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Dynamic Scaling of the Growing Rough Surfaces

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§1. Introduction

The formation of rough surfaces is a rich phenomenon in nature.[1)] In a strict sense, there is almost nothing of a smooth surface but a rough one. Typical examples for the growing rough surfaces include crystal growth,[2)] viscous flow in porous media,[3)] mountain topography,[4)] and bacterial colony.[5)] It is one of very important problems in statistical physics to understand the growing mechanism of such surfaces. In 1985, Family and Vicsek proposed the scaling hypothesis for the growing rough surfaces.[6)] Based on the scaling, a lot of experimental and theoretical studies, such as experiments mentioned above and Kardar-Parisi-Zhang (KPZ)[7)] equation and Edwards-Wilkinson (EW)[8)] equation, have been carried out to confirm the Family-Vicsek scaling behavior.

The Family-Vicsek (FV) scaling is described as follows. Let us consider a rough surface which grows from the substrate of line seeds. Namely, for simplicity, we take a rough surface grown from one-dimensional substrate in two-dimensional space, i.e., (1+1) dimensions. It is straightforward to extend the system to higher dimensions. In general, the standard deviation $w(L,t)$ of height from the substrate is given as follows,

$$w(L,t) = \sqrt{\frac{1}{L}\sum_{i}^{L}(y_i - \bar{h})^2}, \tag{1·1}$$

where L is the width of a strip of substrate, t the time, y_i the surface height at the substrate site x_i. Here $\bar{h}$ is the average height given as

$$\bar{h} = \frac{1}{L}\sum_{i}^{L} y_i. \tag{1·2}$$

Various rough surfaces satisfy the relations

$$w(L,t) \sim t^{\beta}, (t \ll t^{*}), \tag{1·3}$$

$$w(L,t) \sim L^{\alpha}, (t \gg t^{*}), \tag{1·4}$$

where the scaling relation (1·3) means that the standard deviation $w(L,t)$ grows with a power law of time in early stages of growth, while (1·4) means that $w(L,t)$ saturates to a value at some characteristic time t^{*}, and then it exhibits a power law of L in the late stages. This signifies that the growing rough surface is a self-affine fractal. The exponents α and β are called the roughness exponent and growth exponent, respectively. Family and Vicsek thought that

$$t^{*} \sim L^{z}, \quad z = \frac{\alpha}{\beta}. \tag{1·5}$$

where z is called the dynamic exponent.

All these relations are unified systematically in the following dynamic scaling hypothesis:

$$w(L,t) \sim L^{\alpha}\Psi(\frac{t}{L^{z}}), \tag{1·6}$$

where the scaling function $\Psi(x)$ should satisfy the following behavior : $\Psi(x) = x^{\beta}(x \ll 1),\ 1(x \gg 1)$. Using this dynamic scaling hypothesis (the Family-Vicsek dynamic scaling), one can in principle obtain the exponents α and β. However, it is hard to use the Family-Vicsek dynamic scaling because it is very difficult to obtain the values of $w(L,t)$ experimentally and in some case even numerically in early stages of growth. It follows then that values of the exponent β for many growing rough surfaces have not been obtained yet. Any other dynamic scaling approaches are clearly needed to avoid such a difficulty, and obtain reliable values of β for familiar experiments of growing rough surfaces.

1.1. *Alternative Approach to the FV dynamic scaling*[10)]

Our purpose is to examine the scaling exponents α and β by an alternative approach to the FV scaling. The fact is, there is the crossover structure at the characteristic width L^* on w vs. L plot. Instead of t^*, we make use of L^* in the FV scaling. Namely, the FV scaling is modified to the following form of dynamic scaling.

$$w \sim L^{\alpha} \ (L \ll L^*), \tag{1·7}$$

$$w \sim \text{constant} \ (L \gg L^*). \tag{1·8}$$

Then (8) and (9) are unified into a single dynamic scaling form:

$$w \sim t^{\frac{\alpha}{z}} f(\frac{L}{t^{\frac{1}{z}}}), \tag{1·9}$$

$$f(x) = \begin{cases} x^{\alpha} & (x \ll 1) \\ 1 & (x \gg 1). \end{cases} \tag{1·10}$$

Take a few of log-log plots of $w(L,t)$ vs L by suitable times to estimate the value of α, and then we can examine the exponent z by using our dynamic scaling through data collapse.

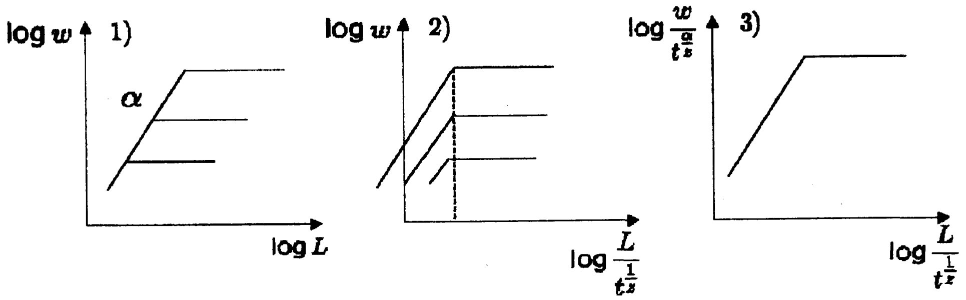

Fig. 1. Illustration of the rescale of modified FV scaling.

§2. (1+1)-d Eden model ··· well-known example

For example, we examine the Eden model[9]. The growth rule is simple : Let us start from the line seed. One of the perimeter sites of cluster is chosen randomly with equal probability and is incorporated into the cluster as its member, and this process is repeated. Since the Eden model is well-known model, it may be the most appropriate one to check the validity of our alternative approach. Figure 2 shows the log-log plots of w vs L at every interval of $t = 20.0$. The slope yield a value of α, $\alpha \simeq 0.5$. To examine a data collapse, we plot $\frac{L}{t^{\frac{1}{z}}}$ against $t^{\frac{\alpha}{z}}$ for values of α and z. Figure 3 shows the estimation of data collapse. Our data tend to fall onto a single

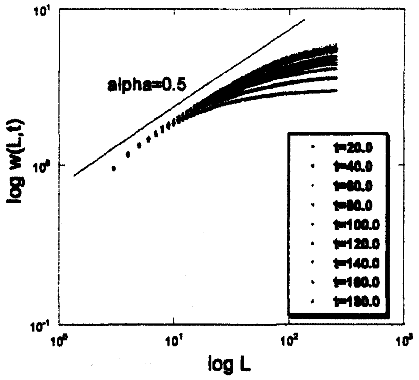

Fig. 2. The log-log plots of $w(L,t)$ vs L for the surfaces of the Eden model at every interval of $t = 20.0$ for System size = 500.

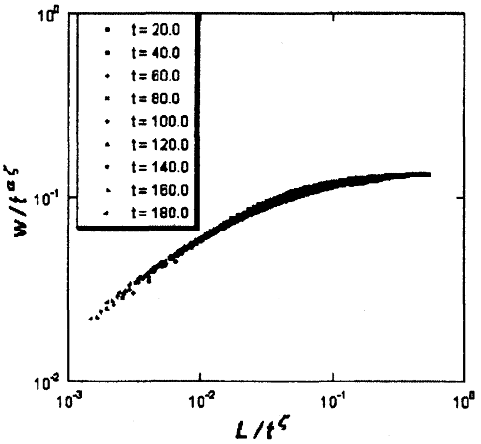

Fig. 3. Results of a data collapse for the Eden model with the logarithmic scale. $z = 1.5$, $\alpha = 0.5$.

curve for the values of the exponent $\alpha \simeq 0.50$ and $z \simeq 1.50$. Hence we obtain the values of the exponents that coincide with the well-known values of the exponents of the Eden model. We conclude that the above fact that we can successfully apply modified dynamic scaling hypothesis to the Eden model assures us of the validity of it.

§3. Directed Percolation Deppining model

Typical model of paper wetting experiment is Directed Percolation Deppining (DPD) model.[11),12)] This model is proposed by Buldyrev *et. al.* and Tang *et. al.*, independently. The growth rule is following. At first, for all sites we block a fraction p of the cells. Let us start line seed. One of nearest neibour site of cluster is chosen randomly without choosing the block site. To obtain no overhang-interface, we impose the rule that all block sites below the cluster site become wet as well. Note that, on this model, the scaling property is characterized by the directed percolation (DP) problem. Then, for p below a critical value p_c the interface propagate without stopping. This phase is called moving phase. On the other hand, for p above p_c the interface is pinning by DP cluster, pinning phase. Let us examine the roughness exponent α of DPD model on moving phase. At First, we take the standard diviation $w(L,t)$ as a function of the width of a strip of the substrate L. Figure 4 shows the log-log plots of $w(L,t)$ vs L for the $p = 0.40$ DPD surfaces. Figure 5 shows w vs

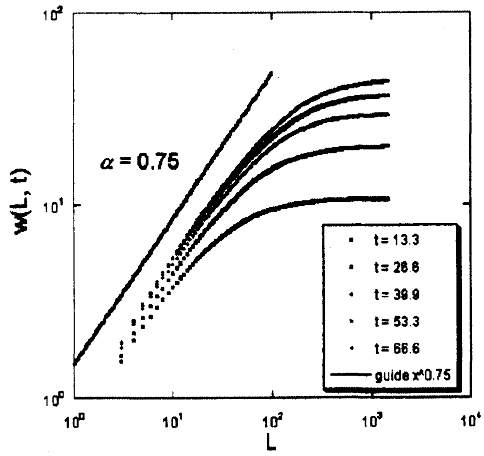

Fig. 4. The log-log plots of $w(L,t)$ vs L for the surfaces of the DPD model for System size = 3000. $p = 0.40 \simeq p_c = 0.47$.

t plot. In Figs. 4 and 5, we obtain the roughness exponent $\alpha \simeq 0.75$ and growth exponent $\beta \simeq 0.85$. In Figs. 4 and 5, there are the crossover structure. In this structure, we expect the exisitence of a characteristic length, which changes the nature of interface from $\alpha > 0.50$ into $\alpha = 0.5$, i.e. not self-affine interfaces. Next,

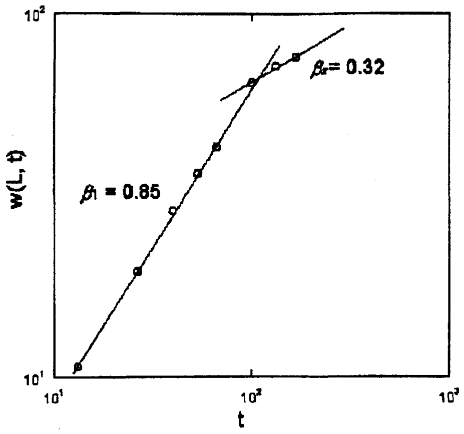

Fig. 5. The log-log plots of $w(L,t)$ vs t for the surfaces of the DPD model for System size = 3000. $p = 0.40, alittlebelowp_c (= 0.47)$.

we try the data collapse by plotting the quantity $w/t^{\alpha/z}$ against $L/t^{\frac{1}{z}}$ by varying z and α. Figure 6 shows the results of the data collapse for DPD model with the logarithmic scale. From Fig. 6, we obtain the value of $z \simeq 1.0$ and $\alpha \simeq 0.90$. These

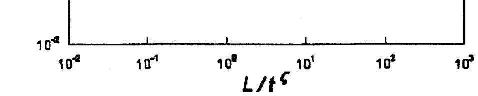

Fig. 6. Results of a data collapse for the $p = 0.40$ DPD model with the logarithmic scale. $z = 1.0$, $\alpha = 0.90$.

values is different from the values by "graph" method. This difference is from the reason for the crossover structure in Figs. 4 and 5. This structure is out of validity of the FV scaling hypothesis. But at least, owing to very nice data collapse we can conjecture the existence of a kind of dynamic scaling in such interfaces.

§4. Multi-affinity for DPD model

If quenched noise leads to power-law distributed noise, so the interface has a hierarchy of local roughness exponent. Just as multifractal, we call it multi-affine interface.[13] Determining multi-affinity, we measure the qth order correlation function defined by the following:

$$C_q(x,t) = \langle |h(x',t') - h(x'+x,t'+t)|^q \rangle_{x',t'}. \tag{4·1}$$

The scaling exponent α_q and β_q is defined by the following relation:[14]

$$C_q(x,0) \sim x^{q\alpha_q}, \tag{4·2}$$

$$C_q(0,t) \sim t^{q\beta_q}. \tag{4·3}$$

If α_q or β_q are dependent of degree q, we call its interface multi-affine. If these exponents are independent of q, its interface is self-affine. For example, we examine the DPD model on moving phase. Figure 7 shows the qth order correlation function for DPD model. Since the slope on Fig. 7, i.e. α_q, is dependent on q, the interfaces

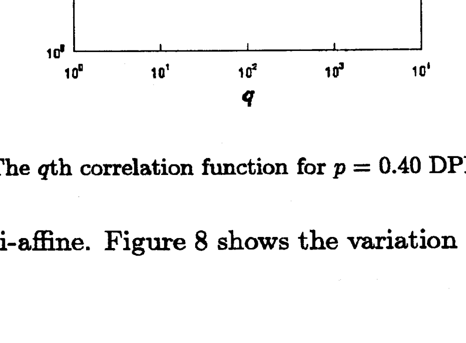

Fig. 7. The qth correlation function for $p = 0.40$ DPD model.

of DPD model are multi-affine. Figure 8 shows the variation with q of the roughness exponent α_q.

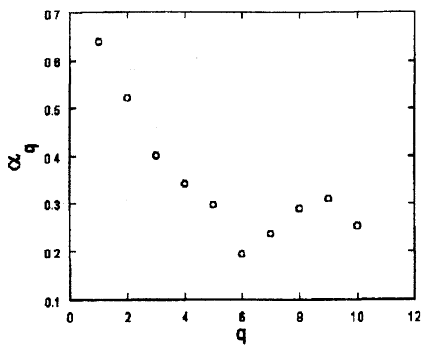

Fig. 8. The α_q spectrum for the DPD model.

§5. Conclusion

In summary, in order to examine the dynamic exponent z, we find an alternative approach to the dynamic scaling hypothesis for the growing rough surface. We have first investigated the self-affinity of the well-known Eden model to check the validity of the alternative approach. Next, we have numerically investigated the self-affinity of the DPD model. We have found that the interface for DPD model was not self-affine, but multi-affine. In future problems, we should investigate the source of multi-affinity. Barabási and Stanley proposed that power-law distributed noise leads to multi-affinity.[15] So, we should measure the noise distribution in order to discuss the Barabási-Stanley suggestion, but it is now under investigation.

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