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A Note on Reflecting Power of Rocks

By

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Abstract

In this paper is firstly described the importance of reflecting power of rocks. A formula giving reflecting power of rocks is then derived from that giving reflecting power of absorbing media by putting $\sigma/\nu=0$. Our formula thus derived gives fair agreements between observed and calculated reflecting powers for granite and basalt specimens in Japan with Na-D line and with angle of incidence equal to 45°. Drude's formula for metallic reflection transformed to the case of $\sigma/\nu=0$ produces discrepancies between observed and calculated values considerably greater than those by our formula.

Introduction

Knowledge of reflecting power of rocks for solar radiation is important from the following several considerations.

1) Reflecting power of rocks for solar radiation combined with their emissivity, specific heat and thermal conductivity will control diurnal and annual changes in surface temperature of bare outcrop of rocks situated at the surfaces not only of our earth but also of the moon and very possibly of other planets near the sun. The diurnal and annual changes of surface temperature combined with thermal diffusivity give rise to the similarly periodic changes of temperature at points beneath the bare outcrop of rocks and a non-uniform distribution of the temperature with respect to depth is produced.

2) Such a non-uniform distribution of temperature developed inside the surface layer beneath outcrop of rocks will produce thermal stresses in the layer. In high mountains and in deserts, where the temperature fluctuation is large, tensile stresses at lowest temperature are sometimes sufficient to produce cracks in rocks.[1] This phenomenon is likely occuring easily at the moon's surface as the diurnal heating and cooling is tremendous; rock flakes

will leave the cracks thus developed and will deposit on relatively lower levels of the lunar surface.

3) Heating of stone walls of stony buildings by absorption of direct solar radiation and of solar radiation reflected from ground surface planted with no grass and heating of rooms by letting in solar radiation reflected from the same ground surface cannot be studied without knowledge of the reflecting power of rocks. It is worth noting that any one of the pencils in diverse directions of diffused solar radiation from the bare ground surface is of a regular reflection with a definite angle of incidence at a point, where this pencil comes from, of the surface of one of numerous sand grains consisting the ground surface.

4) If the surface layer of the rocks contains ulvöspinel-rich titano-magnetites and ilmenite-rich hematites whose Curie points are different with one another in the range of the ordinary temperatures, the diurnal and annual changes of temperature inside the layer should cause the similar periodic changes in the remanet magnetization of the layer; and with this phenomenon the problem of stability of magnetization of rocks may have a close relationship. We may, therefore, expect that differences would be found in magnetization between rock specimens from outcrops sufficiently exposed to solar radiation and those insufficiently done, and not only this but also that the magnetization in *situ* at a given place of the outcrop would unnergo daily and seasonal variations.

5) The most fascinating will be a utilization of the reflecting power of terrestrial rocks for inferring the kinds of rocks which build different parts of the lunar surface, especially whether the rocks forming the bright area are different from those doing the dark.

From the above points of view, we have long been enthusiastic in the determination of reflecting power of various kinds of terrestrial rocks for light in terms of its wave-length and also for total solar radiation. The present paper is the first report of our work.

Reflecting power of absorbing media

In order to derive a formula of reflecting power of rocks, let us make use of the formula of reflecting power of absorbing media which will be derived from that for transparent.

If we assume the incident light consisting of the ordinary light falling upon a boundary of two media to be composed of plane polarized lights with their planes of polarization in all directions, then these lights can be resolved into two kinds of plane polarized light of equal intensity, one polalized in and the other perpendicular to the plane of incidence. If we denote by $r_{//}$ and $r_{\perp}$ respectively the amplitude of the reflected light of the incident one polarized in and that perpendicular to the plane of incidence and if the amplitudes of these two incident lights be taken as unity, then the reflecting power denoted by r^2 of any medium is given by the well-known formula[2),3)].

$$r^2=\frac{1}{2}(r_{//}^2+r_{\perp}^2). \tag{1}$$

According to Fresnel, $r_{//}$ and $r_{\perp}$ for transparent media are calculated by

$$r_{//}=\frac{n_2\cos\theta_e-n_1\cos\theta_d}{n_2\cos\theta_e+n_1\cos\theta_d} \quad and \quad r_{\perp}=\frac{n_1\cos\theta_e-n_2\cos\theta_d}{n_1\cos\theta_e+n_2\cos\theta_d}\,, \tag{2}$$

where θ_e and θ_d are respectively angles of incidence and refraction, n_1 and n_2 indices of refraction respectively of medium I and II, and the incident light is assumed to travel through the medium I.

If we take absorbing media, Maxwell's electro-magnetic theory of light for such media introduces the following relations ;

$$n^2(1-\varkappa^2)=\varepsilon, \quad 2n^2\varkappa=\frac{\sigma}{\nu}\,. \tag{3), (4}$$

In the above formulas, n is index of refraction, $\varkappa$ coefficient of extinction, ε dielectric canstant, σ electric conductivity of the medium under consideration and ν frequency of light. The equation (3) and (4) are derived from $n(1-\sqrt{-1}\,\varkappa)=\sqrt{\varepsilon'}$ where ε' is complex dielectric constant corresponding to real ε and given by $\varepsilon'=\varepsilon-\sqrt{-1}\,\sigma/\nu$.

Further, we have the following well-known law of refraction for absorbing media,

$$\sin\chi=\frac{\sin\theta}{\sqrt{\varepsilon'}}=\frac{\sin\theta}{n\,(1-\sqrt{-1}\varkappa)}\,, \tag{5}$$

where χ and θ stand respectively for θ_d and θ_e in equation (2). Taking the medium I to be air and II an absorbing medium, we have to put $n_1=1$ and $n_2=n(1-\sqrt{-1}\,\varkappa)$. Substituting these in equation (2), we get

$$r_{//}=\frac{n\,(1-\sqrt{-1}\,\varkappa)\cos\theta-\cos\chi}{n\,(1-\sqrt{-1}\,\varkappa)\cos\theta+\cos\chi} \text{ and } r_{\perp}=\frac{\cos\theta-n\,(1-\sqrt{-1}\,\varkappa)\cos\chi}{\cos\theta+n\,(1-\sqrt{-1}\,\varkappa)\cos\chi}\,. \tag{6}$$

Replacing $\cos\chi$ in equation (6) by a quantity expressed in terms of $\sin\theta$ and $n\,(1-\sqrt{-1}\varkappa)$ by means of equation (5), we obtain from equation (1) and the thus rewritten equation (6)

$$r^2=\frac{1}{2}\left[\left\{\frac{n^2(1-\sqrt{-1}\,\varkappa)^2\cos\theta-\sqrt{n^2(1-\sqrt{-1}\,\varkappa)^2-\sin^2\theta}}{n^2(1-\sqrt{-1}\,\varkappa)^2\cos\theta+\sqrt{n^2(1-\sqrt{-1}\,\varkappa)^2-\sin^2\theta}}\right\}^2+\left\{\frac{\cos\theta-\sqrt{n^2(1-\sqrt{-1}\,\varkappa)^2-\sin^2\theta}}{\cos\theta+\sqrt{n^2(1-\sqrt{-1}\,\varkappa)^2-\sin^2\theta}}\right\}^2\right]. \quad (7)$$

Reflecting power of rocks

For rocks electric conductivity σ is very small as compared with frequancy ν and therefore we may put $\sigma/\nu=0$ excepting extremely low values of ν. Therefore, as $n>1$, equations (4) and (3) give us $\varkappa=0$ and $n^2=\varepsilon$. Hence, pulling $\varkappa=\theta$ in equation (7), the reflecting power of rocks is given by

$$r^2=\frac{1}{2}\left[\left\{\frac{n^2\cos\theta-\sqrt{n^2-\sin^2\theta}}{n^2\cos\theta+\sqrt{n^2-\sin^2\theta}}\right\}^2+\left\{\frac{\cos\theta-\sqrt{n^2-\sin^2\theta}}{\cos\theta+\sqrt{n^2-\sin^2\theta}}\right\}^2\right]. \quad (8)$$

This formula coincides with what we can derive from equations (1) and (2) by putting $n_1=1$ and $n_2=n$. Transforming equation (8) into a form convenient to compute the value of reflecting power, we get,

$$r^2=1-\left\{\frac{1}{1+\dfrac{(n^4+1)\cos^2\theta+n^2-1}{2n^2\cos\theta\sqrt{n^2-\sin^2\theta}}}-\frac{1}{1+\dfrac{2\cos^2\theta+n^2-1}{2\cos\theta\sqrt{n^2-\sin^2\theta}}}\right\}. \quad (9)$$

By means of this formula, we can compute values of reflecting power r^2 of various rocks when their values of indices (average) of refraction n and incident angle θ are given.

Relation between dielectric constant and reflecting power for $\theta=0^\circ$

Th. Liebisch and H. Rubens obtained the following formulas expressing the relation between dielectric constant ε of minerals[4] and their reflecting power for $\theta=0^\circ$, which they denote also by r^2:

$$r^2=\frac{(\sqrt{\varepsilon}-1)^2+\varkappa^2}{(\sqrt{\varepsilon}+1)^2+\varkappa^2}. \quad (10)$$

For an ultra-short electric wave of wave-length of 45 cm ($\nu=6.7\times10^8$), W. Schmidt[5] experimentally proved that $\varkappa$ in equation (10) could be neglected and proposed for such a wave the following formula:

$$r_\infty^2=\left(\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}\right)^2. \quad (11)$$

Th. Liebisch and H. Rubens[6] experimentally proved that for carbonate minerals (Calcite, Dolomite, Aragonite, Cerussite) and sulphate minerals (Barite, Celestine, Anglesite), equation (11) was also applicable for an infra-red wave of wave-length of 3×10^6 Å ($\nu=10^{12}$). As obtained in the precading page we have $n^2=\varepsilon$ for rocks and therefore from equation (8) we obtain for them the following formula for $\theta=0^\circ$:

$$r^2=\left(\frac{n-1}{n+1}\right)^2. \tag{12}$$

This equation is theoretically independent of ν excepting so extremely low values of it that the equality of $\varkappa(=\sigma/2n^2\nu)=0$ becomes erroneous (cf. (4)). Equation (12) coincides with W. Schmidt's formula (11), if we employ the equality $n^2=\varepsilon$; and his experimental results mentioned above suggest that validity of $\varkappa=0$ for rocks holds down to the low values of ν as low as 6.7×10^8.

Change of reflecting power with incident angle

Table I. Change of reflecting power with incident angle calculated for $n=1.500$

Incident angle θ	Reflecting power: r^2 (Formula (9))	Reflecting power: r^2 for $n=1.500$
90°	1	1.000 (100.00%)
75°	$1-\left\{\dfrac{1}{1+\dfrac{(2-\sqrt{3})n^4+4n^2-(2+\sqrt{3})}{(\sqrt{6}-\sqrt{2})n^2\sqrt{4n^2-(2+\sqrt{3})}}}+\dfrac{1}{1+\dfrac{4n^2-2\sqrt{3}}{(\sqrt{6}-\sqrt{2})\sqrt{4n^2-(2+\sqrt{3})}}}\right\}$	0.2531 (25.31 %)
60°	$1-\left\{\dfrac{1}{1+\dfrac{n^4+4n^2-3}{2n^2\sqrt{4n^2-3}}}+\dfrac{1}{1+\dfrac{2n^2-1}{\sqrt{4n^2-3}}}\right\}$	0.0893 (8.93%)
45°	$1-\left\{\dfrac{1}{1+\dfrac{n^4+2n^2-1}{2n^2\sqrt{2n^2-1}}}+\dfrac{1}{1+\dfrac{n^2}{\sqrt{2n^2-1}}}\right\}$	0.0504 (5.04%)
30°	$1-\left\{\dfrac{1}{1+\dfrac{3n^4+4n^2-1}{2\sqrt{3}n^2\sqrt{4n^2-1}}}-\dfrac{1}{1+\dfrac{2n^2+1}{\sqrt{3}\sqrt{4n^2-1}}}\right\}$	0.0417 (4.17%)
15°	$1-\left\{\dfrac{1}{1+\dfrac{(2+\sqrt{3})n^4+4n^2-(2-\sqrt{3})}{(\sqrt{6}+\sqrt{2})n^2\sqrt{4n^2-(2-\sqrt{3})}}}+\dfrac{1}{1+\dfrac{4n^2+2\sqrt{3}}{(\sqrt{6}+\sqrt{2})\sqrt{4n^2-(2-\sqrt{3})}}}\right\}$	0.0402 (4.02%)
0°	$\left(\dfrac{n-1}{n+1}\right)^2$	0.0400 (4.00%)

Change of reflecting power with incident angle calculated by equation (9) with $n=1.500$ is given in Table I. A graph representing this change is shown in Fig. I.

On Drude's formula

There is well-known Drude's formula[7] for metallic reflection, and this formula is

$$r^2=\frac{1}{2}\left\{\frac{\left(n-\dfrac{1}{\cos\theta}\right)^2+n^2\varkappa^2}{\left(n+\dfrac{1}{\cos\theta}\right)^2+n^2\varkappa^2}+\frac{(n-\cos\theta)^2+n^2\varkappa^2}{(n+\cos\theta)^2+n^2\varkappa^2}\right\}, \quad (13)$$

which Drude has derived from equation (7) by expanding $[n^2(1-\sqrt{-1}\,\varkappa)^2-\sin^2\theta]^{1/2}$ and its reciprocal in powers of $\sin^2\theta$ and the neglecting the third and higher terms.

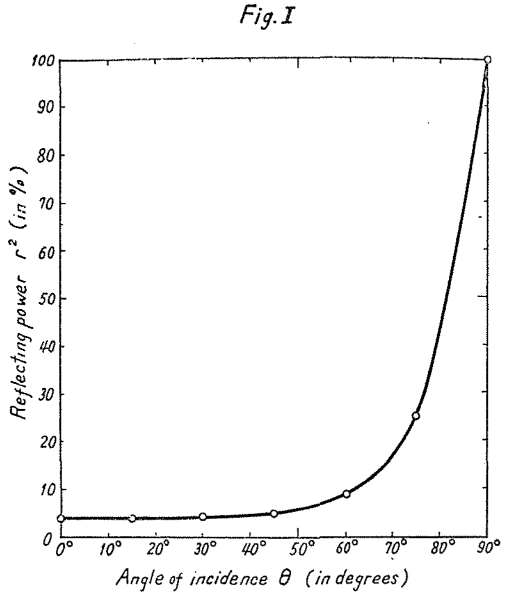

Fig. 1. Change of reflecting power with incident angle obtained by calculation with $n=1.500$

If we intend to use equantion (13) for rocks for which we have assumed $\varkappa=0$ as already mentioned, equation (13) degenerates to

$$r^2=1-2n\cos\theta\left\{\frac{1}{(n\cos\theta+1)^2}+\frac{1}{(n+\cos\theta)^2}\right\}. \quad (14)$$

Average indices of refraction of rocks for Na–D line

Average index of refraction of a rock may be given by a weighted mean of the indices of refractions of the norm-component minerals derived from chemical analysis of this rock.

If indices of refraction of all the norm-component minerals of a rock are denoted by respectively $n_1, n_2, \ldots, n_l$ and their weight percentages by $m_1, m_2, \ldots, m_l$, then the average index of refraction of this rock may be given by the following formula:

$$n=\frac{n_1 m_1+n_2 m_2+\cdots+n_l m_l}{m_1+m_2+\cdots+m_l}. \tag{15}$$

In Table II are shown indices of refraction of several types of igneous rocks in Japan calculated by (15) from their chemical analyses[8),9)].

Table II. Average indices of refraction for Na–D line of several igneous rocks in Japan

Rocks	Average index of refraction n for Na–D line (5893 Å)	Number of analyses	Range	Index of refraction of the specimens whose reflecting powers for $\theta=45°$ are measured
Granite	1.538	3	1.534～1.544	1.537
Rhyolite	1.537	4	1.526～1.564	——
Dacite	1.545	2	1.507～1.583	——
Trachyte	1.492	2	1.489～1.494	——
Andesite	1.488	9	1.411～1.536	——
Basalt	1.463	4	1.441～1.486	1.459
Dolerite	1.468	1	——	——

The numerals in the last column of Table I are the indices of refraction calculated for a granite specimen from Kameoka, Kyoto prefecture, Japan and for a basalt specimen from Genbudo, Hyogo prefecture, Japan, whose reflecting powers for $\theta=45°$ have already been determined by us.

By taking into consideration densities of norm-component minerals, the authors have calculated average indices of refraction by volume percentages of these minerals. The results indicate that the average index by weight percentages is less than that by volume at most by 0.004.

Calculations of reflecting powers for $\theta=0°$ and the corresponding dielectric constants of several igneous rocks

By equation (12) in which n is put equal to the average indices of refraction shown in the second column of Table II, reflecting powers for $\theta=0°$ and for Na–D line are calculated and dielectric constants are also done by $\varepsilon=n^2$ for the same line and the results are shown in Table III.

Comparison between observed and calculated reflecting powers for $\theta=45°$ and for Na–D line of granite and basalt specimens

Reflecting powers for $\theta=45°$ and for Na–D line of a granite specimen

Table III. Calculated values of reflecting powers for $\theta=0°$ and for Na-D line and dielectric constants for the same line of several igneous rocks in Japan

Rocks	Average index of refraction n for Na-D line (5893 Å)	Reflecting power for $\theta=0°$ in %	Dielectric constant ($\varepsilon=n^2$)
Granite	1.538	4.49	2.37
Rhyolite	1.537	4.48	2.36
Dacite	1.545	4.58	2.39
Trachyte	1.492	3.90	2.23
Andesite	1.488	3.84	2 21
Basalt	1.463	3.53	2.14
Dolerite	1.468	3.59	2.16

from Kameoka and also of a basalt specimen from Genbundo have been calculated by two formulas, one being our formula (9) (the formula for $\theta=45°$ in Table I) and the other Drude's (the formula (14)) and the results are compared with the values observed by the present authors[10] as shown in Table IV.

Table IV. Comparison between observed and calculated reflecting powers for $\theta=45°$ and for Na-D line

Rocks	Observed value in %	Refractive indiex n	Calculated values			
			By formula (9)	Obs.-Calc.	By Drude's formula with $\varkappa=0$ (14)	Obs.-Calc.
Granite	5.9	1.537	5.54	+0.4	6.94	−1.0
Basalt	4.1	1.459	4.46	−0.4	5.98	−1.9

From Table IV, it can be seen that for our rock specimens our formula (9) derived from Fresnel's formula (2) gives an agreement between observed and calculated values within errors of ±10 % of the observed values, while Drude's formula produces values 17 to 46 % systematically greater than the observed. Such a great disagreement produced by Drude's formula will be due to the approximation already mentioned by which this formula is derived.

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