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VENEZIANO-TYPE FORMULA FOR KK- AND $K\bar{K}$-SCATTERING AMPLITUDES

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ABSTRACT

The Veneziano model for KK- and $K\bar{K}$-scatterings is discussed by taking into account two Regge trajectories independently, namely ρ-f, and ϕ-f' trajectories. These Veneziano-type scattering amplitudes are available for calculating the ratio of partial decay widths and the S-wave scattering lengths. Finally these results are compared with the experimental data and those calculated by the method of current algebra.

1. Introduction

Recently crossing symmetric Veneziano-type amplitudes [1] for the various scatterings which show Regge asymptotic behaviors in the high energy limit have been constructed with many remarkable and successful results. In particular when there are no external particles with spin as in the case of $\pi\pi$-, πK-, KK-scatterings [2] [3] [4] or there is the only external particle with spin as in the case of $\pi\pi \to \pi\omega$ [1], $\pi\pi \to \pi A_1$, $\pi\pi \to \pi A_2$ [5] [6], $\pi K \to \pi K^*$ and $\pi K \to \pi K_A$ [7] as well as in the case of $\pi A \to BC$ [8], Veneziano-type formulas have been applied. Fairly good results are obtained through Adler's consistency condition and the universal slope of Regge trajectories, namely mass formulas, relations among coupling constants and so forth. These successful achievements for mesons have been promoting further extension of Veneziano-type amplitudes to hadrons as in the case of πN- and KN-scatterings [8] [9] [10] [11], and several predictions have been found in good agreement with the experiments. Especially the author takes much interest in the fact that several form factors have been calculated in good agreement with the experimental data by correlating Veneziano-type scattering amplitudes with partially conserved axial-vector current, current algebra, the field-current identity and the soft-meson methods [7] [12] [13] [14] [15] [16]. Spurred by these remarkable successes, the author means to take himself into a slight adventuresome attempt in the present note.

Veneziano-type amplitudes for KK- and $K\bar{K}$-scatterings have been constructed as mentioned before [3] and several remarkable results are obtained, namely the mass formulae among π-, K-, ϕ-, K^*-, ρ-, f'-, K^{**}- and A_2-mesons and the relations among coupling constants for the vertices: $\rho K\bar{K}$, $\omega K\bar{K}$, $\phi K\bar{K}$, $f K\bar{K}$, $A_2 K\bar{K}$ and $f' K\bar{K}$ by assuming the universal slope for Regge trajectories and Adler's consistency condition, although $K\bar{K}$ S-wave scattering lengths calculated are entirely different from the results predicted by the method of current algebra. In that letter [3], however, two different trajectories are needed by the request of two conditions 1) crossing symmetry and 2) conservation of Strangeness, namely ρ- f and ϕ- f' trajectories. It also was shown because of taking these different trajectories into account that ω-A_2 trajectory should be associated and degenerated with ρ-f trajectory.

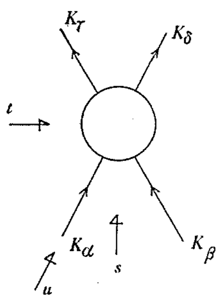
In this present note Veneziano-type amplitudes for KK- and K$\bar{\text{K}}$-scatterings are presented by taking account of ρ-f and ϕ-f' trajectories independently in which there exist poles at points corresponding to resonances. Next it is intended to lead the ratio of partial decay widths and the S-wave scattering lengths by means of these Veneziano-type amplitudes for KK- and K$\bar{\text{K}}$-scatterings and the results obtained in the reference [3].

2. Scattering Amplitudes

The crossing-symmetric Veneziano-type representations for KK- and K$\bar{\text{K}}$-scattering amplitudes are presented, which have simple poles at points corresponding to resonances on Regge trajectories and show asymptotic Regge behaviors in the high energy limit. First of all KK-scattering amplitude is decomposed as follows;

$$\begin{aligned} T_{\gamma\alpha,\delta\beta}(s,t,u) &= \delta_{\alpha\gamma}\,\delta_{\beta\delta}\,A(s,t,u)+\delta_{\beta\gamma}\,\delta_{\alpha\delta}\,B(s,t,u) \\ &= \frac{1}{2}\,\delta_{\alpha\gamma}\,\delta_{\beta\delta}\,\{A^{(0)}(s,t,u)+A^{(1)}(s,t,u)\} \\ &\quad -\frac{1}{2}\,\delta_{\alpha\delta}\,\delta_{\beta\gamma}\,\{A^{(0)}(s,t,u)-A^{(1)}(s,t,u)\} \end{aligned} \tag{2.1}$$

, where superscript numbers show isospin states.



Therefore,

$$\begin{aligned} A^{(0)}(s,t,u) &= A(s,t,u)-B(s,t,u) \\ A^{(1)}(s,t,u) &= A(s,t,u)+B(s,t,u) \end{aligned} \tag{2.2}$$

On the other hand according to crossing symmetry for the exchanges $t \leftrightarrow u$ and $\gamma \leftrightarrow \delta$,

$$T_{\gamma\alpha,\delta\beta}(s,t,u)=T_{\delta\alpha,\gamma\beta}(s,u,t)\,.$$

Therefore,

$$A(s,t,u)=B(s,u,t)\,. \tag{2.3}$$

Taking KK-scattering into the s-channel, $A^{(0)}(s,t,u)$ is odd under $t \leftrightarrow u$ crossing from (2.2) and (2.3), while $A^{(1)}(s,t,u)$ is even. Since amplitudes $A^{(0)}(s,t,u)$ and $A^{(1)}(s,t,u)$ can't include s as an explicit variable, we can write down the following Veneziano-type amplitudes for KK-scattering in the s-channel.

$$\begin{aligned} A_s^{(0)}(s,t,u) &= \pm\beta^{(0)}(\alpha(t)-\alpha(u))\,B(1-(t),1-\alpha(u)) \\ A_s^{(1)}(s,t,u) &= \beta^{(1)}\,\{C\pm(\alpha(t)+\alpha(u))\}\,B(1-\alpha(t),1-\alpha(u)) \end{aligned} \tag{2.4}$$

In these equations $B(1-\alpha(t), 1-\alpha(u))$ is the ordinary B-function,

$$B(1-\alpha(t),1-\alpha(u))=\frac{\Gamma(1-\alpha(t))\,\Gamma(1-\alpha(u))}{\Gamma(2-\alpha(t)-\alpha(u))}\,.$$

$\alpha(t)$ and $\alpha(u)$ are Regge trajectory functions, C is the constant number which is determined by Adler's PCAC consistency condition [17] and parameters $\beta^{(0)}$ and $\beta^{(1)}$ can be related to the various coupling constants and masses of resonance-particles by taking the poles at $\alpha(t)=1$ of K$\bar{\text{K}}$-scattering amplitude in the t-channel and comparing them with the corresponing Born terms.

The K$\bar{\text{K}}$-scattering amplitudes in the t and u-channels are always related to the s-channel amplitudes through the crossing matrices, for example;

$$\begin{pmatrix} A_t^{(0)} \\ A_t^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} A_s^{(0)} \\ A_s^{(1)} \end{pmatrix}$$

$$A_t^{(0)}(s,t,u) = \frac{1}{2}[\pm\beta^{(0)}(\alpha(t)-\alpha(u))B(1-\alpha(t),1-\alpha(u))$$
$$+3\beta^{(1)}\{C\pm(\alpha(t)+\alpha(u))\}B(1-\alpha(t),1-\alpha(u))]$$
$$A_t^{(1)}(s,t,u) = -\frac{1}{2}[\pm\beta^{(0)}(\alpha(t)-\alpha(u))B(1-\alpha(t),1-\alpha(u))$$
$$-\beta^{(1)}\{C\pm(\alpha(t)+\alpha(u))\}B(1-\alpha(t),1-\alpha(u))]\,. \qquad (2{\cdot}5)$$

The universal slope b for Regge trajectories is also assumed in this note, which meets the following equation,

$$2b\,(m_\rho^2-m_\pi^2)=1\,. \qquad (2{\cdot}6)$$

I. In the case of taking ρ-f trajectory as Regge trajectory.

$$\alpha_\rho(t)=1+b\,(t-m_\rho^2)$$

First of all according to Adler's consistency condition

$$A_s^{(1)}(m_K^2, m_K^2, m_K^2)=\beta_\rho^{(1)}[C_\rho\pm\{2+2b(m_K^2-m_\rho^2)\}]$$
$$\times B(1-\alpha_\rho(m_K^2),\ 1-\alpha_\rho(m_K^2))=0\,.$$

Since $\quad B(1-\alpha_\rho(m_K^2),\,1-\alpha_\rho(m_K^2))$ is not zero,

$$C_\rho\pm\{2+2b(m_K^2-m_\rho^2)\}=0$$
$$C_\rho=\mp\left(2+\frac{m_K^2-m_\rho^2}{m_\rho^2-m_\pi^2}\right)\,. \qquad (2{\cdot}7)$$

Secondly in order to find relations between parameters $\beta_\rho^{(0)}$, $\beta_\rho^{(1)}$ and coupling constants $g_{\rho KK}$, $g_{\sigma KK}$ we choose the effective Hamiltonians as follows;

$$H=g_{\rho KK}\,\rho_\mu^0\,(\bar{K}\,\partial_\mu K-K\,\partial_\mu\bar{K})$$
$$H=g_{\sigma KK}\,\sigma\,K\bar{K}\,, \qquad (2{\cdot}8)$$

where σ-meson is regarded as the first daughter of ρ-meson.
Using these Hamiltonians, the Born terms for resonance poles, ρ- and σ-mesons, are obtained as follows;

$$\frac{g_\sigma^2{}_{KK}}{t-m_\sigma^2}+\frac{4g_\rho^2{}_{KK}\,\mathbf{qq}'}{t-m_\rho^2} \qquad \mathbf{qq}'=\mathbf{q}^2\cos\theta_t\quad, \qquad (2{\cdot}9)$$

where **q** is the momentum of incoming particle and **q**′ is that of outgoing particle in the center of mass system. On the other hand Veneziano-type amplitude in the t-channel, namely for $K\overline{K}$-scattering, is expressed as follows;

$$\frac{1}{2}A_t^{(0)}(s,t,u)+\frac{1}{2}A_t^{(1)}(s,t,u)$$
$$=\beta_\rho^{(1)}\left[C_\rho\pm\{\alpha_\rho(t)+\alpha_\rho(u)\}\right]B(1-\alpha_\rho(t),1-\alpha_\rho(u))\ . \qquad (2\cdot10)$$

Taking the pole terms at $\alpha_\rho(t)=1$ of the amplitude (2·10) and comparing them with the Born terms for ρ- and σ-meson poles (2.9),

$$\mp\frac{1}{2}\beta_\rho^{(1)}m_\rho^2=g_\sigma^2{}_{KK} \qquad \mp2\beta_\rho^{(1)}=4g_\rho^2{}_{KK}\ .$$

Therefore

$$\beta_\rho^{(1)}=\mp2g_\rho^2{}_{KK}=\mp2g_\sigma^2{}_{KK}m_\rho^{-2} \qquad (2\cdot11)$$

(the order of signs is the same as above equations).

On the other hand, ρ-f trajectory should be degenerated with ω-A_2 trajectory from (2·5) in these representations, too.

Therefore

$$g_\rho^2{}_{KK}=g_\omega^2{}_{KK}$$

and the relations between $\beta_\rho^{(0)}$ and $\beta_\rho^{(1)}$ can be found from (2·5) and (2·10) as follows;

$$\beta_\rho^{(0)}=\pm\beta_\rho^{(1)} \qquad (2\cdot12)$$

(the order of signs is arbitrary).

II. In the case of taking ϕ-f' trajectory.

$$\alpha_\phi(t)=1+b(t-m_\phi^2)$$

By the quite similar methods to the case of ρ-f trajectory, the same results in form can be obtained

$$C_\phi=\mp\left(2+\frac{m_K{}^2-m_\phi^2}{m_\rho^2-m_\pi^2}\right) \qquad (2\cdot13)$$

$$\beta_\phi^{(1)}=\mp2g^2_{\phi KK}=\mp2g^2_{\eta_{0+}KK}m_\phi^{-2} \qquad (2\cdot14)$$

(the order of signs is the same as above equations)

$$\beta_\phi^{(0)}=\pm\beta_\phi^{(1)} \qquad (2\cdot15)$$

(the order of signs is arbitary),

η_{0+}-meson is regarded as the first daughter of ϕ-meson.
Similarly to the case I, it is natural that there should be a Regge trajectory with isospin 1 which is associated and degenerated with ϕ-f' trajectory, though it has not yet been confirmed.

3. The partial decay widths

The partial decay width is given in the following form through Regge Pole theory,

$$\frac{\Gamma}{2} = -\frac{\mathbf{q}_r}{(2l+1)\pi} \cdot \frac{R}{m_r} \left(\frac{d\, R_e\alpha(t)}{d\,E}\right)^{-1}_{E=E_r}$$
$$= -\frac{\mathbf{q}_r}{(2l+1)\pi} \cdot \frac{R}{2bm_r} \qquad (3\cdot1)$$

, where $\alpha(t)$ is Regge trajectory function, R the residue function at resonance poles, b the universal slope for Regge trajectories, m_r the mass of resonance particle, E_r the energy of resonance particle, and $\mathbf{q}_r$ the momentum of K-meson in the center of mass system.

Therefore, taking f-, A_2-, ϕ-, and f' -mesons as resonance particles in K$\overline{\text{K}}$-scattering, the partial decay widths are represented as follows;

$$\frac{\Gamma(f\to K\overline{K})}{2} = -\frac{\mathbf{q}_f{}^5}{5\pi}\ \frac{1}{m_f^2}\ (\mp 2\beta_\rho{}^{(1)})$$
$$\frac{\Gamma(A_2\to K\overline{K})}{2} = -\frac{\mathbf{q}^5_{A_2}}{5\pi}\ \frac{m^2_{A_2}}{1}\ (\mp 2\beta_\rho{}^{(1)})$$
$$\frac{\Gamma(\phi\to K\overline{K})}{2} = -\frac{\mathbf{q}_\phi{}^3}{3\pi}\ \frac{1}{2bm_\phi^2}\ (\mp 2\beta_\phi{}^{(1)})$$
$$\frac{\Gamma(f'\to K\overline{K})}{2} = -\frac{\mathbf{q}_{f'}{}^5}{5\pi}\ \frac{1}{m_{f'}^2}\ (\mp 2\beta_\phi{}^{(1)})$$

The ratio among $\Gamma(f\to K\overline{K})$, $\Gamma(A_2\to K\overline{K})$ and $\Gamma(f'\to K\overline{K})$ is calculated as follows;

$$\Gamma(f\to K\overline{K}) : \Gamma(A_2\to K\overline{K}) : \Gamma(f'\to K\overline{K})$$
$$= \frac{g^2_{\rho KK}\mathbf{q}^5_f}{m_f^2} : \frac{g^2_{\omega KK}\mathbf{q}^5_{A_2}}{m^2_{A_2}} : \frac{g^2_{\phi KK}\ \mathbf{q}^5_{f'}}{m^2_{f'}} = 1.0 : 1.1 : 5.3 \quad , \qquad (3\cdot3)$$

where the following relations among coupling constants are used [3];

$$g^2_{\rho KK} = g^2_{\omega KK} = \frac{1}{2}\, g^2_{\phi KK} \qquad g^2_{A_2KK} = g^2_{fKK} = \frac{1}{2}\, g^2_{f'KK}$$

Furthermore the ratio, $\Gamma(\phi\to K\overline{K})$: $\Gamma(f'\to K\overline{K})$ also is decided as follows;

$$\Gamma(\phi\to K\overline{K}) : \Gamma(f'\to K\overline{K}) = 0.066 : 1.00 \qquad (3\cdot4)$$

4. The S-wave scattering lengths

The KK and K$\overline{\text{K}}$ S-wave scattering lengths can be calculated by means of eqs. (2·4), (2·5) and the above-obtained results (2·7) (2·11) (2·13) (2·14) (2·15), since the S-wave scattering length is given by the following formula

$$a^{(I)} = -\frac{1}{16\pi W}\ R_e[A^{(I)}(s,\ t,\ u;\ l=0)\,]\,. \qquad (4\cdot1)$$

W is the total energy of this scattering in the center of mass system.

I. In the first case, namely taking ρ-f trajectory.
Concerning the KK S-wave scattering lengths, as $t=u=0$

$$A_s^{(0)}=0 \qquad A_s^{(1)}=2.61\, g^2_{\rho KK} \quad ,$$

where is adopted the following value for coupling constant [3],

$$\frac{g^2_{\rho KK}}{4\pi} \approx 2.1 \, .$$

Therefore, from eq. (4.1)

$$a_s^{(0)}(KK)=-\frac{1}{16\pi\cdot 2m_K} A_s^{(0)}=0$$

$$a_s^{(1)}(KK)=-\frac{1}{16\pi\cdot 2m_K} A_s^{(1)}=-0.68 m_K^{-1} \, . \tag{4.2}$$

Concerning the $K\overline{K}$ S-wave scattering lengths, as $t=4m_K^2$ and $u=0$,

$$A_t^{(0)}=2.60\, g^2_{\rho KK} \qquad A_t^{(1)}=-0.52\, g^2_{\rho KK} \, ,$$

Similarly,

$$a_t^{(0)}(K\overline{K})=-\frac{1}{16\pi\cdot 2m_K} A_t^{(0)}=-0.68\, m_K^{-1}$$

$$a_t^{(1)}(K\overline{K})=-\frac{1}{16\pi\cdot 2m_K} A_t^{(1)}=0.14\, m_K^{-1} \, . \tag{4.3}$$

II. In the second case, namely taking ϕ-f' trajectory.
By the similar method to above case,

$$A_s^{(0)}=0 \, , \; A_s^{(1)}=1.03\, g^2_{\phi KK}$$

$$a_s^{(0)}(KK)=0$$

$$a_s^{(1)}(KK)=-0.54\, m_K^{-1} \tag{4.4}$$

$$A_t^{(0)}=-45.0\, g^2_{\phi KK} \qquad A_t^{(1)}=9.00\, g^2_{\phi KK}$$

$$a_t^{(0)}(K\overline{K})=23.6\, m_K^{-1}$$

$$a_t^{(1)}(K\overline{K})=-4.73\, m_K^{-1} \, . \tag{4.5}$$

5. Results and further discussions

The crossing-symmetric Veneziano-type amplitudes for KK- and $K\overline{K}$-scatterings have been constructed by choosing two conceivable Regge trajectories independently. Besides the ratio of partial decay widths and the KK and $K\overline{K}$ S-wave scattering lengths have been calculated through these scattering amplitudes as mentioned above. In this section these results are investigated closely in comparison of each case with the experimental data and those predicted by the method of current algebra.

At first concerning the ratio of partial decay widths, the experimental ratio of decay modes $\phi \to K\overline{K}$ and $f' \to K\overline{K}$ is as follows;

$$\Gamma(\phi \to K\overline{K}):\; \Gamma(f' \to K\overline{K})=0.062:\; 1.00$$

This value has a good agreement with the result (3·4) which is evaluated theoretically.

On the other hand the experimental widths of f-, A_2- and f' -mesons are 154MeV, 85MeV and 73MeV respectively, and further the fraction of decay mode $f' \to K\overline{K}$ is measured to be 72% [18]. By making use of these measured data and the calculated result of the ratio among decay modes: $f \to K\overline{K}$, $A_2 \to KK$, $f' \to K\overline{K}$ (3·3), the fractions of decay modes $f \to K\overline{K}$ and $A_2 \to K\overline{K}$ can be predicted to be 6.4% and 13% respectively. One of these predicted results, namely in the case of $f \to K\overline{K}$, is considerably coincident with the measured value (about 5%), but the other, namely in the case of $A_2 \to K\overline{K}$, is not so good and larger in comparison with the measured value (about 5%).

The reason for this discrepancy of the latter from the measured value should be guessed chiefly that the dominant decay mode of A_2-meson involves a vector meson with spin 1, namely $A_2 \to \rho\pi$. On the other hand the dominant decay mode of f- meson does not involve any particle with spin except for pseudoscalar mesons. Therefore it is not a good and discreet method that the linearly rising ω-A_2 trajectory with the universal slope is applied in making KK- and K$\overline{\text{K}}$-scattering amplitudes, and this trajectory should not be regarded simply as degenerated with ρ- f trajectory. This fact may tell the incompleteness of Veneziano theory or crossing symmetry. The discussion about this point will be taken under consideration later.

Next, the S-wave scattering lengths are down for discussion.

In the s-channel, $a_s^{(0)}$ (KK)'s are all zero, while there exist the slight difference between the values of two S-wave scattering lengths $a_s^{(1)}$ (KK)'s. Nevertheless their values, on the whole, are consistent with the current algebra predictions $a_s^{(1)}$ (KK)$=-0.54$ m_K^{-1} [3] [20] [21].

On the other hand the S-wave scattering lengths in the t-channel calculated separately in two cases, $a_t^{(0)}$ (K$\overline{\text{K}}$)'s and $a_t^{(1)}$ (K$\overline{\text{K}}$)'s are considerably different from one another, and what is more, they are incompatible with the current algebra predictions [3] [21]. The reason why the K$\overline{\text{K}}$ S-wave scattering lengths in this note are very different from the current algebra predictions seems to be firstly because Veneziano-type scattering amplitude contains not only the possible resonances on the leading trajectory with masses around K$\overline{\text{K}}$ threshold but also those on its parallel daughter trajectories which similarly have masses near K$\overline{\text{K}}$ threshold and secondly because they have been calculated by assuming Adler's consistency condition in the KK- and K$\overline{\text{K}}$-scatterings, too.

On the other hand the reason why the KK S-wave scattering lengths have fairly good agreement with the values by means of the current algebra method in spite of similarly assuming Adler's consistency condition seems to be chiefly because there are no relevant resonances on Regge trajectories which can be adoped in the s-channel owing to the conservation of Strangeness.

Thus, although the KK S-wave scattering lengths in two cases are nearly equal, it would be taken for granted that the K$\overline{\text{K}}$ S-wave scattering lengths are very different from one another. If that is the case, which should be considered to be more well-grounded than other? Judging from the ratio of partial decay widths calculated (3.3) and the experimental data, the resonance particles on ϕ-f' trajectory seem to play their important parts in K$\overline{\text{K}}$ scattering and become the main elements to decide the K$\overline{\text{K}}$ S-wave scattering lengths. Therefore $a_t^{(0)}$ $(K\overline{K})=23.6$ m_K^{-1} and $a_t^{(1)}$ $(K\overline{K})=-4.73$ m_K^{-1} seem to be the most approached values to true $K\bar{K}$ S-wave scattering lengths in comparison with other values, and then $a_s^{(1)}(KK)=-0.54\ m_K^{-1}$ seems to be the most agreeable value to the KK S-wave scattering length because of crossing symmetry. But still these values of $a_t^{(0)}(K\bar{K})$ and $a_t^{(1)}(K\bar{K})$ may be far much larger than usual, and that have not yet been established definitely by the experiment. The large $K\bar{K}$ S-wave scattering lengths, however, have been confirmed and can be admitted roughly by analogy with the diverse experimental data [18] [20] [21] [22]. For all practical purposes, in order to get reliable $K\bar{K}$ S-wave scattering lengths, they should be discussed taking into account simultaneously not only three trajectories adopted here but also those which consist of the resonance particles able to decay into $K\bar{K}$. The discussions about this problem will be made before long through the experimental data.

Next, what we must pay attention to is whether PCAC Adler's consistency condition may be applied in constructing the present scattering amplitudes. It is well known that the method of soft-pion limit have given fairly good results, but since the mass of kaon is much larger than that of pion, the application similar to soft-pion limit in KK- and $K\bar{K}$-scatterings, namely soft-kaon limit, is not easily acceptable, nevertheless there are a few examples that the soft-kaon method stands well in Veneziano model [3] [23]. Therefore it is natural that there should exist ambiguities in the results calculated in this note on account of the simple application of soft-kaon limit. With regard to pros and cons of the application of soft-kaon limit or this applicable boundary, further discussion will be performed in future by the author with more experimental informations about KK and $K\bar{K}$-scatterings and the theoretical results by the method of soft-kaon limit.

Now, at last it is noticeable that the validity of crossing symmetry has been shown in the course of writing down the present note. That is to say, although how to choose the signs $\pm$ in the amplitudes (2·4) is arbitrary, the S-wave scattering lengths always show the same values.

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$\pi_N 1^-(0^+)\to K\bar{K}$, $(5.0\sim13.0 m_K^{-1})$, R_e part$=-2.3$F,
$(6.3\pm2.5)m_K^{-1}$, I_m part$=0.5$F,
$\eta_{0^+}0^+(0^+)\to K\bar{K}$, $(\pm2.8\pm0.5i)m_K^{-1}$,
$S^*\to K_S K_S$, $(5.0\sim13.0)m_K^{-1}$.

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$a_s^{(0)}(KK)=0$, $a_s^{(1)}(KK)=-0.53\ m_K^{-1}$.

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$a_s^{(1)}(KK)=-0.68\ m_K^{-1}$, $a_t^{(1)}(K\bar{K})=0.68\ m_K^{-1}$,
these are calculated by Khuri's method.
$R_e\, a_t^{(0)}(K\bar{K})=2.04\ m_K^{-1}$,
this is calculated through the combination between Weinberg's method and the forward dispersion relations.
$R_e\, a_t^{(0)}(K\bar{K})\approx(5.0\sim 8.3)\ m_K^{-1}$,
$I_m\, a_t^{(0)}(K\bar{K})\approx(0.3\sim 0.5)\ m_K^{-1}$,
these are introduced from the results by R.I. Hess *et al.*

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$R_e\, a_t^{(1)}(K\bar{K})$ $(8.8\pm 2.5)m_K^{-1}$, $(5.0\pm 2.5)\ m_K^{-1}$, $(-22.5\pm 7.5)\ m_K^{-1}$, $(-5.8\pm 1.0)\ m_K^{-1}$,
$|R_e\, a_t^{(1)}(K\bar{K})|\approx 6.3\pm 2.5 m_K^{-1}$, good fit.
We have therefore no access to the sign of the scattering length $R_e\, a_t^{(1)}(K\bar{K})$.
$a^{(1)}\approx(-5.75\pm 1.3i)\ m_K^{-1}$.

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$R_e\, a_t^{(0)}(K\bar{K})\approx(5.0\sim 15.0)\ m_K^{-1}$.

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