Memoirs of the Faculty of Science, Kyoto University, Series of Physics, Astrophysics, Geophysics and Chemistry, Vol. XXXV, No. 1, Article 5, 1977.

ELECTRO-MAGNETIC CORRECTIONS TO PROTON-PROTON SCATTERING AT LOW ENERGY

BY

Kenich IMAI

· Department of Physics, Faculty of Science, Kyoto University, Kyoto

(*Received January 25, 1975*)

ABSTRACT

Electro-magnetic corrections including the finite electro-magnetic structure effects to the proton-proton scattering in the low energy region are calculated by means of constructing the "one photon exchange potential". Except the vacuum polarization effect, the electro-magnetic corrections cause negligibly small effects on the "central" p-wave phase shift, $^3\Delta_c$. But the contributions to the $^1S_0$ phase shift and the p-p polarization can not be neglected, compared with the present experimental accuracies.

1. Introduction

The differential cross section in p-p scattering below 10 MeV have been measured with high accuracies, 0.1~0.5 %, at several laboratories[1)~5)]. Two phase shifts, $^1S_0$ phase shift and the central p-wave phase shift, $^3\Delta_c$, can be almost uniquely determined from these data[6)].

The great accuracy of the data allows one to extract many important informations on the nuclear force. It has been pointed out that the $^3\Delta_c$ is the important parameter to get the information on the "scalar" meson exchange or two-pion exchange mechanism between two nucleons[7)].

But a correct estimation of the electro-magnetic (EM) corrections to the ordinary Coulomb interaction is required in order to get informations on the nuclear force precisely from the proton-proton scattering data. It is well known that the vaccum polarization (VP) effect between two protons contributes to the p-p cross section seriously in this energy region[8)].

The VP effect on p-p scattering is reviewed briefly and some numerical estimations of the effect are shown in the next section.

The other EM corrections to p-p scattering have been neglected in most analyses, although the correction to the scattering length was discussed in connection with the charge independence of the nuclear force[9)] and the contribution from the magnetic $L \cdot S$ and tensor interaction was discussed at higher energies[21)].

These contributions to p-p scattering in this energy region are discussed in section 3. Here the ME potential between two protons are deduced on the analogy of one boson exchange potential, according to the procedure of Kiang, Machida and Nogami[10)]. The effect of the EM structure of a proton is included in this potential. Phase shifts due to this EM potential are calculated using the plane wave born approximation.

The contribution to the $^3\Delta_c$ was found to be negligiblly small at the present experimental accuracy. But the contribution to $^1S_0$ phase shift can not be neglected. The effect of the electro-magnetic $L\cdot S$ interaction to the polarization in p-p scattering was found to be important in this energy region.

2. Vacuum polarization effect

Vacuum polarization (VP) effect on the p-p scattering has been studied by several authors[8),9),11)~13)].

The VP potential to first roder in α was first derived by Uehling[14)] and is given by,

$$V_{vp}(r)=\frac{2\alpha}{3\pi}I(r)\frac{e^2}{r},\quad I(r)=\int_1^\infty e^{-2\kappa r\xi}\left(1+\frac{1}{2\xi^2}\right)\frac{(\xi^2-1)^{1/2}}{\xi^2}d\xi \tag{1}$$

with α the fine structure constant and κ^{-1} the Compton wave length of the electron. The phase shifts caused by the VP potential is given by the equation,

$$\tan\tau_l=-\frac{4\alpha}{3\pi}\eta\int_0^\infty F_l(r)I(r)S_l(r)\frac{dr}{r} \tag{2}$$

with $\eta=e^2/\hbar v$, the Coulomb parameter. In eq. (2), $S_l(r)$ is the regular solution of the radial wave equation in the Coulomb plus VP potential and has the asymptotic form,

$$S_l(r)\xrightarrow[Kr\to\infty]{} F_l(r)+\tan\tau_l G_l(r) \tag{3}$$

where $F_l(r)$ and $G_l(r)$ are the regular and irregular Coulomb functions respectively.

In the first order purturbation theory, eq. (2) becomes

$$\tau_l^B=-\frac{4\alpha}{3\pi}\eta\int_0^\infty\frac{F_l^2(r)I(r)}{r}dr\,. \tag{4}$$

The numerical calculation of eq.(4) was done by Heller[13)]. Analytic expressions of eq. (4), expanded in η, were derived for τ_0^B by Durand[12)] and for τ_1^B by Eriksen, Foldy and Rarita and they agree remarkably with the numerical calculation of eq.(4) above 1 MeV. Sher, Signell and Heller[9)] showed that results of the numerical calculation of eq.(2) agreed with the Born result τ_l^B to within 1 %. And the error of the plane wave Born approximation is less than 4 % above 1 MeV. Usually errors of the nuclear phase shifts determined from present experiments are larger than 10 % of the VP phase shift, so the Born approximation are sufficient to calculate the VP phase shifts.

The VP phase shifts below 10 MeV are shown in Fig 1. As seen in Fig 1, many partial waves must be taken into account to evaluate the VP effect on p-p scattering. It is expected from the function $I(r)$, which gives the potential a range of the order of the Compton wave length of the electron, though it decreases exponentially to zero for $\kappa r\to\infty$.

The VP amplitude is given by,

$$f_{vp}(\theta)=\sum_{l=0}^\infty(2l+1)e_{l,0}\tau_l P_l(\cos\theta) \tag{5}$$

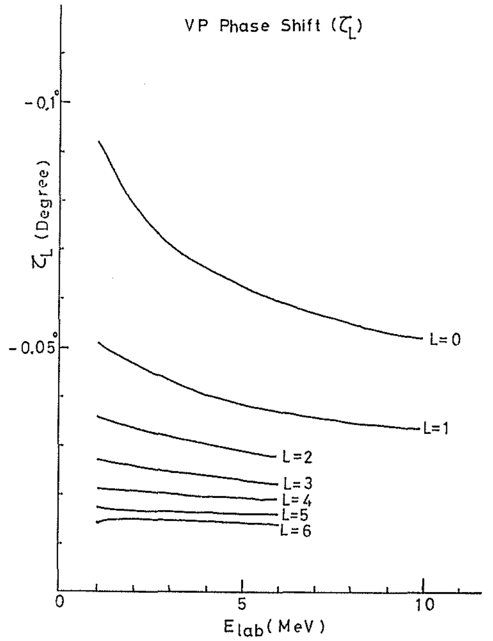

Fig. 1. The VP phase shifts in the Born approximation[13]

where $e_{l,0}$ the Coulomb phase factor Durand derived the first three terms in an expansion of $f_{vp}(0)$ in powers of η. The VP contribution to the *p-p* cross section can be evaluated using the following equation.

$$\Delta\sigma_{vp}=\sigma_T-\sigma_{N+C}$$
$$\sigma_T=\frac{1}{4}|f^S_{vp}+f^S_C+(e^{2i\delta_0^N}-1)e^{2i\tau_0}|^2+\frac{3}{4}|f^a_{vp}+f^a_c|^2$$
$$\sigma_{N+C}=\frac{1}{4}|f^N_C+(e^{2i\delta_0^N}-1)|^2+\frac{3}{4}|f^a_c|^2 \qquad (6)$$

, where the f^s_c and f^a_c the symmetric and antisymmetric Coulomb amplitude and δ_0^N the $^1S_0$ phase shift.

The $\Delta\sigma_{vp}/\sigma_T$ was calculated at 8 MeV using the available value of $^1S_0$ phase shift. As shown in Fig. 2, the angular dependence of the $\Delta\sigma_{vp}/\sigma_T$ is similar to the $\Delta\sigma_{\Delta c}/\sigma_T$, the contribution from the $^3\Delta_c$. So the VP correction is very important to extract the $^3\Delta_c$ from the *p-p* scattering data.

3. Electro-Magnetic potential and its contribution to *p-p* scattering

There are several methods to obtain EM potential between two nucleons[15~17]. Sher, Signell and Heller[9] obtained the EM potential including the EM structure effects of a nucleon, following to the method of Barker and Glover[16]. Recently, Kiang, Machida and Nogami[10] derived the *EM* potential in the *S*-state as the one photon exchange potential which is the limit of one vector meson exchange potential with the meson mass $m\rightarrow 0$. Their results agreed with that of Sher, Signell and

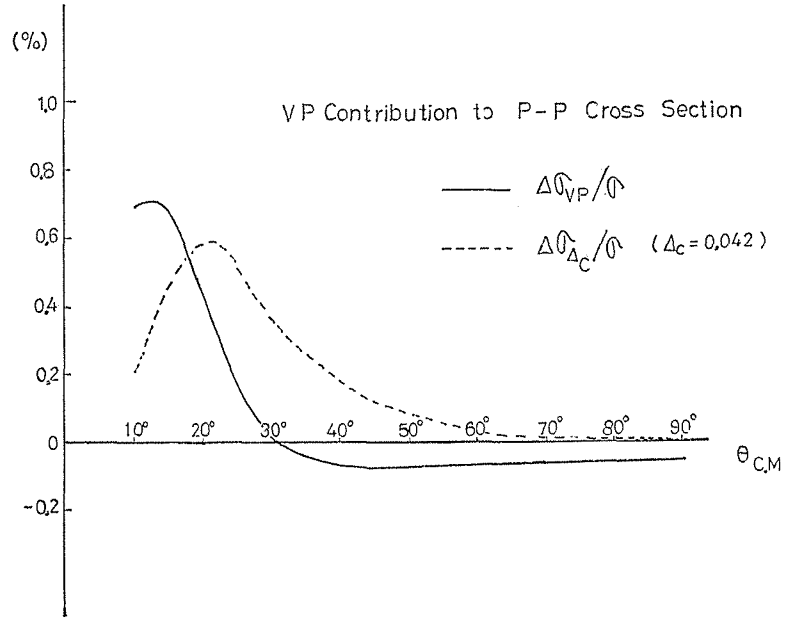

Fig. 2. The VP contribution to the cross section in $p-p$ at 8 MeV. The solid line shows $\Delta\sigma_{vp}/\sigma$. The dashed line shows $\Delta\sigma_{\Delta c}/\sigma$ with ${}^3\Delta_c=0.042°$

Heller for *p-p*. According to this procedure, the full *EM* potential between two protons is deduced. And its effects on the *p-p* scattering at low energies are evaluated here

The *EM* potential (the one photon exchange potential) can be obtained from the one vector meson exchange potential with $m\to 0$, $g_v\to 4\pi e$ and $f_v/m\to 4\pi\mu'\mu_0$, where g_v and f_v/m are the coupling constants of the vector meson and the nucleon, and μ' is the anomalous magnetic moment in a unit of μ_0, nuclear magneton. The one boson exchange potential with non-static corrections were studied in detail by Hoshizaki, Lin and Machida[18].

The *EM* potential in a nomentum space is given to the order of M^{-2} as follows.

$$V=V_0+V_1(i\vec{s}\cdot\vec{q}\times\vec{p})+V_2(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})+ V_3(\vec{\sigma}_1\cdot\vec{p})(\vec{\sigma}_2\cdot\vec{p})+V_4(\vec{\sigma}_1\cdot\vec{q}\times\vec{p})(\vec{\sigma}_2\cdot\vec{q}\times\vec{p})+ V_5(\vec{\sigma}_1\cdot\vec{\sigma}_2) \quad (7)$$

$$V_0=4\pi e^2(1+\vec{p}^2/M^2)/q^2+4\pi\mu_0^2(-1-2\mu'+\mu'^2q^2/4M^2)$$
$$V_1=4\pi\mu_0^2(6+8\mu'-3\mu'^2q^2/2M^2)/q^2$$
$$V_2=4\pi\mu_0^2\{1+2\mu'+\mu'^2(1-q^2/4M^2-p^2/2M^2)\}/q^2$$
$$V_3=4\pi\mu_0^2\mu'^2/2M^2 \quad (8)$$
$$V_4=-6\pi\mu'^2\mu_0^2/M^2q^2$$
$$V_5=-4\pi\mu_0^2\{1+2\mu'+\mu'^2(1-q^2/4M^2-p^2/2M^2)\}$$

$$\vec{p}=(\vec{p}_f+\vec{p}_i)/2, \quad \vec{q}=\vec{p}_f-\vec{p}_i, \quad \vec{s}=(\vec{\sigma}_1+\vec{\sigma}_2)/2$$

where $\vec{p}_i$ and $\vec{p}_f$ are the initial and final momentum of the proton and M is the proton mass and the relation $e/2M=\mu_0$ is used. The potential including the *EM* structure of the proton can be derived by putting $e\rightarrow eF_{1p}(q^2)$ and $\mu'\rightarrow\mu' F_{2p}(q^2)$ in eq (8).

$$\begin{aligned}
V_0&=4\pi e^2F_{1p}^2(1+\vec{p}^2/M^2)/q^2+4\pi\mu_0^2(-F_{1p}^2-2\mu' F_{1p}F_{2p}+\mu' q^2F_{2p}^2/4M^2)\\
V_1&=4\pi\mu_0^2(6F_{1p}^2+8\mu' F_{1p}F_{2p}-3\mu'^2q^2F_{2p}^2/2M^2)/q^2\\
V_2&=4\pi\mu_0^2\{F_{1p}^2+2\mu' F_{1p}F_{2p}+\mu'^2F_{2p}^2(1-q^2/4M^2-p^2/2M^2)\}/q^2\\
V_3&=4\pi\mu_0^2\mu'^2F_{2p}^2/2M^2\\
V_4&=-6\pi\mu'^2\mu_0^2F_{2p}^2/M^2q^2\\
V_5&=-4\pi\mu_0^2\{F_{1p}^2+2\mu' F_{1p}F_{2p}+\mu'^2F_{2p}^2(1-q^2/4M^2-p^2/2M^2)\} \qquad (9)
\end{aligned}$$

, where $F_{1p}(q^2)$ and $F_{2p}(q^2)$ are the Dirac and Pauli form factor of the proton and related to the empirical proton form factor G_{Ep},

$$F_{1p}(q^2)=(1+(1+\mu')q^2/4M^2)\,(1+q^2/4M^2)^{-1}G_{Ep} \qquad (10)$$

$$\begin{aligned}
F_{2p}(q^2)&=F_{1p}(q^2)\,(1+(1+\mu')q^2/4M^2)^{-1}\\
G_{Ep}&=(1+1.24q^2/M^2)^{-2}\,. \qquad (11)
\end{aligned}$$

Eq.(11) is well established by experiments of electron-proton scattering to several GeV^2.

By the Fourier transformation of eq.(9), one can obtain the *EM* potential in a coordinate space.

For the present purpose, the term of the order of p^2/M^2 and q^2/M^2 can be neglected and the form factor can be approximated to

$$F_{1p}=F_{2p}=G_{Ep}$$

, and G_{Ep} is approximated to $(1+5q^2/M^2)^{-1/2}$ for a simplicity of the calculations. This approximation is not valid in the inner region of the potential, but it does not change the mean square radius of the charge and magnetic distribution. In this low energy region, it will be sufficient.

The potential in the coordinate space is given as follows,

$$\begin{aligned}
V^{EM}(r)=&e^2(1-e^{-ar})/r-(1+2\mu')\mu_0^2a^2e^{-ar}/r\\
&-(6+8\mu')\mu_0^2\{r^{-1}-(a+r^{-1})e^{-ar}\}r^{-2}\cdot(\vec{L}\cdot\vec{S})\\
&-(1+\mu')^2\mu_0^2\{r^{-3}-(r^{-2}+ar^{-1}+a^2/3)e^{-ar}/r\}S_{12}\\
&-(1+\mu')^2\mu_0^2a^2r^{-1}e^{-ar}(\vec{\sigma}_1\cdot\vec{\sigma}_2) \qquad (12)
\end{aligned}$$

$$S_{12}=3(\vec{\sigma}_1\cdot\vec{r})\,(\vec{\sigma}_2\cdot\vec{r})/r^2-(\vec{\sigma}_1\cdot\vec{\sigma}_2)$$

where $a=\frac{M}{\sqrt{5}}$ specifies the range of the potential due to the effect of the *EM* structure of the proton. For the point protons, this potential can be simplified as

$$V(r)=e^2/r-\mu_0(1+2\mu')\delta(r)-(6+8\mu')\mu_0^2r^{-3}(\vec{L}\cdot\vec{S})$$
$$-(1+\mu')^2\mu_0^2r^{-3}\cdot S_{12}-\frac{2}{3}(1+\mu')^2\mu_0^2(\vec{\sigma_1}\cdot\vec{\sigma_2})\delta(r)\,. \tag{13}$$

Eq.(13) agreed with the results of reference 15~17.

The EM potential except the ordinary Coulomb potential is so small that the plane wave Born approximation is expected to be sufficient to evaluate the phase shifts due to this potential, as it works well for the VP interaction.

$$\delta_l^{EM}=-Mk^{-2}\int_0^\infty j_l^2(\rho)V^{EM}(\rho)\rho^2d\rho \tag{14}$$

with $\rho=kr$.
For the $^1S_0$ state, the EM potential except the ordinary Coulomb interaction is given by,

$$^1V_0^{EM}(r)=-e^2r^{-1}e^{-ar}+(1+2\mu'+2\mu'^2)\mu_0^2r^{-1}a^2e^{-ar}\,. \tag{15}$$

The first term is due to the effect of the charge distribution of the proton and the second is due to the magnetic interaction including the effect of the EM structure. The $^1S_0$ phase shift due to this potential can be easily calculated, using the second kind Legendre function.

$$^1\delta_0^{EM}={}^1\delta_0{}^{\text{charge}}+{}^1\delta^{\text{mag}}$$
$$=\frac{M\alpha}{2k}Q_0(a^2/2k^2+1)-Ma^2\mu_0^2(1+2\mu'+2\mu^2)Q_0(a^2/2k^2+1)/2k\,. \tag{16}$$

This can be approximated at low energy,

$$^1\delta_0{}^{\text{charge}}=\alpha\beta M^2/2a$$
$$^1\delta_0{}^{\text{mag}}=-\alpha\beta(1+2\mu'+2\mu'^2)/8\,, \tag{17}$$

where β the velocity of the incident proton. The numerical results are shown in Fig. 3. The $^1\delta_0{}^{\text{charge}}$ and $^1\delta_0{}^{\text{mag}}$ are larger than errors of the $^1S_0$ phase shifts in the present experimental status[5),9),19)]. The net EM phase shift, $^1\delta_0{}^{EM}$, is comparable to the errors.

For the ^3P_J state, the p-wave phase shift combinations Δ_C, Δ_{LS} and Δ_T (Eq. 18) can be directly calculated from the central, $\vec{L}\cdot\vec{S}$ and tensor potentials, respectively.

$$\Delta_C=\frac{1}{9}({}^3\delta_{10}+3{}^3\delta_{11}+5{}^3\delta_{12})$$
$$\Delta_{LS}=\frac{1}{12}(-2{}^3\delta_{10}-3{}^3\delta_{11}+5{}^3\delta_{12})$$
$$\Delta_T=\frac{5}{72}(-2{}^3\delta_{10}+3{}^3\delta_{11}-{}^3\delta_{12})\,. \tag{18}$$

The central potential in the triplet odd state is given by

$$^3V_c^{EM}=-\frac{e^2}{r}e^{-ar}-(5+10\mu'+2\mu'^2)\mu_0^2a^2e^{-ar}/3r\,. \tag{19}$$

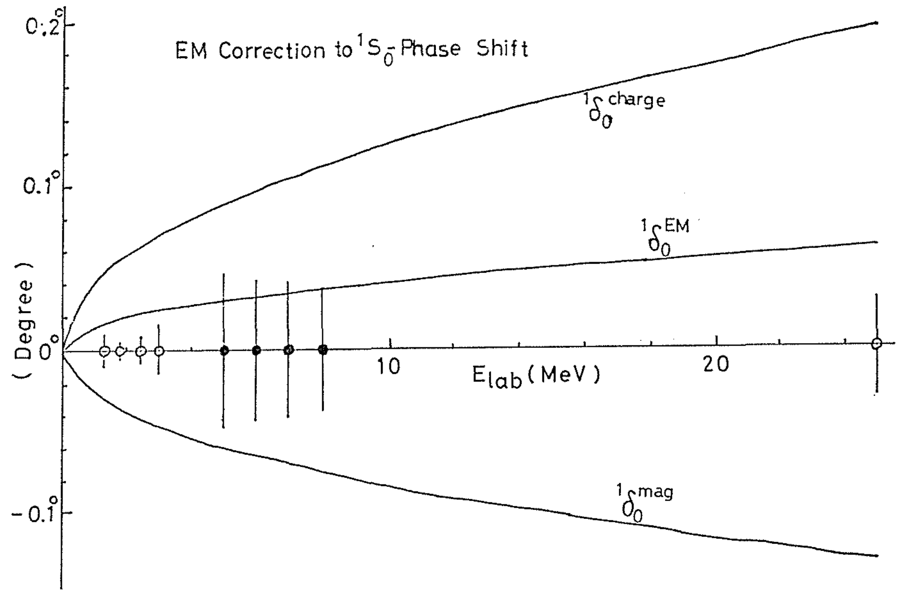

Fig. 3. The EM contribution to the $^1S_0$ phase shift. | indicate the error of the $^1S_0$ phase shift. ○ are from ref. (9) and ● from ref. (5).

This is essentially same at the potential for $^1S_0$ state except the eigen value of the spin-spin operator $(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$. The "central" P-wave phase shift is given by

$$\begin{aligned} {}^3\Delta_c^{EM} &= {}^3\Delta_c^{\text{charge}} + {}^3\Delta_c^{\text{mag}} \\ &= \frac{M}{2k}\{\alpha + a^2\mu_0^2(5+10\mu'+2\mu'^2)/3\}\, Q_1(a/2k^2+1)\,. \end{aligned} \qquad (20)$$

At the low energy, it can be approximated to

$$ {}^3\Delta_c^{EM} = \frac{1}{12}M^4a^{-4}\alpha\beta^3 + \frac{1}{144}M^2a^{-2}\alpha\beta^3(5+10\mu'+2\mu'^2)\,. \qquad (21)$$

The range of the potential due to the EM structure is characterized by the exponential factor $a\left(=\frac{M}{\sqrt{5}}\right)$, which is about half the proton mass, so, roughly speaking, the potential range is twice the Compton wave length of the proton which is fairly shorter than the impact parameter of the P-wave scattering in this energy region. Due to the short range charactor of this potential, the values of ${}^3\Delta_c^{EM}$ are smaller than the errors of ${}^3\Delta_c$ obtained from available data, as seen in Fig. 4. This result shows that the effect from the EM structure to P-wave and higher waves can be neglected. So the finite structure effect can be neglected for the $\vec{L}\cdot\vec{S}$ and tensor potential.

$$\begin{aligned} {}^3V_{LS}^{EM} &= -(6+8\mu')\mu_0^2 \cdot r^{-3}(\vec{L}\cdot\vec{S}) \\ {}^3V_T^{EM} &= -(1+\mu')^2\mu_0^2 r^{-3} \cdot S_{12} \end{aligned} \qquad (22)$$

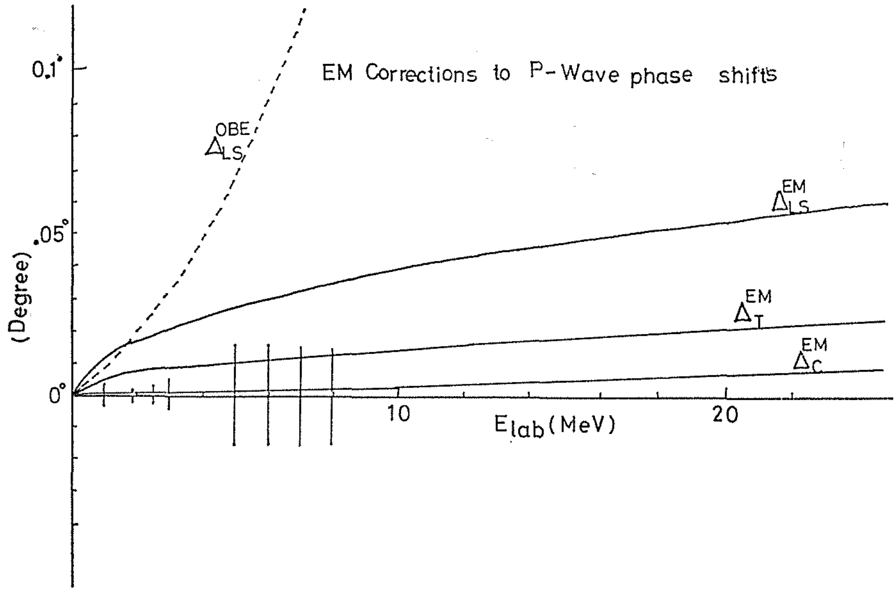

Fig. 4. The EM contribution to the ${}^3P_J$-wave. | indicate the error of the ${}^3\Delta_c$ obtained in ref. (9) and (5) the dashed line show the Δ_{LS}^{OBE} which is calculated by the one-boson exchange model.

The phase shifts ${}^3\Delta_{LS}^{EM}$ and ${}^3\Delta_T^{EM}$ are given as follows,

$$
\begin{aligned}
{}^3\Delta_{LS}^{EM} &= \frac{1}{2}(3+4\mu')\mu_0^2 kM \simeq \alpha\beta(3+4\mu')/16 \\
{}^3\Delta_T^{EM} &= \frac{1}{4}(1+\mu')^2\mu_0^2 kM \simeq \alpha\beta(1+\mu')^2/32 .
\end{aligned}
\tag{23}
$$

The ${}^3\Delta_{LT}^{EM}$ and ${}^3\Delta_T^{EM}$ are also shown in Fig. 4. The ${}^3\Delta_T^{EM}$ is several % of the ${}^3\Delta_T$ by one-pion exchange but the ${}^3\Delta_{LS}^{EM}$ is comparable to the ${}^3\Delta_{LS}$ by one-obson exchange below 3MeV.

The $\vec{L}\cdot\vec{S}$ and tensor potential (eq.(22)) has a rather long range tail, compared with the potential due to the *EM* structure of the proton. So, in this case, higher wave phase shifts must be taken into account to evaluate the contributions from these potential to the cross section and polarization in *p-p* scattering.

The phase shifts due to the $\vec{L}\cdot\vec{S}$ potential (eq.22) can be obtained for the state $|JL\rangle$ as

$$
\begin{aligned}
{}^{LS}\delta_{L\cdot J}^{EM} &= K\langle LJ|\vec{L}\cdot\vec{S}|LJ\rangle/L(L+1) \\
&= \begin{cases} -K/L & (J=L-1) \\ -K/L(L+1) & (J=L) \\ K/(L+1) & (J=L+1) \end{cases} \\
K &= (3+4\mu')\mu_0^2 kM .
\end{aligned}
\tag{24}
$$

If ${}^{LS}\Delta_{LJ}^{EM}$ is small, this $\vec{L}\cdot\vec{S}$ contribution to the scattering matrix can be written as follows.

$$\delta M_{10}^{LS} = -\delta M_{01}^{LS} = \frac{\sqrt{2}\,K}{k} \sum_{L\,\mathrm{odd}}^{\infty} P_L^1(\theta) \frac{2L+1}{L(L+1)}$$
$$= \frac{\sqrt{2}\,K}{k \sin\theta} \tag{25}$$
$$\delta M_{11}^{LS} = \delta M_{00}^{LS} = \delta M_{1-1}^{LS} = 0$$

, where the notation of M_{ij} is given in the appendix of reference (20). The contribution to the cross section and the polarization is as follows.

$$\Delta\sigma^{LS}(\theta) \simeq \frac{1}{2}\{|\delta M_{01}^{LS}|^2 + |\delta M_{10}^{LS}|^2\} = K^2/k^2 \sin^2\theta$$
$$\Delta(I_0 P)^{LS} \simeq \frac{2K}{k \sin\theta} I_m[f_c^a(\theta)]\,. \tag{26}$$

The $\Delta\sigma^{LS}(\theta)$ is negligible small, for example $\sim 5\mu$b/sr at $\theta_{cm} = 10°$, but the contribution to the polarization can not be neglected because of the interference term with the Coulomb amplitude.
The polarization due to the *EM* interaction at 8MeV is shown in Fig. 5.
It will be not impossible to measure the polarization with an accuracy of 0.1 % absolute in this energy region today.

In the case of the tensor potential, it can be easily verified that the interference term with the Coulomb amplitude also vanishes for the cross section, if the phase shifts is small enough to be able to replace $(1\text{-}e^{2i\delta})$ to $-2i\delta$. The *EM* tensor contribution to the cross section was found to be the same order as the $L\cdot S$ contribution.

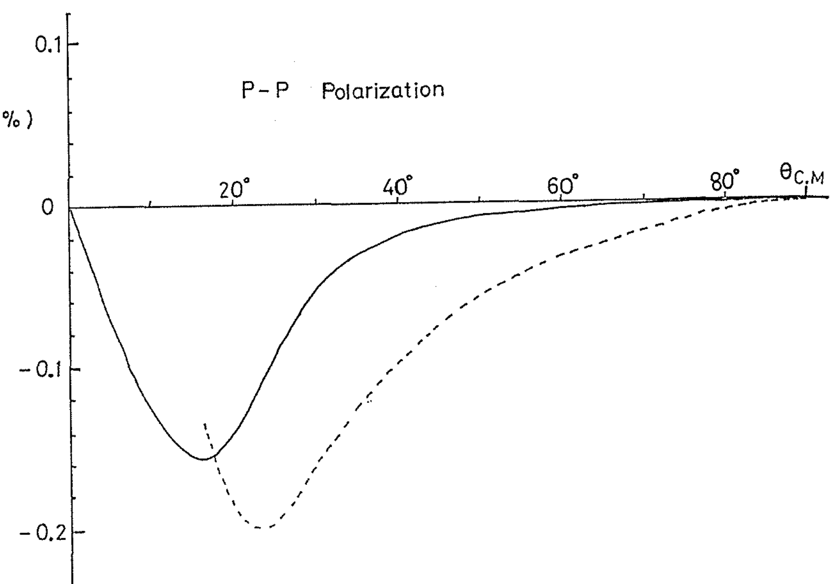

Fig. 5. The EM $\vec{L}\cdot\vec{S}$ contribution to the $p-p$ polarization at 8 MeV. The dashed line shows the polarization calculated with the available phase shift set.[5]

4. Final remarks

For the estimation of the $^3\Delta_c$ in the *p-p* scattering below 10 MeV, only the *VP* effect is important at the present experimental accuracies. The differential cross sections in this energy region are less sensitive to the $^3\Delta_{LS}$ and $^3\Delta_T$, so, in the phase shift analysis[5),6)], they are assumed by the one pion exchange model or one-boson exchange model. The contribution of the $\vec{L}\cdot\vec{S}$ and tensor parts of the *EM* interaction can be neglected for the cross section, but for the polarization the effect of $\vec{L}\cdot\vec{S}$ part is important. Accurate measurments of the polarization are expected in this energy region.

In addition to the *VP* effect, the effect of the *EM* structure of the proton on the $^1S_0$ phase shift is comparable to the experimental accuracies. But the *EM* structure effect is importnat in the inner region of the nuclear force. So the effect of the wave distortion by the nuclear force must be taken into consideration. The plane wave Born approximation gives a very rough estimation of the *EM* structure effects on the $^1S_0$ phase shift.

The *VP* effect decreases at higher energies, but the *EM* contributions to the phase shifts increase. So, when the accuracy of the data is improved, its effects must be considered even in the higher energy region.

ACKNOWLEDGMENT

The author would like to thank Prof. K. Nishimura and Dr. N. Tamura for their suggestions and criticism.

He also wishes to thank Prof. S. Machida for his valuable suggestion.

REFERENCES

1) L. H. Johnston and D. E. Young, Phys. Rev. **116** (1959) 989.
2) D. J. Knecht, P. F. Dahl and S. Messelt, Phys. Rev. **148** (1966) 1031.
3) R. J. Slobodrian, H. E. Conzett, E. Shield and W. F. Tivol, Phys. Rev. **174** (1968) 1122.
4) N. Jarmier, J. H. Jett, J. L. Detch, Jr., and R. L. Hutson, Phys. Rev. **C3** (1971) 10.
5) K. Imai, K. Nisimura, H. Sato and N. Tamura, to be published
6) H. P. Noyes and H. M. Lipinski, Phys. Rev. **162** (1967) 884.
7) K. Imai, K. Nisimura, H. Sato and N. Tamura, Bulletines of the Institute of Chemical Research Kyoto Univ. vol. 48 (1974)
8) L. L. Foldy and E. Eriksen, Phys. Rev. **98** (1955) 775.
9) M. S. Sher, P. Signell and L. Heller, Annals of Phys. **58** (1970) 1.
10) D. Kiang, S. Machida and Y. Nogami, Canadian Journal of Phys. **51** (1973) 1120.
11) E. Eriksen, L. L. Foldy and W. Rarita, Phys. Rev. **103** (1956) 781.
12) L. Durand, Phys. Rev. **108** (1957) 1597.
13) L. Heller, Phys. Rev. **120** (1960) 627.
14) E. A. Uehling, Phys. Rev. **48** (1935) 55.
15) J. Schwinger, Phys. Rev. **78** (1950) 135.
16) W. A. Barker and F. N. Glover, Phys. Rev. **99** (1955) 317.
17) G. Breit, Phys. Rev. **106** (1957) 314.
18) N. Hoshizaki, I. Lin and S. Machida, Prog. Theor. Phys. **26** (1961) 880.
19) M. H. MacGregor, R. A. Arndt and R. M. Wright, Phys. Rev. **182** (1969) 1714.
20) N. Hoshizaki, Supple. of Prog. Theor. Phys. **42** (1968) 107.
21) G. Breit and H. M. Ruppel, Phys. Rev. **127** (1962) 2123.