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Kyoto University
Peer-to-Peer Energy Transfer by Power Gyrators Based on Time-Variable-Transformer Concept

Daniel Kiss, Takashi Hisakado, Member, IEEE, Tohlu Matsushima, Member, IEEE, and Osami Wada, Member, IEEE

Abstract—A control strategy based on matching the source and load changes of the order of milliseconds, called peer-to-peer energy transfer, is introduced. This energy transfer enables a decoupled energy transfer system in common bus networks. To realize the transfer with a pair of two-port circuits, a power gyrator is derived from the phasor-based model of a bidirectional ac/dc converter, based on the concept of a time-variable transformer. Power-gyrator timing synchronization is achieved by communication, and a peer-to-peer energy transfer system is developed. Experimental and simulation results are compared, and it is demonstrated that peer-to-peer energy transfer can be used for decoupling common bus voltage networks.

Index Terms—Decoupled system, peer-to-peer energy transfer, power gyrator, time-variable transformer (TVP), timing synchronization.

I. INTRODUCTION

In recent years, microgrids have been extensively researched [1], [2] for addressing the challenges posed by the expansion of distributed generation and energy storage systems, to use them to the utmost, while improving the reliability of the legacy grid. A significant challenge in microgrid research is the determination of control strategies that are reliable, efficient, and applicable in both grid-connected and islanded modes. Most sources and batteries are connected through an inverter, with the ac side connected to a common bus, creating a network of parallel inverters.

The droop method, originally introduced in [3], is based on the operation of parallel synchronous generators because uninterruptible power supply is the most commonly used control strategy for microgrids [4]. The original droop control has been modified in several ways to accommodate the lower voltages of microgrids. Guerrero et al. in [5] use the P–V droop instead of the traditional P–ω droop; in [6] they use a virtual inductance derived from the phasor-based model of a bidirectional ac/dc converter, based on the concept of a time-variable transformer. Power-gyrator timing synchronization is achieved by communication, and a peer-to-peer energy transfer system is developed. Experimental and simulation results are compared, and it is demonstrated that peer-to-peer energy transfer can be used for decoupling common bus voltage networks.

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more similar to a current source. Thereby, they act as current sources, and are not considerably affected by the changes in the bus voltage, rendering them suitable for peer-to-peer energy transfer. Further, timing synchronization is introduced between the converters by communication and energy-transfer matching, in the order of milliseconds, is realized.

The remainder of this paper is organized as follows. In Section II, the peer-to-peer energy transfer system is described in detail. In Section III, a phasor-based model of the bidirectional ac/dc converter is presented. In Section IV, the realization of the gyrator using voltage–current hybrid control is described. In Section V, the converter used in our experiments is presented. In Section VI, our experimental and modeling results are presented, and in Section VII, the conclusions are drawn.

II. PEER-TO-PEER ENERGY TRANSFER

The peer-to-peer energy transfer system is a network of variable voltage sources with internal impedances, which are the Thevenin equivalent circuits of sources and loads connected in a star architecture, as shown in Fig. 1. The voltage sources are synchronized such that their output current has the same phase as the voltage at the point of common coupling, making the power factor unity. As observed in [26] and [27], a system sampled with the ac frequency is a good approximation of the actual operation. In this case, the system can be described by phasors that are discretized in time. This system can be described as follows:

\[ \dot{V}_{\text{com}}(k) = \dot{V}_i(k) - Z_i \dot{I}_i(k), \quad \text{where} \quad i = 1, 2, \ldots, N \]  

and

\[ \dot{V}_{\text{com}}(k) = \sum_{i=1}^{N} \frac{V_i(k)}{Z_i} - \sum_{i=1}^{N} \frac{\dot{V}_i(k)}{Z_i} \]  

where \( k \) is the sampled moment in time, \( \dot{V}_{\text{com}}(k) \) is the voltage at the common point at time \( k \), \( \dot{V}_i(k) \) is the voltage of the \( i \)th ideal voltage source, and \( Z_i \) is the internal impedance. From (2), the coupling coefficient \( \gamma_i \), which shows the effect of \( \dot{V}_i \) on \( \dot{V}_{\text{com}} \), can be derived as

\[ \gamma_i = \frac{1}{\sum_{i=1}^{N} \frac{1}{Z_i}}, \quad \text{where} \quad \gamma_i = \frac{1}{\sum_{i=1}^{N} \frac{1}{Z_i}}. \]  

Assume that the next voltage \( \dot{V}_i(k+1) \) of each member of the network depends only on \( \dot{V}_i(k) \) and \( \dot{V}_{\text{com}}(k) \), then

\[ \dot{V}_i(k+1) = f_i(\dot{V}_i(k), \dot{V}_{\text{com}}(k)), \quad \text{where} \quad i = 1, 2, \ldots, N \]  

and there is at least one steady state where if \( \dot{V}_{\text{com}}(k) \) does not change, the voltages of the modules also do not change. The steady state is denoted by an upper index “SS.” Then

\[ \dot{V}^\text{SS}_i(k+1) = \dot{V}^\text{SS}_i(k), \]

if \( \dot{V}^\text{SS}_{\text{com}}(k+1) = \dot{V}^\text{SS}_{\text{com}}(k), \) \( i = 1, 2, \ldots, N. \)  

Assume that the system is in the steady state. Then, two voltage sources of the network are selected with voltages \( V_i \) and \( V_2 \), and their voltages are changed by \( \Delta V_1 \) and \( \Delta V_2 \), respectively. Using the coupling coefficients from (3), we obtain

\[ \Delta \dot{V}_{\text{com}}(k) = \gamma_1 \Delta V_1 + \gamma_2 \Delta V_2 \]

where \( \Delta \dot{V}_{\text{com}}(k) \) is the deviation from the steady-state common point voltage \( \dot{V}^\text{SS}_{\text{com}}(k) \). From (5), if \( \dot{V}^\text{SS}_{\text{com}}(k) = 0 \), the rest of the system remains unaffected. To satisfy this condition

\[ \frac{\Delta \dot{V}_1(k)}{Z_1} = \frac{\Delta \dot{V}_2(k)}{Z_2} \]

must be satisfied. If both sides of (7) are multiplied by \( \dot{V}^\text{SS}_{\text{com}}(k) \), an equation of powers is obtained, expressing the peer-to-peer energy transfer, where * denotes the complex conjugate. Hence, if the output powers of the network members are changed by the same amount and at the same time, they can be decoupled from the rest of the system. This superposition of a system on another is called peer-to-peer energy transfer.

III. PHASOR-BASED DESCRIPTION OF A BIDIRECTIONAL AC/DC CONVERTER

A. Bidirectional AC/DC Converter as a Two-Port Network

In the previous section, only the Thevenin equivalent of each member was used to model the peer-to-peer energy transfer. However, in our system, each member is a battery, connected to the ac bus through a full-bridge ac/dc converter shown in Fig. 2. To realize a peer-to-peer energy transfer system, a phasor-based model of the converter must be derived. For this, the converter is treated as a two-port network, and the phasor model of ac/dc conversion is presented.

In Fig. 2, switches \( Q_1-Q_4 \) are controlled by pulsewidth modulation (PWM) signals and are assumed to be ideal; when the voltage \( v_1 \) of the dc-side capacitor and the current \( i_2 \) of the ac-side inductor are given, their states and the values of \( v_2 \) and \( i_1 \) in Fig. 2 are listed in Table I.

Fig. 1. Model of the peer-to-peer energy transfer system using variable voltage sources. Each network member, which is a source or load, is represented by its Thevenin equivalent circuit. The voltage phasors change in discrete time.

Fig. 2. Each member of the peer-to-peer energy transfer in Fig. 1 consists of dc battery and full-bridge ac/dc converter. The converter is modeled by a two-port network and the dc battery is modeled by voltage source \( E_i \) and internal resistance \( r_i \).
TABLE I
POSSIBLE STATES OF QU−Q4, AND THE VOLTAGES AND CURRENTS IN THESE STATES

<table>
<thead>
<tr>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$Q_4$</th>
<th>$v_3$</th>
<th>$i_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>$v_1$</td>
<td>$i_2$</td>
</tr>
<tr>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
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<td>$v_1$</td>
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</tr>
</tbody>
</table>

The time spent in each state within the PWM period ($T_{\text{pwm}}$) is denoted by $\tau_1$, $\tau_2$, $\tau_3$, and $\tau_4$. Then, the duty ratio $\alpha$ can be defined as

$$\alpha = \frac{\tau_1 - \tau_2}{T_{\text{pwm}}},$$

where $T_{\text{pwm}} = \tau_1 + \tau_2 + \tau_3 + \tau_4$. (8)

In (8), $\alpha$ is between $-1$ and $1$. If the voltage $v_1$ of the dc-side capacitor and the current $i_2$ of the ac-side inductor are approximately constant over one period $T_{\text{pwm}}$ of the PWM, then the relationship between the averaged voltages and currents $\bar{v}_1$, $\bar{v}_2$, $\bar{i}_1$, $\bar{i}_2$ can be expressed as

$$\begin{align*}
\bar{v}_1 &= \alpha \bar{v}_1 \\
\bar{i}_1 &= \alpha \bar{i}_2.
\end{align*}$$

In (9) and (10), if the voltage and current changes are considerably slower than the frequency of the PWM signal, their relationship can be expressed as functions of time. It can be expressed in the following matrix form:

$$ \begin{bmatrix} \bar{v}_2(t) \\ \bar{i}_2(t) \end{bmatrix} = \begin{bmatrix} \alpha(t) & 0 \\ 0 & 1/\alpha(t) \end{bmatrix} \begin{bmatrix} \bar{v}_1(t) \\ \bar{i}_1(t) \end{bmatrix}. $$

Equation (11) is the TVT equation, first introduced in [24]. The TVT is an element, similar to an ideal transformer; however, the duty ratio, which can change with time, is used instead of the turns ratio.

### B. Phasor Model of the Converter

As the value of $\alpha$ can be changed with time, it can be set to follow the function

$$\alpha_{\text{ac}}(t) = \alpha_{\text{peak}} \cos(\omega t + \phi) = \alpha_{\text{peak}} |\tilde{\alpha}| e^{j\phi} e^{j\omega t}$$

where $\alpha_{\text{peak}}$ is a value between 0 and 1, $\omega$ is the angular frequency of the ac signal, $\phi$ is the phase, $j$ is the imaginary unit, and $|\cdot|$ denotes the real part of . The secondary side is set as the ac side. Then, the ac current becomes

$$\bar{i}_2(t) = \bar{I}_{\text{ac}} \cos(\omega t + \phi).$$

For the dc side, values averaged over one period of the following ac signal are used:

$$ I_{\text{dc}} = \frac{1}{T_{\text{ac}}} \int_0^{T_{\text{ac}}} \bar{i}_1(t) \, dt. $$

Then, $I_{\text{dc}}$ can be expressed as

$$ I_{\text{dc}} = \frac{1}{T_{\text{ac}}} \int_0^{T_{\text{ac}}} \alpha_{\text{peak}} \cos(\omega t + \delta) I_{\text{ac}} \cos(\omega t + \phi) \, dt $$

$$ = \frac{1}{2} \alpha_{\text{peak}} I_{\text{ac}} \cos(\delta - \phi).$$

Replacing the peak values with the rms values (henceforth, rms values will be used), $\alpha = \alpha_{\text{peak}} / \sqrt{2} \text{ and } I_{\text{ac}} = I_{\text{ac}} / \sqrt{2}$; $I_{\text{dc}}$ can be expressed as

$$ I_{\text{dc}} = \alpha I_{\text{ac}} \cos(\delta - \phi) = \Re[\tilde{\alpha}_0^*]$$

where $\tilde{\alpha}_0 = \alpha e^{j\phi}$ and $I_{\text{ac}} = I_{\text{ac}} e^{j\phi}$.

Set $I_{\text{dc}}$ as the phase reference ($\phi = 0$ and $I_{\text{ac}}$ is a real number) gives

$$ I_{\text{dc}} = \Re[\tilde{\alpha}_0^*] = \Re[\tilde{\alpha}] I_{\text{ac}}. $$

Assuming that the dc-side voltage $\bar{v}_1(t)$ is approximately constant, the dc-side averaged voltage $V_{\text{dc}}$ is defined as

$$ V_{\text{dc}} = \frac{1}{T_{\text{ac}}} \int_0^{T_{\text{ac}}} \bar{v}_1(t) \, dt. $$

Then, the ac-side voltage phasor $\tilde{V}_{\text{ac}}$, defined by $\tilde{v}_2(t) = \Re[\tilde{V}_{\text{ac}} e^{j\omega t}]$, can be expressed as

$$ \tilde{V}_{\text{ac}} = \tilde{\alpha} V_{\text{dc}}. $$

Equations (17) and (19) express the connection between the ac and dc sides using phasors and averaged values. These values can change in discrete time $k$. They can be expressed in a matrix form as follows:

$$ \begin{bmatrix} \tilde{V}_{\text{ac}}(k) \\ I_{\text{ac}}(k) \end{bmatrix} = \begin{bmatrix} \tilde{\alpha}(k) & 0 \\ 0 & 1/\Re[\tilde{\alpha}(k)] \end{bmatrix} \begin{bmatrix} V_{\text{dc}}(k) \\ I_{\text{ac}}(k) \end{bmatrix}. $$

As $\tilde{\alpha}$ is an rms-valued phasor, its amplitude can only change between 0 and $1/\sqrt{2}$. Equation (20) expresses the ac/dc converter as a TVT, which changes its $\tilde{\alpha}$ in discrete time.

### C. Possible TVT-Based Linear Two Ports

In (20), a general matrix of ABCD parameters is included, where $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, and $\tilde{D}$ are complex constants, as

$$ \begin{bmatrix} \tilde{V}_{\text{ac}}(k) \\ I_{\text{ac}}(k) \end{bmatrix} = \begin{bmatrix} \tilde{\alpha}(k) & 0 \\ 0 & 1/\Re[\tilde{\alpha}(k)] \end{bmatrix} \begin{bmatrix} V_{\text{dc}}(k) \\ I_{\text{ac}}(k) \end{bmatrix} $$

$$ = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} \begin{bmatrix} V_{\text{dc}}(k) \\ I_{\text{ac}}(k) \end{bmatrix}. $$

Then, the complex power can be expressed as

$$ \tilde{A} \tilde{C}^* V_{\text{ac}}^2(k) + (\tilde{A} \tilde{D}^* + \tilde{B} \tilde{C}^*) V_{\text{dc}}(k) I_{\text{dc}}(k) + \tilde{B} \tilde{D}^* I_{\text{ac}}^2(k) $$

$$ = \tilde{V}_{\text{ac}}(k) \tilde{I}_{\text{ac}}^2(k). $$

Additionally, the power can be expressed using $\alpha$, where $\delta = \angle \tilde{\alpha}$ as follows:

$$ V_{\text{ac}}(k) I_{\text{ac}}(k) = \frac{\alpha e^{j\phi}}{|\tilde{\alpha}| \cos \delta} V_{\text{dc}}(k) I_{\text{dc}}(k) $$

$$ = \frac{e^{j\phi}}{\cos \delta} V_{\text{dc}}(k) I_{\text{dc}}(k). $$

To make (22) and (23) equal, regardless of $I_{\text{dc}}(k)$ and $V_{\text{dc}}(k)$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, and $\tilde{D}$ must satisfy the following conditions:

$$ \tilde{A} \tilde{C}^* = \tilde{B} \tilde{D}^* = 0 $$

$$ \tilde{A} \tilde{D}^* + \tilde{B} \tilde{C}^* = \frac{e^{j\phi}}{\cos \delta}. $$
They have the following two possible solutions:

\[
\begin{bmatrix}
\hat{A} & \hat{B} \\
\hat{C} & \hat{D}
\end{bmatrix} = \begin{bmatrix}
\hat{A} & 0 \\
0 & 1/\Re[\hat{A}]
\end{bmatrix}
\quad (26)
\]

\[
\begin{bmatrix}
\hat{A} & \hat{B} \\
\hat{C} & \hat{D}
\end{bmatrix} = \begin{bmatrix}
0 & \hat{B} \\
1/\Re[\hat{B}] & 0
\end{bmatrix}.
\quad (27)
\]

As (26) is the case of a transformer with a constant duty ratio, it is a step back from the original TVT. Equation (27) exhibits a gyrator-like characteristic; hence, \( \hat{B} = \beta \) is set, where \( \beta \) is the gyrator impedance. Substituting (27) in (21), \( \beta \) can be expressed in terms of \( \dot{\alpha} \) as

\[
\beta = \dot{\alpha}(k)\frac{V_{\text{dc}}(k)}{I_{\text{dc}}(k)}. \quad (28)
\]

It is to be noted that \( \angle \beta = \angle \dot{\alpha} \). A TVT-based gyrator has already been introduced in [25]; however, it was based on a continuous-time approach, whereas ours is based on phasors.

**D. Comparison of the Transformer and Gyrator**

An important property of the gyrator is that it changes a connected one-port element to its dual, i.e., a capacitor is changed into a coil, as often observed in integrated circuit applications. In our case, the gyrator can be used to change the characteristics of the battery, similar to changing a voltage source into a current source. If a battery is connected to the dc side of the converter as shown in Fig. 3, and the converter operates as a transformer with a constant \( \dot{\alpha} \), and its amplitude is limited between 0 and \( 1/\sqrt{2} \), the secondary-side current–voltage characteristic becomes

\[
\dot{V}_{\text{com}}(k) = \dot{\alpha}_i E_i - (\dot{\alpha}_i \Re[\dot{\alpha}_i] r_i + \dot{Z}_i') \dot{I}_i(k) \quad (29)
\]

where \( r_i \) is the internal resistance of the \( i \)th battery, \( E_i \) is the voltage of the \( i \)th battery, and \( \dot{Z}_i' \) is the impedance, which represents the switching losses and reactive components in the \( i \)th converter. The \( \dot{V}_i \) and \( \dot{Z}_i \) in (1) correspond to \( \dot{\alpha}_i E_i \) and \( \dot{\alpha}_i \Re[\dot{\alpha}_i] r_i + \dot{Z}_i' \) in (29), respectively.

If it is operated as a gyrator, the characteristic becomes

\[
\dot{V}_{\text{com}}(k) = \frac{\beta}{r_i} E_i - \left( \frac{\beta \Re[\beta]}{r_i} + \dot{Z}_i' \right) \dot{I}_i(k). \quad (30)
\]

It is to be noted that the current phasor \( \dot{I}_i \) is a real number and its sign indicates whether the battery is a source (positive) or load (negative). Assuming that the values of \( r_i \) and \( \dot{Z}_i' \) are small, a large \( |\beta| \) is selected. The two characteristics of \( \dot{I}_i - |\dot{V}_{\text{com}}| \) in (29) and (30) for a positive, real \( \dot{I}_i \) are illustrated in Fig. 4.

The purpose of peer-to-peer energy transfer is to decouple the network members from the common voltage \( \dot{V}_{\text{com}} \). By controlling the converters as gyrators, they can be rendered less sensitive to the changes in the common voltage. Further in the paper, it is demonstrated that the effect of the changes on the converters can be reduced with timing synchronization.

**IV. REALIZATION OF A POWER GYRATOR FOR PEER-TO-PEER ENERGY TRANSFER**

**A. Gyrator Realization by Voltage–Current Hybrid Control**

It is not easy to directly use (28) to realize the gyrator because the system is discretized with at least one ac period between the sensing of the voltages and currents, and the change in \( \dot{\alpha}_i \). To overcome this, voltage–current hybrid control is used, which tries to control both the voltage and current simultaneously by changing \( \dot{\alpha}_i \). The feedback equation is

\[
|\dot{\alpha}_i(k + 1)| = |\dot{\alpha}_i(k)| + \frac{K_i'[k]}{E_i} \left\{ \dot{V}_{\text{com}}^T - \dot{V}_{\text{com}}(k) + R_{\beta i} (\dot{I}_i^T - \dot{I}_i(k)) \right\}. \quad (31)
\]

Here, \( \dot{V}_{\text{com}}^T \) is the target common voltage, \( \dot{I}_i^T \) is the target ac current of the \( i \)th module for the peer-to-peer energy transfer, and \( R_{\beta i} \) is a real weight factor that decides whether the characteristic is more similar to a current or voltage source. \( K_i'[k] \) is the gain and \( E_i \) is the voltage of the connected battery, used for normalizing the value in parentheses. Because the phase of \( \dot{\alpha}_i(k) \) is controlled such that \( \dot{V}_{\text{com}}(k) \) and the currents \( \dot{I}_i(k) \) have a unity power factor, it is assumed that \( \dot{V}_{\text{com}}^T, \dot{I}_i^T, \dot{V}_{\text{com}}(k), \text{ and } \dot{I}_i(k) \) are real numbers.

If \( \dot{\alpha}_i(k + 1) = \dot{\alpha}_i(k) \) is set, the steady-state characteristic of hybrid control can be shown as

\[
\dot{V}_{\text{com}}^{SS} = \dot{V}_{\text{com}}^T + R_{\beta i} \dot{I}_i^T - R_{\beta i} \dot{I}_i^{SS}. \quad (32)
\]
In (32), $\dot{V}_{\text{com}}^T + R_{\beta i} \dot{I}_i^T$ is the V-axis intersect and $-R_{\beta i}$ is the slope. If they are set equal to the intersect and slope, respectively, of (30), a gyrator in the steady state can be approximately realized as

$$R_{\beta i} = \left| \frac{\dot{\beta}_i \Re[\dot{\beta}_i]}{r_i} + Z_i' \right|, \quad \dot{V}_{\text{com}}^T + R_{\beta i} \dot{I}_i^T = \left| \frac{\dot{\beta}_i}{r_i} E_i \right|. \quad (33)$$

Because $\dot{\beta}_i \approx \Re[\dot{\beta}_i]$ and $|Z_i'| \ll |\dot{\beta}_i^2/r_i|$ are satisfied, $\dot{\beta}_i$ and $R_{\beta i}$ in (33) can be approximately calculated by

$$|\dot{\beta}_i| \approx \frac{E_i/\dot{I}_i^T + \sqrt{(E_i/\dot{I}_i^T)^2 - 4V_{\text{com}}^T/r_i^2}}{2}, \quad R_{\beta i} \approx \frac{\dot{\beta}_i^2}{r_i}. \quad (34)$$

As both $\dot{V}_{\text{com}}^T$ and $\dot{I}_i^T$ lie on the characteristic lines (32), they can be set as target values when realizing a gyrator with hybrid control.

**B. Hybrid-Control Gain**

In the previous section, the setting of the target values and weight factor of the hybrid control were explained. In this section, the gain setting for reducing the transients in the common voltage during peer-to-peer energy transfer is explained. Ideally, the convergence of the current to the target value must be described by the simple relationship

$$\dot{I}_i(k+1) - \dot{I}_i(k) = \Delta \dot{I}_i = K_i (\dot{I}_i^T - \dot{I}_i(k)). \quad (35)$$

If the convergence follows (35), only the value of the constant gain $K_i$ between the two converters of the peer-to-peer energy transfer must be shared for maintaining equal currents even during transients. Hence, the relationship between $\Delta \dot{I}_i$ and $(\dot{I}_i^T - \dot{I}_i(k))$ is derived. As $V_{\text{com}}(k)$ is constant during the peer-to-peer energy transfer, this assumption is made, and the relationship between $\Delta \dot{\alpha}_i$ and $\Delta \dot{I}_i$ is derived, where $\Delta \dot{\alpha}_i = \dot{\alpha}_i(k+1) - \dot{\alpha}_i(k)$. From (29), the relationship between $\dot{\alpha}_i(k)$ and $\dot{I}_i(k)$ as is follows:

$$\dot{\alpha}_i(k)(r_i \Re[\dot{\alpha}_i(k)] + Z_i') \dot{I}_i(k) = \dot{\alpha}_i(k) E_i - \dot{V}_{\text{com}} \quad (36)$$

and is the same for $k+1$, using $\Delta \dot{I}_i$ and $\Delta \dot{\alpha}_i$ as

$$((\dot{\alpha}_i(k) + \Delta \dot{\alpha}_i)(r_i \Re[\dot{\alpha}_i(k)] + Z_i') (\dot{I}_i(k) + \Delta \dot{I}_i) = (\dot{\alpha}_i(k) + \Delta \dot{\alpha}_i) E_i - \dot{V}_{\text{com}}. \quad (37)$$

Subtracting (36) from (37), and rearranging it to express $\Delta \dot{I}_i$ gives, (38) and (39) shown at the bottom of this page, where the approximation $\Delta \dot{\alpha}_i \ll \dot{\alpha}_i$ is used because the small changes of $\alpha_i$ generate large changes of $\dot{I}_i$ in the system.

$$\Delta \dot{I}_i = \frac{\Delta \dot{\alpha}_i E_i - (\dot{\alpha}_i(k) \Re[\Delta \dot{\alpha}_i] + \Re[\dot{\alpha}_i(k)] \Delta \dot{\alpha}_i + \Delta \dot{\alpha}_i \Re[\Delta \dot{\alpha}_i]) r_i \dot{I}_i(k)}{Z_i' + \dot{\alpha}_i(k) \Re[\dot{\alpha}_i(k)] r_i + \dot{\alpha}_i(k) \Re[\Delta \dot{\alpha}_i] r_i + \Delta \dot{\alpha}_i \Re[\Delta \dot{\alpha}_i] r_i} \quad (38)$$

$$\approx \frac{\Delta \dot{\alpha}_i E_i - (\dot{\alpha}_i(k) \Re[\Delta \dot{\alpha}_i] + \Re[\dot{\alpha}_i(k)] \Delta \dot{\alpha}_i) r_i \dot{I}_i(k)}{Z_i' + \dot{\alpha}_i(k) \Re[\dot{\alpha}_i(k)] r_i} \quad (39)$$

Using approximations $\dot{\alpha}_i \approx \Re[\dot{\alpha}_i], \Delta \dot{\alpha}_i \approx |\dot{\alpha}_i(k+1) - \dot{\alpha}_i(k)|, \Delta \dot{\alpha}_i \approx |\dot{\alpha}_i(k+1) - \dot{\alpha}_i(k)|, Z_i' \approx |Z_i'|$, and (31)

$$\Delta \dot{I}_i \approx \frac{E_i - 2|\dot{\alpha}_i(k)| r_i \dot{I}_i(k)}{|Z_i'| + |\dot{\alpha}_i(k)|^2 r_i} \left( |\dot{\alpha}_i(k+1) - \dot{\alpha}_i(k)| \right) \left( |\dot{\alpha}_i(k+1) - \dot{\alpha}_i(k)| \right) \approx \frac{K_1 \Re[\dot{\alpha}_i(k)] r_i \dot{I}_i(k)}{E_i (|Z_i'| + |\dot{\alpha}_i(k)|^2 r_i)} \left( \dot{I}_i^T - \dot{I}_i(k) \right). \quad (40)$$

To make (41) similar to (35), $K_1'$ must be set as a function of $k$ as

$$K_1'(k) = \frac{E_i (|Z_i'| + |\dot{\alpha}_i(k)|^2 r_i)}{R_{\beta i} (E_i - 2|\dot{\alpha}_i(k)| r_i \dot{I}_i(k))}. \quad (42)$$

If $K_1'(k)$ is set to follow the above equation, the transients can be reduced during the peer-to-peer energy transfer.
V. Peer-to-Peer Energy Transfer System Module

In Figs. 5 and 6, the circuit diagram of the ac/dc converter module used in our experiments is depicted. The power circuit consists of four SiC MOSFETs (Rohm SCT2080KE), which are controlled by 60-kHz PWM signals sent through half-bridge drivers (HBD, IRS21844). The capacitor and coil values are $C_{dc} = 471 \mu F$, $L_{dc} = 125 \mu H$, $C_{ac} = 9.4 \mu F$, and $L_{ac} = 1.4 mH$. The ac-side outputs a 60-Hz sinusoidal wave and is implemented by a lookup table of the duty ratio $\alpha_{ac}$.

The microcontroller measures the voltage and current values $V_{com}$, $I_1$, $V_{dc}$, $I_{dc}$ with a 10-bit 60-kHz analog to digital converter, where the digital values are accumulated over $T_{ac}$; the phasor amplitude of the ac values and the averaged values are calculated. The modules communicate with each other through a ZigBee wireless communication module connected through a 9600-bits universal asynchronous receiver/transmitter (UART). For accurate gain matching, the values of $E_i$ and $r_i$ must be measured. $E_i$ is measured as the dc-side voltage when the dc-side current is zero. Then, $r_i$ is calculated by

$$r_i = \frac{E_i - V_{dc}}{I_{dc}}. \quad (43)$$

Each converter output is synchronized with the common voltage. The synchronization algorithm relies on the zero-cross of the signal. The converter measures the time between the zero-crosses and sets its output period equal to the sensed period. After the frequencies are synchronized, the converter starts adjusting its own phase such that its zero-crosses match the common voltage. This adjustment is done by a simple feedback, where the time difference between the zero-crosses of the common signal and the converter is measured and the phase of the converter is adjusted to reduce this difference.

For peer-to-peer energy transfer, it is necessary to time the changes accurately. The details of our timing synchronization protocol are described in Appendix A, which is based on estimating the communication delays in the system, and is further enhanced using zero-cross sensing.

VI. Modeling and Experiments

A. Network Layout for Peer-to-Peer Energy Transfer

For the modeling and experiments, a network consisting of three network members was used, as shown in Fig. 7. Two of the converters were operated as gyrators and participated in peer-to-peer energy transfer. The module of $\dot{\alpha}_1$ is a source side and the module of $\dot{\alpha}_2$ is a load side. The module $\dot{\alpha}_2$ requests a power $P_1 = -P_2$ to the module $\dot{\alpha}_1$ and both modules change their powers from 0 to $P_1 = V_{com} I_1^T$ and $P_2 = V_{com} I_2^T$ simultaneously. The third was controlled by voltage feedback. In this case, the “rest of the network” is represented by the voltage-controlled converter. As there is only one module, it is a weak and sensitive voltage source, making the possible errors obvious. Nominal parameter values are listed in Table II. That is, the power of the transfer changes from 0 to 400 W with gain $K = 0.5$.

![Fig. 7. Circuit used for modeling. Converters with $\dot{\alpha}_1$ and $\dot{\alpha}_2$ are participating in peer-to-peer energy transfer and are controlled by voltage–current hybrid feedback. $\dot{\alpha}_3$ is calculated using pure voltage feedback.](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{com}^*$</td>
<td>100 V</td>
</tr>
<tr>
<td>$I_{E_i}^*$</td>
<td>4 A</td>
</tr>
<tr>
<td>$E_1, E_2, E_3$</td>
<td>215 V</td>
</tr>
<tr>
<td>$r_1, r_2, r_3$</td>
<td>2.3Ω</td>
</tr>
</tbody>
</table>

The feedback equations are as follows:

$$|\dot{\alpha}_1(k + 1)| = |\dot{\alpha}_1(k)| + \frac{K_{1}(k)}{E_1} \left\{ V_{com}(k) - V_{com}^*(k) \right\} + R_{j1} (I_1 - \dot{I}_1(k))$$

$$|\dot{\alpha}_2(k + 1)| = |\dot{\alpha}_2(k)| + \frac{K_{2}(k)}{E_2} \left\{ V_{com}(k) - V_{com}^*(k) \right\} + R_{j2} (I_2 - \dot{I}_2(k))$$

$$|\dot{\alpha}_3(k + 1)| = |\dot{\alpha}_3(k)| + \frac{K_{V}}{E_3} \left( V_{com}(k) - V_{com}^*(k) \right)$$

where $I_1^T = -I_2^T = I_3^T$.

B. Simulation for Peer-to-Peer Energy Transfer

To create a benchmark for comparing our experimental results, the network was modeled in discrete time and was implemented on MATLAB. It was assumed that $V_{com}$ and the currents were synchronized, i.e., they have a unity power factor. $\dot{\alpha}_i$ was calculated using the feedback equations (44)–(46). Using $\dot{\alpha}_i$, the voltages and currents were calculated by solving the circuit equations. The details of the calculations are provided in Appendix B. Only the simulation steps are depicted here. They are as follows:

1) The initial parameters of the model, such as $E_i$, $r_i$, and $Z_i'$, and the initial value of $\dot{\alpha}_i(0)$ are set. Using these
parameters, the initial values of $\dot{I}_1(0)$ and $\dot{V}_{\text{com}}(0)$ are calculated, additionally.

2) The target real values $\dot{V}_{\text{com}}^T$ and $\dot{I}^T$ are set, and the appropriate gyrator impedance $R_{ji}$ is calculated by (34), for the modules participating in peer-to-peer energy transfer.

3) The new $(k + 1)$ values of $|\alpha_i|$ are calculated based on feedback equations (44)–(46).

4) Using $|\alpha_i(k + 1)|$, the values of $\dot{V}_{\text{com}}(k + 1)$ and $\dot{I}_i(k + 1)$ are calculated.

5) Using $|\alpha_i(k + 1)|$, $\dot{V}_{\text{com}}(k + 1)$, and $\dot{I}_i(k + 1)$, the phases of $\dot{\alpha}_i(k + 1)$ are calculated.

6) Repeat steps 3)–5), a set number of times.

C. Experiment 1: Impedance Measurement

To use (42) for setting the hybrid control gain, the impedances $\dot{Z}_1'$ and $\dot{Z}_2'$ of the converters are needed. To measure this, our experimental results are compared and values that provide the closest match are selected. The results are shown in Fig. 8. In Fig. 8(a), the $\dot{Z}_1'$ and $\dot{Z}_2'$ values were set to zero for both the model and the module control algorithm, respectively. The filled dots indicate the experimental results and the white dots, the calculated results by the model. The modeling and experimental results show different results because the impedances $\dot{Z}_1'$ and $\dot{Z}_2'$ were not considered. In Fig. 8(b), the values of $\dot{Z}_1'$ and $\dot{Z}_2'$ were adjusted such that they match the experimental results as much as possible. These values were $\dot{Z}_1' = \dot{Z}_2' = 1.4 + 0.5j \Omega$. In the other experiments, these values were used for the model as well as the experiments.

D. Experiment 2: Peer-to-Peer Energy Transfer

In this experiment, $\dot{Z}_1'$ and $\dot{Z}_2'$ are used to calculate the gain matching of the hybrid control. The results are shown in Fig. 9, where the power $P_i$ is defined by $P_i = \dot{I}_i \dot{V}_{\text{com}}$ because the power factor equals to 1. The modeling and experimental results are close, demonstrating that the previously calculated $\dot{Z}$ is accurate. Peer-to-peer energy transfer is successful because the change in output power did not affect the common voltage considerably. The differences between the model and measurement are due to the inaccuracies in the control of the converter, such as the voltage and current measurement errors, and the low granularity of the discrete values used by the microcontroller.

E. Experiment 3: Comparison of Peer-to-Peer Energy Transfer With and Without Gain Matching

In this experiment, the importance of gain matching during peer-to-peer energy transfer is demonstrated. In the previous experiments, as the batteries were similar even without gain matching, the results may have been good; however, in these two experiments, $r_1$ is changed to approximately 9.0 Ω by inserting
Fig. 10. Transients during peer-to-peer energy transfer, when gain matching is turned off. A resistor is inserted between the first module and its battery to change the resistance to 90Ω. As the changes between the modules are not matched, the transients are significant. (a) Voltage at the common point $V_{\text{com}}$. (b) Power of each module $P_1$, $P_2$, and $P_3$ defined by $V_{\text{com}} I_1$, $V_{\text{com}} I_2$, and $V_{\text{com}} I_3$, respectively, because of the unity power factor. The filled dots indicate the experimental results and the white dots, the results calculated by the model.

It can be seen that there is a significant change in voltage and the output power of the voltage source, signaling that peer-to-peer energy transfer was not achieved in the beginning. In the next experiment, gain matching was turned on. The results are shown in Fig. 11. In this case, the transients are considerably smaller. The powers are imperfectly matched as the assumption that $\Delta I$ is small does not hold in this case because a relatively high gain of 0.5 is set. The transient can be further reduced by selecting a smaller gain; however, this would result in slower convergence.

**F. Experiment 4: Peer-to-Peer Energy Transfer During Timing Desynchronization**

In this experiment, the importance of timing synchronization is demonstrated. The experimental setup is similar to those used previously; however, the changes are not synchronized in time, i.e., there is a time difference of five ac periods between the changes. The results are shown in Fig. 12. The timing desynchronization caused large transients because the converters changed their outputs at different times, causing unbalance in the system. As our system is simple, the converters finally reached their target values; however, in a more complicated system, such large

Fig. 11. Transients during peer-to-peer energy transfer with gain matching. A resistor was connected between the first converter and its battery. The transients are considerably smaller than those in Fig. 10 and can be further reduced by setting a lower gain. (a) Voltage at the common point $V_{\text{com}}$. (b) Power of each module $P_1$, $P_2$, and $P_3$ defined by $V_{\text{com}} I_1$, $V_{\text{com}} I_2$, and $V_{\text{com}} I_3$, respectively, because of the unity power factor. The filled dots indicate the experimental results and the white dots, the calculated results by the model.

Fig. 12. Transients during timing desynchronization. There is a five-ac-period time difference between the two modules. The transients are large, indicating the imbalance in the system. (a) Voltage at the common point $V_{\text{com}}$. (b) Power of each module $P_1$, $P_2$, and $P_3$ defined by $V_{\text{com}} I_1$, $V_{\text{com}} I_2$, and $V_{\text{com}} I_3$, respectively, because of the unity power factor. The filled dots indicate the experimental results and the white dots, the calculated results by the model.
transients might cause some of the system elements to break down.

VII. CONCLUSION

A peer-to-peer energy transfer system was introduced, in which two network members match their changes in the order of milliseconds, for decoupling the power network from the common bus. A phasor-based model of the bidirectional ac/dc converter was formulated as a two-port representation, and used for deriving a gyrator using the TVT concept. It was demonstrated that the gyrator can be realized in the steady state using voltage–current hybrid control, and a gain-matching method was proposed for the transients of the peer-to-peer energy transfer.

In addition, a timing synchronization protocol was introduced for peer-to-peer energy transfer communication, based on the estimation of the communication delays between the members of the network. The transients of the peer-to-peer energy transfer were modeled and compared with the experimental results. From these results, the possibility of peer-to-peer energy transfer was confirmed.

APPENDIX A

TIMING-SYNCHRONIZATION PROTOCOL

A. Constituents of the Communication Delay Between Two Modules

The communication delay between two modules can be divided into three parts, as shown in Fig. 13.

1) $T_{S\text{-int}}$ is the time taken to generate a message in the sender-module microcontroller ($\mu C_S$) and send it to the ZigBee module ($ZB_S$) through a UART. This delay time can be measured during design and saved as a constant.

2) $T_{R\text{-int}}$ is the time to transfer the message from the ZigBee ($ZB_R$) to the microcontroller ($\mu C_R$), decode, and act on it. This delay time can be measured during design and saved as a constant.

3) $T_{\text{air}}$ is the time between $T_{S\text{-int}}$ and $T_{R\text{-int}}$, which includes message modulation, wireless transfer, and demodulation. As this time depends upon the distance between the modules, the models of the ZigBee modules, etc., it must be estimated for each message.

B. Protocol for Timing Synchronization

To accurately synchronize the timing of the two modules, the communication delay between them is estimated. As every module in the system is synchronized to the same ac signal, the zero-cross of the ac signal is used to improve accuracy. The timing-synchronization protocol is shown in Fig. 14.

1) The sender module generates a synchronization-start order and transfers it through a UART. At time $t_1$, the UART transfer is complete and the sender microcontroller starts a timer. $ZB_S$ sends a message, which has an acknowledge (ACK) request, causing $ZB_R$ to send back an ACK message, without communicating with its microcontroller.

2) At $t_2$, $ZB_S$ receives the ACK, and when the first byte is transferred through the UART to $\mu C_S$, it stops its timer. The value of timer $T_{\text{ACK}}$ is used to estimate $T_{\text{air}}$ because $T_{\text{ACK}} \approx 2T_{\text{air}}$.

3) After $\mu C_R$ receives the synchronization-start order, it sends its own internal delay $T_{R\text{-int}}$ to the sender module.

4) When $\mu C_S$ receives $T_{R\text{-int}}$, it calculates the delay and sends the start order such that it arrives approximately at a minus-to-plus zero-cross [$t_{\text{goal}}$, as shown in Fig. 15(b)].

5) From $t_3$, when the transfer of the message through the UART is complete, the sender module waits for $T_{\text{wait}} = T_{R\text{-int}} + T_{\text{ACK}}/2$. At $t_{\text{S-ready}}$, it switches to a standby state waiting for the next minus-to-plus zero-cross.

6) When $\mu C_R$ receives and deciphers the start order at $t_{R\text{-ready}}$, it starts waiting for the next plus-to-minus zero-cross.

7) At the next plus-to-minus zero-cross, both modules change their output simultaneously.
The experimental results are shown in Fig. 15. Using the protocol, the changes were timed, in the order of milliseconds. Using the zero-cross of the common voltage, the timing difference was several microseconds.

APPENDIX B
MODELING DETAILS

In our model, the power factor at the common point is unity, implying that the phases of the currents and \( V_{\text{com}} \) are all zero. Then, the relationship between \( V_{\text{com}} \) and \( I_1 \) can be expressed as follows:

\[
\dot{V}_{\text{com}} = \dot{\alpha}_i E_i - (\dot{\alpha}_i \Re(\dot{\alpha}_i) r_i + \dot{Z}_i') \dot{I}_i(k), \quad \text{where} \quad i = 1, 2, 3.
\]  

(47)

These equations can be written separately for the real and imaginary parts as

\[
\dot{V}_{\text{com}} = \Re(\dot{\alpha}_i) E_i - (\Re(\dot{\alpha}_i) r_i + \Re[\dot{Z}_i']) \dot{I}_i
\]

(48)

\[
0 = \Im(\dot{\alpha}_i) E_i - (\Im(\dot{\alpha}_i) \Re(\dot{\alpha}_i) r_i + \Im[\dot{Z}_i']) \dot{I}_i
\]

(49)

Equations (48) and (49) are solved and substituted in

\[
|\alpha_i| = \Re(\dot{\alpha}_i)^2 + \Re[\dot{Z}_i']^2
\]

(50)

Using a symbolic math package, it is rearranged into the following equation, such that \( I_i \) is expressed as a function of \( \dot{V}_{\text{com}} \):

\[
p_{i4} \dot{I}_i^4 + p_{i3} \dot{I}_i^3 + p_{i2} \dot{I}_i^2 + p_{i1} \dot{I}_i + p_{i0} = 0
\]

(51)

\[
p_{i4} = |\alpha_i|^4 r_i^2 |\dot{Z}_i'|^2 + 2 |\alpha_i|^2 r_i \Re[\dot{Z}_i']^3
\]

\[
+ 2 |\alpha_i|^2 r_i |\dot{Z}_i'|^3 \Re[\dot{Z}_i'] + 2 |\dot{Z}_i'|^2 \Re[\dot{Z}_i']^2 + \Im[\dot{Z}_i']^4
\]

(52)

\[
p_{i3} = 2 |\alpha_i|^2 r_i^2 |\dot{Z}_i'| |V_{\text{com}} + 6 |\alpha_i|^2 r_i \Re[\dot{Z}_i'] |V_{\text{com}} + 4 |\alpha_i|^2 |\dot{Z}_i'|^2 V_{\text{com}}
\]

\[
+ 2 |\alpha_i|^2 r_i \Im[\dot{Z}_i']^3 V_{\text{com}} + 4 |\alpha_i|^2 \Re[\dot{Z}_i'] |V_{\text{com}}^2 + 4 |\alpha_i|^2 |\dot{Z}_i'|^2 V_{\text{com}}^2
\]

(53)

\[
p_{i2} = |\alpha_i|^4 r_i^2 V_{\text{com}} - |\alpha_i|^2 r_i \Re[\dot{Z}_i']^2 E_i^2 - |\alpha_i|^2 \Im[\dot{Z}_i']^2 E_i^2
\]

\[
+ 6 |\alpha_i|^2 r_i \Re[\dot{Z}_i'] |V_{\text{com}} + 6 |\alpha_i|^2 |\dot{Z}_i'|^2 V_{\text{com}} + 2 \Im[\dot{Z}_i'] |V_{\text{com}}^2
\]

(54)

\[
p_{i1} = -2 |\alpha_i|^2 |\dot{Z}_i'| E_i^2 V_{\text{com}} + 2 |\alpha_i|^2 r_i \dot{V}_{\text{com}} + 4 |\alpha_i|^2 |\dot{Z}_i'| V_{\text{com}}^3
\]

(55)

\[
p_{i0} = -|\alpha_i|^2 E_i^2 V_{\text{com}} + V_{\text{com}}^4.
\]

(56)

Solving (51) for \( I_i \), four roots for each \( \dot{I}_i \) are obtained. An expression is obtained, where the only unknown is \( V_{\text{com}} \) by using the following:

\[
I_1 + I_2 + I_3 = 0.
\]

(57)

Equation (57) is solved using a numerical solver function for every combination of \( \dot{I}_i \) roots. To select the correct root, the resulting \( V_{\text{com}} \) value is substituted into (51) and a solution where every \( \dot{I}_i \) and \( V_{\text{com}} \) has a real component only, and \( V_{\text{com}} \) is nonzero is selected. There is only one such solution.

REFERENCES


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