E-04 | 2.42164E-04 | 8.34919E-08 | 8.34919E-08 |
| 7.50E-01 | 1.72391E-01 | 4.04606E-01 | 2.11564E-01 | 5.04895E-02 | 1.72391E-01 | 9.29226E-01 | 9.29226E-01 | 9.29226E-01 | 9.29226E-01 | 9.29226E-01 | 3.20907E-03 | 3.20907E-03 |
| 1.00E-02 | 4.04422E-01 | 1.03947E-01 | 1.13316E-01 | 2.32328E-01 | 4.04422E-01 | 1.23391E-02 | 1.23391E-02 | 1.23391E-02 | 1.23391E-02 | 1.23391E-02 | 4.27438E-02 | 4.27438E-02 |
| 5.00E-02 | 5.15247E-01 | 1.03947E-01 | 1.13316E-01 | 2.32328E-01 | 5.15247E-01 | 4.35563E-05 | 4.35563E-05 | 4.35563E-05 | 4.35563E-05 | 4.35563E-05 | 4.70192E-01 | 1.59603E-02 |
| 1.00E-01 | 3.84941E-02 | 7.87939E-02 | 1.04454E-05 | 1.70993E-01 | 3.84941E-02 | 1.26280E-09 | 1.26280E-09 | 1.26280E-09 | 1.26280E-09 | 1.26280E-09 | 6.45497E-07 | 6.45497E-07 |
| 5.00E-01 | 3.84941E-02 | 1.49411E-02 | 5.83166E-05 | 1.80933E-04 | 3.84941E-02 | 2.71306E-09 | 2.71306E-09 | 2.71306E-09 | 2.71306E-09 | 2.71306E-09 | 1.44403E-06 | 1.44403E-06 |
| 1.00E-01 | 4.68393E-02 | 1.32164E-02 | 1.32164E-02 | 1.32164E-02 | 4.68393E-02 | 1.32164E-02 | 1.32164E-02 | 1.32164E-02 | 1.32164E-02 | 1.32164E-02 | 1.90797E-09 | 8.83911E-06 |
| 1.35E-01 | 5.08471E-02 | 2.12321E-02 | 2.39585E-06 | 8.28371E-04 | 5.08471E-02 | 2.39585E-06 | 2.39585E-06 | 2.39585E-06 | 2.39585E-06 | 2.39585E-06 | 7.03340E-09 | 3.59338E-07 |
| 2.50E-01 | 5.08682E-02 | 2.12321E-02 | 2.97170E-06 | 1.17756E-04 | 5.08682E-02 | 1.17756E-04 | 1.17756E-04 | 1.17756E-04 | 1.17756E-04 | 1.17756E-04 | 1.54556E-11 | 1.77567E-10 |
| 5.00E-01 | 5.08731E-02 | 2.16590E-02 | 2.97166E-06 | 1.07534E-05 | 5.08731E-02 | 1.07534E-05 | 1.07534E-05 | 1.07534E-05 | 1.07534E-05 | 1.07534E-05 | 1.26771E-11 | 1.81043E-10 |

NUMERICAL TABLES ON THE RANDOM FORCE

Table 7.

| y = 0.0 | | y = 0.2 | | y = 0.4 | | y = 0.6 | | y = 0.8 | | y = 1.0 | |
|----------|-------------|--------------|--------------|--------------|-------------|----------|-------------|--------------|--------------|--------------|--------------|
| x | log E12 | E221 | log E221 | E222 | log E222 | x | log E12 | E221 | log E221 | E222 | log E222 |
| 1.00E-04 | 0.00000E+00 | 0.00000E+00 | -2.89220E-01 | 0.00000E+00 | 0.00000E+00 | 1.00E-04 | 1.19680E-29 | 1.19680E-01 | -2.89220E-01 | 2.56457E-30 | -2.95101E-01 |
| 5.00E-04 | 0.00000E+00 | 0.00000E+00 | -2.46292E-01 | 0.00000E+00 | 0.00000E+00 | 5.00E-04 | 6.54498E-15 | -1.81414E+01 | 9.34998E-25 | -2.46292E-01 | -2.46982E-01 |
| 1.00E-03 | 0.00000E+00 | 0.00000E+00 | -2.19220E-01 | 0.00000E+00 | 0.00000E+00 | 1.00E-03 | 1.04720E-13 | -1.89800E+01 | 1.19680E-22 | -2.19220E-01 | -2.25910E-01 |
| 5.00E-03 | 0.00000E+00 | 0.00000E+00 | -1.70292E-01 | 0.00000E+00 | 0.00000E+00 | 5.00E-03 | 6.54498E-11 | -1.01814E+01 | 9.34998E-18 | -1.70292E-01 | -1.76982E-01 |
| 1.00E-02 | 0.00000E+00 | 0.00000E+00 | -1.49220E-01 | 0.00000E+00 | 0.00000E+00 | 1.00E-02 | 1.04720E-09 | -8.79797E+00 | 1.19680E-15 | -1.49220E-01 | -1.55910E-01 |
| 5.00E-02 | 0.00000E+00 | 0.00000E+00 | -1.00292E-01 | 0.00000E+00 | 0.00000E+00 | 5.00E-02 | 6.54498E-07 | -6.18142E+00 | 9.34998E-11 | -1.00292E-01 | -1.06982E-01 |
| 1.00E-01 | 0.00000E+00 | 0.00000E+00 | -7.92226E-01 | 0.00000E+00 | 0.00000E+00 | 1.00E-01 | 1.04684E-05 | -4.98012E+00 | 1.19681E-08 | -7.92226E-01 | -8.59099E-00 |
| 5.00E-01 | 0.00000E+00 | 0.00000E+00 | -5.13979E-01 | 0.00000E+00 | 0.00000E+00 | 5.00E-01 | 4.07792E-04 | -3.85956E+00 | 7.27768E-06 | -5.13801E-00 | 1.56492E-06 |
| 1.00E-01 | 0.00000E+00 | 0.00000E+00 | -3.04745E-01 | 0.00000E+00 | 0.00000E+00 | 5.00E-01 | 6.44368E-03 | -2.19099E+00 | 9.17979E-04 | -3.03726E-00 | 1.99901E-04 |
| 5.00E-01 | 0.00000E+00 | 0.00000E+00 | -1.82682E-01 | 0.00000E+00 | 0.00000E+00 | 7.50E-01 | 3.11873E-02 | -1.49545E+00 | 1.56760E-02 | -1.81614E+00 | 3.29759E-03 |
| 1.00E-00 | 0.00000E+00 | 0.00000E+00 | -9.76850E-01 | 0.00000E+00 | 0.00000E+00 | 1.00E+00 | 9.74694E-02 | -1.01113E+00 | 1.09974E-01 | -9.58711E-01 | 2.51553E-02 |
| 5.00E-00 | 0.00000E+00 | 0.00000E+00 | 1.56217E-00 | 0.00000E+00 | 0.00000E+00 | 5.00E+00 | 2.66960E+00 | 4.26447E+01 | 4.42772E+01 | 1.64618E+00 | 1.25064E+01 |
| 1.00E+00 | 0.00000E+00 | 0.00000E+00 | 3.68238E+03 | 0.00000E+00 | 0.00000E+00 | 5.00E+00 | 1.75017E+01 | 1.24308E+00 | 1.94723E+03 | 3.28942E+00 | 7.43959E+02 |
| 5.00E+00 | 0.00000E+00 | 0.00000E+00 | 3.66602E+03 | 0.00000E+00 | 0.00000E+00 | 7.50E+00 | 3.21115E+01 | 1.51939E+00 | 9.59421E+00 | 3.98021E+00 | 4.68660E+03 |
| 1.00E+01 | 0.00000E+00 | 0.00000E+00 | 3.90297E+03 | 0.00000E+00 | 0.00000E+00 | 1.00E+01 | 4.11589E+01 | 1.61446E+00 | 1.98683E+04 | 4.29816E+00 | 1.05939E+04 |
| 5.00E+01 | 0.00000E+00 | 0.00000E+00 | 4.00773E+04 | 0.00000E+00 | 0.00000E+00 | 1.35E+01 | 4.44393E+01 | 1.64777E+00 | 2.94733E+04 | 4.46943E+00 | 1.73899E+04 |
| 1.00E+01 | 0.00000E+00 | 0.00000E+00 | 4.03102E+04 | 0.00000E+00 | 0.00000E+00 | 2.50E+01 | 4.50031E+01 | 1.65242E+00 | 3.55730E+04 | 4.52526E+00 | 2.08714E+04 |
| 5.00E+01 | 0.00000E+00 | 0.00000E+00 | 4.03102E+04 | 0.00000E+00 | 0.00000E+00 | 5.00E+01 | 4.50033E+01 | 1.65242E+00 | 3.55264E+04 | 4.52529E+00 | 2.08827E+04 |
| y = 0.5 | | y = 0.7 | | y = 0.9 | | y = 1.1 | | y = 1.3 | | y = 1.5 | |
| x | log E12 | E221 | log E221 | E222 | log E222 | x | log E12 | E221 | log E221 | E222 | log E222 |
| 1.00E-04 | 5.23599E-13 | -1.72810E+01 | 1.19680E-29 | -2.89220E-01 | 6.41141E-31 | 1.00E-04 | 1.65576E-17 | -1.67810E+01 | 1.19680E-29 | -2.89220E-01 | 6.41141E-30 |
| 5.00E-04 | 3.27549E-15 | -1.44851E+01 | 9.34998E-25 | -2.46292E-01 | 5.00892E-26 | 5.00E-04 | 1.05485E-14 | -1.39351E+01 | 9.34998E-25 | -2.46292E-01 | 5.00892E-25 |
| 1.00E-03 | 5.23599E-14 | -1.39810E+01 | 1.19680E-22 | -2.19220E-01 | 6.41141E-24 | 1.00E-03 | 1.05485E-13 | -1.27810E+01 | 1.19680E-22 | -2.19220E-01 | 6.41141E-23 |
| 5.00E-03 | 3.27549E-11 | -1.04851E+01 | 9.34997E-18 | -1.70292E-01 | 5.00892E-19 | 5.00E-03 | 1.05485E-10 | -9.58420E+00 | 9.34998E-18 | -1.70292E-01 | 5.00892E-18 |
| 1.00E-02 | 5.23599E-10 | -9.24810E+00 | 1.19680E-15 | -1.49220E-01 | 6.41141E-17 | 1.00E-02 | 1.65576E-09 | -8.78100E+00 | 1.19680E-15 | -1.49220E-01 | 6.41141E-16 |
| 5.00E-02 | 3.27549E-07 | -6.48516E+00 | 9.34864E-11 | -1.00292E-01 | 5.00891E-12 | 5.00E-02 | 1.05485E-06 | -5.9853E+00 | 9.34998E-11 | -1.00292E-01 | 5.00891E-11 |
| 1.00E-01 | 5.23599E-06 | -5.28121E+00 | 1.19680E-08 | -7.92226E-01 | 6.41141E-10 | 1.00E-01 | 1.65576E-05 | -4.78107E+00 | 1.19680E-08 | -7.92226E-01 | 6.41141E-09 |
| 5.00E-01 | 2.96282E-04 | -3.69116E+00 | 7.26632E-06 | -5.13979E-01 | 3.91191E-07 | 5.00E-01 | 4.45791E-04 | -3.18991E+00 | 7.29130E-06 | -5.13791E-00 | 3.91277E-06 |
| 1.00E+01 | 3.20371E-03 | -2.49446E+00 | 9.11094E-04 | -3.04044E-00 | 4.92965E-05 | 5.00E+01 | 1.02626E-02 | -1.98862E+00 | 9.26087E-04 | -3.03535E-00 | 5.00313E-04 |
| 5.00E+01 | 1.27039E-02 | -1.80384E+00 | 1.50042E-02 | -1.82379E-00 | 8.46280E-04 | 7.50E+01 | 5.11320E-02 | -1.28971E+00 | 1.59894E-02 | -1.80717E-00 | 8.52176E-03 |
| 1.00E+00 | 4.75162E-02 | -1.32316E+00 | 1.06762E-01 | -9.71584E-01 | 6.24451E-03 | 1.00E+00 | 1.59100E-01 | -7.98112E-01 | 1.14224E-01 | -9.42241E-01 | 6.34474E-02 |
| 5.00E+00 | 1.18672E-00 | 7.43499E-02 | 3.87799E-01 | 1.56856E+00 | 2.94133E+00 | 5.00E+00 | 4.95150E+00 | 6.94797E-01 | 1.72697E+00 | 3.40313E+01 | 1.53186E+00 |
| 1.00E+00 | 6.65040E-00 | 8.21540E-01 | 1.38795E-03 | 3.14237E+00 | 1.52079E+02 | 5.00E+00 | 4.09472E+01 | 1.61222E+00 | 3.10909E+01 | 3.49265E+03 | 3.39717E+00 |
| 5.00E+00 | 1.11872E-01 | 1.04877E+00 | 5.72496E-02 | 3.75777E-00 | 7.94252E+02 | 7.50E+00 | 9.25248E+01 | 1.96228E+00 | 1.95122E+04 | 4.29031E+00 | 1.85298E+04 |
| 1.00E+01 | 1.50560E-01 | 1.11151E+00 | 1.04982E-04 | 4.01867E+00 | 1.67769E+03 | 1.00E+01 | 1.28407E+02 | 2.10959E+00 | 4.92124E+04 | 4.69207E+00 | 5.26159E+04 |
| 5.00E+01 | 1.36545E-01 | 1.11350E+00 | 1.38795E-04 | 4.14234E+00 | 2.47491E+03 | 5.00E+01 | 1.48279E+02 | 2.17135E+00 | 8.71799E+00 | 4.94042E+00 | 1.03276E+05 |
| 1.00E+01 | 1.27208E-01 | 1.13769E+00 | 1.49195E-04 | 4.17375E+00 | 2.77971E+03 | 5.00E+01 | 1.52505E+02 | 2.18612E+00 | 1.11859E+05 | 5.04867E+00 | 1.42681E+05 |
| 5.00E+01 | 1.37308E-01 | 1.13770E+00 | 1.49206E-04 | 4.17379E+00 | 2.78021E+03 | 5.00E+01 | 1.52551E+02 | 2.18614E+00 | 1.12003E+05 | 5.04942E+00 | 1.42898E+05 |

Table 8.

Table 7. (continued)

| x | log E12 | E221 | log E221 | E222 | log E222 | x | 0.0001 | 0.0005 | 0.0010 | 0.0050 | 0.0100 | 0.0500 | 0.1000 | 0.2500 | 0.5000 | |
|----------|-------------|-------------|-------------|--------------|-------------|--------------|--------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1.00E-04 | 2.34460E-17 | 1.46505E-01 | 1.15680E-29 | -2.89220E-01 | 1.28228E-29 | -2.89220E-01 | 0 | 0.0112834 | 0.0252271 | 0.0356766 | 0.0796557 | 0.1124629 | 0.2481704 | 0.3452792 | 0.5304999 | 0.6836959 |
| 1.00E-04 | 1.46505E-17 | 1.46505E-01 | 1.46505E-25 | -2.40292E-01 | 1.00178E-24 | -2.39992E-01 | 1 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-05 | 2.34460E-13 | 1.46505E-01 | 1.46505E-13 | -2.11920E-02 | 1.28228E-22 | -2.11890E-01 | 2 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-05 | 1.46505E-10 | 9.34982E-18 | 1.46505E-18 | -1.70292E-01 | 1.00178E-17 | -1.69992E-01 | 3 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-02 | 2.34460E-09 | 1.46505E-01 | 1.46505E-15 | -1.49220E-01 | 1.28228E-15 | -1.47622E-01 | 4 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-02 | 1.46505E-06 | 5.83642E-01 | 9.34982E-11 | -1.00292E-01 | 1.00178E-10 | -8.9993E-01 | 5 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-01 | 2.34460E-05 | 1.46505E-01 | 1.3973E-08 | -7.82920E-01 | 1.28228E-08 | -7.82920E-01 | 6 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 2.50E-01 | 9.34982E-04 | 3.69893E-01 | 7.3003E-06 | -5.11365E-01 | 7.8281E-06 | -5.11365E-01 | 7 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-01 | 1.46505E-02 | 1.46505E-01 | 9.34982E-04 | -3.03057E-01 | 1.00178E-03 | -3.03057E-01 | 8 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 7.50E-01 | 7.35971E-02 | 1.46505E-01 | 1.3881E-02 | -1.60030E-01 | 1.70915E-02 | -1.76722E-01 | 9 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-03 | 2.30772E-01 | 6.26812E-01 | 1.17016E-01 | -9.2953E-01 | 1.27748E-01 | -8.9364E-01 | 10 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 2.50E-03 | 9.0791E-01 | 6.26812E-01 | 6.26812E-01 | 1.79865E-01 | 7.34411E-01 | 1.86594E-01 | 11 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-03 | 8.56088E-01 | 1.92352E-01 | 4.94291E-03 | 5.69402E-01 | 6.74201E-03 | 3.88852E-01 | 12 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 7.50E-03 | 2.41309E-02 | 2.28562E-01 | 4.11103E-04 | 6.13953E-01 | 6.42877E-04 | 4.80813E-01 | 13 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-01 | 3.91882E-02 | 2.59245E-01 | 1.21321E-05 | 5.11823E-01 | 2.28374E-05 | 5.39528E-01 | 14 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.25E-01 | 5.11122E-02 | 2.70852E-01 | 2.96678E-05 | 5.47292E-01 | 5.72289E-05 | 5.72762E-01 | 15 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 2.50E-01 | 5.60352E-02 | 2.74846E-01 | 4.98317E-05 | 5.67990E-01 | 1.02310E-06 | 6.04988E-01 | 16 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-01 | 5.60323E-02 | 2.74839E-01 | 4.81404E-05 | 5.68251E-01 | 1.02326E-06 | 6.01383E-01 | 17 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-02 | 3.21532E-09 | 1.19680E-01 | 1.19680E-22 | -2.11920E-01 | 2.00357E-17 | -2.11920E-01 | 18 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-02 | 3.21532E-09 | 1.19680E-01 | 1.19680E-15 | -1.49220E-01 | 2.00357E-15 | -1.49220E-01 | 19 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-02 | 2.06970E-06 | 5.68409E-01 | 9.34982E-11 | -1.00292E-01 | 2.00357E-10 | -9.69820E-01 | 20 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-01 | 3.31152E-05 | 4.49797E-01 | 1.19679E-08 | -7.92198E-01 | 2.56497E-08 | -7.92198E-01 | 21 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 2.50E-01 | 1.20351E-03 | 2.88232E-01 | 7.3003E-06 | -5.11365E-01 | 1.56528E-05 | -4.80541E-01 | 22 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 5.00E-01 | 2.06970E-02 | 1.68422E-01 | 9.34982E-04 | -3.03057E-01 | 2.00357E-03 | -2.69821E-01 | 23 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 7.50E-01 | 1.04687E-01 | 9.34982E-01 | 1.59584E-02 | -1.79701E-01 | 3.42271E-02 | -1.46553E-01 | 24 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |
| 1.00E-03 | 3.30504E-01 | 4.80824E-01 | 1.19395E-01 | -9.23015E-01 | 2.56535E-01 | -5.91193E-01 | | | | | | | | | | |
| 2.50E-03 | 1.20011E-01 | 1.00111E-01 | 7.06501E-01 | 1.84936E-01 | 2.18904E-01 | | | | | | | | | | | |
| 5.00E-03 | 1.79958E-02 | 2.29518E-01 | 3.87619E-01 | 3.87619E-01 | 4.24667E-01 | | | | | | | | | | | |
| 7.50E-03 | 6.47109E-02 | 2.81088E-01 | 8.66078E-04 | 4.84773E-01 | 2.79595E-04 | 5.39716E-01 | | | | | | | | | | |
| 1.00E-01 | 1.34552E-03 | 3.12883E-01 | 3.93317E-05 | 5.59496E-01 | 1.09954E-06 | 6.04121E-01 | | | | | | | | | | |
| 1.35E-01 | 2.23782E-03 | 3.3483E-01 | 1.30636E-06 | 6.11606E-01 | 4.02466E-06 | 6.60473E-01 | | | | | | | | | | |
| 2.50E-01 | 2.88167E-03 | 3.47146E-01 | 3.66268E-06 | 6.25211E-01 | 1.26760E-07 | 7.10298E-01 | | | | | | | | | | |
| 5.00E-01 | 2.99248E-03 | 3.47603E-01 | 3.70026E-06 | 6.26823E-01 | 1.32356E-07 | 7.12505E-01 | | | | | | | | | | |

y = 10.0

y = 5.0

Table 8. (continued)

| x | 0.7500 | 1.0000 | 2.5000 | 5.0000 | 7.5000 | 10.0000 | 13.5200 | 25.0000 | 50.0000 |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| N | | | | | | | | | |
| 0 | 0.7793286 | 0.8427008 | 0.9746527 | 0.9984346 | 0.9998925 | 0.9999923 | 0.9999998 | 1.0000000 | 1.0000000 |
| 1 | 0.3177297 | 0.4275933 | 0.8282029 | 0.9814339 | 0.9981834 | 0.9998303 | 0.0000042 | 1.0000000 | 1.0000000 |
| 2 | 0.0869302 | 0.1508550 | 0.5841198 | 0.9247648 | 0.9896377 | 0.9987503 | 0.9999440 | 1.0000000 | 1.0000000 |
| 3 | 0.0176903 | 0.0401596 | 0.3400368 | 0.8114265 | 0.9640006 | 0.9944303 | 0.9996722 | 1.0000000 | 1.0000000 |
| 4 | 0.0028532 | 0.0085324 | 0.1656917 | 0.6495148 | 0.9090640 | 0.9820876 | 0.9986224 | 0.9999999 | 1.0000000 |
| 5 | 0.0003804 | 0.0015041 | 0.0688334 | 0.4696128 | 0.8175031 | 0.9546593 | 0.9954683 | 0.9999994 | 1.0000000 |
| 6 | 0.0000432 | 0.0002263 | 0.0248069 | 0.3060656 | 0.6926472 | 0.9047897 | 0.9877149 | 0.9999970 | 1.0000000 |
| 7 | 0.0000043 | 0.0000297 | 0.0078736 | 0.1802601 | 0.5485828 | 0.8280673 | 0.9715879 | 0.9999880 | 1.0000000 |
| 8 | 0.0000004 | 0.0000034 | 0.0022292 | 0.0963897 | 0.4045184 | 0.7257707 | 0.9425162 | 0.9999578 | 1.0000000 |
| 9 | 0.0000000 | 0.0000004 | 0.0005690 | 0.0470542 | 0.2774027 | 0.6054218 | 0.8962752 | 0.9998689 | 1.0000000 |
| 10 | 0.0000000 | 0.0000000 | 0.0001322 | 0.0210881 | 0.1770482 | 0.4787387 | 0.8304669 | 0.9996352 | 1.0000000 |
| 11 | 0.0000000 | 0.0000000 | 0.0000281 | 0.0087234 | 0.1053664 | 0.3580882 | 0.7457309 | 0.9990787 | 1.0000000 |
| 12 | 0.0000000 | 0.0000000 | 0.0000055 | 0.0033474 | 0.0586174 | 0.2531747 | 0.6461108 | 0.9978688 | 1.0000000 |
| 13 | 0.0000000 | 0.0000000 | 0.0000010 | 0.0011970 | 0.0305680 | 0.1692439 | 0.5383618 | 0.9954492 | 1.0000000 |
| 14 | 0.0000000 | 0.0000000 | 0.0000002 | 0.0004005 | 0.0149850 | 0.1070729 | 0.4304531 | 0.9909683 | 1.0000000 |
| 15 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0001259 | 0.0069249 | 0.0641964 | 0.3298375 | 0.9832427 | 1.0000000 |
| 16 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000373 | 0.0030248 | 0.0365341 | 0.2420748 | 0.9707821 | 1.0000000 |
| 17 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000104 | 0.0012520 | 0.0197691 | 0.1701626 | 0.9519023 | 1.0000000 |
| 18 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000028 | 0.0004923 | 0.0101891 | 0.1146052 | 0.9249312 | 0.9999999 |
| 19 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000007 | 0.0001843 | 0.0050107 | 0.0740033 | 0.8884837 | 0.9999997 |
| 20 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000002 | 0.0000658 | 0.0023551 | 0.0458527 | 0.8417562 | 0.9999992 |
| 21 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000225 | 0.0010597 | 0.0272870 | 0.7847715 | 0.9999981 |
| 22 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000073 | 0.0004572 | 0.0156122 | 0.7185101 | 0.9999953 |
| 23 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000023 | 0.0001894 | 0.0085969 | 0.6448864 | 0.9999893 |
| 24 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000007 | 0.0000755 | 0.0045609 | 0.5665633 | 0.9999763 |
| 25 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000002 | 0.0000289 | 0.0023337 | 0.4866418 | 0.9999500 |
| 26 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000001 | 0.0000107 | 0.0011528 | 0.4082873 | 0.9998984 |
| 27 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000038 | 0.0005503 | 0.3343680 | 0.9998010 |
| 28 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000013 | 0.0002541 | 0.2671687 | 0.9996238 |
| 29 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000004 | 0.0001136 | 0.2082219 | 0.9993130 |
| 30 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000001 | 0.0000492 | 0.1582670 | 0.9987863 |
| 31 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000207 | 0.1173203 | 0.9979228 |
| 32 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000084 | 0.0848230 | 0.9965521 |
| 33 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000033 | 0.0598250 | 0.9944434 |
| 34 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000013 | 0.0411698 | 0.9912961 |
| 35 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000005 | 0.0276516 | 0.9867347 |
| 36 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000002 | 0.0181317 | 0.9803103 |
| 37 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000001 | 0.0116112 | 0.9715097 |
| 38 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0072642 | 0.9597755 |
| 39 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0044415 | 0.9445364 |
| 40 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0026550 | 0.9252464 |
| 41 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0015522 | 0.9014315 |
| 42 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0008878 | 0.8727389 |
| 43 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0004970 | 0.8389829 |
| 44 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0002724 | 0.8001829 |
| 45 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0001463 | 0.7565874 |
| 46 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000769 | 0.7086802 |
| 47 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000397 | 0.6571671 |
| 48 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000201 | 0.6029428 |
| 49 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000099 | 0.5470415 |

Table 12.

Table of x

$\mu = 4.500E-03$

| δ | 2R \odot | | 1AU | |
|----------|-------------|-------------|-------------|-------------|
| | 4.51E-08 pc | | 4.85E-06 pc | |
| σ | x | log x | x | log x |
| 1 | 4.98891E+04 | 4.69801E+00 | 4.63918E+02 | 2.66644E+00 |
| 5 | 1.99557E+03 | 3.30007E+00 | 1.85567E+01 | 1.26850E+00 |
| 10 | 4.98891E+02 | 2.69801E+00 | 4.63918E+00 | 6.66441E-01 |
| 15 | 2.21729E+02 | 2.34582E+00 | 2.06186E+00 | 3.14258E-01 |
| 20 | 1.24723E+02 | 2.09595E+00 | 1.15979E+00 | 6.43808E-02 |

$\mu = 1.800E-02$

| δ | 2R \odot | | 1AU | |
|----------|-------------|-------------|-------------|-------------|
| | 4.51E-08 pc | | 4.85E-06 pc | |
| σ | x | log x | x | log x |
| 1 | 1.99557E+05 | 5.30007E+00 | 1.85567E+03 | 3.26850E+00 |
| 5 | 7.98226E+03 | 3.90213E+00 | 7.42268E+01 | 1.87056E+00 |
| 10 | 1.99557E+03 | 3.30007E+00 | 1.85567E+01 | 1.26850E+00 |
| 15 | 8.86918E+02 | 2.94788E+00 | 8.24742E+00 | 9.16318E-01 |
| 20 | 4.98891E+02 | 2.69801E+00 | 4.63918E+00 | 6.66441E-01 |

$\mu = 7.200E-02$

| δ | 2R \odot | | 1AU | |
|----------|-------------|-------------|-------------|-------------|
| | 4.51E-08 pc | | 4.85E-06 pc | |
| σ | x | log x | x | log x |
| 1 | 7.98226E+05 | 5.90213E+00 | 7.42268E+03 | 3.87056E+00 |
| 5 | 3.19290E+04 | 4.50419E+00 | 2.96907E+02 | 2.47262E+00 |
| 10 | 7.98226E+03 | 3.90213E+00 | 7.42268E+01 | 1.87056E+00 |
| 15 | 3.54767E+03 | 3.54994E+00 | 3.29897E+01 | 1.51838E+00 |
| 20 | 1.99557E+03 | 3.30007E+00 | 1.85567E+01 | 1.26850E+00 |

Table of y

| α/V | $V_* = 2.0$ | | $V_* = 5.0$ | | $V_* = 10.0$ | | $V_* = 20.0$ | |
|------------|-------------|--------------|-------------|--------------|--------------|--------------|--------------|-------------|
| | y | log y | y | log y | y | log y | y | log y |
| 1 | 8.00000E+00 | 9.03090E-01 | 5.00000E+01 | 1.69897E+00 | 2.00000E+02 | 2.30103E+00 | 8.00000E+02 | 2.90309E+00 |
| 5 | 3.20000E-01 | -4.94850E-01 | 2.00000E+00 | 3.01030E-01 | 8.00000E+00 | 9.03090E-01 | 3.20000E+01 | 1.50515E+00 |
| 10 | 8.00000E-02 | -1.09691E+00 | 5.00000E-01 | -3.01030E-01 | 2.00000E+00 | 3.01030E-01 | 8.00000E+00 | 9.03090E-01 |
| 15 | 3.55556E-02 | -1.44909E+00 | 2.22222E-01 | -6.53213E-01 | 8.88889E-01 | -5.11525E-02 | 3.55556E+00 | 5.50907E-01 |
| 20 | 2.00000E-02 | -1.69897E+00 | 1.25000E-01 | -9.03090E-01 | 5.00000E-01 | -3.01030E-01 | 2.00000E+00 | 3.01030E-01 |