On local symbols and the reciprocity law for foliated dynamical systems on 3-manifolds: research announcement (Algebraic Number Theory and Related Topics 2017)

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By

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Abstract

This is a research announcement of the joint work with Jyunheong Kim, Takeo Noda and Yuji Terashima [KMNT]. It is motivated by the question, posed by Deninger, on finding out analogies for 3-manifolds of the Hilbert reciprocity law for number fields along the line of arithmetic topology. We introduce local symbols and show the reciprocity law for a 3-dimensional foliated dynamical system.

1. 3-dimensional foliated dynamical systems

We introduce the notion of a 3-dimensional foliated dynamical system and recall some related notions and facts, following [D1] $\sim$ [D7] and [Ko1], [Ko2].

Definition 1.1. We define a 3-dimensional *foliated dynamical system*, called an *FDS* for short, by a triple $\mathfrak{S} = (M, \mathcal{F}, \phi)$, where
(1) M is a connected, closed, smooth 3-manifold,
(2) $\mathcal{F}$ is a complex foliation by Riemann surfaces on M,
(3) ϕ is a smooth dynamical system on M,
and these data must satisfy the following conditions:
(i) there are finite number of compact leaves $L_1, \ldots, L_r$ such that $\phi^t(L_i) = L_i$ for any i and t, and any orbit of the flow ϕ is transverse to leaves in $M \setminus \cup_{i=1}^r L_i$.

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(ii) each diffeomorphism ϕ^t of M maps any leaf to a leaf.

We denote by $\mathcal{P}_{\mathfrak{S}}$ the set of closed orbits and by $\mathcal{P}_{\mathfrak{S}}^{\infty}$ the set of non-transverse compact leaves $L_1, \ldots, L_r$. We also write γ_i^{∞} for L_i. We set $\overline{\mathcal{P}_{\mathfrak{S}}} := \mathcal{P}_{\mathfrak{S}} \sqcup \mathcal{P}_{\mathfrak{S}}^{\infty}$.

Remark 1.2. A foliated dynamical system $\mathfrak{S} = (M, \mathcal{F}, \phi)$ may be regarded as a geometric analogue of $\overline{\mathrm{Spec}(\mathcal{O}_k)} = \mathrm{Spec}(\mathcal{O}_k) \cup \{\text{infinite primes}\}$ for the ring $\mathcal{O}_k$ of integers of a number field k. Here the set $P_{\mathfrak{S}}$ of closed orbits corresponds to the set of finite primes and the set of compact non-transversal leaves $\gamma_1^{\infty}, \ldots, \gamma_r^{\infty}$ corresponds to the set of infinite primes. So the analogy is closer if $\mathcal{P}_{\mathfrak{S}}$ is a countably infinite set. We give such examples in Section 2.

Let $\mathfrak{S} = (M, \mathcal{F}, \phi)$ be an FDS and let $M_0 := M \setminus \cup_{i=1}^{r} \gamma_i^{\infty}$. Let $T\mathcal{F}$ denote the subbundle of the tangent bundle TM_0 whose total space is the union of the tangent spaces of leaves. Let $\dot{\phi}^t = \frac{d}{dt}\phi^t$ be the vector field on M_0 which generates the flow ϕ.

Definition-Proposition 1.3. *Notations being as above, there exists uniquely the smooth* 1*-form* $\omega_{\mathfrak{S}}$ *on* M_0 *satisfying*

$$\omega_{\mathfrak{S}}|_{T\mathcal{F}} = 0, \;\; \omega_{\mathfrak{S}}(\dot{\phi}^t) = 1 \;\text{ and }\; d\omega_{\mathfrak{S}} = 0.$$

We call $\omega_{\mathfrak{S}}$ the *canonical* 1*-form* of $\mathfrak{S}$.

Definition 1.4. Let $\mathfrak{S} = (M, \mathcal{F}, \phi)$ be an FDS. The de Rham cohomology class of the canonical 1-form $[\omega_{\mathfrak{S}}] \in H^1(M_0; \mathbb{R}) = \mathrm{Hom}(H_1(M_0; \mathbb{Z}), \mathbb{R})$ defines the *period homomorphism*

$$[\omega_{\mathfrak{S}}] : H_1(M_0; \mathbb{Z}) \longrightarrow \mathbb{R}; \;\; [\ell] \mapsto \int_{\ell} \omega_{\mathfrak{S}}.$$

We define the *period group* of $\mathfrak{S}$, denoted by $\Lambda_{\mathfrak{S}}$, to be the image of $[\omega_{\mathfrak{S}}]$.

Definition 1.5. Let $\mathfrak{S} = (M, \mathcal{F}, \phi)$ be a FDS and let $M_0 := M \setminus \mathcal{P}_{\mathfrak{S}}^{\infty}$. An *FDS-meromorphic function* on M is defined to be a smooth map $f : M_0 \to \mathbb{P}^1(\mathbb{C}) = \mathbb{C} \cup \{\infty\}$ satisfying the following conditions:
(1) f restricted to any leaf is a meromorphic function,
(2) the zeros and poles of f lie along finitely many closed orbits.

2. Examples of FDS's

Example 2.1 (Surface bundle over S^1 and Anosov flow). Let Σ be a closed Riemann surface of genus ≥ 1 and let φ be an automorphism of Σ of pseudo-Anosov type. Let

$M(\Sigma;\varphi)$ be the mapping torus define by

$$M(\Sigma;\varphi) = (\Sigma \times \mathbb{R})/(z, \theta+1) \sim (\varphi(z), \theta)$$

and let

$$\varpi : M \to S^1 = \{w \in \mathbb{C}\,;\, |w| = 1\};\ \ \varpi([z,\theta]) := \exp(2\pi i \theta)$$

be the fibration. Let $\mathcal{F} := \{\varpi^{-1}(w)\}_{w \in S^1}$ and let ϕ be the suspension flow defined by

$$\phi^t([z,\theta]) := [z, \theta + t].$$

Then $\mathfrak{S} := (M(\Sigma;\varphi), \mathcal{F}, \phi)$ is an FDS. The pseudo-Anosov property implies that $\mathcal{P}_{\mathfrak{S}}$ is a countable infinite set.

Remark 2.1.1. An FDS of surface bundle over S^1 may be regarded as an analogue of a projective smooth curve C over a finite field $\mathbb{F}_q$, where the 2-dimensional foliation corresponds to the geometric fiber $C \otimes \overline{\mathbb{F}_q}$ and the monodromy φ corresponds to the Frobenius automorphism in $\mathrm{Gal}(\overline{\mathbb{F}_q}/\mathbb{F}_q)$.

Example 2.2 (Reeb foliation on S^3 and the horse-shoe flow). Let $S^3 = V_1 \cup V_2$ be the Heegaard splitting of genus one, where V_i is the solid torus $D^2 \times S^1$. Consider the Reeb foliation on each V_i so that we obtain the foliation $\mathcal{F}$ on S^3 ([CC; 1.1], [Ta; §16]). We define the dynamical system ϕ as follows:
(i) First, we consider a flow on S^3 whose orbits are transverse to any leaf of the Reeb foliation. Then there is the only one closed orbit $\gamma_i = \{0\} \times S^1$ in each Reeb component V_i. We note that around γ_i, the flow is the suspension flow of the contraction map $z \mapsto az$ $(0 < a < 1)$.
(ii) Next, we replace the contraction map in (i) by Smale's horse-shoe map ([Sm]) φ and replace the flow around γ_i by the suspension of φ. Let ϕ be the resulting flow on all of S^3.
Thus we have an FDS $\mathfrak{S} = (S^3, \mathcal{F}, \phi)$ with the only one non-transverse leaf ∂V_i. The property of the horse-shoe map implies that $\mathcal{P}_{\mathfrak{S}}$ is a countable infinite set ([KH; Cor. 2.5.1]).

Example 2.3 (Open book decomposition). Let M be a closed 3-manifold. It is known that M contains a fibered link L, namely, there is a fibration $\varpi : M \setminus V(L)^o \to S^1$, where $V(L)^o$ is an open tubular neighborhood of L. We have the foliation on $M \setminus V(L)^o$ given by fibers of ϖ (leaves are transverse to the boundary $\partial V(L)$). The structure this induces on M is called an open book decomposition ([Ca]). We fill in $V(L)$ with the Reeb component and tubularize (spin) the foliation on $M \setminus V(L)^o$ around $\partial V(L)$ to obtain the foliation $\mathcal{F}$ on all of M. We define the flow on $M \setminus V(L)^o$ by the suspension of the monodromy φ of the fibration ϖ. Here we suppose that φ is pseudo-Anosov (e.g., L is a hyperbolic knot). The flow on $V(L)^o$ is defined to be the one transverse to any leaf of the Reeb foliation. Thus we have an FDS $\mathfrak{S} = (M, \mathcal{F}, \phi)$ with the only one non-transverse leaf $\partial V(L)$ and countably infinite $\mathcal{P}_{\mathfrak{S}}$.

3. Smooth Deligne cohomology

We recall smooth Deligne cohomology groups. A basic reference is [Br].

Let $\mathfrak{S} = (M, \mathcal{F}, \phi)$ be an FDS. Let X be a submanifold of M obtained by removing $\mathcal{P}_{\mathfrak{S}}^{\infty}$ and finitely many closed orbits. Let $\mathcal{A}^i$ denote the sheaf of $\mathbb{C}$-valued smooth i-forms on X. Let Λ be a subgroup of the additive group $\mathbb{R}$. For $n \in \mathbb{Z}$ and $n \geq 0$, we set $\Lambda(n) := (2\pi\sqrt{-1})^n \Lambda$.

Definition 3.1. We define the *smooth Deligne complex* $\Lambda(n)_{\mathscr{D}}$ on X by

$$\Lambda(n)_{\mathscr{D}} : \ \Lambda(n) \to \mathcal{A}^0 \stackrel{d}{\to} \mathcal{A}^1 \stackrel{d}{\to} \cdots \stackrel{d}{\to} \mathcal{A}^{n-1},$$

where $\Lambda(n)$ is put in degree 0 and d denotes the differential. The *smooth Deligne cohomology groups* are defined to be the hypercohomology groups of $\Lambda(n)_{\mathscr{D}}$, denoted by $H^q_{\mathscr{D}}(M; \Lambda(n))$ for $q \geq 0$:

$$H^q_{\mathscr{D}}(X; \Lambda(n)) := \mathbb{H}^q(X; \Lambda(n)_{\mathscr{D}}).$$

In particular, when Λ is the period group $\Lambda_{\mathfrak{S}}$, we call $H^q_{\mathscr{D}}(X; \Lambda_{\mathfrak{S}}(n))$ the *FDS-Deligne cohomology groups* of $\mathfrak{S}$.

We compute the smooth Deligne cohomology groups as Čech hypercohomology groups of an open covering $\mathcal{U} = \{U_a\}_{a \in I}$ of X with coefficients in $\Lambda(n)_{\mathscr{D}}$

$$H^q_{\mathscr{D}}(M; \Lambda(n)) = \mathbb{H}^q(\mathcal{U}; \Lambda(n)_{\mathscr{D}}),$$

where the open covering $\mathcal{U}$ is taken so that all non-empty intersections $U_{a_0 \cdots a_j} := U_{a_0} \cap \cdots \cap U_{a_j}$ are contractible. So a Čech cocycle representing an element of $H^n_{\mathscr{D}}(M; \Lambda(n))$ is of the form

$$(\lambda_{a_0 \ldots a_n}, \theta^0_{a_0 \ldots a_{n-1}}, \ldots, \theta^{n-1}_{a_0}) \in \check{C}^n(\mathcal{U}, \Lambda(n)) \oplus \check{C}^{n-1}(\mathcal{U}, \mathcal{A}^0) \oplus \cdots \oplus \check{C}^0(\mathcal{U}, \mathcal{A}^{n-1})$$

which satisfies the cocycle condition

$$\delta(\theta^0_{a_0 \ldots a_{n-1}}) + (-1)^n \lambda_{a_0 \ldots a_n} = 0, \ \ \delta(\theta^i_{a_0 \ldots a_{n-1-i}}) + (-1)^{n-i} d\theta^{i-1}_{a_0 \ldots a_{n-i}} = 0 \ (i \geq 1),$$

where δ is the Čech differential with respect to the open covering $\mathcal{U}$.

Example 3.2. Let f be an FDS-meromorphic function on M whose zeros and poles are lying along closed orbits $\gamma_1, \dots, \gamma_N$. Let $X := M_0 \setminus \cup_{i=1}^N \gamma_i$. Let $\log_a f$ denote a branch of $\log f$ on U_a. Then the Čech cocycle

$$(n_{a_0 a_1}, \log_{a_0} f),\ n_{a_0 a_1} = (\delta \log f)_{a_0 a_1} = \log_{a_1} f - \log_{a_0} f \in \mathbb{Z}(1)$$

determines the cohomology class of $H^1_{\mathscr{D}}(X; \mathbb{Z}(1))$, by which we denote $c(f)$.

Example 3.3. Fix a base point $p_0 \in X$. For each $U_a (a \in I)$, we choose a point $p_a \in U_a$ and a path γ_a from p_0 to p_a. For $p \in U_a$, we set

$$f_{\omega_{\mathfrak{S}}, a}(p) := 2\pi\sqrt{-1} \int_{\gamma_p \cdot \gamma_a} \omega_{\mathfrak{S}},$$

where γ_p is a path from p_a to p inside U_a. Then the Čech cocycle

$$(\lambda_{a_0 a_1}, f_{\omega_{\mathfrak{S}}, a_0}),\ \lambda_{a_0 a_1} = (\delta f_{\omega_{\mathfrak{S}}})_{a_0 a_1} \in \Lambda_{\mathfrak{S}}(1)$$

defines the cohomology class of $H^1_{\mathscr{D}}(X; \Lambda_{\mathfrak{S}}(1))$, by which we denote $c(\omega_{\mathfrak{S}})$.

Definition 3.4. Let $0 < n \le 3$. We define the *n-curvature homomorphim*

$$\Omega : H^n_{\mathscr{D}}(X; \Lambda(n)) \longrightarrow \mathcal{A}^n(X)$$

by

$$\Omega(c)|_{U_a} := d\theta_a^{n-1}$$

for $c = [(\lambda_{a_0 \dots a_n}, \dots, \theta_{a_0}^{n-1})]$.

When Λ is a subring of $\mathbb{R}$, the smooth Deligne cohomology groups are equipped with the product on the smooth Deligne complexes

$$\Lambda(n)_{\mathscr{D}} \otimes \Lambda(n')_{\mathscr{D}} \longrightarrow \Lambda(n+n')_{\mathscr{D}} \tag{3.5}$$

defined by

$$x \cup y = \begin{cases} xy & \deg(x) = 0, \\ x \wedge dy & \deg(x) > 0 \text{ and } \deg(y) = n', \\ 0 & \text{otherwise.} \end{cases} \tag{3.6}$$

For our purpose, we extend the product (3.5) for the case where Λ is a subring of $\mathbb{R}$ and Λ' is a Λ-submodule of $\mathbb{R}$ as follows. Namely, by the same formula as in (3.6), we have the product

$$\Lambda(n)_{\mathscr{D}} \otimes \Lambda'(n')_{\mathscr{D}} \longrightarrow \Lambda'(n+n')_{\mathscr{D}}$$

which induces

$$H^n_{\mathscr{D}}(X;\Lambda(n)) \otimes H^{n'}_{\mathscr{D}}(X;\Lambda'(n')) \longrightarrow H^{n+n'}_{\mathscr{D}}(X;\Lambda'(n+n')). \tag{3.7}$$

4. Holonomies of Deligne cocycles

We recall holonomy integrals of smooth Deligne cocycles, following [GT1], [GT2] and [Te].

As in Section 3, let $\mathfrak{S} = (M, \mathcal{F}, \phi)$ be an FDS and let X be a submanifold of M obtained by removing $\mathcal{P}_{\mathfrak{S}}$ and some finitely many closed orbits. For $1 \leq n \leq 3$, let $c \in H^n_{\mathscr{D}}(X;\Lambda(n))$ and let Y be an $(n-1)$-dimesnional closed submanifold of X. We shall define a paring $\int_Y c$, which takes values in $\mathbb{C}$ mod $\Lambda(n)$, as follows.

First, we fix an open covering $\mathcal{U} = \{U_a\}_{a\in I}$ such that all non-empty intersections $U_{a_0\cdots a_j} := U_{a_0} \cap \cdots \cap U_{a_j}$ are contractible and choose a Čech representative cocycle $(\lambda_{a_0\ldots a_n}, \theta^0_{a_0\ldots a_{n-1}}, \ldots, \theta^{n-1}_{a_0})$ of c. Second, we choose a smooth finite triangulation $K = \{\sigma\}$ of Y and an index map $\iota : K \to I$ satisfying $\sigma \subset U_{\iota(\sigma)}$. For $i = 0, \ldots, n-1$, we define the set $F_K(i)$ of flags of simplices

$$F_K(i) := \{\vec{\sigma} = (\sigma^{n-1-i}, \ldots, \sigma^{n-1}) \,|\, \sigma^j \in K, \dim \sigma^j = j, \sigma^{n-1-i} \subset \cdots \subset \sigma^{n-1}\}.$$

Then, we define

$$\int_Y c := \sum_{i=0}^{n-1} \sum_{\vec{\sigma}\in F_K(i)} \int_{\sigma^{n-1-i}} \theta^{n-1-i}_{\iota(\sigma^{n-1})\iota(\sigma^{n-2})\ldots\iota(\sigma^{n-1-i})} \mod \Lambda(n).$$

The following theorems were proved in [Te].

Theorem 4.1. *This definition is independent of all choices.*

Theorem 4.2. *If there is an n-dimensional submanifold Z of X whose boundary is $\partial Z = Y$, we have*

$$\int_Y c = \int_Z \Omega(c) \mod \Lambda(n).$$

5. Hilbert type reciprocity law

Let $\mathfrak{S} = (M, \mathcal{F}, \phi)$ be an FDS. Let f and g be FDS-meromorphic functions on M whose zeros and poles lie along $\gamma_1, \ldots, \gamma_N \in \mathcal{P}_{\mathfrak{S}}$. We set $X := M_0 \setminus \cup_{i=1}^N \gamma_i$. For $\gamma \in \overline{\mathcal{P}_{\mathfrak{S}}}$, let $V(\gamma)$ denote a tubular neighborhood of γ and we denote by $T(\gamma)$ the boundary of $V(\gamma)$.

As in Examples 3.2, we have the smooth Deligne cohomology classes

$$c(f) = [(m_{a_0a_1}, \log_{a_0} f)],\ c(g) = [(n_{a_0a_1}, \log_{a_0} g)] \in H^1_{\mathscr{D}}(X; \mathbb{Z}(1))$$

and, as in Example 3.3, we have the FDS-Deligne cohomology class

$$c(\omega_{\mathfrak{S}}) = [(\lambda_{a_0a_1}, f_{\omega_{\mathfrak{S}},a_0})] \in H^1_{\mathscr{D}}(X; \Lambda_{\mathfrak{S}}(1)).$$

By the product in (3.7) applied to the case that $\Lambda = \mathbb{Z}$ and $\Lambda' = \Lambda_{\mathfrak{S}}$, we have the FDS-Deligne cohomology class

$$c(f) \cup c(g) \cup c(\omega_{\mathfrak{S}}) \in H^3_{\mathscr{D}}(X; \Lambda_{\mathfrak{S}}(3)).$$

Definition 5.1. We define the *local symbol* $\langle f, g\rangle_\gamma$ by

$$\langle f, g\rangle_\gamma := \int_{T(\gamma)} c(f) \cup c(g) \cup c(\omega_{\mathfrak{S}}) \mod \Lambda_{\mathfrak{S}}(3).$$

We note that the integral of the r.h.s. is finite, since $T(\gamma)$ is compact.

Theorem 5.2. (Hilbert type reciprocity law). *We have*

$$\sum_{\gamma \in \overline{\mathcal{P}_{\mathfrak{S}}}} \langle f, g\rangle_\gamma = 0 \mod \Lambda_{\mathfrak{S}}(3).$$

Theorem 5.2 follows from Theorem 4.2 ($Z = X \setminus (\cup_{i=1}^r V(\gamma_i^\infty)^o \cup \cup_{i=1}^N V(\gamma_i)^o)$, $Y = \cup_{i=1}^r T(\gamma_i^\infty) \cup \cup_{i=1}^N T(\gamma_i)$) and that the 3-curvature $\Omega(c(f) \cup c(g) \cup c(\omega_{\mathfrak{S}})) = 0$.

Remark 5.3. (1) Definition 5.1 and Theorem 5.2 may be regarded as natural extensions to a 3-dimensional foliated dynamical system of the reciprocity law for tame symbols on a Riemann surface ([Bl], [BM], [Dl]).
(2) Stelzig ([St]) introduced local symbols and showed the reciprocity law for surface bundles over S^1. Our results generalize his results for FDS's by means of holonomy integrals of Deligne cohomology.

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