

Equivalent circuit of interleaved air-core toroidal transformer derived from analogy with coupled transmission lines

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Abstract An air-core transformer is a suitable device for megahertz-class power converters. This letter presents an equivalent circuit of a fabricated interleaved air-core toroidal transformer. In order to determine the modeling parameter of the transformer, the electrical characteristics are discussed on the basis of the frequency responses measured from 0.1 to 110 MHz. As a result, it is found that the fabricated transformer is dominated by the mutual capacitance in addition to the self- and mutual inductance. The proposed equivalent circuit is distributed parameter model. This is derived from analogy with the coupled transmission lines. The equivalent circuit shows an excellent agreement with the experiment. These results conclude that the importance of the mutual capacitance to be implemented as a distributed parameter.

Keywords: air-core transformer, equivalent circuit, parasitic capacitance, coupled transmission lines

Classification: Power devices and circuits

1. Introduction

Megahertz operation of power converters has been interested [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Power converters are strongly required to reduce their size and weight. The size of power converters is governed by the volume of the passive components. Their volumes depend on the amount of the energy handled in a switching cycle. The increase of the switching frequency can reduce the energy stored in a switching cycle. Therefore, high-frequency switching above megahertz permits to fabricate power converters with smaller passive components [13].

In the frequency range above megahertz, air-core configurations in magnetic components become the popular option [14, 15, 16, 17, 18, 19, 20, 21]. This is because the iron losses of the conventional magnetic materials such as Mn-Zn and Ni-Zn are not practically negligible in that frequency range. Since the air-core magnetic components do not use a magnetic material, it is necessary to wind the winding closely to confine the magnetic flux inside themselves. This is also essential for improving the coupling coefficient of an air-core transformer. However, the closely wound winding leads to the increase of the parasitic capacitance. A previous study reported that the current distribution in a toroidal inductor becomes nonuniform because the current flowing through the parasitic capacitance is not negligible in the

DOI: 10.1587/elex.17.20200262 Received July 29, 2020 Accepted October 2, 2020 Publicized October 15, 2020 Copyedited November 10, 2020 high-frequency region [22]. This nonuniformity will occur in the transformer as well. In addition, the nonuniformity is related to the wavelength-dependent phenomena such as $\lambda/4$ resonance. This implies that the transformer has to be treated considering the wavelength-dependent phenomena, in the frequency range that the parasitic capacitance is not negligible.

The circuit simulation is useful to investigate the operation of the power converters considering the wavelengthdependent phenomena. For the circuit simulation, the equivalent circuit of the transformer is indispensable. In the frequency range up to several hundred kilohertz, the equivalent circuits are derived on the basis of the resonant frequencies or the electrostatic energy stored in the transformer [23, 24, 25, 26, 27]. These equivalent circuits are the lumped model. The lumped model is based on the assumption that the voltage and current distributions are uniform in the windings. This is not suitable to investigate the wavelength-dependent phenomena because the phenomena are caused by the nonuniformity. Whereas, the distributed model takes into account the spatial and the time variation of the voltage and current. This model allows to discuss the nonuniformity of the voltage and current distribution in windings. Therefore, the distributed model of the transformer is crucial to investigate the wavelength-dependent phenomena.

The transformer's parasitic capacitances are classified into three types of capacitances, which are the mutual capacitance C_m , winding self capacitances, C_p and C_s , and winding-to-ground capacitances, C_{pg} and C_{sg} . If all capacitances are taken into account in the distributed model, the equivalent circuit becomes complex. In particular, it is difficult to separate these three on the basis of the impedance obtained from windings. Therefore, it is important for constructing the equivalent circuit to identify the dominant parasitic capacitances in the applied frequency range.

This letter presents an equivalent circuit of a fabricated interleaved air-core toroidal transformer. This topology is suitable for confining the magnetic flux. We measure and discuss the frequency responses of the transformers to identify the modeling parameters. Through these results, we derive the equivalent circuit of the transformer from analogy with the coupled transmission lines. The proposed equivalent circuit shows an excellent agreement with the experiment.

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Fig. 1 Interleaved air-core toroidal transformer: (a) schematic diagram, and photographs of top view of (b) narrow-slit transformer, and (c) wide-slit transformer.

 Table I
 List of physical dimensions and number of turns for fabricated interleaved air-core toroidal transformer

| Outer diameter [mm] | 50.0 |
|-----------------------------|------|
| Inner diameter [mm] | 24.0 |
| Height [mm] | 11.2 |
| Number of turns per winding | 15 |

2. Interleaved air-core toroidal transformer

2.1 Physical layout

The interleaved air-core toroidal transformer is fabricated by printed circuit boards (PCBs) and pin headers. Fig. 1 (a) shows the schematic of the transformer. The physical dimensions and the number of turns are explained in Table I. Two types of transformers are fabricated to investigate the effect of the parasitic capacitance in the transformer. As shown in Figs. 1 (b) and (c), the top and bottom PCB slit widths are different for both transformers. One is set at 0.15 mm, and the other is set at 0.9 mm. Hereafter we call them the narrow-slit and wide-slit transformer, respectively.

2.2 Electrical characteristics

This subsection discusses the electrical characteristics of the transformer on the basis of the frequency responses. Then, we identify the parameter that dominates the characteristics of the transformer in the frequency range measured in this letter.

The frequency responses are measured using an impedance analyzer (Keysight Technologies, 4294A, measurable frequency range: 40 Hz to 110 MHz) with a test fixture (Keysight Technologies, 16047E). Open and short calibrations are performed accordingly. The transformers is excited in the frequency range from 0.1 to 110 MHz.

Fig. 2 shows the schematics of the measurement setup. We measure eight kinds of frequency responses for both the transformers. The frequency response in Fig. 2 (b) is measured from the opposite port in Fig. 2 (a). These frequency responses are essentially the same measurements. Similarly, Fig. 2 (c) corresponds to (d), Fig. 2 (e) to (f), and Fig. 2 (g)



Fig. 2 Schematics of measurement setup: (a) $Z_{PP',open}$, (b) $Z_{SS',open}$, (c) $Z_{PP',short}$, (d) $Z_{SS',short}$, (e) $Z_{PS'}$, (f) $Z_{SP'}$, (g) Z_{PS} , and (h) $Z_{PS'}$.

to (h).

Figs. 3 and 4 show the measured frequency responses of the narrow- and wide-slit transformer. The impedances in the above-mentioned relationship are shown with the blue and red lines in the same figure. It can be seen that the blue line is identical to the red line in every figure. This shows that both the transformers are symmetrical for the measured port. Hereinafter, we discuss based on $Z_{PP',open}$, $Z_{PP',short}$, $Z_{PS'}$, and Z_{PS} .

In Figs. 3 (a) and (b), it is seen that the impedances, $Z_{\text{PP',open}}$ and $Z_{\text{PP',short}}$, are linearly increased with reference to logarithm of the frequency range below 20 MHz. We obtained similar results in the wide-slit transformer. These impedances correspond to the inductances of the transformer. Therefore, from these impedances, we can estimate the self-inductances, L_p and L_s , and the mutual inductance L_m by Eqs. (1) and (2).

$$L_{\rm p} = L_{\rm s} = L \left(Z_{\rm PP', open} \right) \tag{1}$$

$$L_{\rm m} = k L_{\rm p}, k = \sqrt{1 - \frac{L \left(Z_{\rm PP', short}\right)}{L \left(Z_{\rm PP', open}\right)}} \tag{2}$$

Where *k* denotes the coupling coefficient and L(Z) the inductance component estimated from the impedance *Z*. The inductance components of $Z_{PP',open}$ and $Z_{PP',short}$ are estimated with the least-squares method. Several resonances are confirmed. These resonances show that the transformers are affected by the parasitic capacitances.

The impedances, $Z_{PS'}$ and Z_{PS} , are linearly decreased with reference to logarithm of the frequency range below 10 MHz in (c) and (d) of Figs. 3 and 4. These impedances correspond to the parasitic capacitances in the transformers. In order to identify the dominant parasitic capacitances, we measured the frequency responses of the air-core toroidal inductors. These inductors are made by removing the secondary wind-



Fig. 3 Measured frequency responses of narrow-slit air-core toroidal transformer: (a) $Z_{PP',open}$, $Z_{SS',open}$, (b) $Z_{PP',short}$, $Z_{SS',short}$, (c) $Z_{PS'}$, $Z_{SP'}$, and (d) Z_{PS} and $Z_{P'S'}$. The impedances are shown with the blue and red lines. The resonant frequencies are indicated with green lines.



Fig. 4 Measured frequency responses of wide-slit air-core toroidal transformer: (a) $Z_{PP',open}$, $Z_{SS',open}$, (b) $Z_{PP',short}$, $Z_{SS',short}$, (c) $Z_{PS'}$, $Z_{SP'}$, and (d) Z_{PS} and $Z_{P'S'}$. The impedances are shown with the blue and red lines. The resonant frequencies are indicated with green lines.

ing from the transformers as shown in Fig. 5. Hereinafter, we call them the narrow-slit and wide-slit inductor, respectively. The frequency responses of these inductors are not affected by the mutual capacitance because there is not the secondary winding. Fig. 6 shows the frequency responses of the inductors. The blue line shows the frequency response of the



Fig. 5 Photographs of top view of (a) narrow-slit inductor and (b) wide-slit inductor.



Fig. 6 Frequency responses of narrow- and wide-slit air-core toroidal inductor. The blue line shows the impedance of the narrow-slit toroidal inductor and the red line the impedance of the wide-slit toroidal inductor.

 Table II
 List of estimated parameters of interleaved air-core toroidal transformers

| Parameter | Narrow-slit | Wide-slit |
|-----------------------------------|-------------|-----------|
| $L_{\rm p}, L_{\rm s} [{\rm nH}]$ | 547 | 564 |
| $L_{\rm m}$ [nH] | 337 | 322 |
| k | 0.615 | 0.570 |
| <i>C</i> _m [pF] | 76.7 | 43.0 |

narrow-slit inductor and the red line the wide-slit inductor. Fig. 6 indicates that the effects of the winding self capacitance and winding-to-ground capacitance can be negligible because no clear resonance is confirmed in the measured frequency range. Therefore, $Z_{PS'}$ and Z_{PS} correspond to the mutual capacitance. We estimate the capacitance components of $Z_{PS'}$ and Z_{PS} with the least-squares method from (c) and (d) of Figs. 3 and 4. As a result, we obtained the same values of the mutual capacitance estimated from $Z_{PS'}$ and Z_{PS} . This result supports the validity of the measurement in this letter. The difference between the frequency responses of $Z_{PS'}$ and Z_{PS} in the high-frequency region was caused by the changing polarity of the transformer coupling.

The estimated parameters are summarized in Table II. The self- and mutual inductances are estimated to be almost the same in both the transformers, respectively. While the mutual capacitance is different as approximately twice. Therefore, the mutual capacitance of the interleaved air-core toroidal transformer strongly depends on the slit width.

The resonant frequencies are indicated with the green lines. The frequency responses show similar characteristics between the narrow- and wide-slit transformer. However, all resonant frequencies of the wide-slit transformer are higher than the narrow-slit transformer. That is because the narrow-slit transformer's mutual capacitance is twice as large as the wide-slit transformer. Therefore, it is important for high-frequency equivalent circuit to consider the mutual capacitance in the interleaved air-core toroidal transformer.

 Table III
 List of calculated values of interleaved air-core toroidal transformers regarding coupled transmission lines

| Parameter | Narrow-slit | Wide-slit |
|-------------------------------------|--------------------|----------------------|
| $L_{p'}, L_{s'}$ [nH/m] | 739 | 761 |
| $C_{\rm m'}$ [pF/m] | 104 | 58.0 |
| $v_{\rm p}, v_{\rm s} [{\rm m/s}]$ | 1.14×10^8 | 1.50×10^{8} |

3. Equivalent circuit of interleaved air-core toroidal transformer

This section derives the high-frequency equivalent circuit of the interleaved air-core toroidal transformer from analogy with the coupled transmission lines [28]. Then, we demonstrate the validity of the proposed equivalent circuit through comparisons of the measurement with the circuit simulation.

From the results of Section III, it was found that the mutual capacitance was dominant in the interleaved air-core toroidal transformers. Since the mutual capacitance is distributed between the primary and secondary windings, the transformer viewed from the port P–S' can be regarded as the coupled transmission lines. Therefore, the propagation velocity v are given by Eq. (3).

$$v = \frac{1}{\sqrt{L_{\rm p'}C_{\rm m'}}} = \frac{1}{\sqrt{L_{\rm s'}C_{\rm m'}}}$$
(3)

Where $L_{p'}$ denotes the self-inductance of the primary winding per unit length, $L_{s'}$ the self-inductance of the secondary winding per unit length, and $C_{m'}$ the mutual capacitance between the primary and secondary windings per unit length. The calculated values using the winding length ($\ell = 741$ mm) are listed in Table III.

A circuit element can be regarded as lumped element when the physical dimensions are enough smaller compared to the wavelength. The general consensus is when the dimension is smaller than about 1/20 of the wavelength [29]. The relationship between the wavelength λ , frequency f, and propagation velocity v is given by Eq. (4).

$$v = f\lambda \tag{4}$$

The wavelength is calculated as 1.05 m when the transformer is excited at 110 MHz. Therefore, the transformers cannot be represented until 110 MHz by the lumped element equivalent circuit as shown in Fig. 7 (a) (hereinafter referred as lumped model).

This letter represents the distributed parameter model of the transformer with the lumped element approximation. Fig. 7 (b) shows the equivalent circuit of an arbitrary section of the transformer. Assuming that the self- and mutual inductances and the mutual capacitance are uniformly distributed along the length of the windings, we can derive the equivalent circuit by connecting the equivalent circuits of an arbitrary section, considering the transformer structure. The number of the transformer divisions N is set at 16, so that the symmetry and Eq. (5) are satisfied.

$$N \ge \frac{20\ell}{\lambda} \tag{5}$$

The equivalent circuit is obtained as shown in Fig. 7 (c) (hereinafter referred as distributed model). The parameter



Fig. 7 Schematics of equivalent circuit of interleaved air-core toroidal transformer: (a) lumped model, (b) an arbitrary section, and (c) distributed model.

 Table IV
 List of parameter values of equivalent circuit in interleaved air-core toroidal transformers

| Parameter | Narrow-slit | Wide-slit |
|------------------------------------|-------------|-----------|
| $L_{\rm pd}, L_{\rm sd} [\rm nH]$ | 34.20 | 35.25 |
| $L_{\rm md}$ [nH] | 21.03 | 20.09 |
| k | 0.615 | 0.570 |
| $C_{\rm md}$ [pF] | 2.40 | 1.34 |

values are listed in Table IV.

We verify distributed model through comparisons with circuit simulations. The circuit simulations are conducted using a commercial circuit simulator (SIMetrix Technologies, SIMetrix Ver. 7.2). Figs. 8 and 9 show the comparison of the measurement with the circuit simulation for the narrow- and wide-slit transformer, respectively. The black lines denote the measured results, the red dotted lines the circuit simulation obtained from the distributed model, and the blue dotted lines the circuit simulation obtained from the lumped model. The distributed model represents the frequency responses accurately. The difference between the measurement and circuit simulation to the lumped model gradually increases in the frequency range above 10 MHz. That is because the frequencies, which are not less than 1/20 of the wavelength, are calculated to be 7.7 MHz in the narrow-slit transformer and 10.1 MHz in the wide-slit transformer, respectively. In addition, the lumped model do not represent the third resonant frequency in Figs. 8 (c) and 9 (c). These results indicate the limit of the lamped model at high-frequency region.

4. Conclusion

This letter discussed the electrical characteristics of an interleaved air-core toroidal transformer and derived the equivalent circuit from analogy with the coupled transmission lines. It was found that the frequency responses of the transformers were mainly affected by the mutual capacitance depending on the slit width. The transformer can be modeled on the basis of analogy with the coupled transmission lines when the winding self capacitance and winding-toground capacitance are negligible. The derived distributed equivalent circuit represented the frequency responses accurately, whereas the lumped model was not able to rep-



Fig. 8 Comparisons of measurement with circuit simulation for narrowslit interleaved air-core toroidal transformer. The black lines show the measured results, the red dotted lines the circuit simulation based on the distributed model, and the blue dotted lines the circuit simulation based on the lumped model.



Fig. 9 Comparisons of measurement with circuit simulation for wide-slit interleaved air-core toroidal transformer. The black lines show the measured results, the red dotted lines the circuit simulation based on the distributed model, and the blue dotted lines the circuit simulation based on the lumped model.

resent the high-frequency characteristics. Both equivalent circuits were constructed with the same parameter values estimated from the low-frequency characteristics. Only the circuit topology is different in both models. In addition, the proposed equivalent circuit did not use the high-frequency characteristics such as the resonant frequency. This is inherently different from methods such as Foster's reactance theorem [30], which uses high-frequency characteristics in order to construct the frequency response. These results show the importance of the mutual capacitance to be implemented as a distributed parameter.

Wavelength-dependent phenomena such as $\lambda/4$ resonance and directivity appear in the coupled transmission lines. The equivalent circuit was derived from analogy with the coupled transmission lines. The fact implies that the phenomena cannot avoid due to the switching of the power converter. That is because there are many high-frequency components during switching transients.

We conclude that the proposed equivalent circuit enables the circuit simulation in the frequency range above megahertz. We also expect the model can clarify the relationship between the high-frequency components and the spatial magnetic flux distribution in air-core transformers.

Acknowledgments

This work was partially supported by the Program on Open Innovation Platform with Enterprises, Research Institute and Academia of Japan Science and Technology Agency, Crossministerial Strategic Innovation Promotion Program of New Energy and Industrial Technology Development Organization, and JSPS KAKENHI grant number 20H02151.

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