

Temperature of Granite Magma of Kitashirakawa-Granite,
Kyoto, Japan, as Estimated from a Heat Conduction
Theory of Formation of Wollastonite

By

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Abstract

By Kitashirakawa-Granite intruded between Mt. Daimonji and Mt. Hiei, is metamorphosed the Palaeozoic formation at Mt. Daimonji to produce a contact mineral of wollastonite in lenticular limestone in the Palaeozoic formation at Gobessho east of Mt. Daimonji. In this paper, the question of the temperature of granite magma required to produce the wollastonite is discussed in terms of a heat conduction theory and Gibbs free energy; and if the intrusive level is assumed to be several thousands of meter, the temperature is estimated to fall between 1100°C and 1800°C.

On a geological survey

The author made a geological survey of the area under consideration; Fig. 1 shows the geological map of the region. The Palaeozoic formations at Mt. Hiei and Mt. Daimonji are metamorphosed by the contact of Kitashirakawa-Granite. Shale and sandstone in the formations are metamorphosed into hornfels, and chert into quartzite. In the hornfels at Mt. Daimonji, contact minerals of cordierite and andalusite are found. The limestones cropping out at Gobessho, which are small and lenticular in form, are also metamorphosed into crystalline, and include the following contact minerals: a great quantity of wollastonite, and beside it vesuvianite, garnet, diopside and prehnite. As to the temperature needed to produce wollastonite many studies have preceded the present. Applying the results of these studies to the wollastonite in the Gobessho limestone, a temperature of the magma of Kitashirakawa-Granite required to produce the wollastonite by thermal contact will be estimated from the stand point of a heat conduction theory. In doing this, firstly we should know 1) the intrusive shape of the granite and 2) the distances between the limestones including the wollastonite and the contact plane between the granite and the Palaeozoic. And especially the survey in the southern contact (Mt. Daimonji) should be carefully made.

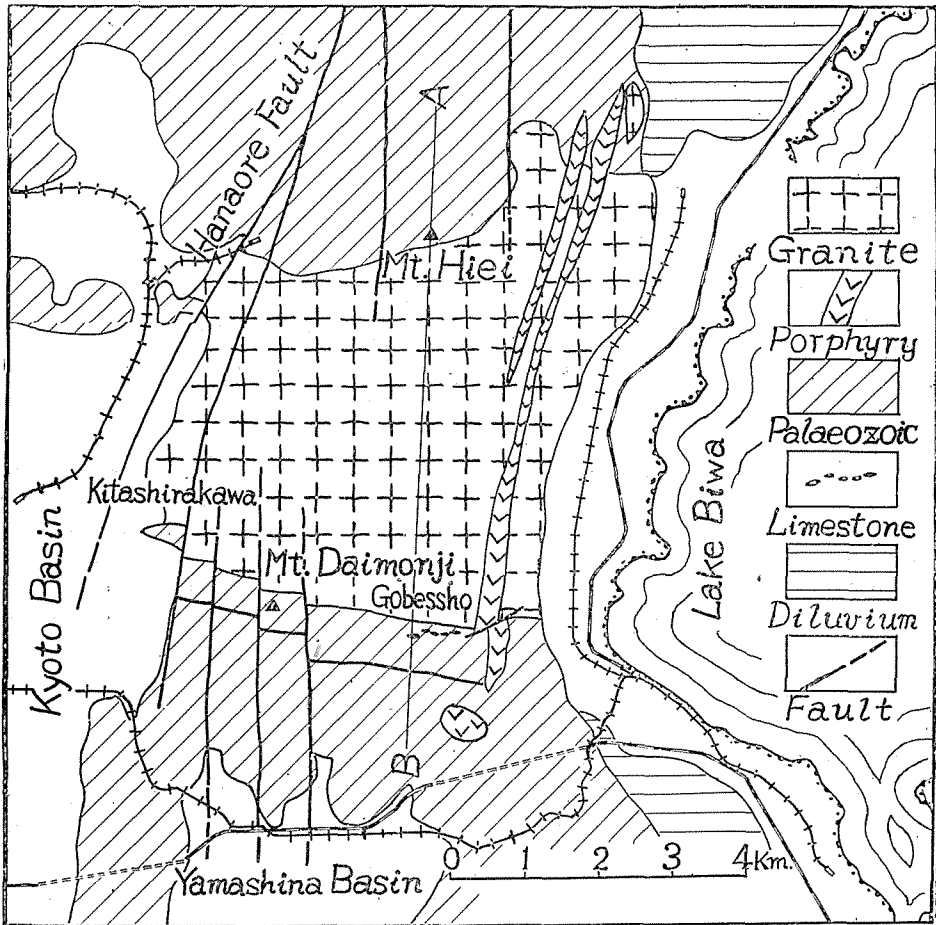


Fig. 1. Geological map

In the northern contact (Mt. Hiei) hornfels extends comparatively widely, while in the southern it discontinues at an E-W fault running parallel to the contact, which is cut by a number of N-S faults parallel to the famous Hanaore Fault described by S. Nakamura¹⁾. The E-W fault seems to have been developed by lateral tensile stress due to cooling of the granite. The fault plane, which is vertical, seems to run perpendicular to the tensile stress, that is, parallel to the contact plane. Considering also that in the southern contact the Palaeozoic is of a vertical dip and an E-W strike, the contact plane may be assumed to be vertical. In Fig. 2 is shown a N-S section of the region.

As to the age of dykes of porphyries occurring at the eastern part of this

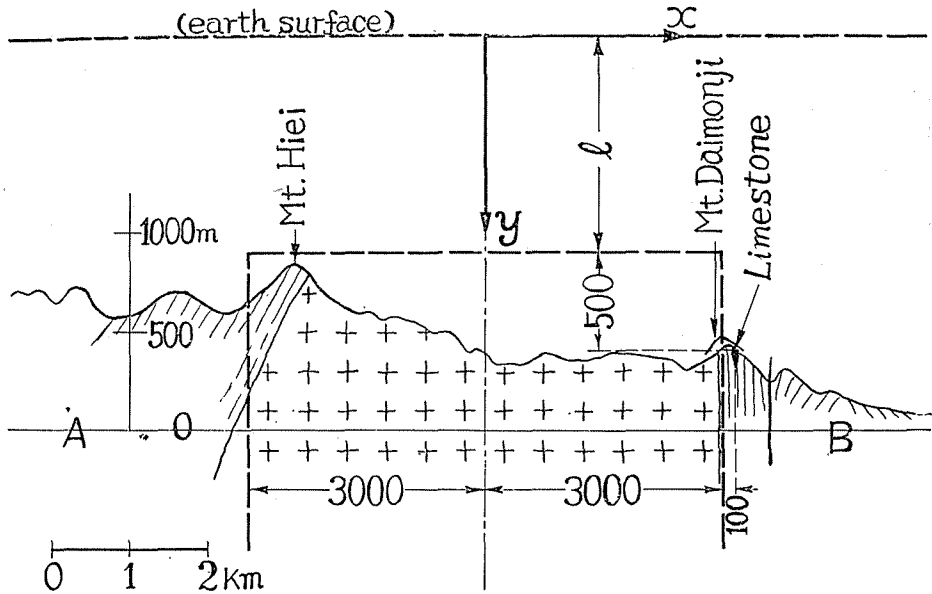


Fig. 2. N-S section through Mt. Hiei and assumed intrusive shape of Kitashirakawa-Granite

granite mass, T. Hiki³⁾ concludes that the intrusion of these dykes is later than that of the granite. Hence the dykes may be put outside our present consideration dealing with the thermal contact of the granite. The geologic age of the intrusion of the granite is generally accepted as Cretaceous.

Now let our inspection be extended to outside of this locality. Kitashirakawa-Granite underlies an anticline of the Palaeozoic, and this anticline seems to cross Kyoto Basin and continue to Hozugawa-Anticline (Nishiyama Mountainland, western part of Kyoto) as described by S. Matsushita³⁾. Therefore it may be assumed that this granite mass has intruded under this anticline which extends long from east to west.

Assumption 1. The intrusive shape of the granite mass is, therefore, likely as shown in Fig. 2; x -axis is taken horizontally in the direction of south, y -axis vertically downward, extension of the granite mass, assumed to be two dimensional, $-3000m < x < 3000m$ and $l < y < \infty$, where l is an intrusive level of the granite.

Gobesho limestones crop out at five places in the range of distances of 80 to 200 m from the contact. Since the wollastonite is also found in the farthest limestone from the contact, at the place at a distance of 200 m from it there may have occurred at one time a temperature higher than that required to produce the wollastonite. Though the contact plane in the south seems generally to be vertical as mentioned above, we can not know if the contact plane at

Gobessho is locally vertical, that is, we cannot know exactly the distances between the limestones and the contact plane. Hence, let us assume that up to the distance as far as $100 + \alpha m$ from the contact the temperatures were higher than that required for the production of the wollastonite, and employ the next assumption.

Assumption 2. The position of the farthest limestone in which the wollastonite could be produced is indicated by the coordinates $(x, y) = (3100 + \alpha m, l + 500m)$.

Production of wollastonite by thermal metamorphism of limestone including silica

The limestones in the contact zone appear to have undergone processes of the following four stages:

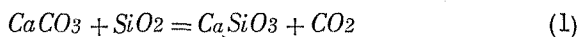
First stage; both rock pressure and temperature increase by sinking of geosyncline of the Palaeozoic formation.

Second stage; temperature increases by a magma intrusion.

Third stage; Temperature decreases by the cooling of the intruded magma.

Fourth stage; both rock pressure and temperature decrease owing to upheaval of the ground and erosion of its surface.

For the chemical reaction



Danielsson⁴⁾ gives the following Gibbs free energy difference:

$$\Delta G = \Delta G^\circ + RT \ln p + RT \int_1^p \left(\frac{Z-1}{p} \right)_r dp + \frac{2.42}{100} \int_1^\pi \Delta v_r dp, \quad (2)$$

where R is the gas constant $1.985 \text{ cal/}^\circ K$, T temperature in $^\circ K$, p pressure of CO_2 gas in atm., π rock pressure in atm., Z the factor of compressibility of CO_2 gas, ΔG° Gibbs free energy difference at 1 atm. given by

$$\Delta G^\circ = 21216 - 34.24 T - 0.99 \ln T + 3.84 \cdot 10^{-3} T^2 - 0.17 \cdot 10^5 T^{-1} \quad (3)$$

$$\text{and } \Delta v_r = \text{Molvol wollastonite} - \text{Molvol calcite} - \text{Molvol quartz}, \quad (4)$$

$$\text{where he put } \Delta v_r(p, T) = \Delta v = -19.68 \text{ cm}^3/\text{mol}. \quad (4')$$

And the condition of the equilibrium of the reaction, $\Delta G = 0$, is given by

$$\log p_{CO_2} = -\frac{\Delta G^\circ}{2.303 RT} - \frac{1}{2.303} \int_1^p \left(\frac{Z-1}{p} \right)_r dp + \frac{0.104\pi}{T}. \quad (5)$$

In Fig. 3 are shown the p - T diagrams for $\pi = 0, 3000, 5000$ atm. and $\pi = p_{CO_2}$.

In the state of the equilibrium which should have occurred in the limestone, the maximum mol number n of the four molecules is given by

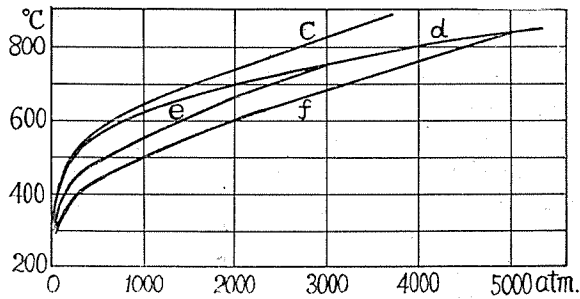


Fig. 3. p-T diagram of the equilibrium $\text{CaCO}_3 + \text{SiO}_2 = \text{CaSiO}_3 + \text{CO}_2$ by Danielsson
c : $\pi = 0$, *d* : $\pi = p_{\text{CO}_2}$, *e* : $\pi = 3000 \text{ atm.}$, *f* : $\pi = 5000 \text{ atm.}$

$$n = \frac{p_{\text{CO}_2} V}{ZRT} \quad (6)$$

in which p_{CO_2} is the equilibrium pressure given by (5) and V the total volume of pores in the limestone occupied by all the CO_2 gas molecules, which consists of both those produced by the reaction and those existing prior to the reaction. If there is none of the latter, (6) is just equal to the mol number of the four molecules. Comparing $\Delta v = -19.68 \text{ cm}^3/\text{mol}$ with mol-volume of wollastonite $= 40.0 \text{ cm}^3/\text{mol}$ in (4), it can be seen that the porosity increases by half as much as the volume of wollastonite produced. In Table I are given the maximum quantity of produced wollastonite for different temperatures and the increase of the corresponding porosity, assuming the initial porosity prior to the reaction to be 1% and 3%. In the last column is shown the quantity of SiO_2 needed for producing wollastonite, assuming the density of the limestone to be 2.5 gm/cm^3 ⁵⁾. In this calculation the *c*-curve in Fig. 3 was used and $Z=1$ (ideal gas) was assumed in (6). The average porosity of limestone is 3.0~15% and that of marble is 0.5~1.0%⁶⁾. It may be considered that our limestone was marble prior to the second stage, as both temperature and rock pressure were high at such a depth that granite intruded. In any case the porosity of our limestone must have been slight. Therefore the initial porosity was taken to be 1% or 3%.

From Table I we know that the porosity increases rapidly with temperature. High rock pressure, however, will not allow such an increasing of porosity. If the porosity is kept at a constant value, the quantity of the wollastonite product is far less than that in Table I and likely as shown in Table II. The state which would occur practically would be that which combine the conditions of Table I and II.

In the case of Table I, at high temperatures the CO_2 gas may be able to move from pore to pore in limestone. But in our case the CO_2 gas can hardly escape from a lenticular body of the limestone, because it is surrounded by dense compact sedimentary rocks. In the case of Table II, the CO_2 gas will

be unable to move freely from pore to pore in the limestone, and can hardly escape. In the second stage, since the mother rock will have a property of plasticity because of being exposed to high temperature and pressure, fissures or cracks will hardly occur in the rock. Perhaps the fissures would have developed in the later half of the third stage by stresses due to cooling or in the next fourth stage by stresses due to upheaval. We can hardly expect the CO_2 gas escapes in great quantity along the fissures thus developed, because the equilibrium pressure of CO_2 gas is already low owing to the decrease in temperature. That the wollastonite is found in great quantity may be an evidence that the CO_2 gas escaped from the limestone. For dealing with escape of gases existing in pores of sedimentary rocks, let us employ the next assumption.

Table I

Temp. °C	Equilibrium pressure of CO_2 gas atm.	Maximum wollastonite product volume %	Porosity in limestone %	Silica needed for wollastonite product weight %
360	10	0.008	1.0	0.0046
465	100	0.067	1.0	0.040
650	1000	0.71	1.3	0.41
740	2000	1.9	1.9	1.10
820	3000	4.0	3.0	2.41
360	10	0.023	3.0	0.014
465	100	0.20	3.0	0.12
650	1000	2.1	4.0	1.24
740	2000	5.7	5.7	3.30
820	3000	12.0	9.0	7.23

Table II

360	10	0.008	1.0	0.0046
465	100	0.067	1.0	0.040
650	1000	0.53	1.0	0.32
740	2000	0.96	1.0	0.58
820	3000	1.3	1.0	0.80
360	10	0.023	3.0	0.014
465	100	0.20	3.0	0.12
650	1000	1.6	3.0	0.95
740	2000	2.9	3.0	1.74
820	3000	4.0	3.0	2.41

Assumption 3. As long as the pressure of the gases in the pores is less than rock pressure, the gases are confined in the pores, and if it is higher, the gases escape from the pores until the pressure becomes equal to the rock pressure.

The pressure expressed here is not a partial pressure of CO_2 gas, but the total pressure of the gases existing in the pores.

Distribution of radioactive matter in the contact zone in this locality is investigated by J. Hatsuda and S. Nishimura². From their research it is shown that the radioactive contamination in the mother rock (shale and sandstone) due to the contact of granite magma almost disappears beyond a distance of 5 m from the contact. This fact will show that a diffusion of matter in the mother rock can hardly take place even through geological time, and our Assumption 3 may be considered verified.

Using Assumption 3 the mechanism of the production of wollastonite will be considered as follows:

First stage: Until sedimentary rocks become compact and dense, gases in pores of the rocks may escape comparatively freely. When deposition of sediments occurs, the pores of the sediments are filled with water. Next, the pores will be partially occupied by CO₂ gas and methane gas, etc. produced from decomposition of organic matters and also by such CO₂ gas that may be produced from limestone and calcareous sediments by the action of humic acid from the similar decomposition. During sinking of the sediments into the earth, a rise in temperature occurs and the generation of gases still continues, and the volume of the water existing in the pores will decrease as that of vapor H₂O and the gases (CO₂, CH₄, etc.) increases. At last, the decomposition of organic matters will cease entirely. Until this time, the pressure of the CO₂ gas perhaps is higher than its equilibrium pressure in equation (5), and therefore the molecule of CaSiO₃ is not yet produced in limestone in the sedimentary rocks, as $\Delta G > 0$ from (2) and (5). Danielsson⁴) gives the following equations,

$$\left(\frac{\partial \Delta G}{\partial \pi}\right)_{T,p} = \frac{2.42}{100} \Delta v (< 0), \quad \left(\frac{\partial \Delta G}{\partial p}\right)_{T,\pi} = \frac{RT}{p} (> 0), \quad (7), (8)$$

$$\left(\frac{\partial \Delta G}{\partial T}\right)_{p,\pi} = -35.23 - 0.99 \ln T + 7.68 \cdot 10^{-3} T + R \ln p (< 0), \quad (9)$$

where (7) is negative from (4'), (8) positive of course and (9) negative in an ordinary range of temperatures and pressures. Therefore, as both the rock pressure and the temperature increase by the sinking of the geosyncline and the pressure of CO₂ gas decreases by escaping from the pores, ΔG decreases because

$$\delta \Delta G = \left(\frac{\partial \Delta G}{\partial \pi}\right)_{T,p} \delta \pi + \left(\frac{\partial \Delta G}{\partial p}\right)_{T,\pi} \delta p + \left(\frac{\partial \Delta G}{\partial T}\right)_{p,\pi} \delta T, \quad (10)$$

is negative from (7), (8) and (9). After this, the sediments will become compact and dense and all the gases in the pores can hardly escape and therefore become confined to obey the law of $pV = nRT$. In this case $\delta \Delta G$ is, therefore, given by

$$\delta \Delta G = \left(\frac{\partial \Delta G}{\partial \pi}\right)_{T,p} \delta \pi + \left\{ R + \left(\frac{\partial \Delta G}{\partial T}\right)_{p,\pi} \right\} \delta T. \quad (11)$$

As $\delta \Delta G$ given by (11) is also negative, ΔG still continues decreasing.

Therefore, in the first stage in which the value of ΔG is positive and decreasing as described above two cases are possible, i) one in which the value of ΔG remains greater than zero and the wollastonite is not yet produced owing to high pressure of pre-existing CO_2 gas, and ii) the other in which the value of ΔG reaches zero and the wollastonite at last begins to appear.

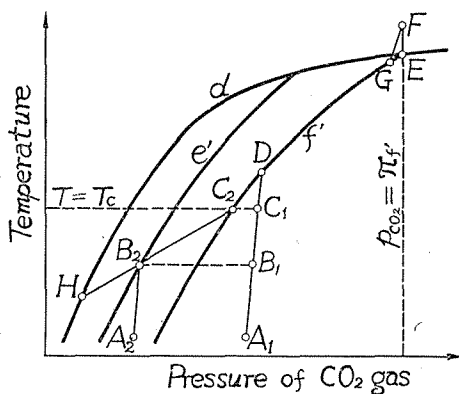


Fig. 4 Schematic diagram of process of wollastonite formation

The earth temperature in and around the limestone at this end is denoted by T_c . The end of the first stage is represented by C_1 in the case i) ($\Delta G > 0$) or by C_2 in the case ii) ($\Delta G = 0$) and these two points lie on the line $T = T_c$. The path letting the

reaction system reach C_1 would be $\overrightarrow{A_1 B_1 C_1}$ and that C_2 would be $\overrightarrow{A_2 B_2 C_2}$. The pressure of CO_2 gas in A_2 must have been lower than that in A_1 . Along the

path $\overrightarrow{B_2 C_2}$ the rock pressure gradually increases due to the sinking and it may be assumed that the equilibrium takes place at all points on this path.

Second stage: As soon as the second stage begins, the temperature must rise due to the intrusion of granite magma but the rock pressure remains constant and we get $\delta T > 0$ and $\delta \pi = 0$ and therefore $\delta \Delta G$ becomes negative from (11). ΔG at C_1 will decrease and reach zero value, that is, an equilibrium D will occur on the curve f' . In this stage the reaction system will move along

the path $\overrightarrow{C_1 D E}$ or $\overrightarrow{C_2 D E}$. On the path $\overrightarrow{D E}$ and $\overrightarrow{C_2 D E}$ the mol number of the CO_2 gas must obey the equation (6), while those of vapour and the other gases are assumed constant. If the condition occurs to which Table I is subject, the pressures of the vapour and the other gases decrease in spite of the increase of temperature, because the rate of increase of the volume of the pores is larger than that of the temperature (cf. Table I), as for vapour, for instance, we have $p_{\text{H}_2\text{O}} = n_{\text{H}_2\text{O}} RT/V$. If the condition occurs to which Table II is subject, the pressures increase with increase of temperature, but the rates of increase of these pressures

In Fig. 4 is schematically shown the transformation of the reaction system. The curves d , e' and f' represent the equilibria given by the reaction equation (1) for different values of rock pressure π ; the curve d corresponds to $\pi = p_{\text{CO}_2}$, those such as e' and f' to constant rock pressures and the rock pressure increases in the order from d to f' , and the rock pressure $\pi_{f'}$ in f' is one corresponding to the depth of the limestone $l+500\text{m}$ when the geosynclinal sinking stops which is the end of the first stage. The earth

are far less than that of p_{CO_2} . Therefore, in these two cases the pressures due to the vapour and the other gases will be negligible as composed with p_{CO_2} . Accordingly, we may consider that the pressure in Assumption 3 is almost equal to p_{CO_2} in the equilibrium state at high temperatures, and that when the equilibrium reaches E where $p_{CO_2} = \pi_f'$ the escape of gases from the pores begins.

On the other hand, if we consider the wollastonite product, it increases as shown in Table I or II, if a sufficient quantity of SiO_2 is given. At the state E , molecules of wollastonite are produced equal in number to those of CO_2 gas escaping. Hereafter these molecules of wollastonite produced by the escape of CO_2 gas will be excluded from the system in equilibrium of the chemical reaction (1), under the assumption that CO_2 gas is not introduced into the pores from outside. Silica in limestone being used up by production of the wollastonite, the system will move from E to F as p_{CO_2} will remain equal to π_f' due to escape of CO_2 gas. The state F corresponds to the maximum temperature which the limestone will be exposed to.

Third stage: The system moves along the paths \overrightarrow{FG} and $\overrightarrow{GDC_2}$. On the path $\overrightarrow{GDC_2}$, the wollastonite product reduces.

Fourth stage: The system moves along the path $\overrightarrow{C_2B_2H}$, and the wollastonite product will reduce again. Therefore, there will remain those molecules of wollastonite that have been produced by the escape of CO_2 gas and they are those which build the wollastonite crystals we find in our limestone at the present. If no escape of CO_2 gas had occurred in the above process, the system would have returns to A_1 or to A_2 and no wollastonite would have been found in the limestone.

Therefore, it may be concluded that the temperature required to make wollastonite molecules thus remained is higher than the equilibrium temperature for $p_{CO_2} = \pi_f'$, that is, the temperature at E (Fig. 4). In Table III are given the temperatures at E for different depths $l + 500$ m, by assuming the density of mother rock (the Palaeozoic formation) to be 2.5 gm/cm^3 for the calculation of the rock pressure π .

Table III Temperature required for production of wollastonite

Depth ($l+500$) m	3500	5500	8000	10500	13500
Rock pressure π atm.	875	1370	2000	2620	3380
Temp. required to produce $CaSiO_3$ °C	600	650	700	740	770

Earth temperature at Cretaceous age before granite intrusion

The Cretaceous age dates from 60 to 120 millions of years ago, but it is

not so distant in time when compared with age of the earth's crust, 1600~3200 millions of years. Therefore, the distribution of the earth temperature θ in the Cretaceous period may be obtained by making use of the two formulas of Jeffreys⁹⁾, by which he has calculated the distribution of the earth temperature at the present. Of these formulas let us use one based upon the assumption of exponential decrease

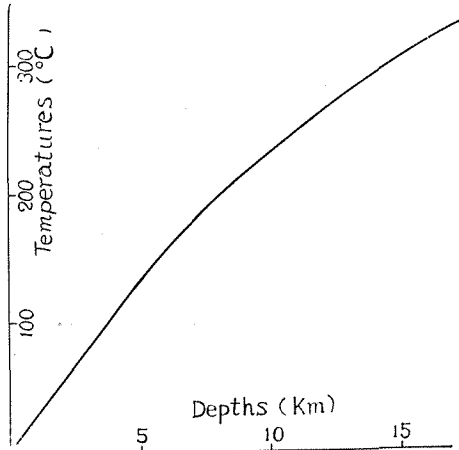


Fig. 5. Distribution of the earth temperature in the Cretaceous period

of radioactivity, and let us employ in this formula $t = 4.73 \times 10^{10}$ sec (= 1500 millions of years) which is the interval of time between the Cretaceous period and the date of birth of primitive solid crust, which is assumed by Jeffreys to be 1600 millions of years ago from the present. Thus we get Fig. 5.

For simplicity of calculation, the linear distribution given by tangents of the curve in Fig. 5 at different depths $l+500$ m will be used as substitute for the earth temperature, as the curve is almost linear. If the tangent is represented by an equation,

$$\theta = A + By, \quad (12)$$

the constants A and B are obtained as shown in Table IV for different values of $l+500$ m which is the depth y .

Table IV Constants A and B in equation (12)

$(l+500)$	m	3500	5500	8000	10500	13500
A	°C	0	20	44	65	94
B	°C/m	0.0263	0.0229	0.0190	0.0163	0.0144

An estimate of a temperature of granite magma

For simplicity's sake, let the next assumptions be set up.

Assumption 4. To form the intrusive shape (Fig. 2), it does not take much time and we assume the initial condition to be

$$(u)_{t=0} = \begin{cases} U & \text{for the granite magma,} \\ = A + By & \text{for the mother rock,} \end{cases} \quad (13)$$

where U is the granite magma temperature to be found and t the time since the intrusion of granite magma.

Assumption 5. The following is disregarded: **5.1** convection current in the magma, **5.2** heat of solidification of the magma, **5.3** heat of metamorphism of the mother rock, and **5.4** differences in specific heat c , density ρ , and thermal conductivity k between the granite and the mother rock.

We have a differential equation of heat conduction,

$$\frac{\partial u}{\partial t} = \kappa^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (14)$$

where $\kappa^2 = k/c\rho$ (κ^2 : thermal diffusivity), subject to the boundary condition

$$(u)_{y=0} = A \quad (15)$$

and the initial condition (13). If we put $u = v + w$, we are to find v and w , which satisfy the differential equation of the same type as (14), subject to the boundary conditions,

$$(v)_{y=0} = A, \quad (w)_{y=0} = 0 \quad (16), (17)$$

and the initial conditions,

$$\left. \begin{aligned} (v)_{t=0} &= A + By, \\ (w)_{t=0} &= 0 \quad \text{for the mother rock,} \\ &= U - (A + By) \quad \text{for the granite magma.} \end{aligned} \right\} (19)$$

From the boundary and initial conditions of v , we can see that v is the earth temperature when the granite magma is assumed to be replaced by the mother rock. As the change of this earth temperature is very slow as compared with those of the temperatures of the mother rock and granite magma, we may approximately put,

$$v = A + By, \quad (20)$$

being independent of t . The solution for w is, as will be proved in the Appendix,

$$\begin{aligned} w = & \frac{1}{4} \left(\operatorname{Erf} \frac{b-x}{2\kappa\sqrt{t}} + \operatorname{Erf} \frac{b+x}{2\kappa\sqrt{t}} \right) \left\{ (U-A) \left(\operatorname{Erf} \frac{l+y}{2\kappa\sqrt{t}} - \operatorname{Erf} \frac{l-y}{2\kappa\sqrt{t}} \right) \right. \\ & \left. - By \left(2 - \operatorname{Erf} \frac{l-y}{2\kappa\sqrt{t}} - \operatorname{Erf} \frac{l+y}{2\kappa\sqrt{t}} \right) - \frac{2\kappa B\sqrt{t}}{\sqrt{\pi}} \left(\operatorname{Exp} \frac{-(l-y)^2}{4\kappa^2 t} - \operatorname{Exp} \frac{-(l+y)^2}{4\kappa^2 t} \right) \right\} \quad (21) \end{aligned}$$

where b ($=3000$ m) is half the width of the intruded magma.

Thermal diffusivity κ^2 for shale, sandstone and granite are 0.004, 0.014 and 0.013 cm^2/sec^9 respectively, that is, 12.6, 44.3 and 41.1 m^2/year . Let us roughly take $\kappa^2 = 36 = 6^2 \text{m}^2/\text{year}$. Negrecting the small terms involved in (21) for $0 < t < 10000$ years (this time interval suffices our purpose as will be seen in Fig. 6), the temperature u is given by,

$$u = A + By + \frac{1}{4} \left(1 + \operatorname{Erf} \frac{b-x}{2\kappa\sqrt{t}} \right) \times \left\{ (U - A - By) \left(1 - \operatorname{Erf} \frac{l-y}{2\kappa\sqrt{t}} \right) - \frac{2\kappa B\sqrt{t}}{\sqrt{\pi}} \operatorname{Exp} \frac{-(l-y)^2}{4\kappa^2 t} \right\}. \quad (22)$$

From (22) we can determine such a temperature U that the maximum value of u at the position $(x, y) = (3100 + \alpha m, l + 500m)$ defined in Assumption 2 is equal to the temperature (Table III) to produce the wollastonite. Here let us put $\alpha = 0$, then the temperature U takes values shown in Table V. And the

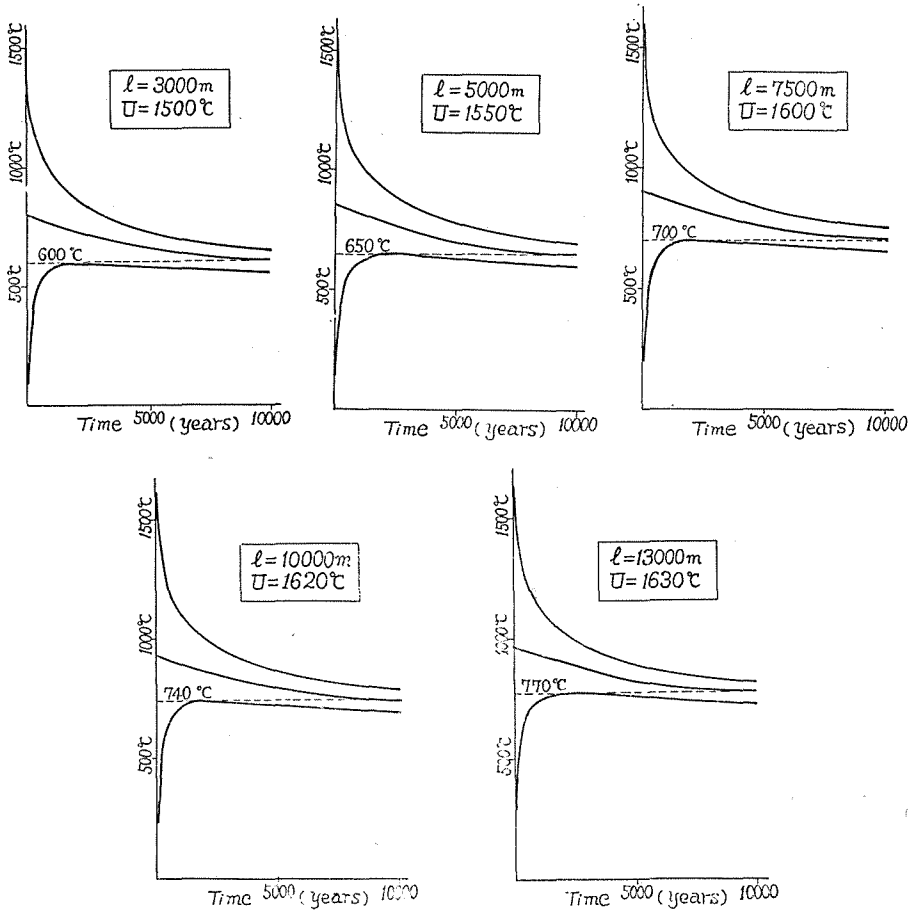


Fig. 6. Temperature u computed by (22)
 Upper-curve : at $(x, y) = (2900\text{m } l + 500\text{m})$
 Middle-curve : at $(x, y) = (3000\text{m } l + 500\text{m})$
 Lower-curve : at $(x, y) = (3100\text{m } l + 500\text{m})$

changes of temperature with respect to time at $(x, y) = (2900m, l + 500m)$, $(3000m, l + 500m)$ and $(3100m, l + 500m)$ are graphically shown in Fig. 6 for different intrusive levels.

Table V. Magma temperature U for $\alpha = 0$.

Intrusive level l m	3000	5000	7500	10000	13000
Magma temperature U °C	1500	1550	1600	1620	1630

Comments on Assumptions employed

1. On Assumption 5.2

In Assumption 5.2 we disregarded the heat generated during solidification of magma, which may be assumed to be large as compared with the heat possibly absorbed by the mother rock to undergo metamorphism. Here let us investigate the effect of the former heat. For doing this, one-dimensional problem shown in Fig. 7 is adopted, where an intrusive level of granite magma l is taken as 5000m. Let the heat generated per unit volume per decrease in temperature of 1°C be denoted by $L(u, \xi)$. As we have

1) $k \frac{\partial^2 u}{\partial \xi^2} d\xi dt =$ heat quantity accumulating within a column with unit area of cross-section and height $d\xi$ during time interval dt ,

2) $-L(u, \xi) du d\xi = -L(u, \xi) \frac{\partial u}{\partial t} d\xi dt =$ heat quantity generated in the column by a temperature increase du , and

3) $c\rho du d\xi = c\rho \frac{\partial u}{\partial t} d\xi dt =$ heat quantity used to increase the temperature of the column,

we get the following differential equation of heat conduction;

$$\left(1 + \frac{L(u, \xi)}{c\rho} \right) \frac{\partial u}{\partial t} = \kappa^2 \frac{\partial^2 u}{\partial \xi^2}. \tag{23}$$

The boundary condition and the initial condition are as be mentioned in the previous section,

$$\left. \begin{aligned} (u)_{\xi=0} = A, (u)_{t=0} = A + By \text{ for } 0 < \xi < l \text{ (mother rock),} \\ = U \text{ for } l < \xi < \infty \text{ (granite magma).} \end{aligned} \right\} \tag{24}$$

In this calculation the magma temperature U is taken to be 1200°C, which is

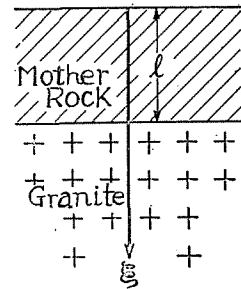


Fig. 7. Model for equation (23)

lower than the results got in Table V.

To estimate the rough order of magnitude of L , let us consider the granite to consist of quartz only. The latent heat of high quartz melting and that of high-low quartz inversion are 210 and 10.5 *joule/gm* respectively and the density of quartz 2.65 *gm/cm³*¹⁰. And let us further assume that the latent heat is generated uniformly in the temperature range of 1100°C to 500°C during which the solidification takes place. The decrease in temperature during the solidification being 600°C, $L(u, \xi)$ takes values,

$$L(u, \xi) = \begin{cases} 1 & \text{joule/cm}^3 \text{ } ^\circ\text{C for } \xi > l \text{ and } 1100 > u > 500, \\ 0 & \text{joule/cm}^3 \text{ } ^\circ\text{C for } \xi < l, u > 1100 \text{ and } 500 > u. \end{cases} \quad (25)$$

Moreover $c\rho = 2$ *joule/cm³ } ^\circ\text{C}* will be adopted, using the data for rocks, $c = 0.7 \sim 0.95$ *joule/gm } ^\circ\text{C}* and $\rho = 2.5$ *gm/cm³*¹¹.

Equation (23) subject to (24) will be solved by the perturbation method. If we put,

$$u = u_0 + u_1, \quad L(u, \xi)/c\rho = L_0 + L_1(u, \xi), \quad (26), (27)$$

in which L_0 is the mean value of $L(u, \xi)/c\rho$ and from a rough calculation we assume $L_0 = 0.1$, (23) and (24) can be separated into the following equations:

$$\left. \begin{aligned} \kappa^2 \frac{\partial^2 u_0}{\partial \xi^2} - (1 + L_0) \frac{\partial u_0}{\partial t} &= 0, \\ (u_0)_{\xi=0} &= A, \quad (u_0)_{t=0} = A + B\gamma \quad \text{for } 0 < \xi < l, \\ &= U \quad \text{for } l < \xi < \infty, \end{aligned} \right\} \quad (28)$$

$$\text{and} \quad \left. \begin{aligned} \kappa^2 \frac{\partial^2 u_1}{\partial \xi^2} - \left(1 - L_0 + L_1(u_0, \xi)\right) \frac{\partial u_1}{\partial t} &= L_1(u_0, \xi) \frac{\partial u_0}{\partial t}, \\ (u_1)_{\xi=0} &= 0, \quad (u_1)_{t=0} = 0, \end{aligned} \right\} \quad (29)$$

where $L_1(u_0, \xi) \frac{\partial u_1}{\partial t}$ in the left hand side of equation (29) may be neglected as a small term, but here it is retained. Resolving u_0 into two terms, one being independent and the other dependent of time, just as we previously have put $u = v + w$, the solution for u_0 becomes,

$$u_0 = A + B\xi + \frac{1}{2} \left\{ (U - A) \left(\text{Erf} \frac{l + \xi}{2p\sqrt{t}} - \text{Erf} \frac{l - \xi}{2p\sqrt{t}} \right) - B\xi \left(2 - \text{Erf} \frac{l + \xi}{2p\sqrt{t}} - \text{Erf} \frac{l - \xi}{2p\sqrt{t}} \right) + \frac{2pB\sqrt{t}}{\sqrt{\pi}} \left(\text{Exp} \frac{-(l - \xi)^2}{4p^2t} + \text{Exp} \frac{-(l + \xi)^2}{4p^2t} \right) \right\}, \quad (30)$$

where $p^2 = \kappa^2 / (1 + L_0)$. As for the solution for u_1 a numerical calculation by the calculus of finite differences is available, and this result is shown in Table VI. Regrettably, the calculation of u_1 could not be carried on for $t > 3200$ years owing to the accumulation of errors.

In Fig. 8 the temperature distribution for $t = 2400$ years is shown as compared with that in the case of no latent heat. It will be concluded from Fig.

Table VI Temperature u_1

t years $(\xi-l)$ m	0	400	800	1200	1600	2000	2400	2800	3200
-900	0	0	0	0	0	0	0	2.0	
-780	0	0	0	0	0	0	2.8	—	
-660	0	0	0	0	0	4.0	—	7.3	
-540	0	0	0	0	5.8	—	9.7	—	
-420	0	0	0	8.4	—	12.8	—	15.5	
-300	0	5.5	11.5	—	16.6	—	18.8	—	
-180	0	17.0	—	21.0	—	22.5	—	23.2	
-60	0	23.6	25.8	—	26.2	—	26.2	—	26.5
60	0	37.1	—	31.3	—	29.8	—	29.8	
180	0	-0.8	31.4	—	31.1	—	31.5	—	
300	0	-1.5	—	20.3	—	27.3	—	29.7	
420	0	0	-3.1	—	14.6	—	20.8	—	
540	0	0	0	-2.8	—	5.2	—	14.5	
660	0	0	0	0	-2.3	—	1.0	—	
780	0	0	0	0	0	-1.8	—	-0.6	
900	0	0	0	0	0	0	-1.3	—	

8 that the latent heat generated by solidification of magma delays cooling of the granite and the mother rock and does not bring about a significant temperature increase in the mother rock. Considering effect of the heat possibly absorbed by metamorphism of the mother rock, the temperature in the mother rock will be a little less than the value shown by the full line in the figure.

2. On Assumption 2

If we employ $\alpha = 100$ which corresponds to the observed distance 200 m of the wollastonite-bearing limestone farthest from the contact, the equation (22) gives us that the temperatures of magma are about 300°C higher than those in Table V in which $\alpha = 0$. If we employ $\alpha = -100$ (the limestone is in contact with the magma), they are about 400°C lower than those in Table V.

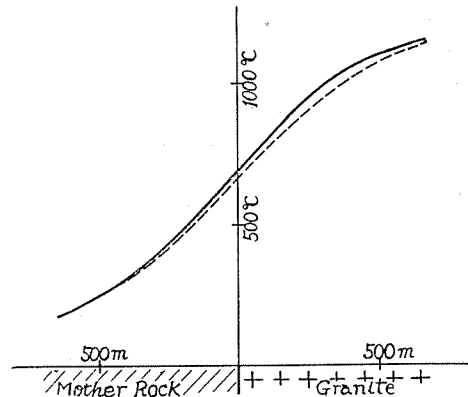


Fig. 8. Temperature u computed by (26), (30) and Table VI for $t=2400$ years.
 Full line: latent heat is taken into consideration
 Dotted line: latent heat is disregarded

3. On Assumption 4

The assumption 4, an assumption of instantaneous formation of the intrusive shape, should be replaced from practical point of view by another assumption that formation of the intrusive shape has needed a certain time interval. This assumption implies movement of the contact plane between the magma and the mother rock towards the mother rock side possibly due to enlargement of the magma mass; and this movement, if occurred, should ultimately stop. For the movement of the contact plane two cases may be considered, one accompanying no appreciable melting of the mother rock and the other considerable.

The first case has no essential difference against Assumption 3, because the temperature u determined by equation (22) depends upon the distance from the contact plane $b-x$ and not upon the width of the shape of the magma $2b$.

However, in the second case, if we set the initial time when the above movement stops, the initial condition will be such that the temperature of the magma in the vicinity of the final contact plane is considerably lower than the initial magma temperature U due to latent heat of fusion and the temperature of the mother rock in the vicinity of the same plane is greater to some extent than temperature prior to the magma intrusion. Such a temperature distribution may be approximately substituted by a temperature distribution which will be obtained from equation (22) by giving to t a certain positive value and by increasing the distance of the limestone from the contact plane $x-b$ by an amount equal to the distance melted. This increase of $x-b$ necessarily gives to the required magma temperature U a value greater than that in the case of instantaneous formation of the intrusive shape.

4. On Assumption 5.1

In Assumption 5.1 convection current is disregarded. However, if we take it into consideration, the convection current would accelerate cooling of the magma and supply more heat on the mother rock letting its heating penetrate farther. But the temperature required for producing wollastonite at the farthest position $x=3100+\alpha m$ is a given quantity. Therefore, it seems that when the convection is taken into consideration, the magma temperature U should be taken less than that calculated by disregarding convection current.

A large number of measurements of the viscosity of melts of natural igneous rocks⁽²⁾ shows that the viscosity of melts of rocks decreases with a rate extremely greater than that with which temperature of the melts increases. This fact indicates that the higher the temperature of the magma, the more and more intense the convection current. Therefore, the higher the magma temperature obtained by disregarding convection current, the larger and larger the correction to lower this temperature, when the convection current is taken into consideration.

The convection current seems to play an important part in the determination of the magma temperature, but owing to that the structure of the convection current is unknown, a numerical computation to find value of the correction mentioned above will be a difficult task.

Conclusion

Temperature of magma is evidently higher than its melting point at which the magma solidifies. This temperature of magma can be found by thermal metamorphism the mother rock has undergone, and may hardly be done by making use of minerals in the solidified magma itself. In other words, the recorder of the temperature of magma is rather in the mother rock in contact with the solidified magma than in it.

This paper stands upon the above point of view. It is now generally accepted that temperature of granite magma is as low as 790~900°C. The result obtained in the comment 2 indicates that the temperature of granite magma varies from 1800°C to 1930°C for $\alpha=100$ m and from 1100°C to 1230°C for $\alpha=-100$ m as the intrusive level l does from 3000 to 13000m. Our field observation locates the farthest wollastonite-bearing limestone at a distance of 200 m from the contact and this distance gives $\alpha=100$ m. The case of $\alpha=-100$ m would locate the farthest limestone just at the contact plane and therefore is not consistent with our observation. But this case may be an approximate equivalent of the case in which immediately beneath the farthest wollastonite-bearing limestone lies a contact plane. Existence of such a contact plane may not be impossible. Therefore, if we could assume the least intrusive level to be several thousands of meters, the lowest temperature of granite magma would be from 1100°C to 1800°C. These temperatures are considerably greater than the temperature 750~900°C generally accepted as mentioned above. The p - T equilibrium diagram of SiO_2 thermodynamically obtained by Mosesman and Pitzer¹³⁾ shows that for the pressures corresponding to the depths in which granite intrudes quartz is stable but tridymite and cristobalite unstable. Consulting with this, it does not seem that the above obtained least temperatures 1100°C to 1800°C are unplausibly high values.

As mentioned in the comment 4, the effect of convection current in the magma is unknown, and therefore the present author will not strongly persist in the above values of temperature. And he considers that the similar investigations should be done making use of other contact minerals in the neighbourhood of our contact plane.

Appendix

In the following is given the derivation of the equation (21) which is the solution of a differential equation of heat conduction satisfying the boundary

condition (17) and the initial condition (19). Here let the initial condition (19) be denoted by $f(x, y)$. $f(x, y)$ being symmetrical for x , it is represented with the Fourier integral,

$$f(x, y) = \frac{2}{\pi} \int_0^{\infty} d\alpha \int_0^{\infty} f(\lambda, y) \cos \alpha x \cos \alpha \lambda d\lambda,$$

and $f(\lambda, y)$ is represented with

$$f(\lambda, y) = \frac{2}{\pi} \int_0^{\infty} d\beta \int_0^{\infty} f(\lambda, \mu) \sin \beta y \sin \beta \mu d\mu,$$

satisfying the boundary condition (17). Therefore we can represent $f(x, y)$ with

$$f(x, y) = \frac{4}{\pi^2} \int_0^{\infty} d\alpha \int_0^{\infty} d\beta \int_0^{\infty} \int_0^{\infty} f(\lambda, \mu) \cos \alpha x \cos \alpha \lambda \sin \beta y \sin \beta \mu d\lambda d\mu. \quad (31)$$

The particular solution of the differential equation of heat conduction is, through a simple calculation,

$$w \propto e^{-\kappa^2(\alpha^2 + \beta^2)t} \cos \alpha x \sin \beta y,$$

which satisfies (17). In order that the general solution of w satisfies the initial condition (31), w is given by the following equation;

$$w = \frac{4}{\pi^2} \int_0^{\infty} d\alpha \int_0^{\infty} d\beta \int_0^{\infty} \int_0^{\infty} e^{-\kappa^2(\alpha^2 + \beta^2)t} f(\lambda, \mu) \cos \alpha x \cos \alpha \lambda \sin \beta y \sin \beta \mu d\lambda d\mu. \quad (32)$$

Using the formula,

$$\int_0^{\infty} e^{-a^2x^2} \cos bx dx = \frac{\sqrt{\pi}}{2a} e^{-\frac{b^2}{4a^2}},$$

(32) is transformed to,

$$w = \frac{1}{4\kappa^2\pi t} \int_0^{\infty} \int_0^{\infty} f(\lambda, \mu) \left(e^{-\frac{(\lambda-x)^2}{4\kappa^2t}} + e^{-\frac{-(\lambda+x)^2}{4\kappa^2t}} \right) \left(e^{-\frac{-(\mu-y)^2}{4\kappa^2t}} - e^{-\frac{-(\mu+y)^2}{4\kappa^2t}} \right) d\lambda d\mu. \quad (33)$$

The equation (21) will be derived from this equation.

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