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EMISSION-LINE INTENSITIES OF THE HYDROGEN ATOM

BY

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ABSTRACT

The total intensities of H_α, H_β and P_α lines emitted from a finite uniform plane-parallel atmosphere are calculated for all combinations of electron temperatures of 6000, 7000, 8000, 10000, 15000, 20000°K, total number densities of 10^{10}, 10^{11}, 10^{12}, 10^{13} cm^{-3}, and geometric thicknesses of 1000, 10000, 100000 km. The turbulent velocity is taken as 10 km/sec. The atmosphere, standing vertically upon the solar surface, is illuminated on both sides by photospheric radiation and chromospheric UV radiation. The model atom has four discrete levels and a continuum. The line is assumed to be purely Doppler broadened. Detailed balance in Lyman lines is assumed.

1. Introduction

Balmer lines are very strong in most of visual emission spectra of cosmical objects because hydrogen is the most abundant. For atmospheres which are optically thick in Balmer lines, it is a complicated problem to calculate their intensities. It is necessary to solve simultaneously the transfer equations of several lines and continua with the steady-state equations. With the recent advance of the numerical solution of the transfer equation, several authers have solved the hydrogen multi-level problem (see Athay, Mathis and Skumanich 1968). There are some investigations concerned with a comparison with observations. Hearn (1966, 1967) has calculated the total intensities of Lyman α and β lines emitted from a finite slab and tried to determine the physical conditions of the emitting region. Cuny (1968) has calculated the profiles of Lyman α and β lines for a few models of the chromosphere and further obtained the absorption profiles of H_α and H_β lines assuming detailed balance in Lyman lines and compared them with the observations. In the study of early-type model atmospheres Mihalas and Auer have studied the effect of hydrogen lines and continua on the model atmosphere and obtained the absorption profiles of Balmer lines (see Mihalas, 1970)

However no exact calculation has been made on the emission intensities of Balmer lines as yet. The physical conditions in spicules and prominences are wellknown as compared with those in other cosmical objects showing emission spectra. By comparing the theoretical intensities with their observations, we can understand the properties of Balmer lines better. Such knowledge would be useful for the study of cosmical emission objects whose physical structures are unknown. Thus we calculate Balmer emission intensities in the conditions of chromosphere and prominences.

2. Steady-state equations

Throughout this work we use a model atom with four discrete levels and a continuum. The model atom is governed by Rydberg formula with a Rydberg constant $R_H=$ 109677.576 cm^{-1}. Oscillator strengths are taken from Goldwire (1968). Photoionization cross sections are obtained from the approximation formulae for bound-free gaunt factors given by Mihalas (1967). Following the suggestion of Sampson (1969), we adopt the collisional transition rates used by Peterson and Strom (1969).

The steady-state equations are written in terms of the non-equilibrium factor b_j defined by

$$b_j=N_j/N_j^* \,, \tag{1}$$

where N_j is the population of the level j and N_j^* is the population in the local thermodynamic equilibrium. We then have

$$\begin{aligned} -\sum_i b_i\, C_{ij}+b_j\, [\sum_i (C_{ji}+\Delta_{ji})+\sum_k C_{jk}+C_{jc}+R_{jc}] \\ -\sum_k b_k(C_{kj}+\Delta_{kj})=C_{cj}+R_{cj},\ i<j<k \,, \end{aligned} \tag{2}$$

where

$$\Delta_{ji}=A_{ji}\, g_j \exp\, (h\nu_j/kT_e)\, \delta_{ji} \,, \tag{3}$$

$$R_{jc}=g_j \exp\, (h\nu_j/kT_e) \int_{\nu_j}^{\infty} \frac{4\pi a_j(\nu)}{h\nu} J_\nu\, d\nu \,, \tag{4}$$

$$R_{cj}=g_j \exp\, (h\nu_j/kT_e) \int_{\nu_j}^{\infty} \frac{4\pi a_j(\nu)}{h\nu} \left(\frac{2h\nu^3}{c^2}+J_\nu\right) \exp\, (-h\nu/kT_e) d\nu \,, \tag{5}$$

$$C_{ij}=C_{ji}=C_{ij}^*\, g_j \exp\, (h\nu_j/kT_e) \,, \tag{6}$$

$$C_{jc}=C_{cj}=C_{jc}^*\, g_j \exp\, (h\nu_j/kT_e) \,, \tag{7}$$

where A_{ji} is the Einstein coefficient of spontaneous emission, g_j is the statistical weight of the level j, ν_j is the frequency at the head of the j-th continuum, T_e is the electron temperature, δ_{ji} is the net radiative bracket defined by Thomas (1960), $a_j(\nu)$ is the photoionization cross section from the level j, J_ν is the mean intensity, and C_{ij}^* and C_{jc}^* are the usual collisional excitation and ionization cross sections, respectively.

The total number density N is assumed to be constant across the atmosphere considered. The conservation equation of particles is written as

$$N_e^2 \sum_j b_j\, \phi_j(T_e)+2N_e=N \,, \tag{8}$$

where N_e is the electron density and

$$\phi_j\, (T_e)=\left(\frac{h^2}{2\pi\, m\, kT_e}\right)^{3/2} \frac{g_j \exp\, (h\nu_j/kT_e)}{2U_c} \,, \tag{9}$$

where U_c is the partition function of the proton, which is equal to unity.

3. Transfer equation

The transfer equation of a spectral line is solved on the assumptions that the source function is independent of frequency, that the line is purely Doppler broadened, and that the continuous absorption is negligible. Then the transfer equation is written as

$$\mu \frac{d I_{x,\mu}(\tau)}{d\tau} = X(x)\,[I_{x,\mu}(\tau) - S(\tau)]\,, \tag{10}$$

where $I_{x,\mu}(\tau)$ is the specific intensity, μ is the cosine of the angle between the direction of propagation and the outward normal of the atmosphere, x is the non-dimensional frequency measured from the line center in Doppler widths, and τ is the mean optical depth* related to the geometrical depth z by

$$d\tau = \frac{h\nu_{ij}}{4\pi \Delta\nu_D} N_i^* \mathfrak{B}_{ij}\,[b_i - b_j \exp(-h\nu_{ij}/kT_e)]\,\mathrm{d}z\,, \tag{11}$$

where $\mathfrak{B}_{ij}$ is the Einstein coefficient of absorption and $\Delta\nu_D$ is the Doppler width. The function $X(x)$ is the normalized Doppler profile. The frequency-independent source function $S(\tau)$ is expressed in tems of its own radiation field as

$$S(\tau) = \frac{\int_{-\infty}^{\infty} J_x(\tau)\, X(x) dx + W(\tau)\, B}{1+\epsilon(\tau)}\,, \tag{12}$$

where $\epsilon(\tau)$ and $W(\tau)$ are the parameters which depend on the mean intensities of all lines except the line of interest and all continua, and B is the Planck function. From the definition of the net radiative bracket we have

$$\delta_{ji} = \frac{W}{S} - \epsilon\,. \tag{13}$$

For the continuum transfer equation, the overlap of continua is taken into account. It becomes important as the electron temperature increases and as we go to higher levels. Kawaguchi (1965) has demonstrated that the effect of the Lyman α radiation on photoionizations is negligible. Hence the contribution of all lines to photoionizations is ignored. The transfer equation of the j_1-th continuum in the frequency range $\nu_{j_1} \leqslant \nu \leqslant \nu_{j_1 - 1}$ is written as

$$\mu \frac{d I_{\nu,\mu}(\tau)}{d\tau} = X(\nu,\tau)\,[I_{\nu,\mu}(\tau) - S_\nu(\tau)]\,, \tag{14}$$

where τ is the optical depth at the head of the continuum defined by

$$d\tau = \sum_{j \geqslant j_1} N_j^*\, a_j(\nu_{j_1})\,[b_j - \exp(-h\nu_{j_1}/kT_e)]\, dz\,, \tag{15}$$

and the function $X(\nu, \tau)$ is given by

$$X(\nu, \tau) = \frac{\sum_{j \geqslant j_1} N_j^*\, a_j(\nu)\,[b_j - \exp(-h\nu/kT_e)]}{\sum_{j \geqslant j_1} N_j^*\, a_j(\nu_{j_1})\,[b_j - \exp(-h\nu_{j_1}/kT_e)]}\,. \tag{16}$$

The source function $S_\nu(\tau)$ is expresses in terms of its own radiation field with help of the steady-stae equations.

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* The mean optical depth and thickness are $\sqrt{\pi}$ times as large as those in the line center.

$$S_\nu(\tau)=\frac{\xi_\nu(\tau)\left[\sigma_\nu(\tau)\int_{\nu_{j1}}^{\infty}\frac{4\pi\, a_{j_1}(\nu)}{h\nu}J_\nu(\tau)\,d\nu+\widetilde{W}(\tau)\,B_\nu\right]}{1+\tilde{\varepsilon}_\nu(\tau)}\,, \tag{17}$$

where

$$\xi_\nu(\tau)=\frac{[b_{j_1}-\exp(-h\nu/kT_e)]\sum_{j\geqslant j_1} N_j{}^*\, a_j(\nu)}{\sum_{j\geqslant j_1} N_j{}^*\, a_j(\nu)\,[b_j-\exp(-h\nu/kT_e)]}\,, \tag{18}$$

$$\sigma_\nu(\tau)=\frac{B_\nu}{\int_{\nu_{j1}}^{\infty}\frac{4\pi\, a_{j_1}(\nu)}{h\nu}B_\nu d\nu}\,, \tag{19}$$

and $\tilde{\varepsilon}_\nu(\tau)$ and $\widetilde{W}(\tau)$ depend on the mean intensities of all lines and all continua except the relevant continuum.

Thus, the transfer equations of both the line and continuum lead to a uniform expression. These equations are solved by the difference-equation technique suggested by Feautrier (1964). We introduce discrete ordinates in frequency and angle: frequency points for the line $\{x_i\}$, $i=1,\ldots, n_l$, frequency points for the continuum in the range $\nu_{j_1}\leqslant\nu\leqslant\nu_{j_1-1}$ $\{\nu_i\}$, $i=1,\ldots, n_c$, and angle points $\{\mu_j\}$, $j=1,\ldots, m$, and replace the integrals in equations (12) and (17) by weighted sums over discrete frequencies and angles. A single frequency-angle point is denoted by the index $k=i+n(j-1)$. Let us define

$$\Im_k\equiv\frac{1}{2}\,[I_{\nu_i,\,\mu_j}(\tau)+I_{\nu_i,\,-\mu_j}(\tau)]\,. \tag{20}$$

The transfer equations (10) and (14) are replaced to the following equation

$$\frac{\mu_k}{X_k}\frac{\partial}{\partial\tau}\left(\frac{\mu_k}{X_k}\frac{\partial\Im_k}{\partial\tau}\right)=\Im_k-S_k\,. \tag{21}$$

We further introduce discrete depth points $\{z_l\}$, $l=1,\ldots, p$, such that $0<z_1<\ldots<z_p=H/2$, where H is the geometrical thickness of the atmosphere, and replace the derivative by a difference approximation

$$\frac{\mu}{X}\frac{\partial}{\partial\tau}\left(\frac{\mu}{X}\frac{\partial\Im}{\partial\tau}\right)\approx\frac{\mu}{X^l}\frac{1}{\tau_{l+1}-\tau_{l-1}}\left[\left(\frac{\mu}{X^{l+1}}+\frac{\mu}{X^l}\right)\frac{\Im^{l+1}-\Im^l}{\tau_{l+1}-\tau_l}\right.$$
$$\left.-\left(\frac{\mu}{X^l}+\frac{\mu}{X^{l-1}}\right)\frac{\Im^l-\Im^{l-1}}{\tau_l-\tau_{l-1}}\right]\,. \tag{22}$$

Equation (21) then leads to the vector equation

$$-\boldsymbol{A}_l\,\boldsymbol{J}^{l-1}+\boldsymbol{B}_l\,\boldsymbol{J}^l-\boldsymbol{C}_l\,\boldsymbol{J}^{l+1}=\boldsymbol{D}_l\,, \tag{23}$$

where $\boldsymbol{J}^l$ is the column vector of dimension $n\times m$ with components $\Im_k^l$, $\boldsymbol{A}_l$ and $\boldsymbol{C}_l$ are the diagonal matrices, $\boldsymbol{B}_l$ is the nondiagonal matrix, and $\boldsymbol{D}_l$ is the column vector. At the surface we use the boundary condition of Auer (1967), modified for incident radiation, and obtained the vector equation excluding $\boldsymbol{J}^{-1}$. At the center we usually use the symmetric condition. For very optically thick lines it is replaced by the condition of detailed balance. In either case we obtain the vector equation excluding $\boldsymbol{J}^{p+1}$. Since these vector equations are tridiagonal in form, we can solve them by a standard recursion formula.

For the angle integral we make use of a Gauss-Legendre quadrature formula with one angle point in the range $0 \leqslant \mu \leqslant 1$, i.e., $m=1$. For the line integral over frequency we use a Gauss-Hermite formula with two points in half the line profile, i.e., $n_l=2$. The continuum integral contains some difficulty. The recombination rate has the integrand which varies nearly as $\exp(-h\nu/kT_e)/\nu$. On the other hand, the photoionization rate in the atmosphere which is not very optically thick for the corresponding continuum has the integrand with the color temperature of the incident radiation in place of T_e. Accordingly, we must use frequency points and weights suitable for two different temperatures. Further the interval of integration is finite because we take into account the overlap of continua. Making use of exponential integral, we choose three frequency points and weight, i.e., $n_c=3$, so that the integral formula

$$\int_{\nu_j}^{\nu_{j-1}} \exp(-h\nu/kT_e) \frac{d\nu}{\nu} \approx \sum_{i=1}^{n_c} \exp(-h\nu_i{}^j/kT_e) \frac{w_i{}^j}{\nu_i{}^j} \tag{24}$$

is accurate to less than 4% in the temperature range $5000\,°\mathrm{K} \leqslant T_e \leqslant 20000\,°\mathrm{K}$. Adopted frequency points and weights are given in Table 1.

Table 1. Frequency Points and Weights for Continua

j	i	$\nu_i{}^j$ *	$w_i{}^j$ *
	1	13.36	2.06
1	2	11.87	1.23
	3	11.15	0.412
	1	5.13	2.06
2	2	3.65	1.23
	3	2.92	0.412
	1	2.40	0.655
3	2	1.80	0.522
	3	1.357	0.346
	1	1.109	0.210
4	2	0.917	0.174
	3	0.757	0.149

* in μ^{-1}.

We take the primary depth point $z_1=10^{13}/N$ in the case of detailed balance in Lyman lines and otherwise $z_1=10^9/N$, and partition depth scale into equal intervals of 0.2 in log z. When the mean optical thickness in a line or the optical thickness at the head of a continuum $T<10^{-3}$, we put $\mathfrak{J}_k=E_{0,k}$ throughout the atmosphere, where $E_{0,k}$ is the intensity of incident radiation. Only when $T \geqslant 10^{-3}$, we solve thet ransfer equation. Over the range $\tau<10^{-4}$, $\mathfrak{J}_k$ is taken to be constant for the line and continuum. For the line, detailed balance is assumed at optical depths above a critical value. Cuny (1967) has shown that detailed balance is valid at optical depths $\tau > 100/\epsilon$ in a semi-infinite atmosphere with constant ϵ. Following Cuny, it is found that this criterion can apply to a finite atmosphere also. For greater safety we take the critical optical depth $\tau_{crit}=1000/\epsilon$. At optical depths $\tau > \tau_{crit}$ we put $\delta_{ji}=0$ and $S=WB/\epsilon$. Such a situation occurs in only a few atmospheres with the highest density.

In this study we take into account all overlaps of continua. Overlapping continua of lower order contribute to the scattering term of the source function (17). The contributions are calculated from the intensities obtained in the preceding iteration and used as corrections.

We begin the iteration by putting the net radiative brackets of all lines $\delta_{ji}=0$ and the intensities of all continua $\mathfrak{J}_k=E_{0,k}$ throughout the atmosphere. In the course of one iteration we first treat continua in the order of Lyman, Balmer, Paschen and Bracket continua and then lines in the order of Balmer and Paschen series. Lyman lines are skipped, their net radiative brackets remaining zero throughout the iteration. The iteration scheme is similar in form to used by Cuny (1967) and Ishizawa (1971). The electron density and b_j factors are obtained iteratively from equations (2) and (8) whenever a transfer equation is solved. In an earlier stage of the iteration, corrections are too large. If the full correction is applied, the net radiative bracket sometimes becomes unusually negative so that some of b_j factors become negative. This divergence is suppressed by using a fraction of the computed correction between 0.5 and 0.95. In the multi-level problem of the hydrogen atom, line source functions are sometimes inconsistent even after b_j factors are not affected by further iteration (see Avrett 1968). Convergence is therefore checked in two ways: whether the difference of the b_j factors obtained in the last two iterations is less than 1% and whether the self-consistency of line source functions (see Avrett 1968) is attained to less than 5%. The latter condition is always more severe.

4. Calculated results

Calculations are made for all combinations of $T_e=6000$, 7000, 8000, 10000, 15000, 20000°K, $N=10^{10}$, 10^{11}, 10^{12}, 10^{13} cm^{-3}, and $H=1000$, 10000, 100000 km. The turbulent velocity is taken equal to 10 km/sec.

The intensity of incident radiation is taken as the flux incident on unit area of a vertical surface. The incident fluxes of Balmer lines are calculated using the center-limb variations of central intensities given by White (1962). No data of the center-limb variation of P_α line is available. The central intensity of P_α line at the disk center is obtained from

Table 2. Intensities of Incident Radiation

	λ (Å)	E_0*
H_α	6565	2.46(−6)
H_β	4863	1.55(−6)
P_α	18756	7.21(−6)
	749	1.33(−13)
Lc	842	2.32(−12)
	897	9.20(−12)
	1949	5.74(−8)
Bc	2742	6.43(−7)
	3421	1.78(−6)
	4167	5.11(−6)
Pc	5557	1.14(−5)
	7275	1.48(−5)
	9013	1.54(−5)
Brc	10899	1.55(−5)
	13202	1.49(−5)

* erg cm^{-2} sec^{-1} sterad^{-1} and $\Delta\nu=1$ sec.

Arizona-NASA Atlas (Bijl, Kuiper and Cruikshan 1969). The ratio of the flux incident on unit area of a vertical surface to the central intensity at the disk center is taken equal to a mean of the ratio for the near-by darkening (0.5). The Lyman continuum follows a black body of 6650°K (Tousey, 1963; Pottash, 1964). We use this radiation temperature with a dilution factor of 0.5. Limb darkening and blanketing due to absorption lines are taken into account for other continua. Their data are taken from Allen (1963) and Michard (1950). Absolute intensities are obtained from measures of the solar continuum given by Minnaert (1953) and Houtgast (1968). Adopted intensities of incident radiation are given in Table 2.

The calculated results are given in Tables 3, 4 and 5. The mean electron density $<N_e>$ and the mean popularions of discrete levels $<N_j>$ are given in Table 3. They are simple geometrical means. Half the mean optical thicknesses in lines and half the optical thicknesses at the head of continua $T/2$ are given in Table 4. The total intensities I and the central intensities I_c of lines are given in Table 5. If a line has the self-reversal, its peak intensity I_p also is given. All of these are normally emergent intensities.

Physical discussions on the calculated results and their comparisons with the solar observations will appear elsewhere.

Table 3. Mean Electron Density and Mean Populations of Discrete Levels

T_e(°K)	N(cm^{-3})	H(km)	$<N_e>$	$<N_1>$	$<N_2>$	$<N_3>$	$<N_4>$
6000	10^{10}	1000	4.65(9)	7.01(8)	5.01(2)	2.08	6.57(−1)
		10000	4.65(9)	7.00(8)	5.01(2)	2.08	6.57(−1)
		100000	4.62(9)	7.54(8)	4.95(2)	2.06	6.49(−1)
	10^{11}	1000	3.11(10)	3.77(10)	2.18(4)	9.08(1)	2.88(1)
		10000	1.30(10)	7.39(10)	5.19(3)	2.16(1)	6.84
		100000	7.05(9)	8.59(10)	1.37(3)	5.77	1.81
	10^{12}	1000	3.06(10)	9.39(11)	3.08(4)	1.29(2)	4.12(1)
		10000	2.21(10)	9.56(11)	1.25(4)	5.25(1)	1.73(1)
		100000	2.14(10)	9.57(11)	1.09(4)	5.29(1)	1.45(1)
	10^{13}	1000	6.96(10)	9.86(12)	1.17(5)	4.94(2)	1.56(2)
		10000	6.87(10)	9.86(12)	1.08(5)	5.51(2)	1.50(2)
		100000	7.34(10)	9.85(12)	1.07(5)	1.29(3)	2.91(2)
7000	10^{10}	1000	4.67(9)	6.61(8)	4.61(2)	1.91	6.03(−1)
		10000	4.67(9)	6.63(8)	4.61(2)	1.91	6.03(−1)
		100000	4.65(9)	7.03(8)	4.57(2)	1.90	5.98(−1)
	10^{11}	1000	3.21(10)	3.57(10)	2.14(4)	8.91(1)	2.82(1)
		10000	2.40(10)	5.19(10)	1.24(4)	5.20(1)	1.63(1)
		100000	2.23(10)	5.53(10)	1.04(4)	5.17(1)	1.42(1)
	10^{12}	1000	8.79(10)	8.24(11)	1.64(5)	7.00(2)	2.19(2)
		10000	8.60(10)	8.28(11)	1.53(5)	8.42(2)	2.20(2)
		100000	9.52(10)	8.10(11)	1.48(5)	2.47(3)	5.80(2)
	10^{13}	1000	2.93(11)	9.41(12)	1.73(6)	1.14(4)	2.90(3)
		10000	3.58(11)	9.28(12)	1.70(6)	4.41(4)	1.43(4)
		100000	4.64(11)	9.07(12)	1.65(6)	8.94(4)	4.61(4)
	10^{10}	1000	4.69(9)	6.23(8)	4.32(2)	1.79	5.65(−1)
		10000	4.69(9)	6.24(8)	4.32(2)	1.79	5.65(−1)
		100000	4.70(9)	6.03(8)	4.34(2)	1.80	5.67(−1)

Table 3. (Continued)

T_e(°K)	N(cm^{-3})	H(km)	$\langle N_e \rangle$	$\langle N_1 \rangle$	$\langle N_2 \rangle$	$\langle N_3 \rangle$	$\langle N_4 \rangle$
8000	10^{11}	1000	3.48(10)	3.03(10)	2.45(4)	1.02(2)	3.22(1)
		10000	3.89(10)	2.22(10)	2.96(4)	1.29(2)	3.95(1)
		100000	4.00(10)	2.00(10)	3.00(4)	1.86(2)	4.49(1)
	10^{12}	1000	2.13(11)	5.74(11)	8.57(5)	4.80(3)	1.32(3)
		10000	2.32(11)	5.35(11)	8.12(5)	1.30(4)	3.19(3)
		100000	2.89(11)	4.21(11)	6.38(5)	3.22(4)	1.55(4)
	10^{13}	1000	1.21(12)	7.59(12)	1.15(7)	4.94(5)	2.29(5)
		10000	1.67(12)	6.67(12)	1.01(7)	9.56(5)	6.00(5)
		100000	1.76(12)	6.49(12)	9.79(6)	1.04(6)	6.69(5)
10000	10^{10}	1000	4.76(9)	4.87(8)	4.47(2)	1.85	5.76(−1)
		10000	4.76(9)	4.79(8)	4.46(2)	1.84	5.76(−1)
		100000	4.84(9)	3.24(8)	4.39(2)	1.82	5.69(−1)
	10^{11}	1000	4.35(10)	1.29(10)	4.59(4)	1.92(2)	5.96(1)
		10000	4.76(10)	4.74(9)	4.23(4)	1.91(2)	5.71(1)
		100000	4.91(10)	1.77(9)	3.88(4)	2.80(2)	6.50(1)
	10^{12}	1000	4.47(11)	1.07(11)	2.76(6)	3.61(4)	8.99(3)
		10000	4.73(11)	5.34(10)	1.54(6)	7.12(4)	2.94(4)
		100000	4.86(11)	2.82(10)	8.21(5)	7.63(4)	4.98(4)
	10^{13}	1000	4.58(12)	8.41(11)	2.44(7)	3.51(6)	2.62(6)
		10000	4.67(12)	6.57(11)	1.92(7)	3.55(6)	2.79(6)
		100000	4.69(12)	6.12(11)	1.79(7)	3.45(6)	2.72(6)
15000	10^{10}	1000	4.98(9)	4.65(7)	6.30(2)	2.59	7.87(−1)
		10000	4.98(9)	4.62(7)	6.28(2)	2.58	7.84(−1)
		100000	4.98(9)	4.17(7)	5.98(2)	2.46	7.48(−1)
	10^{11}	1000	4.97(10)	5.37(8)	6.37(4)	2.72(2)	8.36(1)
		10000	4.98(10)	4.73(8)	5.90(4)	2.84(2)	8.06(1)
		100000	4.99(10)	2.15(8)	4.03(4)	3.23(2)	7.26(1)
	10^{12}	1000	4.97(11)	6.17(9)	3.24(6)	7.51(4)	2.05(4)
		10000	4.99(11)	2.41(9)	1.20(6)	7.46(4)	3.50(4)
		100000	5.00(11)	6.66(8)	4.61(5)	5.74(4)	4.24(4)
	10^{13}	1000	4.99(12)	2.42(10)	1.20(7)	2.30(6)	2.06(6)
		10000	5.00(12)	7.32(9)	6.11(6)	1.79(6)	1.75(6)
		100000	5.00(12)	3.61(9)	4.99(6)	1.66(6)	1.65(6)
20000	10^{10}	1000	5.00(9)	5.82(6)	5.32(2)	2.19	6.65(−1)
		10000	5.00(9)	5.82(6)	5.31(2)	2.18	6.65(−1)
		100000	5.00(9)	5.76(6)	5.28(2)	2.18	6.61(−1)
	10^{11}	1000	5.00(10)	6.27(7)	5.07(4)	2.21(2)	6.83(1)
		10000	5.00(10)	6.20(7)	5.00(4)	2.41(2)	6.95(1)
		100000	5.00(10)	5.33(7)	4.27(4)	3.78(2)	8.30(1)
	10^{12}	1000	5.00(11)	7.80(8)	2.41(6)	5.90(4)	1.59(4)
		10000	5.00(11)	5.80(8)	1.09(6)	7.64(4)	3.75(4)
		100000	5.00(11)	2.36(8)	3.78(5)	5.24(4)	4.01(4)
	10^{13}	1000	5.00(12)	5.67(9)	8.95(6)	1.84(6)	1.76(6)
		10000	5.00(12)	2.38(9)	3.99(6)	1.29(6)	1.37(6)
		100000	5.00(12)	6.71(8)	2.48(6)	1.00(6)	1.11(6)

Table 4. Half the Mean Optical Thicknesses in Lines and Half the Optical thicknesses at the Heads of Continua

T_e(°K)	N(cm^{-3})	H(km)	$\tau/2$						
			Hα	Hβ	Pα	Lc	Bc	Pc	Brc
6000	10^{10}	1000	1.97(−2)	2.73(−3)	2.55(−4)	2.21(−1)	3.48(−7)	2.21(−9)	5.08(−10)
		10000	1.97(−1)	2.73(−2)	2.55(−3)	2.21	3.48(−6)	2.21(−8)	5.08(−9)
		100000	1.95	2.70(−1)	2.53(−2)	2.38(1)	3.44(−5)	2.19(−7)	5.02(−8)
	10^{11}	1000	8.61(−1)	1.19(−1)	1.11(−2)	1.19(1)	1.52(−5)	9.66(−8)	2.18(−8)
		10000	2.04	2.83(−1)	2.64(−2)	2.34(2)	3.60(−5)	2.30(−7)	5.24(−8)
		100000	5.42	7.49(−1)	7.09(−2)	2.71(3)	9.54(−5)	6.15(−7)	1.40(−7)
	10^{12}	1000	1.22	1.68(−1)	1.57(−2)	2.96(2)	2.14(−5)	1.36(−7)	3.06(−8)
		10000	4.92	6.80(−1)	6.43(−2)	3.02(3)	8.66(−5)	5.59(−7)	1.27(−8)
		100000	4.19(1)	5.79	6.65(−1)	3.02(4)	7.38(−4)	5.66(−6)	1.15(−7)
	10^{13}	1000	4.61	6.37(−1)	6.05(−2)	3.11(3)	8.12(−5)	5.26(−7)	1.20(−7)
		10000	4.27(1)	5.91	6.93(−1)	3.11(4)	7.53(−4)	5.90(−6)	1.20(−6)
		100000	4.22(2)	5.85(1)	1.68(1)	3.11(5)	7.46(−3)	1.42(−4)	3.12(−5)
7000	10^{10}	1000	1.75(−2)	2.42(−3)	2.26(−4)	2.09(−1)	3.20(−7)	2.06(−9)	5.20(−10)
		10000	1.75(−1)	2.42(−2)	2.26(−3)	2.09	3.20(−6)	2.06(−8)	5.20(−9)
		100000	1.73	2.39(−1)	2.24(−2)	2.22(1)	3.17(−5)	2.05(−7)	5.15(−8)
	10^{11}	1000	8.11(−1)	1.12(−1)	1.05(−2)	1.13(1)	1.49(−5)	9.59(−8)	2.41(−8)
		10000	4.68	6.47(−1)	6.13(−2)	1.64(2)	8.58(−5)	5.60(−7)	1.40(−7)
		100000	3.95(1)	5.46	6.24(−1)	1.75(3)	7.23(−4)	5.58(−6)	1.25(−6)
	10^{12}	1000	6.19	8.56(−1)	8.25(−2)	2.60(2)	1.14(−4)	7.55(−7)	1.90(−7)
		10000	5.78(1)	8.00	1.02	2.61(3)	1.06(−3)	9.11(−6)	2.00(−6)
		100000	5.58(2)	7.75(1)	3.06(1)	2.56(4)	1.03(−2)	2.76(−4)	6.99(−5)
	10^{13}	1000	6.54(1)	9.04	1.39	2.97(3)	1.20(−3)	1.24(−5)	2.81(−6)
		10000	6.37(2)	8.87(1)	5.19(1)	2.93(4)	1.18(−2)	5.03(−4)	1.79(−4)
		100000	6.13(3)	8.60(2)	9.04(2)	2.86(5)	1.16(−1)	1.07(−2)	6.39(−3)
8000	10^{10}	1000	1.58(−2)	2.18(−3)	2.03(−4)	1.97(−1)	3.00(−7)	1.94(−9)	5.28(−10)
		10000	1.58(−1)	2.18(−2)	2.03(−3)	1.97	3.00(−6)	1.94(−8)	5.27(−9)
		100000	1.58	2.19(−1)	2.05(−2)	1.90(1)	3.01(−5)	1.96(−7)	5.30(−8)
	10^{11}	1000	8.93(−1)	1.23(−1)	1.16(−2)	9.58	1.70(−5)	1.11(−7)	3.05(−8)
		10000	1.08(1)	1.49	1.47(−1)	7.02(1)	2.05(−4)	1.40(−6)	3.72(−7)
		100000	1.09(2)	1.51(1)	2.20	6.32(2)	2.08(−3)	2.02(−5)	4.41(−6)
	10^{12}	1000	3.13(1)	4.32	5.57(−1)	1.81(2)	5.95(−4)	5.24(−6)	1.32(−6)
		10000	2.95(2)	4.09(1)	1.54(1)	1.69(3)	5.65(−3)	1.45(−4)	3.94(−5)
		100000	2.28(3)	3.20(2)	3.22(2)	1.33(4)	4.46(−2)	3.84(−3)	2.15(−3)
	10^{13}	1000	4.12(2)	5.77(1)	5.08(1)	2.39(3)	8.03(−3)	5.84(−4)	3.04(−4)
		10000	3.52(3)	5.00(2)	8.49(2)	2.10(4)	7.09(−2)	1.18(−2)	8.40(−3)
		100000	3.41(4)	4.85(3)	9.07(3)	2.05(5)	6.91(−1)	1.28(−1)	9.38(−2)
10000	10^{10}	1000	1.53(−2)	2.11(−3)	1.97(−4)	1.54(−1)	3.10(−7)	2.04(−9)	6.22(−10)
		10000	1.52(−1)	2.11(−2)	1.97(−3)	1.51	3.10(−6)	2.04(−8)	6.21(−9)
		100000	1.50	2.07(−1)	1.93(−2)	1.02(1)	3.05(−5)	2.00(−7)	6.04(−8)
	10^{11}	1000	1.57	2.16(−1)	2.04(−2)	4.08	3.18(−5)	2.14(−7)	6.87(−8)
		10000	1.44(1)	2.00	2.04(−1)	1.50(1)	2.94(−4)	2.11(−6)	6.17(−7)
		100000	1.32(2)	1.83(1)	3.12	5.61(1)	2.69(−3)	3.08(−5)	7.18(−6)
	10^{12}	1000	9.38(1)	1.30(1)	3.99	3.35(1)	1.92(−3)	4.04(−5)	1.12(−5)
		10000	5.15(2)	7.22(1)	7.02(1)	1.69(2)	1.07(−2)	8.38(−4)	4.09(−4)
		100000	2.71(3)	3.84(2)	6.20(2)	8.91(2)	5.82(−2)	9.49(−3)	7.07(−3)
	10^{13}	1000	7.83(2)	1.12(2)	2.62(2)	2.66(2)	1.74(−2)	4.41(−3)	3.63(−3)
		10000	6.02(3)	8.72(2)	2.55(3)	2.08(3)	1.37(−1)	4.49(−2)	3.87(−2)
		100000	5.59(4)	8.11(3)	2.46(4)	1.94(4)	1.28	4.36(−1)	3.78(−1)
15000	10^{10}	1000	1.88(−2)	2.60(−3)	2.43(−4)	1.47(−2)	4.37(−7)	2.94(−9)	1.02(−9)
		10000	1.87(−1)	2.59(−2)	2.42(−3)	1.46(−1)	4.36(−6)	2.93(−8)	1.02(−8)
		100000	1.78	2.46(−1)	2.31(−2)	1.32	4.15(−5)	2.79(−7)	9.63(−8)
	10^{11}	1000	1.90	2.62(−1)	2.52(−2)	1.70(−1)	4.42(−5)	3.10(−7)	1.09(−7)
		10000	1.76(1)	2.43	2.68(−1)	1.49	4.10(−4)	3.22(−6)	1.05(−6)
		100000	1.20(2)	1.66(1)	3.16	6.80	2.80(−3)	3.61(−5)	9.31(−6)
	10^{12}	1000	9.59(1)	1.33(1)	7.13	1.95	2.26(−3)	8.59(−5)	2.87(−5)
		10000	3.42(2)	4.92(1)	6.15(1)	7.66	8.43(−3)	8.94(−4)	5.00(−4)
		100000	1.30(3)	1.86(2)	3.76(2)	2.11(1)	3.27(−2)	7.28(−3)	6.07(−3)
	10^{13}	1000	3.27(2)	4.73(1)	1.28(2)	7.67	8.59(−3)	2.98(−3)	2.89(−3)
		10000	1.59(3)	2.34(2)	9.05(2)	2.30(1)	4.46(−2)	2.34(−2)	2.43(−2)
		100000	1.27(4)	1.88(3)	8.21(3)	1.14(2)	3.66(−1)	2.18(−1)	2.29(−1)
20000	10^{10}	1000	1.42(−2)	1.97(−3)	1.84(−4)	1.84(−3)	3.69(−7)	2.49(−9)	8.86(−10)
		10000	1.42(−1)	1.97(−2)	1.84(−3)	1.84(−2)	3.69(−6)	2.49(−8)	8.86(−9)
		100000	1.42	1.96(−1)	1.84(−2)	1.82(−1)	3.67(−5)	2.48(−7)	8.81(−8)
	10^{11}	1000	1.36	1.88(−1)	1.84(−2)	1.98(−2)	3.52(−5)	2.53(−7)	9.11(−8)
		10000	1.34(1)	1.85	2.04(−1)	1.96(−1)	3.47(−4)	2.75(−6)	9.29(−7)
		100000	1.14(2)	1.58(1)	3.34	1.68	2.97(−3)	4.26(−5)	1.13(−5)
	10^{12}	1000	6.39(1)	8.89	5.04	2.47(−1)	1.68(−3)	6.74(−5)	2.25(−5)
		10000	2.82(2)	3.98(1)	5.57(1)	1.83	7.61(−3)	9.23(−4)	5.41(−4)
		100000	9.62(2)	1.38(2)	3.01(2)	7.50	2.71(−2)	6.71(−3)	5.79(−3)
	10^{13}	1000	2.18(2)	3.15(1)	8.52(1)	1.79	6.44(−3)	2.42(−3)	2.49(−3)
		10000	9.18(2)	1.35(2)	5.20(2)	7.48	2.93(−2)	1.72(−2)	1.92(−2)
		100000	5.46(3)	8.15(2)	3.38(3)	2.14(1)	1.84(−1)	1.34(−1)	1.54(−1)

Table 5. The Total Intensities, Central Intensities and Peak Intensities of Lines

T_e(°K)	N(cm^{-3})	H(km)	I*			I_c**			I_p**		
			Hα	Hβ	Pα	Hα	Hβ	Pα	Hα	Hβ	Pα
6000	10^{10}	1000	2.16(3)	1.85(2)	4.98(1)	5.65(−8)	3.59(−9)	3.73(−9)	—	—	—
		10000	2.02(4)	1.83(3)	4.98(2)	5.12(−7)	3.55(−8)	3.72(−8)	—	—	—
		100000	1.15(5)	1.65(4)	4.87(3)	2.29(−6)	3.07(−7)	3.63(−7)	—	—	—
	10^{11}	1000	6.98(4)	7.75(3)	2.17(3)	1.60(−6)	1.48(−7)	1.62(−7)	—	—	—
		10000	1.18(5)	1.72(4)	5.13(3)	2.32(−6)	3.19(−7)	3.82(−7)	—	—	—
		100000	1.67(5)	3.86(4)	1.33(4)	2.59(−6)	6.67(−6)	9.86(−7)	—	—	—
	10^{12}	1000	8.85(4)	1.08(4)	3.10(3)	1.93(−6)	2.04(−7)	2.31(−7)	—	—	—
		10000	1.63(5)	3.60(4)	1.22(4)	2.59(−6)	6.29(−7)	9.01(−7)	—	—	—
		100000	2.56(5)	1.06(5)	8.56(4)	2.70(−6)	1.19(−6)	5.78(−6)	2.96(−6)	—	—
	10^{13}	1000	1.61(5)	3.46(4)	1.15(4)	2.60(−6)	6.07(−7)	8.52(−7)	—	—	—
		10000	2.61(5)	1.08(5)	8.77(4)	2.73(−6)	1.21(−6)	5.89(−6)	3.01(−6)	—	—
		100000	4.75(5)	2.17(5)	2.49(5)	2.89(−6)	1.36(−6)	9.27(−6)	6.75(−6)	2.11(−6)	—
7000	10^{10}	1000	1.99(3)	1.71(2)	4.58(1)	5.01(−8)	3.18(−9)	3.29(−9)	—	—	—
		10000	1.88(4)	1.69(3)	4.57(2)	4.59(−7)	3.14(−8)	3.29(−8)	—	—	—
		100000	1.12(5)	1.54(4)	4.50(3)	2.21(−6)	2.76(−7)	3.22(−7)	—	—	—
	10^{11}	1000	6.96(4)	7.61(3)	2.13(3)	1.55(−6)	1.39(−7)	1.53(−7)	—	—	—
		10000	1.67(5)	3.61(4)	1.20(4)	2.58(−6)	6.08(−7)	8.57(−7)	—	—	—
		100000	2.65(5)	1.08(5)	8.52(4)	2.70(−6)	1.19(−6)	5.56(−6)	2.96(−6)	—	—
	10^{12}	1000	1.82(5)	4.52(4)	1.60(4)	2.64(−6)	7.39(−7)	1.13(−6)	—	—	—
		10000	2.94(5)	1.22(5)	1.15(5)	2.77(−6)	1.23(−6)	7.11(−6)	3.25(−6)	1.24(−6)	—
		100000	6.04(5)	2.89(5)	2.89(5)	2.99(−6)	1.46(−6)	9.36(−6)	9.31(−6)	2.99(−6)	—
	10^{13}	1000	3.34(5)	1.43(5)	1.36(5)	2.96(−6)	1.37(−6)	7.98(−6)	3.84(−6)	1.41(−6)	—
		10000	8.37(5)	4.97(5)	3.78(5)	3.32(−6)	1.86(−6)	1.05(−5)	1.44(−5)	6.04(−6)	1.20(−5)
		100000	1.66(6)	1.47(6)	6.76(5)	3.43(−6)	2.10(−6)	1.14(−5)	3.17(−5)	2.19(−5)	2.30(−5)
8000	10^{10}	1000	1.87(3)	1.59(2)	4.29(1)	4.53(−8)	2.86(−9)	2.97(−9)	—	—	—
		10000	1.77(4)	1.58(3)	4.28(2)	4.18(−7)	2.83(−8)	2.97(−8)	—	—	—
		100000	1.11(5)	1.47(4)	4.27(3)	2.14(−6)	2.55(−7)	2.95(−7)	—	—	—
	10^{11}	1000	7.76(4)	8.56(3)	2.42(3)	1.64(−6)	1.52(−7)	1.68(−7)	—	—	—
		10000	2.14(5)	6.66(4)	2.82(4)	2.65(−6)	9.62(−7)	1.91(−6)	2.67(−6)	—	—
		100000	3.41(5)	1.44(5)	1.70(5)	2.78(−6)	1.23(−6)	8.80(−6)	3.62(−6)	1.27(−6)	—
	10^{12}	1000	2.95(5)	1.17(5)	8.09(4)	2.90(−6)	1.31(−6)	5.14(−6)	3.30(−6)	—	—
		10000	6.26(5)	3.09(5)	2.86(5)	3.23(−6)	1.68(−6)	9.72(−6)	8.87(−6)	3.01(−6)	—
		100000	1.51(6)	1.25(6)	6.10(5)	3.49(−6)	2.17(−6)	1.13(−5)	2.89(−5)	1.84(−5)	2.04(−5)
	10^{13}	1000	1.38(6)	1.17(6)	5.71(5)	4.61(−6)	3.72(−6)	1.39(−5)	2.39(−5)	1.43(−5)	1.87(−5)
		10000	2.95(6)	3.39(6)	9.79(5)	4.94(−6)	4.56(−6)	1.55(−5)	5.61(−5)	4.69(−5)	3.13(−5)
		100000	4.18(6)	5.03(6)	1.21(6)	4.99(−6)	4.63(−6)	1.55(−5)	6.78(−5)	5.83(−5)	3.39(−5)
10000	10^{10}	1000	1.93(3)	1.63(2)	4.36(1)	4.37(−8)	2.73(−9)	2.83(−9)	—	—	—
		10000	1.83(4)	1.61(3)	4.34(2)	4.04(−7)	2.70(−8)	2.82(−8)	—	—	—
		100000	1.15(5)	1.48(4)	4.28(3)	2.09(−6)	2.40(−7)	2.77(−7)	—	—	—
	10^{11}	1000	1.19(5)	1.54(4)	4.47(3)	2.15(−6)	2.50(−7)	2.89(−7)	—	—	—
		10000	2.47(5)	8.35(4)	4.00(4)	2.70(−6)	1.06(−6)	2.51(−6)	2.75(−6)	—	—
		100000	4.03(5)	1.71(5)	2.01(5)	2.86(−6)	1.28(−6)	9.00(−6)	4.19(−6)	1.39(−6)	—
	10^{12}	1000	6.10(5)	2.90(5)	2.36(5)	3.75(−6)	2.07(−6)	9.88(−6)	7.37(−6)	2.57(−6)	—
		10000	1.55(6)	1.21(6)	5.61(5)	4.46(−6)	3.33(−6)	1.24(−5)	2.60(−5)	1.43(−5)	1.69(−5)
		100000	2.92(6)	3.33(6)	9.71(5)	4.74(−6)	4.05(−6)	1.40(−5)	5.43(−5)	4.65(−5)	3.23(−5)
	10^{13}	1000	5.16(6)	7.48(6)	1.44(6)	1.18(−5)	1.83(−5)	2.61(−5)	8.56(−5)	8.46(−5)	4.13(−5)
		10000	7.73(6)	1.17(7)	1.81(6)	1.20(−5)	1.88(−5)	2.64(−5)	1.20(−4)	1.23(−4)	4.67(−5)
		100000	1.01(7)	1.54(7)	2.13(6)	1.22(−5)	1.91(−5)	2.66(−5)	1.30(−4)	1.35(−4)	4.79(−5)
15000	10^{10}	1000	2.70(3)	2.23(2)	6.00(1)	5.34(−8)	3.27(−9)	3.40(−9)	—	—	—
		10000	2.52(4)	2.20(3)	5.98(2)	4.85(−7)	3.22(−8)	3.38(−8)	—	—	—
		100000	1.44(5)	1.93(4)	5.65(3)	2.21(−6)	2.72(−7)	3.19(−7)	—	—	—
	10^{11}	1000	1.54(5)	2.13(4)	6.24(3)	2.34(−6)	3.00(−7)	3.52(−7)	—	—	—
		10000	3.06(5)	1.06(5)	5.47(4)	2.79(−6)	1.13(−6)	2.97(−6)	2.90(−6)	—	—
		100000	4.91(5)	2.05(5)	2.23(5)	2.98(−6)	1.34(−6)	8.66(−6)	4.62(−6)	1.48(−6)	—
	10^{12}	1000	1.14(6)	6.12(5)	3.46(5)	5.38(−6)	3.54(−6)	1.10(−5)	1.30(−5)	4.88(−6)	—
		10000	2.45(6)	2.18(6)	7.36(5)	6.45(−6)	5.79(−6)	1.41(−5)	3.50(−5)	2.18(−5)	2.00(−5)
		100000	4.53(6)	5.96(6)	1.30(6)	7.11(−6)	7.70(−6)	1.68(−5)	7.29(−5)	7.01(−5)	3.91(−5)
	10^{13}	1000	8.78(6)	1.50(7)	2.20(6)	2.57(−5)	4.71(−5)	3.83(−5)	1.16(−4)	1.37(−4)	5.74(−5)
		10000	1.38(7)	2.56(7)	2.89(6)	2.68(−5)	5.09(−5)	3.99(−5)	1.92(−4)	2.42(−4)	7.05(−5)
		100000	1.90(7)	3.59(7)	3.49(6)	2.69(−5)	5.12(−5)	4.00(−5)	2.36(−4)	3.02(−4)	7.57(−5)
20000	10^{10}	1000	2.29(3)	1.88(2)	5.06(1)	4.07(−8)	2.48(−9)	2.58(−9)	—	—	—
		10000	2.18(4)	1.87(3)	5.06(2)	3.79(−7)	2.45(−8)	2.57(−8)	—	—	—
		100000	1.41(5)	1.73(4)	5.00(3)	2.04(−6)	2.22(−7)	2.53(−7)	—	—	—
	10^{11}	1000	1.46(5)	1.79(4)	5.10(3)	2.12(−6)	2.29(−7)	2.59(−7)	—	—	—
		10000	3.30(5)	1.07(5)	4.82(4)	2.85(−6)	1.08(−6)	2.37(−6)	2.93(−6)	—	—
		100000	5.85(5)	2.41(5)	2.46(5)	3.11(−6)	1.41(−6)	8.49(−6)	5.08(−6)	1.59(−6)	—
	10^{12}	1000	1.37(6)	7.10(5)	3.54(5)	6.40(−6)	4.27(−6)	1.10(−5)	1.38(−5)	5.13(−6)	—
		10000	3.20(6)	2.97(6)	8.62(5)	7.96(−6)	7.80(−6)	1.50(−5)	3.96(−5)	2.58(−5)	2.11(−5)
		100000	5.73(6)	7.85(6)	1.53(6)	8.81(−6)	1.05(−5)	1.81(−5)	8.10(−5)	8.09(−5)	4.14(−5)
	10^{13}	1000	1.12(7)	2.01(7)	2.74(6)	3.47(−5)	6.83(−5)	4.50(−5)	1.26(−4)	1.57(−4)	6.56(−5)
		10000	1.75(7)	3.51(7)	3.71(6)	3.66(−5)	7.54(−5)	4.75(−5)	2.13(−4)	2.91(−4)	8.57(−5)
		100000	2.49(7)	5.18(7)	4.57(6)	3.72(−5)	7.76(−5)	4.82(−5)	2.92(−4)	4.11(−4)	9.66(−5)

* erg cm^{-2} sec^{-1} sterad^{-1}.

** erg cm^{-2} sec^{-1} sterad^{-1} and $\Delta\nu=1$ sec.

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