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VENEZIANO-TYPE KAON ELECTROMAGNETIC FORM FACTOR

BY

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ABSTRACT

The kaon electromagnetic form factor is expressed by making use of Veneziano-type scattering amplitudes for $K\bar{K}\to K\bar{K}$ and $K\bar{K}\to K_A\bar{K}$, current algebra, the field-current identity and the soft-kaon method. It is taken into consideration whether or not their straightforward applications are reasonable to decide the kaon electromagnetic form factor, through the discussion about its discrepancy in form from those of pion and proton.

§1. Introduction

Recently crossing symmetric Veneziano-type scattering amplitude seems obsolete to lose general popularity. There was, however, one period when it was very available to draw many successful achievements, particularly with the scattering amplitude related to pion (1)(2)(3)(4)(5)(6)(7)(8). In the present note crossing-symmetric Veneziano-type amplitudes for $K\bar{K}\to K\bar{K}$ and $K\bar{K}\to K_A\bar{K}$ scatterings are taken up to express the kaon electromagnetic form factor by relating them with partially conserved axial-vector current, current algebra, the field-current identity and the soft-kaon method, and then the availability of these theories, hypotheses and method is investigated closely once again.

The pion electromagnetic form factor obtained by several theoretical methods is in good agreement with experimental data and also shows the considerable similarity in form to the proton one (9)(10)(11)(12)(13)(14)(15)(16)(17)(18). As a matter of fact, the Veneziano-type pion electromagnetic form factor was able not only to lead nearly to the same conclusion as the above for small momentum transfer squared, but also to show its proportionality to the kaon electromagnetic from factor calculated with Veneziano-type scattering amplitudes for $\pi K\to\pi K$ and $\pi K\to\pi K_A$ (13). Therefore it is natural that the kaon electromagnetic form factor should be expected to behave like those of pion and proton (10)(13). As it is here, however, the results calculated with Veneziano-type scattering amplitudes for $KK\to KK$ and $KK\to KK_A$ are quite different from the pion and proton electromagnetic form factors. Thereupon simple and unconventional attempts are made on the modified kaon mass in relation to the soft-kaon limit on account of securing the same form as the pion and proton electromagnetic form factors.

§2. Scattering Amplitude for $K\bar{K}\to K_A\bar{K}$

The crossing symmetric Veneziano-type scattering amplitudes for

$$K(q)+\bar{K}(-q')\to K_A(k\cdot\varepsilon)+\bar{K}(-p)\,,$$

which show Regge asymptotic behavior in the high energy limit, are presented by using not only ϕ-f' trajectory, but also two mixing ρ-f and ϕ-f' trajectories.

I. In the case of taking only ϕ-f' trajectory.

$$\begin{aligned} T(s,\,t,\,u) &= \gamma_1\{a+\alpha(t)+\alpha(u)\}\,B(1-\alpha(t),\,1-\alpha(u))(p+q\cdot\varepsilon) \\ &\quad -\gamma_1\{\alpha(t)-\alpha(u)\}\,B(1-\alpha(t),\,1-\alpha(u))(p-q\cdot\varepsilon)\,, \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} s &= (p+q)^2 = (k+q')^2\,, \\ t &= (q-q')^2 = (k-p)^2\,, \\ u &= (p-q')^2 = (k-q)^2\,, \\ B(1-\alpha(t),\,1-\alpha(u)) &= \frac{\Gamma(1-\alpha(t))\Gamma(1-\alpha(u))}{\Gamma(2-\alpha(t)-\alpha(u))}\,, \end{aligned}$$

$\alpha(t)=1+b(t-m_\phi^2)$ is ϕ-f' trajectory function with universal slope $2b(m_\rho^2-m_\pi^2)=1$, and a and γ_1 are the constant numbers related to coupling constants and masses of mesons. Adler's PCAC consistency condition for this scattering amplitude, $T(m_K^2,\,m_{K_A}^2,\,m_K^2)=0$ requires the following relation among m_ϕ^2, m_K^2 and $m_{K_A}^2$:

$$2m_\phi^2 = m_K^2+m_{K_A}^2\,. \tag{2.2}$$

Here is chosen the following Hamiltonian densities so that you may find the relations between the constants in this amplitude and the coupling constants of vertices: $\eta_{0^+}KK$, ϕKK, $\eta_{0^+}K_AK$ and ϕK_AK,

$$\begin{aligned} H &= g_{\eta_{0^+}KK}\eta_{0^+}K\bar{K} \\ H &= g_{\eta_{0^+}K_AK}(\partial_\mu\eta_{0^+}K_A\bar{K}-\eta_{0^+}K_A\partial_\mu\bar{K}) \\ H &= g_{\phi KK}\phi_\mu(\bar{K}\partial_\mu K-K\partial_\mu\bar{K}) \\ H &= g_S\phi_\mu K_{A\mu}\bar{K}+g_D\phi_\mu K_{A\nu}\partial_\mu\partial_\nu\bar{K}\,, \end{aligned} \tag{2.3}$$

where g_S and g_D are the so-called S- and D-wave coupling constants for vertex ϕKK_A and η_{0^+}-meson is regarded as the first daughter of ϕ-meson. Calculating Born terms for resonance poles, η_{0^+}- and ϕ-mesons, with these Hamiltonian densities (2.3),

$$\begin{aligned} & g_{\eta_{0^+}KK}g_{\eta_{0^+}K_AK}\frac{(q-q'-p)\cdot\varepsilon}{t-m_{\eta_{0^+}}^2} \\ & +g_Sg_{\phi KK}\frac{(q+q')\cdot\varepsilon}{t-m_\phi^2}-g_Dg_{\phi KK}\frac{2\boldsymbol{q}\boldsymbol{q}'p\cdot\varepsilon}{t-m_\phi^2}\,, \end{aligned} \tag{2.4}$$

where $\boldsymbol{q}$ and $\boldsymbol{q}'$ are the respective momenta of incoming and outgoing particles in the center of mass system. On the other hand, expanding Veneziano-type scattering amplitude (2.1) about η_{0^+}- and ϕ-mesons,

$$-\frac{r_1}{b(t-m_\phi^2)}\{(a+2)(p+q\cdot\varepsilon)+b(3m_K^2+m_{K_A}^2-3m_\phi^2)p\cdot\varepsilon -4b\boldsymbol{q}\boldsymbol{q}'p\cdot\varepsilon\}\,. \quad (2.5)$$

In comparison of these terms with Born terms (2.4),

$$\begin{aligned} &g_S g_{\phi KK} = g_{\eta_0{}^+KK}\, g_{\eta_0{}^+K_AK} \\ &6g_S = (3m_\phi^2-3m_K^2-m_{K_A}^2)g_D \\ &2r_1 = -g_D g_{\phi KK} \\ &a = 2\left\{\frac{g_S}{g_D(m_\rho^2-m_\pi^2)}-1\right\}. \end{aligned} \quad (2.6)$$

II. In the case of taking two mixing ρ-f and ϕ-f' trajectories.

$$\begin{aligned} T(s,t,u) &= r_2\{(A+\alpha_\phi(t)+\alpha_\rho(u))B(1-\alpha_\phi(t),\,1-\alpha_\rho(u)) \\ &\quad +(A+\alpha_\rho(t)+\alpha_\phi(u))B(1-\alpha_\rho(t),\,1-\alpha_\phi(u))\}(p+q\cdot\varepsilon) \\ &\quad -r_2\{(\alpha_\phi(t)-\alpha_\rho(u))B(1-\alpha_\phi(t),\,1-\alpha_\rho(u)) \\ &\quad +(\alpha_\rho(t)-\alpha_\phi(u))B(1-\alpha_\rho(t),\,1-\alpha_\phi(u))\}(p-q\cdot\varepsilon)\,. \end{aligned} \quad (2.7)$$

Adler's PCAC consistency condition for the above scattering amplitude requires the following relation among m_ϕ^2, m_ρ^2, m_K^2 and $m_{K_A}^2$:

$$m_{K_A}^2-m_\phi^2 = m_\rho^2-m_K^2\,. \quad (2.8)$$

While, through quite the same method as the first case, the following relations are introduced:

$$\begin{aligned} &g_{\eta_0{}^+KK}\,g_{\eta_0{}^+K_AK} = g_s g_{\phi KK} \\ &g_{\sigma KK}\,g_{\sigma K_AK} = g_s g_{\rho KK} \\ &6g_S = g_D(m_\rho^2-2m_K^2) \\ &6g_s = g_d(m_\phi^2-2m_K^2) \\ &2r_2 = -xg_D g_{\phi KK} = -yg_d g_{\rho KK} \\ &A+2 = \frac{2}{m_\rho^2-m_\pi^2}\cdot\frac{g_S}{g_D} = \frac{2}{m_\rho^2-m_\pi^2}\cdot\frac{g_s}{g_d} \\ &m_\phi^2-m_\rho^2 = 2\left(\frac{g_s}{g_d}-\frac{g_S}{g_D}\right), \end{aligned} \quad (2.9)$$

where g_s and g_d are the S- and D-wave coupling constants for vertex ρKK_A, and then x and y show the sharing ratio of ϕ-f' and ρ-f trajectories for the present scattering.

§3. The kaon electromagnetic form factor

Here is assumed the following field-current identity:

$$A_\mu^{(-)}(x) = A_\mu^{(4)}(x)-iA_\mu^{(5)}(x) = \sqrt{2}f_K\partial_\mu K(x)+\sqrt{2}f_{K_A}K_{A\mu}(x)\,,$$

where $A_\mu^{(-)}(x)$ is the strangeness-changing axial vector current related to the kaon and axial vector meson fields through the field-current identity assumption, f_K the kaon decay constant and f_{K_A} the constant defined by $\langle 0|A_\mu^{(-)}(0)|K_A\rangle=\sqrt{2}f_{K_A}\varepsilon_\mu$. The matrix element of this axial vector current between one-kaon state and two-kaon state is taken into consideration in connection with the Veneziano-type scattering amplitudes for $K\bar{K}\to K\bar{K}$ (3) (19) and $K\bar{K}\to K_A\bar{K}$ presented in the section 2.

I. In the case of taking only ϕ-f' trajectory.

$$\begin{aligned}
&\langle K^-(-p)|A_\mu^{(-)}(0)|K^+(q),\ K^-(-q')\rangle\\
&=\sqrt{2}f_K\langle K^-(-p)|\partial_\mu K(0)|K^+(q),\ K^-(-q')\rangle\\
&\quad+\sqrt{2}f_{K_A}\langle K^-(-p)|K_{A\mu}(0)|K^+(q),\ K^-(q')\rangle\\
&=\frac{\sqrt{2}if_K\beta_1}{k^2-m_K^2}k_\mu\{C+\alpha(t)+\alpha(u)\}B(1-\alpha(t),\ 1-\alpha(u))\\
&\quad+\frac{\sqrt{2}f_{K_A}r_1}{k^2-m_{K_A}^2}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{K_A}^2}\right)(p+q)_\nu\{a+\alpha(t)+\alpha(u)\}\\
&\qquad\qquad\times B(1-\alpha(t),\ 1-\alpha(u))\\
&\quad-\frac{\sqrt{2}f_{K_A}r_1}{k^2-m_{K_A}^2}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{K_A}^2}\right)(p-q)_\nu\{\alpha(t)-\alpha(u)\}\\
&\qquad\qquad\times B(1-\alpha(t),\ 1-\alpha(u))\,. \qquad (3.1)
\end{aligned}$$

The divergence of the above matrix element is as follows:

$$\begin{aligned}
&\langle K^-(-p)|\partial_\mu A_\mu^{(-)}(0)|K^+(q),\ K^-(-q')\rangle\\
&=-ik_\mu\langle K^-(-p)|A_\mu^{(-)}(0)|K^+(q),\ K^-(-q')\rangle\\
&=\left\{\left(\frac{\sqrt{2}f_K m_K^2}{k^2-m_K^2}+\sqrt{2}f_K\right)\beta_1\{C+\alpha(t)+\alpha(u)\}\right.\\
&\quad-\frac{\sqrt{2}if_{K_A}r_1}{m_K^2}k(p+q)\{a+\alpha(t)+\alpha(u)\}\\
&\quad\left.+\frac{\sqrt{2}if_{K_A}r_1}{m_{K_A}^2}k(p-q)\{\alpha(t)-\alpha(u)\}\right\}B(1-\alpha(t),\ 1-\alpha(u))\,.
\end{aligned}$$

The PCAC relation for kaon:

$$\partial_\mu A_\mu^{(-)}(x)=\sqrt{2}f_K m_K^2 K^-(x)\,,$$

therefore, requires the following condition:

$$\begin{aligned}
&\sqrt{2}f_K\beta_1\{C+\alpha(t)+\alpha(u)\}\\
&\quad-\frac{\sqrt{2}if_{K_A}r_1}{m_{K_A}^2}k(p+q)\{a+\alpha(t)+\alpha(u)\}\\
&\quad+\frac{\sqrt{2}if_{K_A}r_1}{m_{K_A}^2}k(p-q)\{\alpha(t)-\alpha(u)\}=0\,.
\end{aligned}$$

That is to say

$$C = -2 - \frac{m_K^2 - m_\phi^2}{m_\rho^2 - m_\pi^2}$$
$$a = -2 - \frac{2m_K^2 - m_\phi^2}{m_\rho^2 - m_\pi^2} \tag{3.2}$$
$$f_{K_A}\gamma_1 = -if_K\beta_1 \frac{m_{K_A}^2}{4m_K^2} .$$

On the other hand the commutation relation of current algebra and the reduction technique in the soft-kaon limit are applied to the above matrix element (3.1) as follows:

$$\begin{aligned}
&\langle K^-(-p)|A_\mu^{(-)}(0)|K^+(q), K^-(-q')\rangle \\
&\xrightarrow[q\to 0]{} -\frac{i}{\sqrt{2}f_K}\langle K^-(-p)|V_\mu^{(3)}(0)+\sqrt{3}V_\mu^{(8)}(0)|K^-(-q')\rangle \\
&= -\sqrt{2}ibf_K\beta_1 \frac{m_{K_A}^2}{4m_K^2}\cdot\frac{u-2m_K^2}{u-m_{K_A}^2}(p+q')_\mu \frac{\Gamma(1-\alpha(m_K^2))\Gamma(1-\alpha(u))}{\Gamma(2-\alpha(m_K^2)-\alpha(u))} \\
&\quad +\sqrt{2}ibf_K\beta_1 \frac{u-6m_K^2}{4m_K^2}(p-q')_\mu \frac{\Gamma(1-\alpha(m_K^2))\Gamma(1-\alpha(u))}{\Gamma(2-\alpha(m_K^2)-\alpha(u))} .
\end{aligned} \tag{3.3}$$

Here is adopted the conserved part of equation (3.3) in order to decide the electromagnetic form factor $F_K(u)$, since the electromagnetic current must be conserved.

$$F_K(u) = 2bf_K^2\beta_1 \cdot \frac{m_{K_A}^2}{4m_K^2}\cdot\frac{u-2m_K^2}{u-m_{K_A}^2}\cdot\frac{\Gamma(1-\alpha(m_K^2))\Gamma(1-\alpha(u))}{\Gamma(2-\alpha(m_K^2)-\alpha(u))} . \tag{3.4}$$

The correct normalization of $F_K(u)$ at $u=0$, namely $F_K(0)=1$, demands the following relation:

$$bf_K^2\beta_1 = 0.66 .$$

The kaon root mean square radius is calculated to be $\sqrt{\langle r_K^2\rangle}=0.38\times 10^{-13}$ cm, where $\langle r_K^2\rangle$ is related to the kaon electromagnetic form factor in form $\langle r_K^2\rangle = -6F_K'(0)/F_K(0)$.

II. In the case of taking two mixing ρ-f and ϕ-f' trajectories.

$$\begin{aligned}
&\langle K^-(-p)|A_\mu^{(-)}(0)|K^+(q), K^-(-q')\rangle \\
&= \frac{\sqrt{2}if_K\beta_2}{k^2-m_K^2}k_\mu\{(C-\alpha_\phi(t)-\alpha_\rho(u))B(1-\alpha_\phi(t), 1-\alpha_\rho(u)) \\
&\qquad +(C-\alpha_\rho(t)-\alpha_\phi(u))B(1-\alpha_\rho(t), 1-\alpha_\phi(u))\} \\
&\quad +\frac{\sqrt{2}f_{K_A}\gamma_2}{k^2-m_{K_A}^2}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{K_A}^2}\right)(p+q)_\nu \\
&\qquad \times\{(A+\alpha_\phi(t)+\alpha_\rho(u))B(1-\alpha_\phi(t), 1-\alpha_\rho(u)) \\
&\quad +(A+\alpha_\rho(t)+\alpha_\phi(u))B(1-\alpha_\rho(t), 1-\alpha_\phi(u))\} \\
&\quad -\frac{\sqrt{2}f_{K_A}\gamma_2}{k^2-m_{K_A}^2}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{K_A}^2}\right)(p-q)_\nu \\
&\qquad \times\{(\alpha_\phi(t)-\alpha_\rho(u))B(1-\alpha_\phi(t), 1-\alpha_\rho(u)) \\
&\quad +(\alpha_\rho(t)-\alpha_\phi(u))B(1-\alpha_\rho(t), 1-\alpha_\phi(u))\} .
\end{aligned} \tag{3.5}$$

Quite similarly to the first case, taking into consideration both the divergence of this matrix element and the PCAC relation for kaon, the following relations are introduced:

$$
\begin{aligned}
f_{K_A}\gamma_2 &= if_K\beta_2 \frac{m_{K_A}^2}{3m_K^2} \\
C &= 2+\frac{4m_K^2-m_\phi^2-m_\rho^2}{2(m_\rho^2-m_\pi^2)} \\
A &= -2-\frac{6m_K^2-m_\phi^2-m_\rho^2}{2(m_\rho^2-m_\pi^2)}\,.
\end{aligned} \tag{3.6}
$$

Again through the application of the commutation relation of current algebra and the reduction technique in the soft-kaon limit to the matrix element (3.5),

$$
\begin{aligned}
&-\frac{i}{\sqrt{2}f_K}\langle K^-(-p)|\,V_\mu^{(3)}(0)+\sqrt{3}\,V_\mu^{(8)}(0)\,|K^-(-q)\rangle \\
&= -\sqrt{2}if_K\beta_2\cdot\frac{1}{u-m_{K_A}^2}\cdot\frac{m_{K_A}^2}{3m_K^2}\cdot\frac{2u-m_\rho^2+m_\phi^2-6m_K^2}{2(m_\rho^2-m_\pi^2)}\cdot\frac{1}{2}\,(p+q')_\mu \\
&\qquad\times\frac{\Gamma(1-\alpha_\phi(m_K^2))\Gamma(1-\alpha_\rho(u))}{\Gamma(2-\alpha_\phi(m_K^2)-\alpha_\rho(u))} \\
&\quad -\sqrt{2}if_K\beta_2\cdot\frac{1}{u-m_{K_A}^2}\cdot\frac{m_{K_A}^2}{3m_K^2}\cdot\frac{2u-m_\phi^2+m_\rho^2-6m_K^2}{2(m_\rho^2-m_\pi^2)}\cdot\frac{1}{2}\,(p+q')_\mu \\
&\qquad\times\frac{\Gamma(1-\alpha_\rho(m_K^2))\Gamma(1-\alpha_\phi(u))}{\Gamma(2-\alpha_\rho(m_K^2)-\alpha_\phi(u))} \\
&\quad +\sqrt{2}if_K\beta_2\left\{\frac{1}{u-m_K^2}\cdot\frac{3m_K^2-u}{2(m_\rho^2-m_\pi^2)}+\frac{1}{6m_K^2}\cdot\frac{2u-m_\rho^2+m_\phi^2-6m_K^2}{2(m_\rho^2-m_\pi^2)}\right\}(p-q')_\mu \\
&\qquad\times\frac{\Gamma(1-\alpha_\phi(m_K^2))\Gamma(1-\alpha_\rho(u))}{\Gamma(2-\alpha_\phi(m_K^2)-\alpha_\rho(u))} \\
&\quad +\sqrt{2}if_K\beta_2\left\{\frac{1}{u-m_K^2}\cdot\frac{3m_K^2-u}{2(m_\rho^2-m_\pi^2)}+\frac{1}{6m_K^2}\cdot\frac{2u-m_\phi^2+m_\rho^2-6m_K^2}{2(m_\rho^2-m_\pi^2)}\right\}(p-q')_\mu \\
&\qquad\times\frac{\Gamma(1-\alpha_\rho(m_K^2))\Gamma(1-\alpha_\phi(u))}{\Gamma(2-\alpha_\rho(m_K^2)-\alpha_\phi(u))}\,.
\end{aligned} \tag{3.7}
$$

Here again is adopted the conserved part of equation (3.7) as the kaon electromagnetic form factor.

$$
\begin{aligned}
F_K(u) &= bf_K^2\beta_2\cdot\frac{m_{K_A}^2}{3m_K^2}\cdot\frac{2u-m_\rho^2+m_\phi^2-6m_K^2}{u-m_{K_A}^2}\cdot\frac{\Gamma(1-\alpha_\phi(m_K^2))\Gamma(1-\alpha_\rho(u))}{\Gamma(2-\alpha_\phi(m_K^2)-\alpha_\rho(u))} \\
&\quad +bf_K^2\beta_2\cdot\frac{m_{K_A}^2}{3m_K^2}\cdot\frac{2u-m_\phi^2+m_\rho^2-6m_K^2}{u-m_{K_A}^2}\cdot\frac{\Gamma(1-\alpha_\rho(m_K^2))\Gamma(1-\alpha_\phi(u))}{\Gamma(2-\alpha_\rho(m_K^2)-\alpha_\phi(u))}\,.
\end{aligned} \tag{3.8}
$$

The correct normalization of $F_K(u)$ at $u=0$ in this case demands the following relation:

$$bf_K^2\beta_2 = 0.08\,.$$

Similarly the kaon root mean square radius is calculated to be $\sqrt{\langle r_K^2 \rangle} = 0.10i \times 10^{-13}$ cm through this kaon electromagnetic form factor.

III. Another attempts

Another simple attempts are made on the basis of the assumption that the kaon electromagnetic form factor also might be expected to behave like those of pion and proton. It seems to be because the kaon mass is much larger as compared with the pion one that the straighforward application of the soft-kaon method can't lead to favourable results in the case of I and II. This application, therefore, may have influence upon the kaon mass of itself which is found explicitly in the kaon electromagnetic form factor (3.4) and (3.8). The fact is, however, that there is no desirable modified kaon mass to the normal behaviour of the pion and proton form factors; Figures 1,2 and Tables 1,2. Subsequently the other is made in relation to the soft-kaon limit with only ϕ-f' trajectory. This soft-kaon limit is not by the conventional method ($q \to o$), but q remains as an indefinite variable to lead to a desirable kaon electromagnetic form factor. The same expression for the kaon electromagnetic form factor as (3.4) is obtained through the quite similar process to that evolved in the section 3.

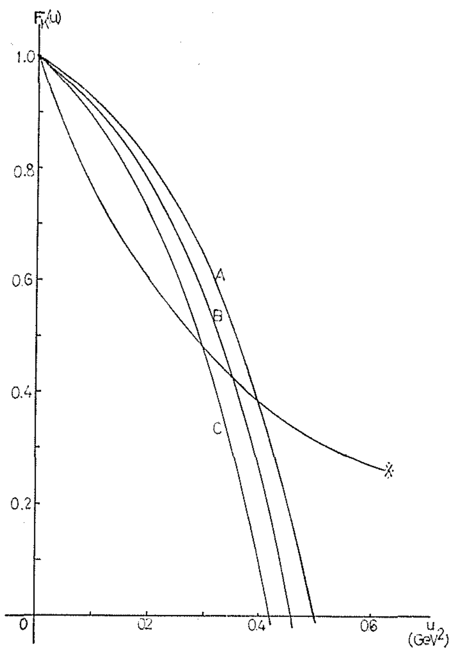

Fig. 1. The graph of the kaon electromagnetic form factor function (3.4). A for the normal value of the kaon mass, $m_K^2 = 0.246$ (GeV²). B and C for the modified values: $m_K^2 = 0.230$ (GeV²) and 0.210 (GeV²) respectively. ※ for the pion or proton electromagnetic form factor on the basis of the experiments.

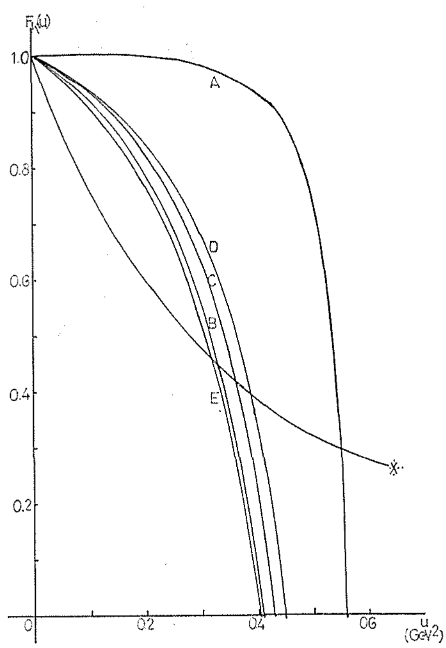

Fig. 2. The graph of the kaon electromagnetic form factor function (3.8). A for the normal value of the kaon mass. B, C, D and E for the modified values: $m_K^2 =$ 0.160 (GeV²), 0.170 (GeV²), 0.700 (GeV²) and 0.720 (GeV²) respectively. ※ for the pion or proton electromagnetic form factor on the basis of the experiments.

Table 1. The root mean square radii for a few values of m_K^2 in the kaon electromagnetic form factor function (3.4). In this connection, the root mean square radius of proton or pion $\sqrt{\langle r^2\rangle}=0.80\pm0.10$ fm or 0.86 ± 0.14 fm.

GeV² / fm	0.246	0.210	0.230
$\sqrt{\langle r_K^2\rangle}$	0.38	0.47	0.42

Table 2. The root mean square radii for a few values of m_K^2 in the kaon electromagnetic form factor function (3.8).

GeV² / fm	0.246	0.160	0.170	0.700	0.720
$\sqrt{\langle r_K^2\rangle}$	0.10 *i*	0.38	0.34	0.32	0.40

$$F_K(u) = 2bf_K^2\beta_3\frac{m_{K_A}^2}{4m_K^2}\cdot\frac{u-2m_K^2}{k^2-m_{K_A}^2}\cdot\frac{\Gamma(1-\alpha(t))\Gamma(1-\alpha(u))}{\Gamma(2-\alpha(t)-\alpha(u))} \tag{3.9}$$

More explicitly by using the correct normalization $F_K(0)=1$,

$$F_K(u) = -1.928(u-0.492)\cdot\frac{\Gamma(1.84-0.883t)\Gamma(0.918-0.883u)}{\Gamma(1.84-0.883(t+u))}.$$

The graph of the above form factor function is drawn in Figure 3 for several t values which seem to give a desirable form, without favourable results; Table 3.

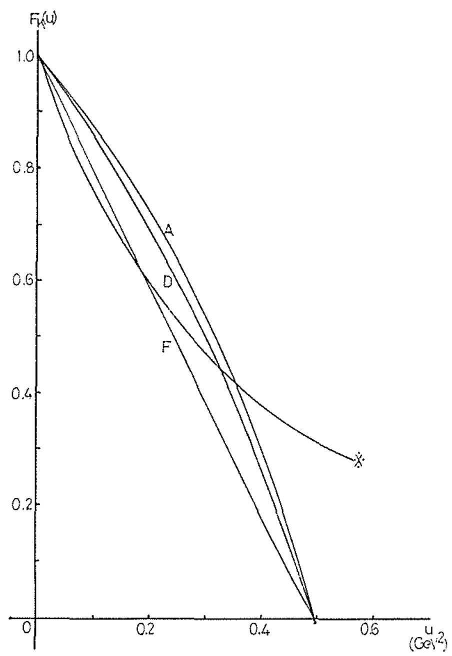

Fig. 3. The graph of the kaon electromagnetic form factor function (3.9). A, B, C, D, E and F for several t values: $t=0.01$(GeV²), 0.10(GeV²), 0.24 (GeV²), 0.36(GeV²), 0.65(GeV²) and 1.00(GeV²) respectively. ※ for the pion or proton electromagnetic form factor on the basis of the experiments.

Table 3. The root mean square radii for several t values in the kaon electromagnetic form factor function (3.9).

GeV² / fm	0.01	0.10	0.24	0.36	0.65	1.00
$\sqrt{\langle r_K^2 \rangle}$	0.51	0.52	0.54	0.56	0.61	0.68

§4. Results and discussions

The pion and nucleon electromagnetic form factors have been discussed in good agreement with experimental data through various theories and methods. They show that the pion and nucleon electromagnetic form factors are hardly different in form and therefore their root mean square radii have nearly the same values. Judging from these conclusions, it may be expected that the kaon electromagnetic form factor and its root mean square radius are nearly equal to those of pion and nucleon in form and value respectively. In the present note these expectations are investigated closely through the Veneziano-type $K\bar{K}\rightarrow K\bar{K}$ and $K\bar{K}\rightarrow K_A\bar{K}$ scattering amplitudes, the PCAC relation for kaon, current algebra, the field-current identity and the soft-kaon method. The above-obtained results, however, are in contradiction to these expectations completely; Figures 1, 2 and Tables 1, 2. It is important to explain clearly what this remarkable difference results from. What must be taken into account is whether Adler's PCAC consistency condition for kaon and the soft-kaon method can be applied with validity. It has been proved that these condition and method are quite available and have got several good results, in particular, in the case that pion is implicated in some form or other because the pion mass is very small. The obtained results show, however, that in the case that pion is not implicated as the present note, although it is shown that the soft-kaon method stands well in a few papers (3)(16)(20), these should not be applied to kaon simply and straightforwardly in the similar way to pion because the kaon mass is much larger than the pion mass. The PCAC relation for kaon, therefore, seems not necessarily to be good approximation in the discussion of kaon physics. Thereupon the modification of Adler's PCAC consistency condition and the soft-kaon method with relation to off-shell kaon mass is introduced, but several simple attempts through the modified kaon mass and the unconventional soft-kaon limit do not cause good results for the kaon electromagnetic form factor and its root mean square radius. It may be true that, bearing serious problem of breaking the conventional energy-momentum equation, these attempts should give a great contradiction to the usual form factor. The author's interest is, however, in this piont with relation to the space-time structure.

On the other hand in spite of containing not only the possible resonances on the leading trajectory but also those on its parallel daughter trajectories, several successes have been achieved through Veneziano-type scattering amplitude, and also the theories, methods and so forth adopted here have given good results as compared with experimental data. Judging from these results and conclusions there may be a point of view that the extension of these theories and methods to kaon physics as in this note is reasonable and acceptable. On the contrary, therefore, if the results obtained here in two cases should be reasonable, you could not help thinking that the main reason for unexpected results might be because kaon is a particle with strangeness. In other words the kaon's strangeness seems likely to give abnormal behaviour of its electromagnetic form factor, although this discussion cannot be carried on adequately since the kaon electromagnetic form factor has not been established through the experimental data. Probably only experimental result can answer these investigations and questions. The author further intends to make other attempts to make this problem clear through the theories and methods applied here or others, particularly in relation to the space-time structure.

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