Cohomology of conical symplectic resolutions

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10/22 2020

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Definition

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A smooth symplectic variety (X, ω) with an action of $\mathbb{S} = \mathbb{C}^*$ is a conical if X is the following coniditions,

- $\phi: X \to X_0 = \operatorname{Spec}\mathbb{C}[X]$ is projective birational.
- \circ $s^*\omega = s^n\omega$ for $s \in \mathbb{C}^*$.
- $\ \ \, \mathbb C^*$ acts on $\mathbb C[X]$ with only non-negative weights and $\mathbb C[X]^{\mathbb C^*}=\mathbb C$

Example

Let X be a crepant resolution of \mathbb{C}^2/G , where a group G is a finite subgroup of $\mathrm{SL}_2(\mathbb{C})$. Then the Hilbert scheme on n-points of X is a conical symplectic resolution.

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Symplectic dual

Consider a conical symplectic resolution X with a Hamiltonian action of $\mathbb{T} = \mathbb{C}^*$, commuting \mathbb{S} , such that $X^{\mathbb{T}}$ is finite.

Let \mathbb{W} be a Weyl group of the group of Hamiltonian symplectic morphisms of X and W be a Namikawa Weyl group of X.

Definition

A symplectic duality from X to $X^!$ consists of

- ① There exists a order reversing bijection from the index set of $X^{\mathbb{T}}$ to the index set of $(X^!)^{\mathbb{T}}$.
- $m{Q}$ $W\simeq \mathbb{W}^!$ and $W^!\simeq \mathbb{W}$
- etc

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Examples

- The Hilbert scheme on n-points of \mathbb{C}^2 is self dual.
- An affine A type quiver variety and other affine A type quiver variety are symplectic dual.
- A hypertoric variety and other hypertoric variety are symplectic dual.

Conjecture

Conjecture (Hikita, 2015)

If $X \to X_0$ and $X' \to X_0'$ are symplectic dual, then we have the following graded algebra isomorphim:

$$H^*(X) \simeq C[X_0^{\mathbb{T}}]$$

$$H^*(X') \simeq C[X_0^T]$$

Theorem (Hikita, 2015)

In the case of the Hilbert scheme on n-points of \mathbb{C}^2 and a hypertoric variety, type A S3-variety, this conjecture is true.

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Case of the framed moduli space

Let M(r, n) be the framed moduli space of torsion free sheaves on \mathbb{P}^2 with rank r and $c_2 = n$. Then M(r, n) is a conical symplectic resolution.

Theorem (Nakajima-Yoshioka, 2003)

we have the following equations:

$$\sum_{d} \dim H^{d}(M(r,n))t^{d} = \sum_{(Y_{1},\cdots,Y_{r})} t^{2(rn-(\sum_{j}jl(Y_{j})))}$$

where $(Y_1 \cdots, Y_r)$ is r—tuples Young diagram running over $\sum |Y_i| = n$, $I(Y_i)$ is a length of Young diagram Y_i

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Main result

Let X_r be a crepant resolution of $\mathbb{C}^2/(\mathbb{Z}/r\mathbb{Z})$.

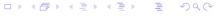
Theorem (Ben-Licata-Proudfoot-Webster, 2014)

 $\operatorname{Hilb}^n(X_r)$ and M(r, n) are symplectic dual.

Theorem (Hatano)

There exists a graded vector space isomorphism

$$H^*(M(r,n))\simeq \mathbb{C}[((\mathrm{Hilb}X_r)_0)^{\mathbb{C}^*}].$$



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