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A STUDY ON THERMOSOLUTAL CONVECTION IN SALINE LAKES

By

Yuki YUSA

Geophysical Research Station, Faculty of Science, Kyoto University

ABSTRACT

Through heat balance and macroscopic thermal model studies of Lake Vanda in Victoria Land, Antarctica based on thermal properties observed by the present author et al. in both the 1970–71 and the 1971–72 austral summers, it has been shown that the lake is balanced thermally with the solar radiation as well as other saline lakes in Victoria Land by considering the partly developed thermosolutal convection (Yusa, 1972 and 1975b).

In this paper, the phenomenon is more closely studied from the point of view of thermosolutal convection. The existing theory for thermosolutal convection based on temperature distribution involves some limits in interpreting the actual thermosolutal system. In place of temperature distribution, heat flow is introduced into the theory, from which is derived a new criterion concerning the onset and the development of thermosolutal convection. With this criterion one can distinguish the zone of thermosolutal convection from the zone under the control of the molecular process, which reasonably explains physical meanings of the step-like structure in temperature and salinity profiles and of the apparent diffusivity throughout the structure zone of the latter. These results give reasonable bases to the unusually high water temperatures and to the general feature of the water temperature distribution appearing under the solar heating of saline waters with stable salinity gradients. Furthermore, it is shown that the oscillation of heat flow observed in Lake Vanda results from thermosolutal convection. It is also suggested that the thermosolutal structure in this lake may be maintained for a fairy long period for the future under the unchanged climatic condition.

I. Introduction

In the large ice-free area of Victoria Land, Antarctica, there are some saline lakes which show fairly higher temperatures than the annual mean air temperature in their surroundings, and many studies on the origin of salts and the cause of high temperatures have been carried out. Concerning the thermal feature, it has been clarified that each saline lake is almost balanced thermally with solar radiation, and that the stable density stratification prevents circulation throughout the entire lake and therefore contributes significantly to the unusually high temperatures. However, in part of Lake Vanda, which is the largest lake in this area and whose bottom temperature reaches up to about 25°C, we must assume the development of convective motion based on some observations such as the existence of distinctive step-like structures in temperature and salinity profiles and the relation between the upward heat flow and the mean temperature gradient.

The study of free convection in fluid was begun by the famous experiment of

Bénard, while Rayleigh (1916) studied theoretically the thermal convection in fluid whose temperature is higher at the lower level and clarified the marginal state of the onset of convective instability and the scale of the convection cell. Since then, Rayleigh's theory has given a base to various studies on convective phenomena.

Stommel, Arons and Blanchard (1956) considered for the first time the possible behaviour of motion in thermosolutal system qualitatively, and suggested that the motion might be greatly influenced by the difference between molecular diffusivities for heat and solute. Since then, many experimental and theoretical investigations have been carried out. The representative theory for thermosolutal convection was developed by Veronis (1965) and Baines and Gill (1969), who extended and applied Rayleigh's theory to the thermosolutal system and obtained the critical condition for the onset of instability by means of linear stability analysis.

The stability theory deduced by them tells whether an infinitesimal disturbance given to the system in rest is amplified or dissipated. In the physical limnology, we may apply it to infer the thermal (thermosolutal) condition of the onset of circulation in lakes or ponds, stably stratifying without any motion initially, due to heating and/or cooling. In general, however, if convective instability occurs in a system, we can only observe the state after the event, as Namekawa (1938) pointed out. In addition, it is often observed that the distribution of temperature or salinity is maintained for rather a long period. In the study of thermal (thermosolutal) structures of lakes or ponds based on such observations, it is required to judge whether the convection is developing or not, but it is impossible to determine this by the existing stability as examined in this paper. So far, many articles have been concerned with estimations and estimating methods of (eddy) diffusivities on the basis of heat flow and temperature distribution. However, in order to know whether the thermal (thermosolutal) structures in lakes are subject to molecular process or to eddy (convection) process, it may be one of the most pertinent ways to deduce a simple criterion including heat flow.

In this paper, we shall try to deduce a criterion based on heat flow data, and to examine its propriety by applying it to thermosolutal systems in some antarctic saline lakes. This treatment will clarify the range where thermosolutal convection may occur and develop. Furthermore, we will consider the changes in temperature and salinity distributions formed in a saline lake on the basis of the obtained result.

II. Saline lakes in Victoria Land, Antarctica

Before considering the thermosolutal convection, we will summarize some features of saline lakes in Victoria Land, which provided the immediate incentive for the present work. The largest ice-free area in Antarctica, referred to as "the Dry Valley", is developing in Victoria Land along the west coast of McMurdo Sound, whose total area amounts to about 4000 km^2. The main part consists of three large glacially eroded valleys, i.e. Victoria, Wright and Taylor Valleys, which run from the coast to the inland. Through reconnaissances since IGY, some lakes and ponds have been discovered there (Kusunoki et al, 1973). Most of them are saline and their surfaces are frozen permanently, with the only exception of small and shallow Don Juan Pond (Wright Valley) which is said not to freeze until $-55°C$ due to its extraordinarily high solute concentration. Below these ice sheets, we can find "lake waters", which show fairly high temperatures in spite of the low air temperature of the surrounding area (about −20°C in the annual mean). These unusually high temperatures have attracted investigators' attention, and various studies have been carried out on the cause of the high temperature as well as the origin of the solute and the genesis of the Dry Valleys.

Fig. 1 is a rough map of the Dry Valleys including locations of saline lakes, and Table 1 shows locations and approximate scales of main saline lakes and ponds. Figs. 2 and 3 are examples of temperature and salinity profiles observed in Lakes Vanda, Bonney East Lobe, Fryxell and Miers respectively. As seen in Fig. 2 water temperatures in these lakes generally rise as depth increases from 0°C just below the ice sheets. The maximum temperatures in Lakes Bonney and Fryxell appear at the middle parts, while those in Lakes Vanda and Miers near the bottom. On the other hand, as seen in Fig. 3, salinities also rise as depth increases, and stratifications in them seem to be gravitationally stable in general. In addition, these lake waters can be hardly disturbed by external factors such as wind, because of their ice sheet coverings. Referring to these conditions, it has been considered that transfers of heat and chemical substances are mainly subject to a molecular process. In Lakes

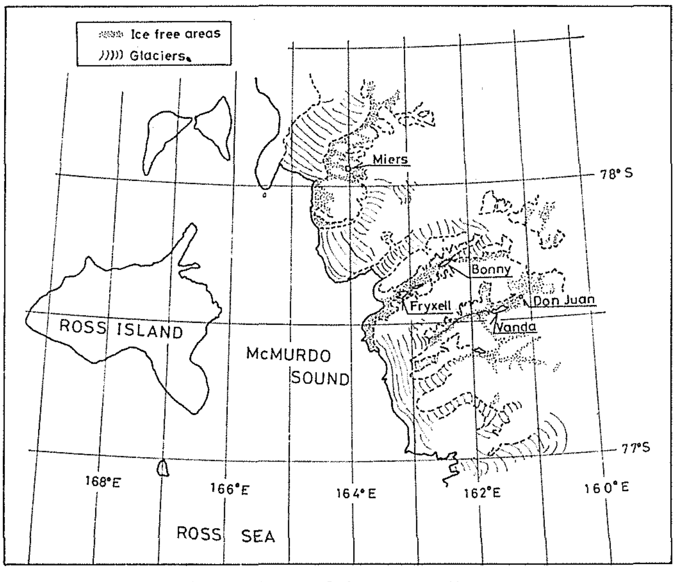

Fig. 1. Map of the Dry Valley.

Table 1. Locations and scales of saline lakes in Victoria Land

Name of Lake	Location		Valley	Length(km)	Width(km)	Depth(m)
Vanda	77°32′S,	161°30′E	Wright	5.6	1.4	66
Bonney (East)	77°43′S,	162°23′E	Taylor	4.82	0.77	33
Fryxell	77°36′S,	163°08′E	Taylor	5	2	17
Miers	78°07′S,	163°54′E	Miers	1.5	0.7	20
Don Juan	77°43′S,	160°10′E	Wright	0.7	0.3	0.1–0.2

Length, width and depth are changeable due to changes of inflow, evaporation etc..

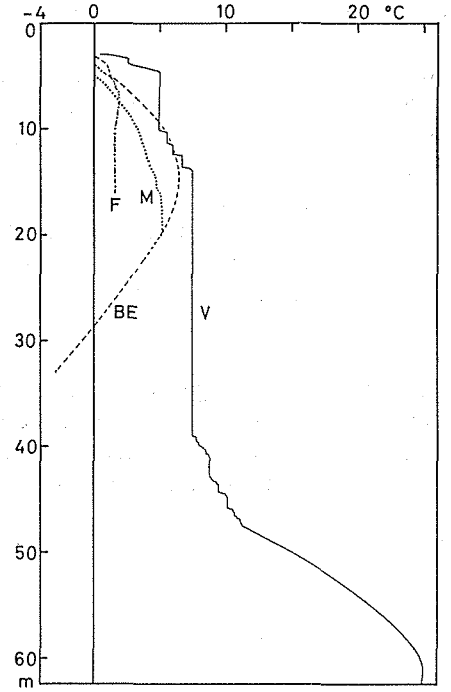

Fig. 2. Temperature profiles in saline lakes, Victoria Land. V: Lake Vanda, BE: Lake Bonney East Lobe, F: Lake Fryxell, M: Lake Miers.

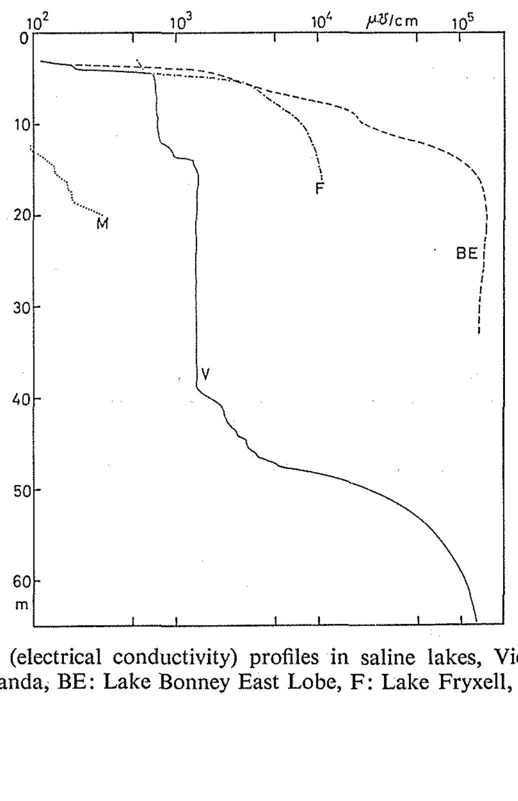

Fig. 3. Salinity (electrical conductivity) profiles in saline lakes, Victoria Land. V: Lake Vanda, BE: Lake Bonney East Lobe, F: Lake Fryxell, M: Lake Miers.

Bonney and Fryxell, their water temperatures and the shapes of their smooth profiles can be explained by the following equation deduced from the assumptions of solar heating and molecular diffusion (Hoare et al, 1964 and 1965; Shirtcliffe and Benseman, 1964).

$$T=-\frac{Q_0}{k\eta}e^{-\eta z}+Cz+E\,, \qquad (1)$$

where T is the temperature at depth z, Q_0 is the intensity of solar radiation past the level $z=0$ (the lower surface of the ice sheet), η is the extinction coefficient of lake water, k is the thermal conductivity of the water, and C and E are constants controlled by the temperature at $z=0$ (0°C in this case) and the amount of heat flowing out of the lake through bottom. Eq. (1) seems to be almost appropriate to the water temperature in Lake Miers, though both profiles of temperature and salinity show step-like structures in the bottom part (Bell, 1967).

Table 2. Heat balance in Lake Vanda during the period from November 13, 1971 to January 3, 1972 (*cal/cm² day*)

	Heat gain			Heat loss+Storage+Advection		
Period	Q_i	Q_a	Q_i+Q_a	Q_e	Q_v+Q_f	$Q_e+Q_v+Q_f$
13/Nov.～3/Dec.	197	47	244	184	81	265
3/Dec.～3/Jan.	190	28	218	171	28	209
Total	193	36	229	176	55	231

Q_i: Net flux radiation, Q_a: Sensible heat exchange with the atmosphere, Q_e: Heat loss by evaporation, Q_v: Change of heat storage, Q_f: Heat exchange by advection.

On the other hand, concerning the cause of the unusually high temperatures (up to 25°C in Lake Vanda), some arguments have been made whether the heat may originate from geothermal* and/or hydrothermal** activities or from the solar radiation (Angino et al, 1963; Wilson and Wellman, 1962; Ragotzkie and Likens, 1964). In order to examine this from the viewpoint of heat balance of the lake, the present author et al observed some thermal properties of Lake Vanda in both the antarctic summer of 1970–71 and 1971–72 (Yoshida et al, 1971; Torii et al, 1972). Table 2 is the result of heat balance analyses for the summer of 1971–72, which shows that the effect of geothermal or hydrothermal activity is negligible and that Lake Vanda is also balanced thermally with solar radiation as well as other lakes (Yusa, 1972). However, how the present structure of temperature was formed is a seperate problem from the heat balance.

Applying Eq. (1) to this lake for trial, a quite different profile from the observed and the maximum temperature beyond 100°C will be deduced. In this sense, the temperature of 25°C should better be interpreted as rather low, though no doubt it is extraordinarily high considering the frigid circumstances.

The apparent thermal diffusivity in the upper part of Lake Vanda was estimated

* Geothermal activity is the subsurface thermal activity due to the heat transfer from the interior of the earth.

** Hydrothermal activity is the subsurface thermal activity being accompanied by thermal water.

to be about 6 times larger than the molecular diffusivity in the 1971–72 summer season according to measurements of the upwards heat flow and the mean temperature gradient. Now, the temperature profile in this part shows a typical step-like structure where isothermal layers (several tens of *cm* to several meters in thickness) and thin sheets with sharp temperature gradients (a few tens of *cm* in thickness) appear by turns vertically; and this structure extends horizontally over the entire lake as shown in Fig. 5 (refer to Fig. 4). It has been considered that such a structure has been formed accompanying thermosolutal convection, which occurs and develops under the combined effect of the destabilization due to the tempera-

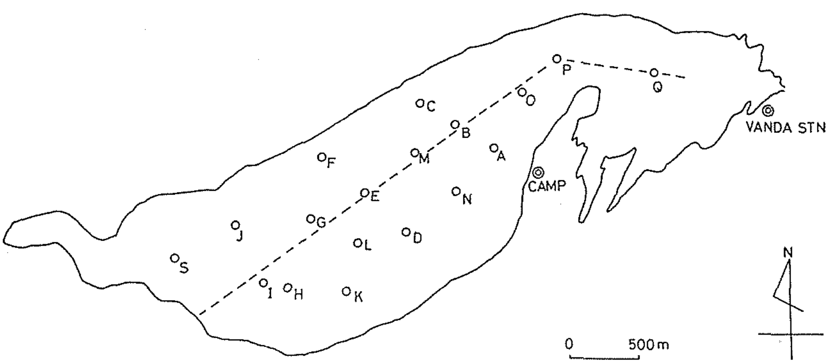

Fig. 4. Lake Vanda and observation stations.

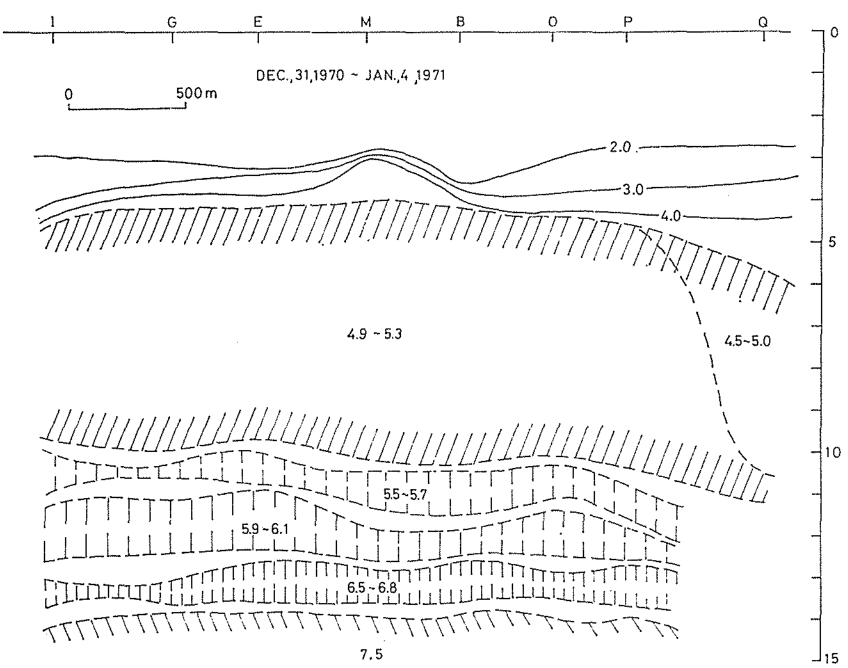

Fig. 5. Cross-sectional distribution of temperature in the upper part of Lake Vanda along the longitudinal axis drawn in Fig. 3, during the period from December 31, 1970 to January 4, 1971.

ture distribution and the stabilization due to salinity distribution (Hoare, 1968; Shirtcliffe and Calhaem, 1968). In fact, an analogous multi-layered stratification was observed in experiments conducted by Turner and Stommel (1964), who heated salt water with a stable salinity gradient (downwards positive gradient) from below.

It is reasonable to suppose that convective motions develop in isothermal layers while molecular processes dominate in thin sheets. This supposition brings the following relation for the mean diffusivity κ_a in the step-like structure zone

$$\kappa_a/\kappa \sim D/\delta \,, \tag{2}$$

(Yusa, 1972), where κ is the molecular diffusivity, D is the thickness of the zone in question and δ is the total thickness of thin sheets. The value of κ_a/κ in the upper step-like structure zone of Lake Vanda during the 1971–72 summer season was calculated to be 4.5 at least, which agrees rather well with the forementioned value obtained from the heat flow and the mean temperature gradient. This suggests the development of thermosolutal convection in the step-like structure zone.

Replacing the step-like structure zone with the distribution of thermal diffusivities, introducing the effect of sometimes occurring lake ice motions, and assuming a thermal (quasi-)steady state, the temperature and its vertical profile in Lake Vanda can be explained by solar heating as shown in Fig. 6 (Yusa, 1975b). (No special geothermal activities were detected in the Dry Valley area according to the temperature measurements in the bottom sediment of Lake Vanda (Yoshida et al. 1971) or according to the geothermal studies of the Dry Valley Drilling Project (Decker, 1974).)

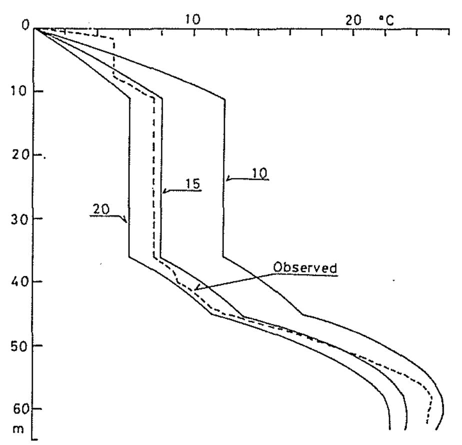

Fig. 6. Macroscopic thermal model by solar heating for Lake Vanda. Numerals show the ratio between the apparent and the molecular diffusivities in the upper part, which involves the effect of lake ice motions occurring sometimes.

III. Previous studies on thremosolutal convection and applications to actual systems

III–1. An outline

As summarized in the former section, each saline lake in Victoria Land is balanced thermally with solar radiation, and also the complicated profile of water temperature in Lake Vanda can be possibly explained by solar heating under the assumption of development of thermosolutal convection. So far, however, we have no examination on thermosolutal conditions in these saline lakes in connection with the thermosolutal convection theory; i.e. whether convection is actually developing or not and whether lakes are under possible conditions for the onset of convection or not. In this section, we will treat the theory developed by Veronis (1965) and Baines and Gill (1969) in outline, and try to examine thermosolutal systems observed in antarctic saline lakes by it. The examination will lead to unacceptable conlcusions that all thermosolutal stratifications are stable; i.e. no convection can occur, and then that even if the system is compulsorily disturbed by a large convective disturbance the initial motion will dissipate finally without a continuous compulsion.

Since Stommel, Arons and Blanchard (1956) pointed out possible and curious phenomena in the thermosolutal system, various studies on free convection in this system (thermosolutal or thermohaline convection) have been carried out both experimentally and theoretically. Those fruits have been summarized by Turner (1973) and Stern (1975). Turner and Stommel (1964) classified thermosolutal systems into four cases. Among them, two cases attract our attention; one is the diffusive regime where both temperature and salinity increase downwards, while another is the finger regime where both of them decrease downwards. Characteristic phenomena in the former regime observed through experiments are as follows; ① the oscillating convection (overstable mode) which appears at the earlier stage of heating the system from below and ② the transient multi-layered stratification (step-like structure in temperature and salinity profiles), while that in the latter system is the development of tall convection cells like "fingers".

Assuming that the density of fluid can be expressed by a linear relation with temperature and salinity changes, thermosolutal convection in a two dimensional Boussinesq fluid (in which variations of fluid properties such as density, viscosity, diffusivity etc. are neglected, but retaining the density variation affecting the gravitational term) may be described by following equations,

$$\rho=\rho_0(1-\alpha T+\beta S)\,, \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0\,, \tag{4}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z}\right)\Delta w=\nu\Delta^2 w-\alpha g\frac{\partial^2 T}{\partial x^2}+\beta g\frac{\partial^2 S}{\partial x^2}\,, \tag{5}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z}\right)T=\kappa\Delta T\,, \tag{6}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z}\right)S=\kappa_s\Delta S\,, \tag{7}$$

where $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$, and variables and constants are described below;

Nomenclatures

ρ_0 : density of fluid at the reference temperature and salinity

T : temperature represented by the difference from the reference state, or period of oscillation
S : salinity represented by the difference from the reference state
ρ : density of fluid at the temperature T and the salinity S
α : coefficient of volume expansion due to unit temperature difference, $\left(-\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)$
β : the proportional density change produced by unit salinity change, $\left(\frac{1}{\rho}\frac{\partial\rho}{\partial S}\right)$
ΔT : temperature difference between the upper and the lower surfaces, positive when higher on the lower surface
ΔS : salinity difference between the upper and the lower surfaces, positive when higher on the lower surface
c : specific heat of fluid
ν : kinetic viscosity
κ : molecular diffusivity for heat
κ_s : molecular diffusivity for salt
h : depth of fluid
g : acceleration of gravity
x : horizontal coordinate
z : vertical coordinate (positive downwards)
t : time
u : horizontal velocity
w : vertical velocity
F_H : amount of heat flow or heating rate from below
$\overline{T}$: horizontal mean of temperature
$\overline{S}$: horizontal mean of salinity
θ : fluctuating part of temperature from the horizontal mean
s : fluctuating part of salinity from the horizontal mean
a : parameter concerning horizontal scale of convection cell
P : Prandtl number (ν/κ)
R_a : Rayleigh number ($\alpha g\Delta Th^3/\nu\kappa$)
R_s : solute Rayleigh number ($\beta g\Delta Sh^3/\nu\kappa$)
τ : κ_s/κ
R_a^h : flux Rayleigh number ($\alpha gF_Hh^4/\rho c\nu\kappa^2$)

Considering a layer of fluid of a depth h with constant temperatures and salinities on the upper and the lower surfaces and extending horizontally without bounds, we conveniently nondimensionalize variables in Eqs. (4)–(7) as follows;

$$\left.\begin{aligned} &x,\ z\sim x^*h,\ z^*h;\ S\sim S^*\Delta S;\ T\sim T^*\Delta T; \\ &u,\ w\sim u^*\frac{\kappa}{h},\ w^*\frac{\kappa}{h};\ t\sim t^*\frac{h^2}{\kappa}, \end{aligned}\right\} \tag{8}$$

where variables with asterisk are nondimensionals.

When both surfaces are dynamically free and profiles of temperature and salinity between them are linear respectively, the following cubic equation bearing on the convective stability of a thermosolutal system in rest will be deduced from nondimensionalized forms of Eqs. (4)–(7) by the perturbation method,

$$\sigma^3+(P+1+\tau)k^2\sigma^2+\left\{(P+P\tau+\tau)-\frac{(R_a-R_s)P}{\eta}\right\}k^4\sigma$$
$$+\left\{\tau+\frac{(R_s-\tau R_a)}{\eta}\right\}Pk^6=0\,, \qquad (9)$$

where $k^2=\pi^2(n^2+a^2)$, $\eta=\pi^4(n^2+a^2)^3/a^2$, πn and πa are vertical and horizontal wavenumbers (n is necessarily an integer and a is an arbitrary constant representing the horizontal scale of a convection cell), and σ is a complex which gives a growth rate and a characteristic of oscillation. Eq. (9) was obtained by Veronis (1965), in which the case of R_a, $R_s>0$ corresponds to the diffusive regime and the case of R_a, $R_s<0$ to the finger regime.

III–2. **The stability analysis**

Baines and Gill (1969) examined Eq. (9) in detail, and obtained some critical relations between R_a and R_s concerning the stability of the system. Now, every saline lake in question shows a stratification belonging to the diffusive regime. So, giving attention to this regime only, we find the following relation (10). For all R_a and R_s, the most unstable mode of disturbance has $n=1$ and $a=1/\sqrt{2}$, when the minimum value of $\eta=(27/4)\pi^4(\fallingdotseq 657)$ is attained.

① $R_a<\frac{P+\tau}{1+P}R_s+(1+\tau)\left(1+\frac{\tau}{P}\right)\frac{27}{4}\pi^4$: stable

② $R_a=\frac{P+\tau}{1+P}R_s+(1+\tau)\left(1+\frac{\tau}{P}\right)\frac{27}{4}\pi^4$: neutral

③ $R_a>\frac{P+\tau}{1+P}R_s+(1+\tau)\left(1+\frac{\tau}{P}\right)\frac{27}{4}\pi^4$: unstable (oscillating)

④ When Eq. (9) has at least one real root, both oscillating and direct modes of instability are possible; this condition can be approximately written as $R_a>R_s+0(R_s^{2/3})$ for large values of R_a and R_s. Especially, when $R_a\geqq\frac{(P+\tau)}{(P+1)\tau^2}R_s$, only the direct mode is possible.

(10)

When an isothermal fluid with a stable salinity gradient is heated from below, condition ③ will be established first by the effect of the rising temperature gradient, and therefore the oscillating instabiilty will occur before everything. This was confirmed expreimentally by Shirtcliffe (1967) and Turner (1968).

Now, let us try to investigate the convective instability in the actual thermosolutal system by the stability criterion shown in (10). In order to examine the local stability at each level, we will conveniently cut the system into thin slices of a certain thickness vertically, and then apply the criterion to each slice. In that case, since values of R_a and R_s for each slice become generally far larger than $(1+\tau)\left(1+\frac{\tau}{P}\right)\frac{27}{4}\pi^4$, the convective instability will occur if the following simplified condition is satisfied, which is deduced from (10).

$$\frac{\alpha \Delta T}{\beta \Delta S} > \frac{P+\tau}{1+P}, \tag{11}$$

where $P+\tau/1+P$ nearly equals 0.9 in the ordinary saline water which has $\tau=0.01$ and $P=7\sim12$. Relation (11) indicates that convective instability can occur even if the stratification is gravitationally stable. This is one of remarkable features of the fluid which consists of two properties with different molecular diffusivities such as heat and salt. By the way, when both diffusivities are almost identical, that is $\tau\sim1$, the convective instability cannot occur unless gravitationally unstable, because the value $P+\tau/1+P$ becomes 1.

III–3. The practical applicability of the stability criterion

—In Lake Vanda— The right figure of Fig. 7 shows a distribution of $\alpha\Delta T/\beta\Delta S$ calculated for each thin slice on the bases of temperature and salinity profiles ob-

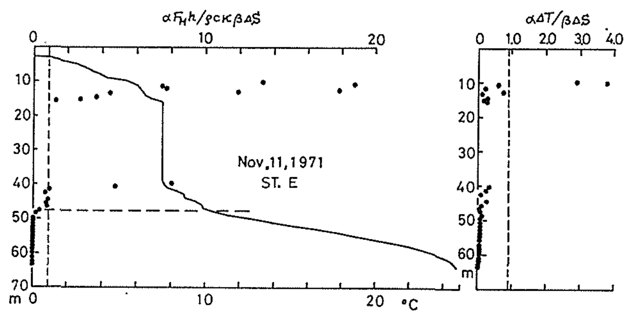

Fig. 7. Distributions of $\alpha\Delta T/\beta\Delta S$ and $\alpha F_H h/\rho c\kappa\beta\Delta S$ in Lake Vanda.

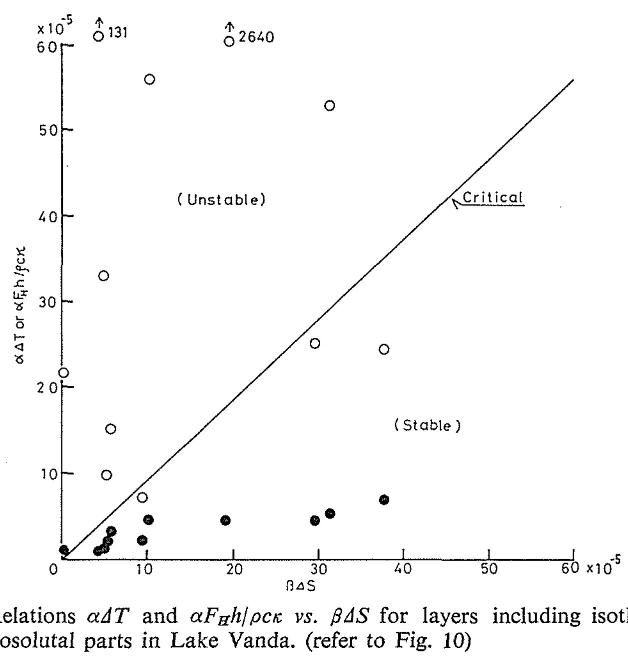

Fig. 8. Relations $\alpha\Delta T$ and $\alpha F_H h/\rho c\kappa$ *vs.* $\beta\Delta S$ for layers including isothermal and isosolutal parts in Lake Vanda. (refer to Fig. 10)

served at Station E (refer to Fig. 4) on November 11, 1971. The vertical broken line in the figure corresponds to the critical value of the onset of instability. Almost all of them fall on the stable side excepting two in the most upper part. For the middle part of the lake, this criterion cannot be applied because of the isothermal and isosolutal conditions.

On the other hand, solid circles in Figs. 8 and 9 are relations between $\alpha\Delta T$ *vs.* $\beta\Delta S$ obtained by aiming at step-like structures shown in Figs. 10 (a) and (b). Fig. 8 shows the relation between adjacent horizontal lines in Figs. 10 (a) and (b); i.e. the relation obtained as including isothermal and isosolutal layer. Notwithstanding, all dots excepting only one situated near the origin fall on the stable side beneath the critical line. Fig. 9 shows the relation for sheets; i.e. the relation obtained from differences of temperature and salinity between adjacent homogeneous layers. All these dots fall on the stable side, too.

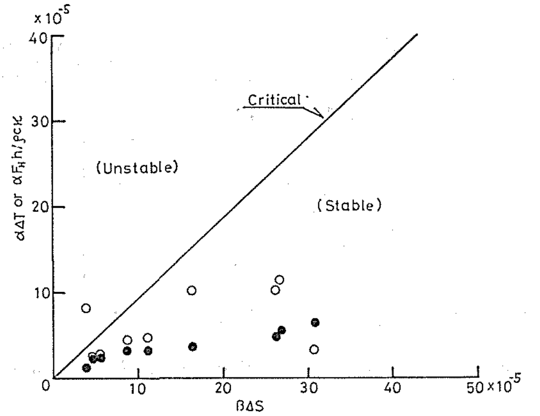

Fig. 9. Relations of $\alpha\Delta T$ and $\alpha F_H h/\rho c\kappa$ *vs.* $\beta\Delta S$ for thin sheets with sharp gradients of temperature and salinity in Lake Vanda. (refer to Fig. 10)

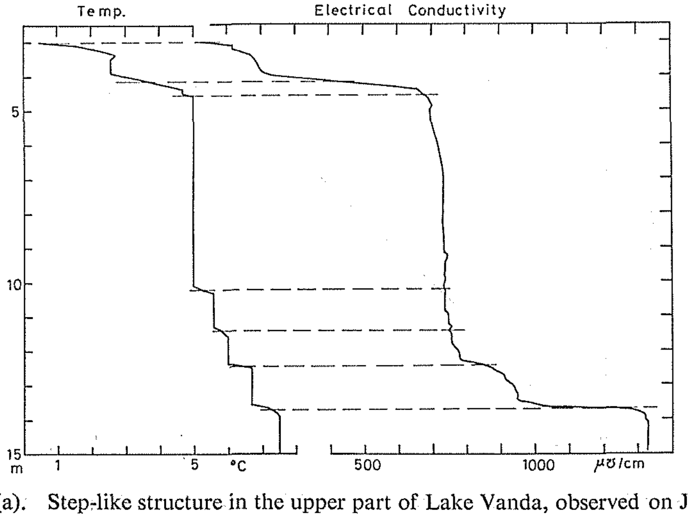

Fig. 10(a). Step-like structure in the upper part of Lake Vanda, observed on January 2, 1971.

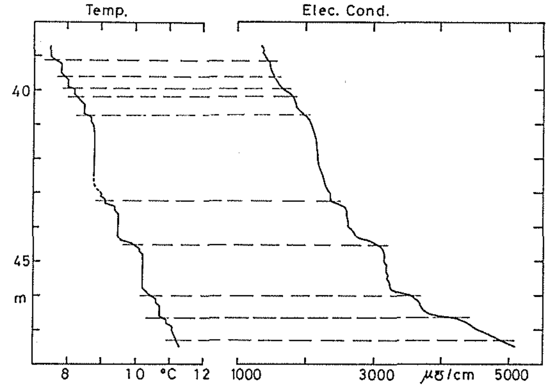

Fig. 10(b). Step-like structure in the lower part of Lake Vanda, observed on January 2, 1971.

These results give support for the previous speculation that there may be no convections in sheets and the deepest part of this lake, but simultaneously deny the development of convections in homogeneous layers. The latter not only makes the physical meaning of the apparent diffusivity in the upper step-like structure zone obscure, but also brings no bases for the assumption in calculating the macroscopic temperature profile shown in Fig. 6.

—In Lake Miers— In Lake Miers, the thermosolutal system belonging to the diffusive regime was observed below 12 meters, where step-like structures of temperature and salinity profiles were developing. These structures are also interpreted as those formed accompanying thermosolutal convection (Bell, 1967), but no studies on them have been made.

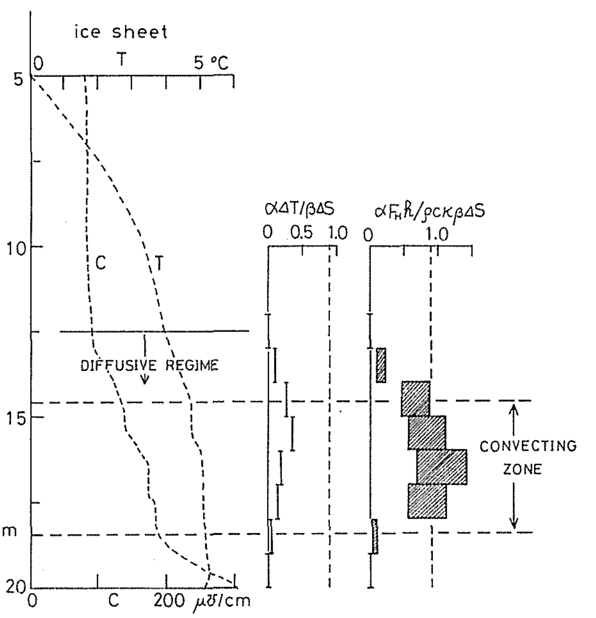

Fig. 11. Profiles of temperature and salinity (after Bell) and distributions of $\alpha\Delta T/\beta\Delta S$ and $\alpha F_H h/\rho c\kappa\beta\Delta S$ in Lake Miers.

The middle plot of Fig. 11 shows the distribution of $\alpha\Delta T/\beta\Delta S$ obtained for each thin slice of 100 cm thick by the stability criterion (11) similar to that in Lake Vanda. All values fall within the stable range, and therefore the onset of convection is impossible.

—In Lake Bonney— Lake Bonney is a representative saline lake in the Dry Valley area as well as Lake Vanda, and has been described as a typical sun-heated lake (Hoare et al, 1964; Shirtcliffe and Benseman, 1964). The thermosolutal stratification belonging to the diffusive regime was observed in the upper part from about 7 to 14 meters deep, but no distinctive step-like structures could be recognized with the only exception of a weak step of salinity profile at about 9 meters deep. Thus, it is considered that the entire lake is under the control of the molecular process. In fact, the observed profile of water temperature can be explained rather well as being due to solar heating and molecular diffusion of heat. Solid circles in the right figure of Fig. 12 show the distribution of $\alpha\Delta T/\beta\Delta S$ for each 50 *cm* thick slice. All of them are far smaller than the critical value, and the criterion (11) seems to be applicable to this lake.

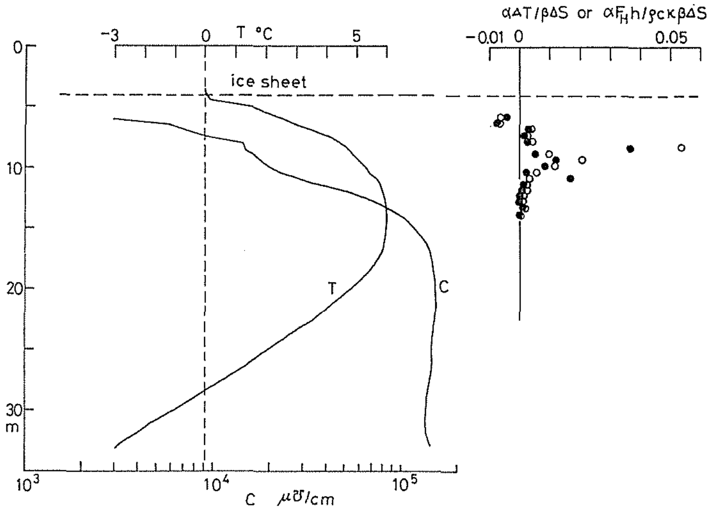

Fig. 12. Profiles of temperature and salinity, and distributions of $\alpha\Delta T/\beta\Delta S$ and $\alpha F_H h/\rho c\kappa\beta\Delta S$ in Lake Bonney, East lobe.

—In Lake Fryxell— The water temperature does not ascend above 4°C in this lake, and the thermosolutal stratification of the diffusive regime has never been observed. Of course, the stratification is gravitationally stable.

—Under the ice island T-3— Other thermosolutal systems belonging to the diffusive regime have been reported, e.g. under the ice island T-3 in the Arctic Ocean (Neshyba et al, 1971), in the bottom water of the Red Sea (Swallow and Crease, 1965) and several brackish lakes in Japan such as Kai-ike and Namako-ike in Koshiki-jima island and Shin-miyo in Miyake-jima island (Arai and Nishizawa, 1974). From among these, we will examine the thermosolutal system under the ice-island T-3.

A thermosolutal system of the diffusive regime, being accompanied by remarkable step-like structures in temperature and salinity profiles, were observed under the ice-island T-3 (84°N, 126°W) in the Arctic Ocean by Neshyba et al (1971). They made a conclusion that those structures had been formed accompanying the thermosolutal convection by referring to Turner's experiment (Turner, 1965).

According to their observation data, the stability $\alpha\Delta T/\Delta\beta S$ is calculated as ranging from 0.067 to 0.5 for each thin sheet with sharp gradients of temperature and salinity, which indicates that each sheet is convectionally stable. On the other hand, however, the value of $\alpha\Delta T/\beta\Delta S$ calculated from their smoothed profiles also falls within the stable range in spite of including the effect of homogeneous layers, i.e. 0.16 for the zone from 240 to 297 meters deep and 0.15 for the zone from 297 to 340 meters deep. Thus, the stability criterion (11) indicates that no convection can occur in this thermosolutal system, which seems to be incongruous with the persistency of the step-like structure.

III–4. **Investigation based on numerical experiments**

As shown through some investigations, the stability criterion (10) or (11), which has been deduced on the basis of temperature and salinity differences between the upper and the lower boundaries of fluid, cannot explain consistently whether an actual thermosolutal system includes convective motion in itself or whether it is possibly a condition for the onset of convection. It may only be able to tell that the stably stratified system is surely stable as in Lake Bonney. This may be inevitable judging from the essential meaning of the existing stability which treats the system in rest. In order to examine the thermosolutal system including convection

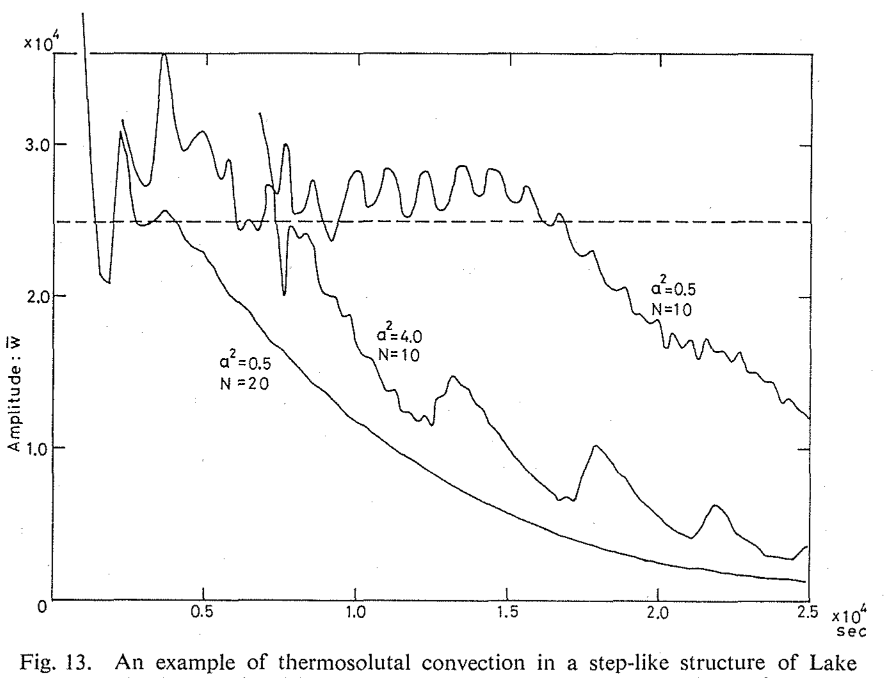

Fig. 13. An example of thermosolutal convection in a step-like structure of Lake Vanda, calculated by Eqs. (4)–(7) under the boundary condition of constant temperatures and salinities on the upper and the lower surfaces. $\bar{w}$ is the rms value of the nondimensional vertical velocity, N the number of terms of Fourier series and a^2 a parameter for the horizontal scale of a convection cell.

itself, we must investigate whether the initial forced convection persists or dissipates under the boundary condition of constant temperature and salinity on both surfaces respectively, even after the forcing has been removed.

Fig. 13 shows examples of change of convective motion for a step-like structure in Lake Vanda, which were obtained by numerical integrations of equations (4)–(7) under an initial forced convection and constant temperatures and salinities on both surfaces. The calculations were carried out by decomposing variables, such as velocity, temperature and salinity, into Fourier series after Herring's method used for the investigation of thermal convection (Herring, 1963). N and a^2 in the figure are the adopted number of Fourier terms and the parameter concerning the horizontal scale of convection cell, respectively. Every rms value of nondimensional vertical velocity w decreases with the lapse of time, as is evident from Fig. 13.

Thus, we cannot help considering that the theory based on temperature and salinity differences is inadequate to treat problems in the convecting system. This may be solely attributable to adopting the temperature difference as a thermal parameter in describing the system, because the temperature difference, in other words the temperature distribution, is established as a final consequence depending on the combined effect of various processes such as heating, cooling, diffusion, convection etc.. So, the thermal (thermosolutal) phenomena in the actual system should be analysed based on an idea involving the primary effect of heating due to solar radiation or heating from below.

IV. Thermosolutal convection based on heat flow

IV–1. Formulation of the problem

Let us consider the local thermosolutal situation in each saline lake by cutting it into thin slices of 50 or 100 *cm* thick as before. Since the water temperature in the diffusive regime rises downwards, each slice is heated from below by the upwards heat flow. As each lake is almost balanced thermally with solar radiation according to the previous studies, the amount of the upwards heat flow passing through the lower boundary of each slice is equal to the amount of solar radiation absorbed by the entire water body lying between the level of the slice in question and that of the maximum temperature. On the other hand, the amount of solar radiation absorbed by the slice itself is generally far smaller than the heat flow from below, except slices situated near the lowest part in question. Thus, we can approximate and replace the solar heating of the lake water with the heating of the thin slice from below. This is analogous to the situation established in the experiment of heating salt water from below. Considering this analogy, we will treat the case in which a layer of salt water depth h with a stable salinity gradient is heated from below with a constant intensity. The temperature at the upper surface is held constant for simplification of the problem. Besides, we will suppose that the salinities at both surfaces are held constant during the period in question, taking into account that the diffusion of salt is extremely slow compared with that of heat as evident from $\tau \sim 0.01$.

Similarly as before, assuming a two dimensional Boussinesq fluid whose density has a linear relation with temperature and salinity, equations (3)–(7) are appropriate here, too.

Supposing that vertical velocities vanish at the upper and the lower surfaces and both surfaces are dynamically free,

$$w=\frac{\partial^2 w}{\partial z^2}=0\,; \quad \text{at} \quad z=0,\, h\,. \tag{12}$$

Resolving temperature and salinity fields into horizontal means and fluctuating parts respectively,

$$\left.\begin{aligned} T&=\overline{T}(z,\,t)+\theta(x,\,z,\,t)\,,\\ S&=\overline{S}(z,\,t)+s(x,\,z,\,t)\,. \end{aligned}\right\} \tag{13}$$

Then, we assume that fluctuating parts have similar modes of the distribution of w,

$$\theta=s=0\,, \quad \text{at} \quad z=0,\, h\,. \tag{14}$$

Consequently, boundary conditions of the respective horizontal mean are written as follows,

$$\left.\begin{aligned} &\overline{T}=\overline{S}=0\,, \quad \text{at} \quad z=0\,,\\ &\rho c\kappa\frac{\partial\overline{T}}{\partial z}=F_H\,, \quad \overline{S}=\Delta S\,, \quad \text{at} \quad z=h\,. \end{aligned}\right\} \tag{15}$$

Variables will be conveniently nondimensionalized as (8), but the temperature must be nondimensionalized with the heating rate F_H instead of the temperature difference ΔT,

$$\overline{T}\sim\overline{T}^*\frac{F_H h}{\rho c\kappa}\,, \quad \theta\sim\theta^*\frac{F_H h}{\rho c\kappa}\,, \tag{16}$$

where variables with asterisk are nondimensionals. By means of this, fundamental equations (4)–(7) are nondimensionalized as follows, but asterisks are omitted for the simplification of expressions.

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0\,, \tag{17}$$

$$\frac{1}{P}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z}\right)\Delta w=\Delta^2 w-R_a^h\frac{\partial^2 T}{\partial x^2}+R_s\frac{\partial^2 S}{\partial x^2}\,, \tag{18}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z}\right)T=\Delta T\,, \tag{19}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z}\right)S=\tau\Delta S\,, \tag{20}$$

where $T=\overline{T}+\theta$, $S=\overline{S}+s$.

Boundary conditions (12), (14) and (15) are also nondimensionalized as,

$$\left.\begin{aligned}&w=\frac{\partial^2 w}{\partial z^2}=\theta=s=0\,, \qquad \text{at } z=0,\,1\,,\\ &\overline{T}=\overline{S}=0\,, \qquad \text{at } z=0\,,\\ &\frac{\partial \overline{T}}{\partial z}=\overline{S}=1\,, \qquad \text{at } z=1\,.\end{aligned}\right\} \tag{21}$$

Assuming that fluctuating parts of velocity, temperature and salinity have periodic distribution in the horizontal, w, θ and s may be expressed by the following Double Fourier series respectively,

$$\left.\begin{aligned}&w=\sum_{n,n'} w_n^{n'}\sin(n\pi z)\cos(n'\pi ax)\,,\\ &\theta=\sum_{l,l'}\theta_l^{l'}\sin(l\pi z)\cos(l'\pi ax)\,,\\ &s=\sum_{p,p'} s_p^{p'}\sin(p\pi z)\cos(p'\pi ax)\,,\end{aligned}\right\} \tag{22}$$

where n, n', l, l', p and p' are integers and a is an arbitrary constant which is a parameter concerning the period in the horizontal. The expression for w in (22) is also appropriate to the free boundary condition. Then, horizontal means of temperature and salinity may be simply written as follows,

$$\left.\begin{aligned}&\overline{T}=z+\sum_{m=1}^{\infty} T_m\sin\left(\frac{2m-1}{2}\pi z\right),\\ &\overline{S}=z+\sum_{r=1}^{\infty} S_r\sin(r\pi z)\,,\end{aligned}\right\} \tag{23}$$

where m and r are integers. Using the equation of continuity, the horizontal velocity u is

$$u=-\sum_{n,n'} w_n^{n'}\left(\frac{n}{an'}\right)\cos(n\pi z)\sin(n'\pi ax)\,. \tag{24}$$

Introducing above expressions (22), (23) and (24) into fundamental equations (18), (19) and (20) will give the set of ordinary differential equations for the Fourier coefficients. However, we will treat the case of a single horizontal wave number, i.e. $n'=l'=p'=1$. In this case, the following equations are obtained for the coefficient of the i-th term of each Fourier series,

$$\begin{aligned}&\frac{1}{P}\frac{dw_i}{dt}+k^2 w_i=\frac{\pi^2 a^2}{k^2}(R_s s_i-R_a^h\theta_i)\,,\\ &\frac{dT_i}{dt}+\left(\frac{2i-1}{2}\pi\right)^2 T_i=\frac{1}{2}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{n+l+i+1}w_n\theta_l\\ &\qquad\cdot\left[(n+l)\left\{\frac{1}{2(n+l-i)+1}+\frac{1}{2(n+l+i)-1}\right\}\right.\\ &\qquad\left.+(l-n)\left\{\frac{1}{2(n-l-i)+1}+\frac{1}{2(n-l+i)-1}\right\}\right],\end{aligned}$$

$$\left.\begin{aligned}
&\frac{d\theta_i}{dt}+k^2\theta_i=\frac{1}{2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(-1)^{n+m+i+1}(2m-1)w_nT_m \\
&\qquad\cdot\left[\frac{1}{2(m+n+i)-1}+\frac{1}{2(m-n-i)-1}\right. \\
&\qquad\left.-\frac{1}{2(m+n-i)-1}-\frac{1}{2(m-n+i)-1}\right]-w_i\,, \\
&\frac{dS_i}{dt}+\tau\pi^2 i^2 S_i=\frac{\pi i}{4}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}w_n s_p(\delta_{n,p+i}++\delta_{n,p-i}-\delta_{n,i-p})\,, \\
&\frac{ds_i}{dt}+\tau k^2 s_i=\frac{1}{2}\sum_{n=1}^{\infty}\sum_{r=1}^{\infty}w_n S_r r(\delta_{n,r-i}+\delta_{n,r+i}+\delta_{n,i-r})-w_i\,, \\
&\qquad\qquad (i=1,\,2,\,3,\,\cdots)\,,
\end{aligned}\right\}\tag{25}$$

where $k^2=\pi^2(i^2+a^2)$, $\delta_{x,y}$ is Kronecker's δ, $\delta_{x,y}=1$ for $x=y$, $\delta_{x,y}=0$ for $x\neq y$.

IV–2. The criterion for the onset and the development of thermosolutal convection

Let us treat the case where the vertical profile of salinity is linear and the initial temperature is zero degrees over the entire system. In this case, all coefficients S_r's in (23) vanish, that is

$$S_r=0\,,\qquad (r=1,\,2,\,3,\,\cdots)\,,\tag{26}$$

and coefficients T's in (23) in (23) have the following initial values,

$$T_m=\frac{8}{\pi^2}\frac{(-1)^m}{(2m-1)^2}\,,\qquad (m=1,\,2,\,3,\,\cdots)\,.\tag{27}$$

Now, we will begin to heat the system from below with a constant intensity continuously. Supposing that the system is at rest, i.e. $w=\theta=s=0$, the system may be developed under the control of the following relations only, which are derived from the second equation in (25),

$$\frac{dT_i}{dt}+\left(\frac{2i-1}{2}\pi\right)^2 T_i=0\qquad (i=1,\,2,\,3,\,\cdots)\,.\tag{28}$$

Consequently, each term of horizontal mean temperature will be obtained from (27) and (28) as follows,

$$T_m=\frac{8}{\pi^2}\frac{(-1)^m}{(2m-1)^2}\exp\left\{-\left(\frac{2m-1}{2}\pi\right)^2 t\right\}\,.\tag{29}$$

Thus, the stability of the system at the time t can be examined by perturbing the system with initial disturbances under the conditions described by (26) and (29). Here, taking up w_1, θ_1 and s_1, which may have the most rapid growth rate, and putting

$$w_1,\,\theta_1,\,s_1\sim e^{\sigma t}\,,\tag{30}$$

the following cubic equation is deduced by substituting (30) into the first, the third and the fifth equations in (25).

$$\sigma^3+(P+1+\tau)k^2\sigma^2+\left\{(P+P\tau+\tau)-\frac{(R_a^hA-R_s)P}{\eta}\right\}k^4\sigma$$
$$+\left\{\tau+\frac{(R_s-\tau R_a^hA)}{\eta}\right\}Pk^6=0\,, \tag{31}$$

where $k^2=\pi^2(1+a^2)$, $\eta=\pi^4(1+a^2)^3/a^2$, and

$$A=1+\frac{128}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{(2m-1)^2(2m+3)(2m-5)}\exp\left\{-\left(\frac{2m-1}{2}\pi\right)^2t\right\}\,, \tag{32}$$

σ is a complex number, whose real and imaginary parts give the growth rate and the characteristic of oscillation of the disturbance respectively. The minimum value of $\eta=(27/4)\pi^4$ is attained when $a=1/\sqrt{2}$ as before.

The form of (31) is quite similar to the equation (9) deduced by Veronis, and therefore the stability criteria summarized by Baines and Gill are also available in

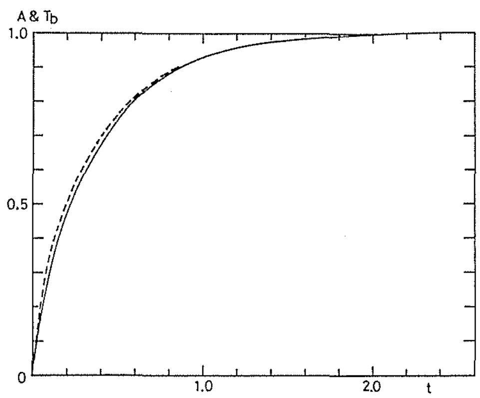

Fig. 14. Time variations of A (solid line) and T_b (broken line), T_b is the bottom temperature.

the present treatment, though the constant R_a must be replaced by the time-dependent function R_a^hA. Excepting in the neighbourhood of $R_s=0$ the stability criteria are represented as follows.

① $R_a^hA<\dfrac{P+\tau}{1+P}R_s+(1+\tau)\left(1+\dfrac{\tau}{P}\right)\dfrac{27}{4}\pi^4$: stable

② $R_a^hA=\dfrac{P+\tau}{1+P}R_s+(1+\tau)\left(1+\dfrac{\tau}{P}\right)\dfrac{27}{4}\pi^4$: neutral

③ $R_a^hA>\dfrac{P+\tau}{1+P}R_s+(1+\tau)\left(1+\dfrac{\tau}{P}\right)\dfrac{27}{4}\pi^4$: unstable (oscillating)

④ When Eq. (31) has at least one real root both oscillating and direct modes of instability are possible; this condition can be approximately written as $R_a^hA>R_s+0(R_s^{2/3})$ for large values of R_a^hA and R_s. Especially, when $R_a^hA\geqq\dfrac{P+\tau}{(P+1)\tau^2}R_s$ only the direct mode is possible.

(33)

When an isothermal salt water with a stable salinity gradient is continuously heated from below with a constant heating rate, the system will shift from situation ① shown in (33) to situations ②, ③, and then ④ in order with the lapse of time, because the time-dependent function A increases monotonically from $A=0(t=0)$ to $A=1(t\rightarrow\infty)$ as shown by the solid line in Fig. 14. Accordingly, if the system should become unstable at last, the convective instability must appear as an oscillating mode before everything. Additionally, in the system where the heating rate is not so large against the salinity gradient, situation ④ may not be attained and only the oscillating mode is possible.

When the convective motion has once occurred and developed due to the attainment of situation ③, distributions of temperature and salinity may deform their shapes, which has been formed by the molecular process till that time. Therefore, strictly speaking, criterion ④ is meaningless, because the assumptions of infinitesimal disturbance and linear salinity profile are not allowed. However, it is thought that the intensity of heating, in other words the magnitude of R_a^h, should control the development of convection in the system; that is whether the convection should end in the oscillating mode only or whether the convection should develop up to the direct mode, but we have only little information about it at present (refer to the section IV–5).

IV–3. **Some characteristics of the new criterion**

We will examine the difference between the existing stability criterion obtained by Veronis et al and the new criterion deduced by the present author. The dimensional temperature difference (ΔT) between the upper and the lower surfaces in every moment is expressed by the nondimensional bottom temperature T_b as,

$$\Delta T=\frac{F_H h}{\rho c\kappa}T_b\,. \tag{34}$$

Consequently, the common Rayleigh number R_a is represented by

$$R_a=\frac{\alpha g h^3}{\nu\kappa}\Delta T=\frac{\alpha g F_H h^4}{\rho c\nu\kappa^2}T_b=R_a^h T_b\,. \tag{35}$$

Thus, the left hand side in (33) is rewritten as.

$$R_a^h A=\frac{A}{T_b}R_a(\leqq R_a)\,. \tag{36}$$

because the relation $A/T_b\leqq 1$ is satisfied in every moment, as evident from the relation between A and T_b shown in Fig. 14; by the way, T_b is calculated from the first equation in (23) and (29). Therefore, when we examine the convective instability of the system in an experiment from the viewpoint of the temperature difference between the upper and the lower surfaces, the actual instability may be observed after the time when the expected difference predicted by the Veronis et al's theory has been attained. Let us carry out, for instance, an experiment in which the instability should be set at the nondimensional time $t=0.1$, when $A=0.3022$ and $T_b=0.357$. Then the stability based on ΔT may predict that instability will occur at the time when $T_b=0.3022$, i.e. $t=0.0725$. This difference is attributable to approximating the nonlinear profile of temperature to the linear.

If the time elapses sufficiently without any disturbances, then the vertical profile of temperature will become linear and the bottom temperature will tend to 1, i.e. the nondimensional to $F_H h/\rho c\kappa$, because of T_m's tending to zero as found from (29). This supposed state may produce the largest temperature difference between the upper and the lower surfaces under the given intensity of heating. However, when the system has a large value of R_a^h, enough to pass through the critical state of the onset of instability, such a supposed state never appears in reality. In this sense, the criterion of $t\rightarrow\infty$ (i.e. $A\rightarrow 1$) may be generally meaningless as a practical "stability". Nevertheless, it is a useful criterion for judging whether the system falls in the unstable range finally or whether the system remains in the stable. Thus, the criterion shown in (33) can be regarded as an expanded thermosolutal criterion including the existing stability; that is it gives the very stability of the system till getting to the critical state of the onset of instability, while that of $A=1$ tells the developmentof thermosolutal convection.

IV–4. The period of the oscillation at the marginal stability

When the thermosolutal system attains the critical state of the onset of instability, an oscillating convection appears. This is one of characteristic phenomena in the thermosolutal system belonging to the diffusive regime. Putting the real part of σ as zero and substituting the marginal relation between R_a^h and R_s (i.e. ② in (33)) into (31), the nondimensional period T^* is obtained as follows,

$$\left.\begin{aligned} &T^*=2\pi/\sigma_i\,, \\ &\sigma_i^2=\frac{a^2}{1+a^2}\frac{P}{1+P}R_s=\frac{1}{3}\frac{P}{1+P}R_s \qquad (\text{for } a^2=0.5)\,, \end{aligned}\right\} \tag{37}$$

where the effect of τ is ignored since τ is far smaller than P.

The corresponding dimensional period T is derived from the relation $T=T^*h^2/\kappa$ as follows,

$$\left.\begin{aligned} &T=2\pi/p_i\,, \\ &p_i^2=\frac{1}{3(1+P)}N_s^2\,, \\ &N_s^2=\beta g\frac{\Delta S}{h}\,, \end{aligned}\right\} \tag{38}$$

where N_s is the Brunt-Bäisälä frequency based on the salinity gradient only.

Such an oscillation must change the direction of flow in each convection cell time-dependently. Since the flow of fluid affects generally heat and solute transfers through the system, both heat and solute released through the upper surface of the fluid should also oscillate. Besides, it is considered that the direction of convective flow does not produce any effect on them, but only the magnitude does. As this oscillating convection has two peaks of the velocity (whose directions are reverse each other) and two zero-velocities during the period of one cycle, both transports of heat and solute have two maxima and two minima during the same period respectively. Consequently, it may be thought that the periods of heat and solute transfers show just half of the oscillating period of the convective motion itself, which will be confirmed by numerical experiments presented in the following section IV–5.

IV–5. Some information on finite amplitude thermosolutal convection in the system of small R_s

The stability theory described before brings no information about behaviour in the system after the onset of instability. This can be investigated by solving the coupled ordinary differential equations given in (25). As we have no analytical solutions for them because of their nonlinearity, we must integrate them numerically under a proper initial condition. In this section, some numerical results in relatively small values of R_a^h and R_s will be introduced. Constants used are follows; $P=7.1$, $\tau=0.01$ and $a=1/\sqrt{2}$.

Figs. 15, 16 and 17 show numerical results in cases of $R_a^h=2000$, 3000 and 4000 respectively where $R_s=1000$ always. Computations were started from the isothermal fluid (zero degree) with a stable and linear profile of salinity. For initial disturbances, all of θ_i's and s_i's were set as zero, while all of w_i's were set as small value of 0.1. The Fourier terms of each variable were truncated to 10. Nu and Nus are nondimensional heat and salt transports through the upper surface, respectively, T_b is the nondimensional temperature at the lower surface and w is the rms value of nondimensional vertical velocity at $x=0$, which is drawn referring to the change of flow direction; i.e. the positive and the negative values correspond to

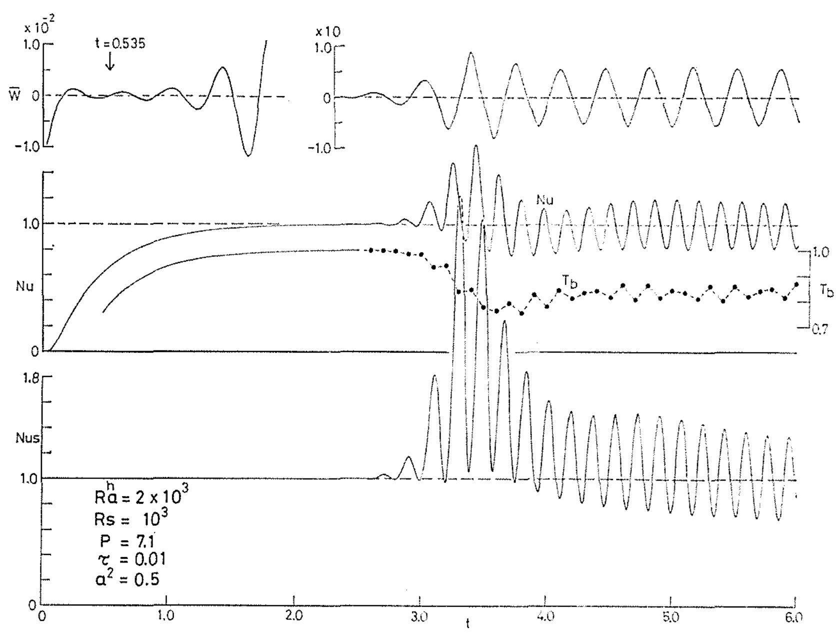

Fig. 15. The convecting behaviour of the thermosolutal system, $R_s=1000$, $R_a^h=2000$, $P=7.1$, $\tau=0.01$ and $a^2=0.5$. Numerical integration of Eq. (25) was started from the isothermal condition, and each Fourier series was truncated to 10 terms respectively. ($\bar{w}$: the rms value of the nondimensional vertical velocity, Nu: the nondimensional heat flow released through the upper surface, Nus: the nondimensional solute flow released through the upper surface, T_b: the nondimensional bottom temperature)

the upflow and the downflow respectively. Arrows in the figures indicate the time of the onset of the marginal instability predicted by the stability analysis. As evident from each figure, disturbance decreases at first; beyond the time marked by the arrow, it begins to increase gradually, and then gets to grow super-exponentially. Of course, it cannot grow without limit due to the restraining effect caused from the nonlinearity, and therefore the system comes to attain a steady motion as observed in each figure. Within the limits of these integrations, oscillating behaviours can be seen in all figures. However, their manifestations are rather different one from the other; i.e. in the case of $R_a^h=2000$, the flow direction reverses itself periodically, and the almost steady oscillation with the period of 0.36 (which agrees well with the period of 0.368 predicted by (37)) seems to continue forever, while in the case of $R_a^h=4000$, no inversions of flow direction can be seen besides the amplitude decreasing with the lapse of time after the motion has fully developed. After a sufficiently long time, the oscillating feature may probably vanish and a steady convection of the direct mode may develop. On the other hand, the oscillating feature in the case of $R_a^h=3000$ is rather complicated as shown in Fig. 16, which shows a combined feature of both aspects of oscillations illustrated above. Consequently, in the system of $R_s=1000$, it may be thought that the second critical state, shifting from ③ to ④ in (33), has a position between $R_a^h=3000$ and 4000.

Reflecting these oscillating behaviours, Nu, Nus and T_b also show oscillating

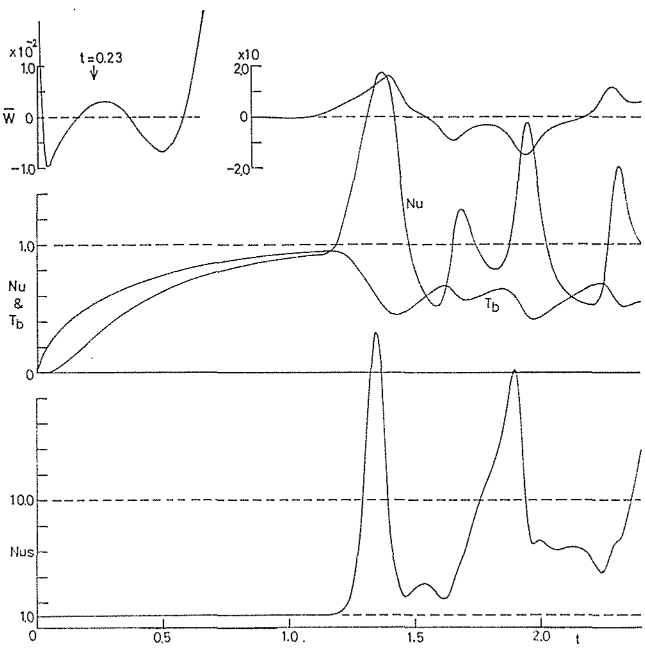

Fig. 16. The convecting behaviour of the thermosolutal system, $R_s=1000$, $R_a^h=3000$, $P=7.1$, $\tau=0.01$ and $a^2=0.5$. Numerical integration of Eq. (25) was started from the isothermal condition, and each Fourier series was truncated to 10 terms respectively. ($\bar{w}$: the rms value of the nondimensional vertcal velocity, Nu: the nondimensional heat flow released through the upper surface, Nus: the nondimensional solute flow released through the upper surface, T_b: the nondimensional bottom temperature)

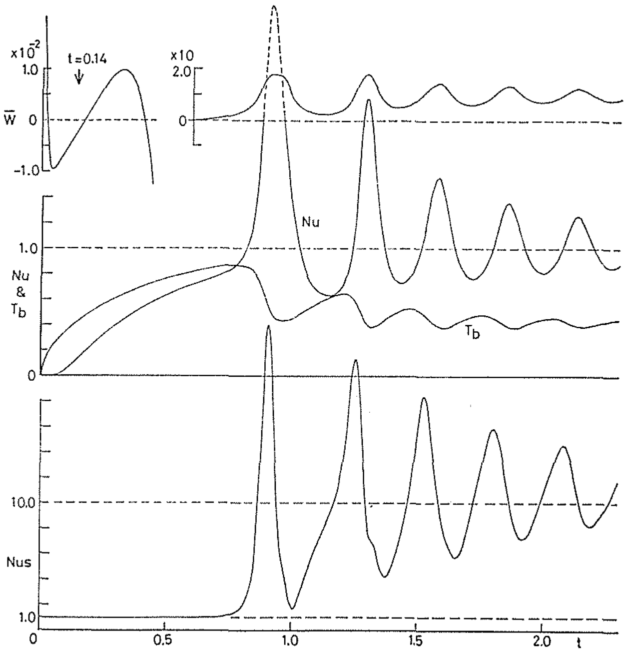

Fig. 17. The convecting behaviour of the thermosolutal system, $R_s=1000$, $R_a^h=4000$, $P=7.1$, $\tau=0.01$ and $a^2=0.5$. Numerical integration of Eq. (25) was started from the isothermal condition, and each Fourier series was truncated to 10 terms respectively. ($\overline{w}$: the rms value of the nondimensional vertical velocity, Nu: the nondimensional heat flow released through the upper surface, Nus: the nondimensional solute flow released through the upper surface, T_b: the nondimensional bottom temperature)

features respectively. It is noteworthy that all of them have just half the period of w in the case of $R_a^h=2000$ (though the period of T_b is not so clear due to the long interval of monitoring), while those in thecase of $R_a^h=4000$ have the same period as w.

Computations were carried out for some cases of larger value of R_b than 4000. The results are summarized in Table 3. Final modes of convective motion predicted based on numerical results are in the last column. Nu, not tabulated in the table, is 1 in every case, which corresponds to the nondimensional intensity of heating from below, i.e. those for $R_a^h=2000$ and 3000 oscillate around 1 forever, while those

Table 3. Numerical results by Eq. (25) under the condition of $R_s=1000$, $P=7.1$, $\tau=0.01$ and $a^2=0.5$. Each Fourier series was truncated to 10 terms, each

R_a^h	T_b	$R_a(=R_a^h T_b)$	$1/T_b$	Nus	remark
2000	0.84	1680	1.19	(1)	oscillating mode
3000	0.56	1680	1.79	(10)	oscillating mode
4000	0.42	1680	2.38	11.12	direct mode
5000	0.38	1900	2.63	11.28	direct mode
10000	0.30	3000	3.33	11.33	direct mode
20000	0.24	4800	4.17	11.49	direct mode

for $R_a^h=4000$ converge to 1 with oscillating around 1. The bottom temperature T_b increases at first due to the relation (28) until the convection fully develops. As the convection dominates, T_b falls due to the release of heat through the upper surface, and then it oscillates around a certain value as in the case of $R_a^h=2000$, or converges to a certain value with oscillation as in the case of $R_a^h=4000$. The values of T_b in the table are such temperatures, which decrease with the increase of R_a. Since the usual Rayleigh number R_a is expressed as $R_a=R_a^h T_b$, the constant value of $R_a=1680$ applys to every case of $R_a^h=2000$, 3000 and 4000. This means that the dimensional temperature becomes constant in the system of small value of R_a^h without regard to the intensity of heating.

Generally, the heat transport in a system is expressed by Nusselt number which is a ratio between the actual transport and the possible transport due to the molecular diffusion under the mean temperature gradient. Nusselt number in the latter sense corresponds to the reciprocal of the nondimensional bottom temperature, $1/T_b$, in the present system. The relationship between the heat transport and the thermal structure has been investigated by means of comparing Nusselt number with Rayleigh number. Therefore, the arrangement of data by such a conventional way leads to a curious result that a number of Nusselt numbers possibly exist for an identical Rayleigh number when the intensity of heating is not so large.

The salt transport *Nus* is an usual salt (solute) Nusselt number. In the case of $R_a^h=2000$, *Nus* is about 1, which means the actual transport is almost same as the molecular transport, while more than 10 times that of salt possibly transferred by the molecular diffusion can be transferred through the system of R_a^h of larger value than 3000.

Fig. 18 shows the numerical result for the case of $R_s=R_a^h=10000$, in which the integration was started from a linear profile of temperature for saving the computing time. As predicted by the stability analysis, this system has an oscillating convection with the inversion of flow direction. Characteristics of w, *Nu*, *Nus* and

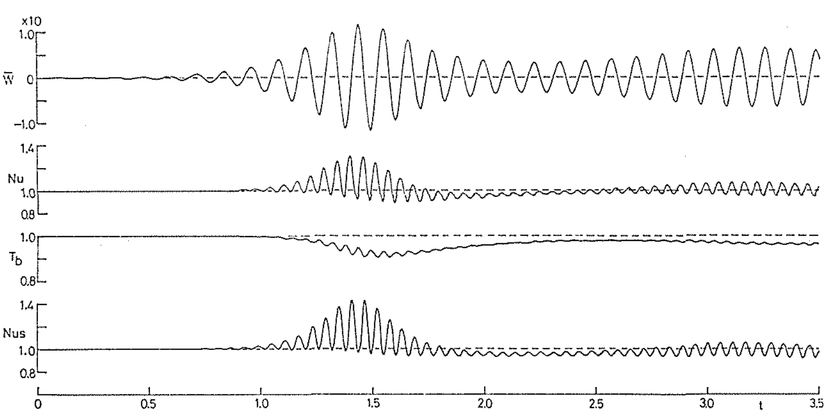

Fig. 18. The convecting behaviour of the thermosolutal system, $R_s=R_a^h=10{,}000$, $P=7.1$, $\tau=0.01$ and $a^2=0.5$. Numerical integration of Eq. (25) was started from the linear temperature profile, and each Fourier series was truncated to 10 terms respectively. ($\bar{w}$: the rms value of the nondimensional vertical velocity, *Nu*: the nondimensional heat flow released through the upper surface, *Nus*: the nondimensional solute flow released through the upper surface, T_b: the nondimensional bottom temperature)

T_b and relations among them are almost similar as in the case of $R_s=1000$ and $R_a^h=2000$, though it is noticeable that the amplitude changes itself time-dependently like "beat" (which is originally a periodic waxing and waning of the sound caused by combination of the two tones of very nearly the same frequency).

V. Analyses of actual thermosolutal systems by the new criterion

Introducing a constant heat flow for the thermal boundary condition at the lower surface has derived a new thermosolutal criterion, which is able to not only examine closely the convective stability of the system, but also judge whether a convection develops or not. In this section, we will apply it to some thermosolutal systems in antarctic saline lakes and examine its applicability.

As similarly done in section III, we will conveniently cut the system into thin slices of 50 or 100 *cm* thick vertically, and then apply the new criterion to each slice. On the occasion of analysing the thermosolutal condition, we will use the criterion of $A=1$, since the present purpose is to examine the development of convective motion in each slice. As each actual system in question has sufficiently large value of R_s and R_a^h in comparison with $(1+\tau)\left(1+\frac{\tau}{P}\right)\frac{27}{4}\pi^4$, the marginal criterion for the development of convection is expressed by the following simplified relation, which is derived from (33),

$$\frac{\alpha F_H h}{\rho c \kappa \beta \Delta S} > \frac{P+\tau}{1+P} (\doteqdot 0.9) , \qquad (39)$$

where $F_H h/\rho c\kappa$ in the left hand side corresponds to the supposed maximum temperature difference between the upper and the lower surfaces. When the left hand side is larger than the right hand side (about 0.9), the system is regarded as including convection in itself. Hereafter, we will tentatively name the left hand side of (39) "the flux-type criterion".

—In Lake Vanda— In Lake Vanda, as mentioned before, a thermosolutal system of the diffusive regime, being accompanied by typical step-like structures of temperature and salinity profiles, has been observed throughout almost the entire lake, excepting the uppermost part with the temperature lower than 4°C. A heat balance analysis and a thermal model study of the lake lead to a conclusion that the heat originates from solar heating (Yusa, 1972 and 1975b). Additionally, the approximate distribution of the water temperature has been almost unchangeable, excluding some changes in details, for these some dozen years. Consequently, it may be not so unreasonable to assume a thermal quasi-steady state for this lake, and then it may be allowed to consider that each thin slice is heated from below by the identical amount of heat with solar radiation absorbed by the entire water body lying below the level of the slice in question.

According to the above considerations, the upwards heat flow F_H across each slice can be approximately estimated as an annual mean by the following relation,

$$F_H = Q_0(e^{-\eta z} - e^{-\eta d}) , \qquad (40)$$

where $Q_0=2.0\times10^{-4} cal/cm^2 sec$, $\eta=3.4\times10^{-4}\ cm^{-1}$, $d=6100\ cm$ and z is the level of each slice measured from the lower surface of the lake ice.

The left figure of Fig. 7 shows the distribution of the flux-type criterion obtained for each thin slice, and the broken line corresponds to the critical value for development of convection. As seen from the outlined profile of water temperature, step-like structures are developing above the 47 meters level. On the other hand, values of flux-type criterion in the shallower part than the latter level are very near or larger than the critical value, while those in the deeper part than that level are very small. By the way, in the middle homogeneous zone, the flux-type criterion becomes infinity since $\Delta S=0$. These results lead to a possible conclusion that the convection may occur or develop in the upper portion above the 47 meters level, while the stratification in the deeper part is quite stable where the molecular process may dominate.

Finally, let us examine the flux-type criterion of each step-like structure. Hollow circles in Fig. 8 are relations between $\alpha F_H h/\rho c\kappa$ and $\beta \Delta S$ obtained including isothermal and isosolutal layer, while those in Fig. 9 are obtained from sheets with steep gradients of temperature and salinity (refer to Figs. 10 (a), (b)). The former are situated near the critical line or in the convective zone far up from it, while the latter fall in the stable zone beneath the critical line excepting only one. Thus, it is concluded that homogeneous layers must be in the convection state, and the sheets must be in the stable state thermosolutally.

These results provide bases for the physical meaning of the apparent diffusivity in the step-like structure zone described in section II and for the macroscopic thermal model of this lake shown in Fig. 6.

—In Lake Miers— Bell (1967) studied thermal characteristics of Lake Miers, and found that it was also balanced thermally with solar radiation. According to his study, the upwards heat flow F_H of this lake is calculated by the following relation,

$$\left.\begin{aligned} F_H &= Q_0 e^{-\eta_1 z} - Q_c\,, \qquad (0 \leqq z \leqq d_1)\,, \\ &= Q_0 e^{-\eta_1 d_1} e^{-\eta_2 (z-d_1)} - Q_c\,, \qquad (d_1 \leqq z \leqq d_2)\,, \\ Q_c &= Q_0 e^{-\eta_1 d_1} e^{-\eta_2 (d_2-d_1)}\,, \end{aligned}\right\} \tag{41}$$

where z is the distance measured from the lower surface of the lake ice, $d_1(=1200$ *cm*) is the thickness of the upper zone with the extinction coefficient $\eta_1(=0.7$ or 1.0×10^{-3} $cm^{-1})$, $d_2(=1400$ *cm*) is the level of the maximum temperature, $\eta_2(=10\times 10^{-3}$ $cm^{-1})$ is the extinction coefficient of the deeper water and Q_0 is the intensity of solar radiation parst the level of $z=0$ which is estimated as 500–800 *cal/cm² year*, i.e. 1.58–2.53×10^{-5} *cal/cm² sec*, based on the thermal data observed by Bell and by Torii et al (1967). Additionally, Q_c is the amount of solar radiation absorbed by the deepest water and the bottom sediment, which is supposed to flow out of the lake through the bottom.

The right figure of Fig. 11 shows the possible distribution of the flux-type criterion obtained for thin slices of 100 *cm* thick. Although various values of the criterion are possible for each slice depending on various combinations among η_1, η_2 and Q_0, they fall in a certain range as shown in the figure. Evidently, every value in the step-like structure zone ranges around the critical value. Therefore, it may be concluded that the latter zone is under the possible condition of the onset and/or the development of thermosolutal convection.

—In Lake Bonney— As mentioned before, Lake Bonney is thermally balanced with the solar radiation, and the upwards heat flow F_H is calculated by the following relation,

$$F_H = Q_0(e^{-\eta z} - e^{-\eta z_c}), \tag{42}$$

where z is the distance measured from the lower surface of the lake ice, $Q_0 = 3.35 \times 10^{-5}$ *cal/cm² sec*, $\eta = 1.2 \times 10^{-3}$ cm^{-1} and $z_c = 1050$ *cm* which is the level of the maximum temperature. Hollow circles in Fig. 12 show the plot of the flux-type criterion. Though they are somewhat larger than the stability $\alpha \Delta T/\beta \Delta S$ represented by solid circles, all of them are far smaller than the critical value. Therefore, it is concluded that the water in Lake Bonney is stably stratified and thermosolutal instability may hardly set because of the strong salinity gradient and the small amount of penetrating radiation, and that heat and salt transfers are mainly controlled by the molecular process.

—Under the ice island T-3— As mentioned in section III, the stability analyses based on the observed profiles of temperature and salinity lead to a conclusion that the stratification under the ice island T-3 is thermosolutally stable. Nevertheless, distinctive step-like structures appear to persist.

Now, the heat transferred upwards through the step-like structure zone may be comparable at least with that transferred across the sheet by molecular conduction, which is estimated at 2.6×10^{-6} *cal/cm² sec*. Regarding this as F_H, the flux-type criterion is calculated as 2.4 for the zone from 240 to 297 meters deep, and as 5.4 for the zone from 297 to 340 meters deep. Thus, it may be concluded that thermosolutal convection is developing in this system.

The existing stability, as treated in section III, has led to a conlcusion that all thermosolutal systems examined in this paper are stably stratified without any convective motions irrespective of the step-like structures. On the contrary, the new thermosolutal criterion has made the following judgement; the step-like structure zones include convective motions in themselves, while the zone with smooth profiles of temperature and salinity are stably stratified. The latter can give reasonable bases to the physical meaning of the apparent diffusivity in the step-like structure zone of Lake Vanda and to the macroscopic thermal models of the antarctic saline lakes. These results distinctively suggest that it is unreasonable to analyse the actual thermosolutal system on the basis of the temperature distribution, which ought to be recognized as the final sequence produced through various processes, and that it becomes possible to make a reasonable interpretation of the actual thermosolutal system just based on the primary effect, such as the heating by solar radiation or the heating from below.

VI. An example of the oscillating thermosolutal convection

When salt water with a stable salinity gradient is heated from below, the thermosolutal convection occurring at first shows an oscillating mode as predicted by the stability analysis. In the case where both magnitudes of the thremal parameter R_a^h and the solutal parameter R_s are identical with each other, the oscillation will persist steadily. Such a mode has been observed in the laboratory (Shirtcliffe, 1967; Turner, 1968), and has been presented in this paper by numerical experiment (refer to section IV–5).

As a link in the chain of the heat balance study of Lake Vanda, the continuous recording of penetrating radiation through the lake ice was conducted during from December 28th to 31st of 1971 with a thermo-junction type heat flux meter, which was set just below the lake ice and showed a high sensitivity to solar radiation. A part of the record obtained is shown in Fig. 19, whose pattern is similar to the solar radiation chart obtained at the ground surface. However, it is conspicuous that a small oscillation with short period is always superposed on the record, though a disturbance around 14 to 15 o'clock took place due to floating clouds. Those periods are distributed in the range of from 9 to 15 minutes, but the most predominant period is 12 minutes. Such an oscillation is quite peculiar to the record obtained under the lake ice. As often mentioned before, the water temperature in this lake increases downwards, which maintains the upwards heat flow through the lake water. Consequently, we cannot help but to consider that the small oscillation in Fig. 19 is the very oscillation of the upwards heat flow.

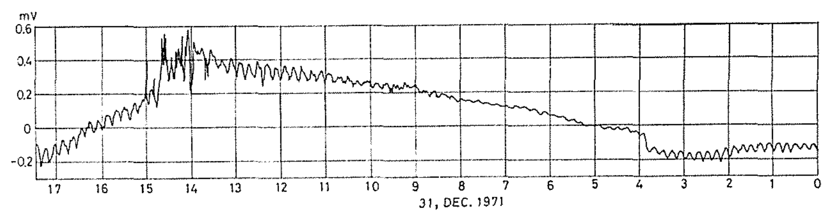

Fig. 19. An example of heat flux chart at the lower surface of the lake ice in Lake Vanda, December 31, 1971.

Table 4. Thermosolutal data and calculated value of flux-type criterion and oscillating period in the uppermost diffusive regime of Lake Vanda during December, 1971

α (1/°C)	0.81×10^{-5}
β (*cm*/μ℧)	4.24×10^{-7}
F_H (*cal*/*cm*^{2} *sec*)	1.37×10^{-4}
$\rho c\kappa$ (*cal*/*cm sec* °C)	1.36×10^{-3}
$\Delta S/h$ (μ℧/*cm*/*cm*)	1.5–2.4
$\alpha F_H h/\rho c\kappa\beta\Delta S$	0.80–1.27
T (*min*)	19.8–26.2

The stratification around the heat flux meter is gravitationally stable, because the water temperature around it is below 4°C and besides the salinity increases downwards. However, as the water temperature rises beyond 4°C at the level of about 25 *cm* below the lake ice, the thermosolutal stratification of the diffusive regime is attained. We will direct our attention to the range of from 25 to 50 *cm* deep below the lake ice; the thermosolutal data of which is tabulated in Table 4, where the upwards heat flow F_H has been obtained from the amount of melting at the lower surface of the lake ice. Based on these date, the flux-type criterion in this range is calculated as 0.80 to 1.27, which is a possible value for the development of the oscillating thermosolutal convection. The oscillating period may be close upon 19.8 to 26.2 minutes as predicted by the relation (38), and therefore the accompanying oscillation of heat flow may be the half period of the convection itself suggested and comfirmed in sections IV–4 and IV–5; that is from 9.9 to 13.1 minutes. The period of the small oscillation seen in Fig. 19, 12 minutes, agrees well with the latter result. Referring to charts obtained on other days, the amplitude changes time-dependently, which resembles the aspect of the beat-like oscillation shown in Fig. 18. Such a resemblance seems to also suggest that the oscillation in Fig. 19 results from the thermosolutal convection of the oscillating overstable mode.

VII. Thermosolutal convection in the Antarctic saline lake—especially focusing on Lake Vanda—

Solute concentration in each saline lake in Victoria Land generally increases downward, and the isosolutal line is almost flat horizontally. Based on such an observation, the following speculation on the origin of the salinity profile in the lake has been presented (Wilson, 1964).

When the climatic change took place at some time in the past, a large quantity of glacial melting water would have flowed on top of a highly concentrated solution or a dried up salt sediment. Since that time the solute has been diffusing upwards.

For example, the salinity profile in the deepest part of Lake Vanda can be explained by the foregoing opinion. According to recent salinity data, it has been deduced that such an inflow occurred about 900 years ago. Unfortunately, we have no data in addition to this enough to discuss what climatic change could have happened all over this ice-free area at that time. Such a problem is beyond the present study, and should be studied in relation to the climatic history of the Antarctic region in the future.

Anyway, since that time when a large quantity of inflow occurred, the lake has been continuously heated by solar radiation. Along the way, a thermosolutal system of the diffusive regime would have been established in the lake. Then, it is understood that the condition beyond the marginal state for the onset of convective instability would have been attained in some parts of the lake, and that thermosolutal convection would have occurred and developed. If no convection occurred, Lake Vanda with its present depth would have an extraordinarily high temperature beyond 100°C (of course, such can never be), and it would take around a thousand years until a thermal quasi-steady state would have been accomplished. In the shallower part of this lake, however, it seems that the convection had occurred in the rather earlier stage of solar heating because of the low salinity and its small gradient (Yusa, 1975a). This means that the mean diffusivity over the whole lake had shifted from the molecular level to the higher level. Owing to this, the water temperature cannot rise so high. Assuming that the present lake level and the present convecting zone have been maintained, it is expected that maximum temperature of about 23°C must appear near the lake bottom in the steady state, which agrees well with the observation (Yusa, 1975b; see Fig. 6 in this paper). In this case, such a steady state may have been attained sufficiently after five hundreds years or so. If so, the foregoing assumption of a thermal quasi-steady state is reasonable for Lake Vanda.

As clarified through the present treatment, the onset or the development of convection in a thermosolutal system is determined by the combined effect of the salinity gradient and the intensity of heating. In the deeper part of Lake Vanda or in Lake Bonney, no convection can occur at the present time because of the small amount of heating due to the absorption of penetrating radiation in comparison with the strong salinity gradient. Since the salinity and its gradient at a certain depth are formed essentially by the diffusion from the bottom as mentioned before, they will change in the future with the progress of the diffusion. Depending on this, the value of the flux-type criterion at each depth will also change, and consequently the possible zone for the onset of convection may change. This means the change in the distribution of eddy diffusivity in a broad sense, which affects the water temperature and the salinity as well as their distributions. As to the deeper part of Lake Vanda where the salinity tends to decrease, the convection zone will stretch downwards from now on, and therefore the water temperature has a tendency to lowering. Supposing the unchangeable lake level and solar radiation, however, it seems that it would take more than 10 thousands years roughly for the attainment of a gentle salinity gradient to raise a convective instability even below 47 meters deep as shown in Fig. 20, which shows a rough prediction of changes in the salinity profile and the flux-type criterion in the deeper part of Lake Vanda. Accordingly, as long as the present lake level is maintained, a general shape of the temperature distribution may last extending over a fairly long period for the future.

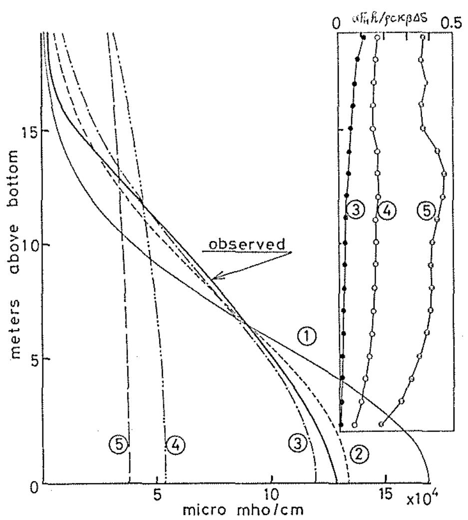

Fig. 20. The calculated profiles of salinity with the observation data, and the predicted change of $\alpha F_H h/\rho c\kappa\beta\Delta S$.
①: 500 years, ②: 800 years, ③: 1,000 years, ④: 5,000 years, ⑤: 10,000 years.

At Lake Vanda, numerous ancient shorelines are visible around the lake, which suggest long-term fluctuation in the lake level. Of course, it is not difficult to suppose that the lake level at some time in the past might be lower than the present one. In fact, the lake level has been having a tendency to rise since 1964. Such a fluctuation in the lake level changes the amount of heating due to solar radiation at each level, i.e. the value of F_H, and therefore it becomes a significant factor affecting the value of the flux-type criterion at each level.

In the deeper (non-convecting) part of Lake Vanda, the change in the salinity profile and the accompanying change in the flux-type criterion are extremely slow as mentioned before. Therefore, the development of the thermosolutal convection zone in the future may be influenced more significantly by the fluctuation of the lake level (lowering in Lake Vanda) than by the change in the salinity profile.

Thus, it is thought that the thermosolutal structures in Antarctic saline lakes have been formed and will be changing on due to the physical process of the onset of thermosolutal convection and due to the fluctuation of the lake level, or climatic change. Especially focusing on Lake Vanda, up to the present the formation process of the thermosolutal structure has been subject to the influence of the former, rather than otherwise, which is determined by the combined effect between the diffusion rate of salt and the heating rate by solar radiation; while in the future it will be influenced rather by climatic change, which clearly appears as the lake level fluctuation.

VIII. **Conclusion**

The results may be summarized as follows;—

A series of thermal phenomena observed in saline lakes in Victoria Land of Antarctica, i.e. ① the unusually high water temperature in comparison with the surrounding air temperature, ② the existence of the step-like structure in the temperature and salinity profile and ③ the oscillation of heat flow through the lake water etc., are resulting from the solar heating of the saline waters with stable salinity gradients.

This result has been deduced from the theory of the thermosolutal convection. During analyses, it was clarified that the existing theory based on the temperature distribution involves some limits in applying itself to the actual thermosolutal system, and so introducing the heat flow produced by heating, which is primary as a thermal property, into the theory was tried. According to this, we have derived a new thermosolutal criterion which has a wide range of applicability including the existing stability criterion in itself, and is useful for interpreting the actual thermosolutal system.

By the help of the present treatment and the new criterion some thermosolutal characteristics in each saline lake in Victoria Land has been clarified. Notwithstanding, the formation process of multi-layered structure accompanying the thermosolutal convection has not been explained yet, and is left as a subject to be studied in the future.

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