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**Mathematical structures of integrable  
systems and their applications**

edited by Shinsuke Iwao

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**Research Institute for Mathematical Sciences  
Kyoto University**

*RIMS Kôkyûroku Bessatsu B78*

*Mathematical structures of integrable systems  
and their applications*

*September 5 ~7, 2018*

*edited by Shinsuke Iwao*

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*Research Institute for Mathematical Sciences*

*Kyoto University, Kyoto, Japan*

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## Preface

This volume is the proceedings of the conference “Mathematical structures of integrable systems and their applications” held at *Research Institute for Mathematical Sciences, Kyoto University* in Japan from September 5 to September 7, 2018. Speakers of the conference contributed their research papers or survey articles for this volume.

The aim of the conference was to connect researchers working on classical/quantum integrable systems with professions who work on various mathematical objects such as algebraic geometry, number theory, probability theory, *etc.* I hope the conference was successful and would like to express my gratitude to everyone who contributed his/her efforts. I also appreciate the help of anonymous referees.

January, 2020

Shinsuke Iwao

## Program

### Mathematical structures of integrable systems and their applications, 2018

RIMS 共同研究（公開型）2018  
「可積分系理論から見える数理構造とその応用」

#### September 5, Wed

- 13:00–13:40 岩尾 慎介  
超離散 KP 方程式を用いた Young 盤の組み合わせ論の証明
- 13:50–14:40 増田 哲  
 $D_7^{(1)}$  型  $q$ -笹野系とその有理解
- 14:50–15:40 野邊 厚  
Chaos in cluster algebras
- 15:50–16:40 井上 玲  
Cluster realization of Coxeter groups and its application

#### September 6, Thu

- 9:30–10:20 瀧 真語  
K3 surfaces and log rational surfaces
- 10:30–11:20 上原 崇人  
K3 曲面の構成と力学系への応用
- 11:40–12:30 松谷 茂樹  
超楕円  $\sigma$  関数による戸田格子の周期解と擬周期解について
- 13:40–14:30 中村 あかね  
Geometric aspects of Generalized Hitchin systems
- 14:50–15:40 佐々田 槇子  
ランダムな初期状態をもつ無限粒子箱玉系と Pitman の定理
- 15:50–16:40 大西 良博  
Weierstrass の  $\sigma$  関数に関する熱方程式論の高い種数の場合への拡張について (J. C. Eilbeck 氏, J. Gibbons 氏, S. Yasuda 氏との共同研究)  
On a generalization of the theory of heat equations for the Weierstrass  $\sigma$  function to higher genus case (joint work with J. C. Eilbeck, J. Gibbons, and S. Yasuda)

#### September 7, Fri

- 10:00–10:50 神吉 雅崇  
離散可積分性判定と互いに素条件
- 11:00–11:50 長井 秀友  
超離散 2 次元戸田方程式とヤコビ恒等式
- 13:00–13:50 坂井 秀隆  
CFT approach to the  $q$ -Painlevé equations (Joint work with M. Jimbo and H. Nagoya)
- 14:00–14:50 鈴木 貴雄  
クラスター代数と高階  $q$ -パンルヴェ系

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