KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.1041

"Bubbles, the U.S. Interest Policy, and the Impact on Global Economic Growth: Reverse Growth Effects of Lower Interest Rates after Bubble Bursting"

Atsushi Motohashi

August 2020

KYOTO UNIVERSITY

KYOTO, JAPAN

Bubbles, the U.S. Interest Policy, and the Impact on Global Economic Growth: Reverse Growth Effects of Lower Interest Rates after Bubble Bursting

Atsushi Motohashi[1]

This Version, August 2020

Abstract

This study analyzes the impact of the U.S. interest rate policy on the global economy. We extend the literature and build a global model consisting of a large country (the U.S.) and many small countries to investigate the mechanism by which economic growth and asset prices accelerate rapidly after a U.S. interest rate reduction. Specifically, we show that a U.S. interest rate reduction not only increases economic growth rates but also expands asset bubbles as long as the bubbles exist in small open economies. We also show, however, that this low interest rate policy has a large side effect, that is, a collapse of the asset bubbles causes a larger drop in the growth rate of small open countries than that in the case without a lower interest rate. This conclusion implies that small countries need to be prepared for overheated asset prices associated with U.S. interest policies.

Keywords: Asset Bubbles; U.S. Interest Rate Policy; Economic Growth; Collapse of Asset Bubbles; Asset Prices

JEL Classification: E32, E44, F43

[1] Graduate School of Economics, Kyoto University, Yoshidahonmachi, Sakyo-ku, Kyoto, 606-8501, Japan

1 Introduction

The U.S. has continued a low-interest rate policy for many years. In December 2008, the Federal Reserve (the Fed) reduced the target rate of federal funds from 0.00 to 0.25% and has continued this notably low-interest rate policy since then. Several international organizations have pointed out the positive and negative effects of this on the global economy. For example, the Bank for International Settlements (BIS, 2019) mentions that this low interest rate policy has enhanced economic activity, especially in the short term, but has also caused other problems such as risk-taking and a high sensitivity of financial markets to policy tightening in the long term. The International Monetary Fund (IMF, 2020) reports that market valuations of equities appear to be stretched and a recession could be deeper and longer than before if risky assets are repriced. These statements underscore the importance of the relationship among this low interest rate policy, risk asset prices, and economic fluctuation risks. In this paper, we focus on the side effects of the low interest policy and its positive effects from a theoretical point of view.

Our study builds a simple model to analyze the impact of the U.S. interest rate policy on risk asset prices and the global economy. In recent years, many studies have investigated large economic movements and the fluctuations of asset prices as the occurrence and collapse of asset bubbles have increased, garnering more attention[2]. Most studies in this vein, however, consider closed economies and do not explain the effects of any external change in interest policies such as an interest reduction or hike by the U.S. central bank.

Studies dealing with asset bubbles in open economies include Olivier (2000), Caballero and Krishnamurthy (2006), Motohashi (2016), and Shimizu (2018). Olivier (2000) shows that rational bubbles do not affect long-run economic growth in small open economies due to the externality of interest rates. Caballero and Krishnamurthy (2006) analyze the effects of asset bubbles in small open economies before and after bubble bursting and show that asset bubbles prevent capital outflow and enhance internal economic growth. However, they do not consider the impact of foreign interest policy changes in a large country, such as the U.S., on these economies. Although Motohashi (2016) and Shimizu (2018) clarify the characteristics of asset bubbles in the global economy by analyzing a small open economy and a two-country model, respectively, their analyses focus on the effects of foreign bubbly assets. Our study differs in that we analyze the effects of changes in the U.S. interest policy on domestic investors' asset holding behavior and domestic economic growth rates in small open economies.

---

[2] Examples include Martin and Ventura (2012), Farhi and Tirole (2012), Hirano et al. (2015), and Hirano and Yanagawa (2017). These studies focus on the incompleteness of financial markets, called "financial friction," and analyze the large economic movements as the occurrence and collapse of asset bubbles. Mitsui and Watanabe (1989) offer the first study in the literature regarding the relationship between the long-run economic growth rate and financial frictions.

Our study makes two main contributions to the literature. First, we show that a U.S. interest rate reduction accelerates economic growth and asset price increases in small countries and a hike decelerates them. This is because the interest rate reduction not only eases entrepreneurial borrowing constraints but also accelerates asset bubble holdings and their prices increase in these small countries. As a result, entrepreneurs are able to borrow more or have more assets to invest in their productive opportunities. This implies that many countries in the global economy may experience high economic growth after a U.S. interest rate reduction. Second, we show that asset bubbles that burst after a U.S. interest reduction have a larger effect, while an interest hike mitigates this. The implication is that an interest reduction encourages entrepreneurs to acquire more bubbly assets and they then lose more assets after the bubble bursts. An interest hike has the opposite effect, consequently, the impact of the bubble bursting is smaller. The conclusion then is that a U.S. interest reduction has the potential to cause large economic movements in the global economy through rising and falling asset prices. We explore the mechanism behind the global overheating in the asset market after a U.S. interest reduction and the effects on global economic growth.

The remainder of this paper is organized as follows. In Section 2, we introduce the basic setup of the model. In Section 3, we define a competitive equilibrium based on the setup and derive the economic growth rate in a small open economy. In Section 4, we analyze the effects of an interest reduction on the economic growth rate before and after bubble bursting. Finally, in Section 5, we summarize our main insights.

2 The Model

We begin by constructing a model to analyze the effects of asset bubbles in a small open economy by extending the models developed by Hirano and Yanagawa (2017) and Hirano et al. (2015). In our model, the international interest rate corresponds to the return on investments in foreign safe assets and is exogenously given as "the U.S. interest rate."

2.1 Basic Model Setup

In the global economy, financial investor portfolios usually include foreign assets as well as domestic ones. Most global portfolios contain government bonds from a large country such as the U.S. A central bank, such as the Federal Reserve, uses the bonds as a monetary policy tool to adjust interest rates. As a result, when the Fed adjusts interest rates, asset prices in other countries are affected where financial investors have these government bonds in their portfolios. Thus, a reduction or hike in interest rates in a representative large country like the U.S. could affect the asset holdings and economic growth in the rest of the world (other small countries).

In our study, the foreign safe assets held by investors in small countries are considered to be U.S. government bonds. Conversely, the internally generated bubbly assets held by these investors are "bubbly assets." To analyze asset bubbles in a general equilibrium framework, we introduce the bubbly asset into the model as a type of security that exceeds its fundamental value.

2.2 Model Structure

We consider a typical entrepreneurial model with financial friction in a discrete-time economy. There is no population growth and the economy has one homogeneous good and a continuum of entrepreneurs. A typical entrepreneur has the following expected discounted utility function:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \, log \, c_t^i\right], \tag{1}$$

where i is the index for each entrepreneur and c_t^i is his or her consumption at date t. The parameter $\beta \in (0,1)$ is the subjective discount factor and $E_0[x]$ is the expected value of x conditional on information at date 0[3].

Each entrepreneur encounters two types of investment projects in every period: high productive investment projects (hereafter, H-projects) and nonproductive (low or negative return) investment projects (hereafter N-projects). Investments produce output. At the beginning of every period, each entrepreneur encounters H-projects (N-projects) with probability p (probability $1-p$), which is exogenous and

---

[3] A log-linear utility function is adopted to analyze the effects of asset bubbles on countries where the ratio of consumption to income is stable. In other words, we focus mainly on countries that have some growth, and third world countries are excluded.

independent across entrepreneurs and constant over time. As a result, the productivity of each entrepreneur's investment portfolio changes over time. Throughout this discussion, an entrepreneur with H-projects (N-projects) is called an H-type (N-type) entrepreneur. The index i indicates the type of entrepreneur $i = \{H, N\}$. The investment technologies (output from each investment project) are expressed by the production function:

$$y_{t+1}^{i} = \alpha_t^i z_t^i, \tag{2}$$

where $z_t^i (\geq 0)$ is the investment level at date t and y_{t+1}^{i} is the output at date $t+1$ produced by the investment. Owing to the linearity of the production function, α_t^i corresponds to the marginal productivity of investments at date t. Since H-projects give high returns to H-types and N-projects give low or negative returns to N-types, α_t^i satisfies $\alpha_t^H > \alpha_t^N$. For simplicity, we assume $\alpha_t^i = \alpha^i$ such that $\alpha^N \leq 1$[4]. Assuming the initial population measure of each type is p and $1-p$ at date 0, the population measure of each type after date 1 is p and $1-p$, respectively.

[4] As in Motohashi (2016), we are able to consider a case where $\alpha^N > 1$. In this setting, adjusting the assumption of the relationship between the return on N-projects and the gross U.S. interest rate, we can obtain the same results as in the present paper.

Each entrepreneur also faces borrowing constraints. He or she can pledge at most a fraction θ of future returns from investments to creditors due to financial friction, as in Kiyotaki and Moore (1997)[5]. Thus, the borrowing constraint is expressed as

$$r_t^{US} b_t^i \leq \theta \alpha^i z_t^i, \tag{3}$$

where r_t^{US} and b_t^i are the gross U.S. interest rate and the amount of borrowing, respectively, at date t. The parameter $\theta \in [0,1]$ corresponds to the degree of imperfection in the financial market and is assumed to be externally given.

Each entrepreneur faces the following flow of funds constraint in every period:

$$c_t^i + z_t^i + P_t x_t^i + gb_t^i = \alpha^i z_t^i + P_t x_{t-1}^i + r_{t-1}^{US} gb_{t-1}^i - r_{t-1}^{US} b_{t-1}^i + b_t^i, \tag{4}$$

where x_t^i is the amount of the bubbly asset purchased by type i entrepreneur and P_t is its price. gb_t^i is the amount of the U.S. government bond purchased by type i entrepreneurs and r_t^{US} is its return.

Once bubbles collapse, the price of the bubbly asset becomes zero. Bubbles survive with a probability of π and a collapse of $1 - \pi$. A lower π value indicates riskier bubbles. Thus, P_t is affected by the risk of collapse. The left-hand side of (4) is the gross expenditure, and the financing of this is expressed by the right-hand side, which is the return on investment and assets in the previous year, plus net borrowing minus debt repayment. Then, the net worth of the entrepreneur

[5] Tirole (2005) also gives the foundations of this setting. We can easily provide a micro-foundation for θ by applying the ideas of Tirole to this model (see Appendix A).

is defined as $e_t^i \equiv \alpha^i z_{t-1}^i + P_t x_{t-1}^i + r_{t-1}^{US} g b_{t-1}^i - r_{t-1}^{US} b_{t-1}^i$ to express the economic implications. The holding of bubbly assets cannot be negative:

$$x_t^i \geq 0. \tag{5}$$

We consider only the case where the U.S. interest rate is positive. Since U.S. government bonds offer an opportunity for asset management in small countries, the interest rate in small countries converges to r_t^{US}. In addition, to exclude the case where entrepreneurs have all their assets in U.S. government bonds, we assume that their investment return does not exceed the marginal productivity of H-projects. Thus, r_t^{US} satisfies the following conditions:

$$1 < r_t^{US} \leq \alpha_t^H. \tag{6}$$

We also assume that $r_t^{US} = r^{US}$ for simplicity and analyze the impact of changes in interest on asset holdings and economic growth accordingly.

3 Market Equilibrium

The previous section provides the basic setup to construct a model analyzing the effects of the U.S. interest policy on the global economy. In this section, we define the competitive equilibrium and derive the economic growth rate of small countries.

3.1 Competitive Equilibrium

The competitive equilibrium is defined as a set of prices $\{r_t^{US} = r^{US}, P_t\}_{t=0}^{\infty}$ and other quantitative economic variables

$\{C_t^H, C_t^N, Z_t^H, Z_t^N, B_t^H, B_t^N, P_t, X_t, GB_t, Y_{t+1}\}_{t=0}^{\infty}$ that satisfy the results of the optimal behavior of entrepreneurs $\{c_t^i, b_t^i, z_t^i, x_t^i, gb_t^i\}_{t=0}^{\infty}$ and market clearing conditions as follows:

1 Each entrepreneur maximizes his/her utility under the following constraints:

$$\max_{c_t^i} E_0\left[\sum_{t=0}^{\infty} \beta^t \log c_t^i\right], \tag{7}$$

$$\begin{gathered} subject\ to\ c_t^i + z_t^i + P_t x_t^i + gb_t^i = e_t^i + b_t^i, \\ r^{US} b_t^i \leq \theta \alpha^i z_t^i\ and\ x_t^i \geq 0. \end{gathered} \tag{8}$$

2 The market-clearing conditions are:

$$C_t^H + C_t^N + Z_t^H + Z_t^N + GB_t^H + GB_t^N = Y_t + r^{US} GB_{t-1}, \tag{9}$$

$$B_t^H + B_t^N = 0, \tag{10}$$

$$X_t = X, \tag{11}$$

where the aggregate consumption, investment, purchasing of the U.S. government bonds, and borrowing and purchasing of bubbly assets of each type of entrepreneur at date t are, respectively, designated as: $\sum_{i \in H_t} c_t^i \equiv C_t^H$, $\sum_{i \in N_t} c_t^i \equiv C_t^N$, $\sum_{i \in H_t} z_t^i \equiv Z_t^H$, $\sum_{i \in N_t} z_t^i \equiv Z_t^N$, $\sum_{i \in H_t} gb_t^i \equiv GB_t^H$, $\sum_{i \in N_t} gb_t^i \equiv GB_t^N$, $\sum_{i \in H_t} b_t^i \equiv B_t^H$, $\sum_{i \in N_t} b_t^i \equiv B_t^N$, $\sum_{i \in H_t} x_t^i \equiv X_t^H$, $\sum_{i \in N_t} x_t^i \equiv X_t^N$.

It is well known that an entrepreneur with the log-linear utility function (1) consumes a fraction $1 - \beta$ of net worth every period:

$$c_t^i = (1 - \beta) e_t^i. \tag{12}$$

3.2 The Investment Function

Next, we consider the investment function of each entrepreneur to derive the output level in equilibrium. N-types prioritize lending their assets to H-types rather than over investing in N-projects because the lending interest rate and the expected return on bubbly assets (reflecting the bursting possibility) exceeds the marginal productivity of N-projects. N-types lend their assets to H-types up to the limit of the borrowing constraint, and then buy bubbly assets using residual assets[6]. N-types, therefore, do not invest in their own production projects. H-types, however, borrow assets from N-types and invest all their assets in H-projects because the marginal productivity of H-projects exceeds the expected returns of the U.S. government bonds and bubbly assets. As a result, H-types are the only entrepreneurs who invest in internal production projects in small countries. Combining the budget and borrowing constraints (8) and (12), the investment function of an H-type is

$$z_t^H = \frac{\beta e_t^H}{1-\frac{\theta\alpha^H}{r^{US}}} = \beta e_t^H \mu(r^{US}), \tag{13}$$

where $\mu(r^{US})$ is defined as $1/\left[1-\left(\frac{\theta\alpha^H}{r^{US}}\right)\right]$. Because βe_t^H represents the savings account of H-types, the function $\mu(r^{US})$ corresponds to his/her multiple investments to owed capital. We call this the "leverage factor of investments."

[6] By introducing a negligible slight cost to buying bubbly assets and U.S. government bonds, N-types prioritize lending their assets to H-types.

Since only H-types invest in internal projects, the investment function of the country is expressed as the aggregate investment of each H- type:

$$Z_t^H = \frac{\beta E_t^H}{1-\frac{\theta \alpha^H}{r^{US}}} = \beta E_t^H \mu(r^{US}). \quad (14)$$

The investment function depends on the net worth of H-types at date t. As mentioned before, H-types at date t come from proportions p of N and H-types at date $t-1$. After borrowing and lending, H-types only invest in H-projects and N-types buy bubbly assets and/or U.S. government bonds. Thus, considering the market clearing condition (10), the net worth of H-types at date t is given by

$$E_t^H = p(Y_t - r^{US} B_{t-1}^H) + p(P_t X + r^{US} GB_{t-1} - r^{US} B_{t-1}^N) = p(Y_t + r^{US} GB_{t-1} + P_t X) = pE_t. \quad (15)$$

As a result, the investment function in (14) is replaced by

$$Z_t^H = \beta p E_t \mu(r^{US}). \quad (16)$$

3.3 The Demand function for Bubbly assets

Here, we consider the demand function for bubbly assets. N-types buy bubbly assets using their remaining savings after lending to H-types. An N-type chooses the optimal amount of x_t^N to realize marginal expected utility from b_t^N, x_t^N and gb_t^N are equalized. By solving the utility maximization problem explained in Appendix B, we can derive the demand function for bubbly assets of an N-Type:

$$P_t x_t^N = \frac{\pi \frac{P_{t+1}}{P_t} - r^{US}}{\frac{P_{t+1}}{P_t} - r^{US}} \beta e_t^N. \tag{17}$$

N-type decisions regarding the amount of holdings of bubbly assets depend on the U.S. interest rate and the bursting probability of asset bubbles. Because only N-types hold bubbly assets, their aggregate demand is derived as

$$P_t X = \frac{\pi \frac{P_{t+1}}{P_t} - r^{US}}{\frac{P_{t+1}}{P_t} - r^{US}} \beta (1-p) E_t. \tag{18}$$

3.4 The Economic Growth Rate

Finally, we consider the economic growth rate in small countries. The aggregate wealth of the countries is

$$\begin{aligned} E_{t+1} &= Y_{t+1} + P_{t+1} X + r^{US} GB_t \\ &= \alpha_t^H Z_t^H + \left(\frac{P_{t+1}}{P_t}\right) P_t X + r^{US} GB_t. \end{aligned} \tag{19}$$

To characterize the economic growth rate, we define the relative size of the residual assets (φ_t) and the growth rate of aggregate wealth (g_t). The relative size of the residual assets indicates the investment ratio of financial assets to domestic savings, which is divided into bubbly assets (η_t) and U.S. government bonds ($1 - \eta_t$). These assets are defined as follows:

$$\varphi_t \equiv \frac{P_t X + GB_t}{\beta E_t} = 1 - \frac{Z_t}{\beta E_t} = 1 - p\mu(r^{US}), \tag{20}$$

$$\eta_t \varphi_t \equiv \frac{P_t X}{\beta E_t} \text{ and } (1-\eta_t)\varphi_t \equiv \frac{GB_t}{\beta E_t}, \tag{21}$$

$$g_t \equiv \frac{E_{t+1}}{E_t}. \tag{22}$$

φ_t is constant over time because the interest rate is given by a constant U.S. interest rate. Entrepreneurs, therefore, determine the ratio of bubbly assets ($\eta_t \varphi$) and the U.S. government bonds to the residual assets that exist in a certain proportion of their savings. From (16) and (18), and these definitions, the growth rate of aggregate wealth (19) and the aggregate demand for bubbly assets can be expressed as

$$g_t = \beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta_t \left[\frac{P_{t+1}}{P_t} - r^{US}\right], \tag{23}$$

$$\frac{P_{t+1}}{P_t} = r^{US} \frac{(1-p)-\eta_t\varphi}{\pi(1-p)-\eta_t\varphi}. \tag{24}$$

Furthermore, from an elementary calculation of the investment function (16), it is clear that the growth rate of the total output ($\widehat{g_t} \equiv Y_{t+1}/Y_t = Z^H_{t+1}/Z^H_t$) equals the growth rate of aggregate wealth (g_t). Thus, we call this the "economic growth rate."

These equations point to important characteristics of bubbly assets in global economies. In the first equation, the first term corresponds to the economic growth rate, which is realized when all assets become real investments. The second term corresponds to the loss of growth opportunities due to the inability to invest in H-projects, which reflects the incompleteness of the financial market; and the third term corresponds to the improvement of the growth rate due to the occurrence of asset bubbles. The second equation confirms that the price of a bubbly asset is higher than the U.S. interest rate due to the premium reflecting bursting risk. The amount of the premium corresponding to the risk of the bubble bursting is displayed as a coefficient of r^{US}. We can easily show that bubbly assets exist under relatively mild conditions, which means that the level of the maximum interest rate should be lower than the level of the equilibrium interest rate in the case of rational bubbles (see Appendix C). By combining (23) and (24), we have the following theorem:

Theorem 1. *The economic growth rate in small countries is expressed as a function of the holding ratio of bubbly assets to their residual assets* (η_t)*, and the economic growth rate becomes an increasing function of the holding ratio. That is, it is given by*

$$g_t(\eta_t) = \beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\eta_t\varphi r^{US}\left[\frac{(1-p)(1-\pi)}{\pi(1-p)-\eta_t\varphi}\right], \tag{25}$$

and $\partial g_t/\partial\eta_t > 0$.

As is clear from the third term of this equation, if entrepreneurs hold more bubbly assets, the total return from their residual assets becomes higher than before because the return on bubbly assets is higher than on U.S. government bonds. In addition, the increase in demand for bubbly assets increases their price. As a result, since the return on bubbly assets improves rapidly, this increases entrepreneurs' investment resources and enhances economic growth. This conclusion is different from that of Hirano et al. (2015), who mention that holding a high level of bubbly assets decreases capital accumulation and economic growth through a decrease in investments. In their model, a closed large country (such as the U.S.) is considered, and the increase in the bubbly assets holding ratio causes the interest rate to rise due to the improvement in domestic financial investment returns. As a result, entrepreneurs cannot borrow assets as readily as before, and investments in H-projects decrease. They call this effect of asset bubbles the "crowd-out effect." However, in our model, since we consider the case of relatively small countries and the interest rate is externally given as the U.S. interest rate, there is no "crowd-out effect;" there are only "crowd-in effects" through improvements in returns on residual assets. As a result, the economic growth rate improves rapidly when entrepreneurs hold more bubbly assets. This is one key difference of our paper from other studies, and, as a result, the changes in the holdings of bubbly assets have a relatively large impact on the economic growth rate in the global economy.

4 Characteristics of Asset Bubbles in the Global Economy

4.1 Stochastic Stationary Equilibrium with Asset Bubbles

Next, we examine the dynamics and stochastic stationary equilibrium with asset bubbles. From the definition of $\eta_t\varphi = \frac{P_t X}{\beta E_t}$, $\eta_t\varphi$ evolves over time as,

$$\eta_{t+1}\varphi = \frac{\frac{P_{t+1}}{P_t}}{\frac{E_{t+1}}{E_t}}\eta_t\varphi. \tag{26}$$

The evolution of the size of the bubbles depends on the relationship between the economic growth rate and the growth rate of the asset bubbles. If an economy has a stable bubble equilibrium, the relative size of the bubbly assets must be constant ($\eta_{t+1}\varphi/\eta_t\varphi$ =1). From equations (24), (25) and (26), we find the condition that $\eta_t\varphi$ should satisfy in a stationary equilibrium as follows:

$$\begin{aligned}\beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta_t r^{US}\left[\frac{(1-p)(1-\pi)}{\pi(1-p)-\varphi\eta_t}\right] \\ = r^{US}\frac{(1-p)-\varphi\eta_t}{\pi(1-p)-\varphi\eta_t}\end{aligned} \tag{27}$$

Solving this equation for the ratio of holdings of bubbly assets, we obtain the following theorem:

Theorem 2. *The ratio of the holdings of bubbly assets in the stochastic stable equilibrium ($\varphi\eta^*$) is expressed as a function of the U.S. interest rate (r^{US}), and is a decreasing function of the U.S. interest rate (r^{US}). That is, it is given by*

$$\varphi\eta^* = (1-p)\frac{\alpha^H[\theta-\beta\pi(\theta-p)]-r^{US}[1-\beta\pi(1-p)]}{\alpha^H[\theta-\beta\pi(\theta-p)+\beta p(1-\theta)(1-\pi)]-r^{US}[1-\beta\pi(1-p)]}, \qquad (28)$$

*where * denotes the stochastic stable equilibrium here and* $\partial\varphi\eta^*/\partial r^{US} < 0$.

From equation (20), $\partial\varphi/\partial r^{US} > 0$. As a result, Theorem 2 implies $\partial\eta^*/\partial r^{US} < 0$. This means that the effect of the U.S. interest policy on the bubbly assets holding ratio is greater than on the amount of residual assets through decreasing leverage. When we consider the U.S. interest reduction case, this eases the H-type borrowing constraint and leverage increases in small countries. This means that residual assets for financial investments decrease. The holding ratio of bubbly assets, however, is rapidly pulled up by N-types, and the ratio of holdings of bubbly assets to their savings increases. In other words, N-types try to gain financial investment returns from limited residual assets and hold riskier assets than before. Such rapid changes in their holdings affect economic growth in these countries.

Inserting equation (28) into (27), we find the economic growth rate in the stochastic steady state.

Theorem 3. *The economic growth rate in the stochastic steady state is expressed as a function of the U.S. interest rate* (r^{US})*, and is a decreasing function of the U.S. interest rate* (r^{US})*. That is, it is given by*

$$g_t^*(r^{US}) = \left(\frac{P_{t+1}}{P_t}\right)^* = \frac{\alpha^H \beta p(1-\theta) r^{US}}{\pi\alpha^H \beta p(1-\theta) - \left[\alpha^H[\theta - \beta\pi(\theta-p)] - r^{US}[1-\beta\pi(1-p)]\right]}$$

$$= \mu(r^{US}) \frac{\alpha^H \beta p(1-\theta)}{1-\beta\pi(1-p)}, \tag{29}$$

$$\textit{and } \partial g_t^*(r^{US})/\partial r^{US} = \partial \left(\frac{P_{t+1}}{P_t}\right)^* / \partial r^{US} < 0.$$

Looking at equation (25), it seems intuitive that the economic growth rate is depressed by a decreasing U.S. interest rate, if endogenous variables are constant. Theorem 3, however, shows contrasting results. This is because the change in the U.S. interest rate affects not only the residual assets ratio (φ) but also the holdings of bubbly assets ratio (η^*). As shown in Theorem 2, the effect of interest rate changes on the holding of bubbly assets ratio (η^*) is greater than on the residual assets ratio (φ). As a result, entrepreneurs will have more bubbly assets than U.S. government bonds. In addition, the prices of bubbly assets also rise as demand increases. The total return from the residual assets, therefore, increases after the U.S. interest rate reduction. Moreover, from equation (20), it is clear that the decline in the residual asset ratio means an increase in the leverage factor of investments. In other words, it eases the borrowing constraints and H-types will be able to invest in more H-projects than before. As a result, there are two enhancing effects on economic growth, which can increase rapidly after a U.S. interest rate reduction. We call these two effects the *asset holding change effect* and *borrowing constraint* *effect*. Since these two effects work in the same direction after a U.S. interest reduction or hike, the economic growth rate fluctuates rapidly in the global economy.

When we consider the U.S. interest hike case, these two effects work in opposite directions. Both the asset holding change effect and borrowing constraint effect depress economic growth, and, at the same time, the bubbly asset price declines.

4.2 Bubble Bursting Effects

Finally, we analyze the effects of bubble bursting on the global economy. The value of the bubbly assets is considered to be zero after the bubble bursts. Using equation (23) and considering P_{t+1} and η^* becomes zero after bubble bursting, the economic growth rate in the global economy after bubble bursting is as follows.

$$g_t^A = \beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) - \beta\varphi\eta^* r^{US},$$

$$g_t^{NB} = \beta\alpha^H - \beta\varphi(\alpha^H - r^{US}),$$

$$\Delta g_t = g_t^A - g_t^*(r^{US}) = -\beta\varphi\eta^* \left(\frac{P_{t+1}}{P_t}\right)^*,$$

where A is an index indicating an economy after bubble bursting, NB is an index indicating an economy with no bubbles, and Δg_t is defined as the change in the economic growth rate before and after the bubble bursting. Examining the case of interest reduction, we obtain the following theorem:

Theorem 4. *The changes in the economic growth rate before and after bubble bursting (Δg_t) is an increasing function of the U.S. interest rate (r^{US}).*

Proof

$\frac{\partial \Delta g_t}{\partial r^{US}} = -\beta\left[\frac{\partial \varphi\eta^*}{\partial r^{US}}\left(\frac{P_{t+1}}{P_t}\right)^* + \varphi\eta^* \frac{\partial\left(\frac{P_{t+1}}{P_t}\right)^*}{\partial r^{US}}\right]$. From Theorems 2 and 3, we know that

$\frac{\partial \varphi\eta^*}{\partial r^{US}} < 0$ and $\frac{\partial\left(\frac{P_{t+1}}{P_t}\right)^*}{\partial r^{US}} < 0$. As a result, we have $\frac{\partial \Delta g_t}{\partial r^{US}} > 0$.

As mentioned in Theorem 3, a U.S. interest rate reduction pulls up the economic growth rate through two main effects: the asset holding change effect and borrowing constraint effect. As a result, the economic growth rates in small countries are accelerated after the U.S. interest rate reduction, before the bubbles burst. N-types, however, could lose more assets as they held more bubbly assets than before. This theorem implies that high economic growth under low U.S. interest rates has the potential to lead to huge economic fluctuations in the global economy (see Figure 1). However, a U.S. interest hike has the opposite effect on the global economy. Although a U.S. interest hike could slow the global economy, this theorem shows that it mitigates potential economic fluctuation risks (see Figure 2). In summary, if the U.S. adopts a low interest rate policy, small countries are exposed to potential economic fluctuation risks. This theorem also implies that small countries need to be prepared for overheated asset prices associated with U.S. interest policies.

Figure 1: US interest rate reduction case

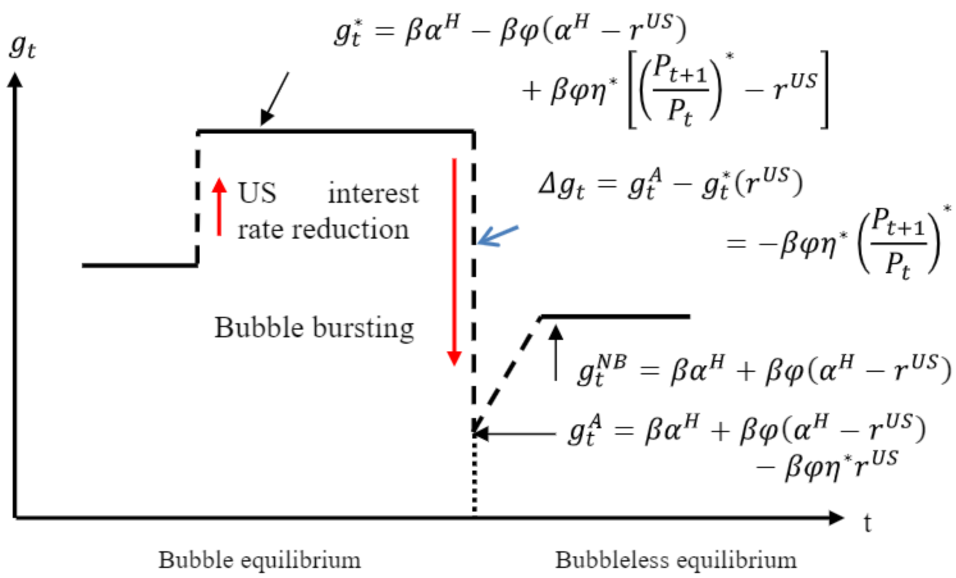
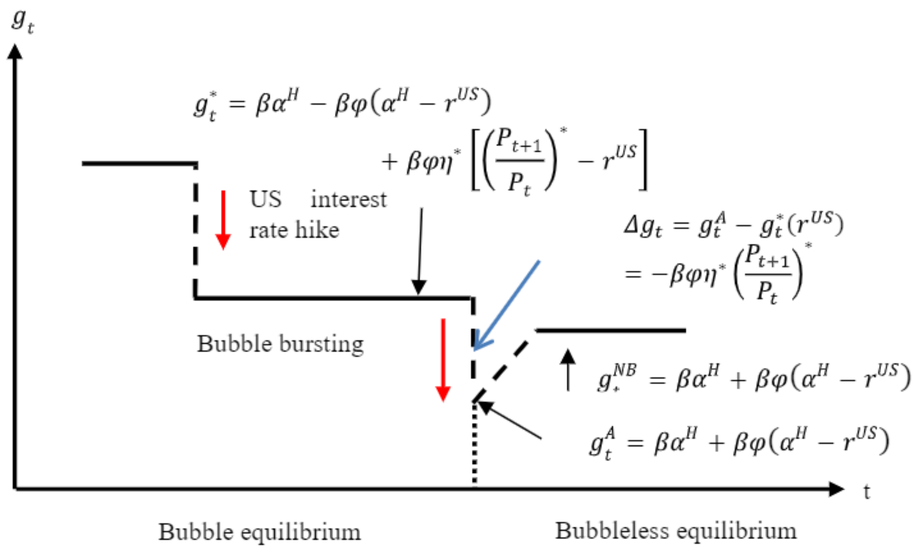


Figure 2: US interest rate hike case



5 Concluding Remarks

We introduce stochastic crashes and U.S. government bonds into the model of Hirano and Yanagawa (2017) and extend the model to examine the effects of the U.S. interest policy on the global economy. Thus, we analyze the impact of the U.S. interest rate on the global economy.

Our study makes several contributions. First, we show that a U.S. interest rate reduction accelerates global economic growth and asset price increases while a hike decelerates them similarly. This is because a U.S. interest rate reduction not only eases entrepreneurial borrowing constraints (the borrowing constraint effect) but also accelerates asset bubbly holdings (the asset holding change effect). Since these effects work in the same direction, entrepreneurs are able to obtain more assets than before to invest in productive opportunities. This result implies that more countries may simultaneously experience high economic growth after an interest reduction by the U.S.

Second, we show that the effect of bursting asset bubbles is larger after a U.S. interest reduction, while a U.S. interest hike mitigates this. Since a U.S. interest reduction encourages entrepreneurs to acquire more bubbly assets at higher prices than before, they tend to lose more assets after the bubble bursts. A U.S. interest hike has an opposite effect and the impact of the bubble bursting is smaller.

The implication is that high economic growth under low U.S. interest rates has the potential to lead to huge economic fluctuations, but a U.S. interest hike will have the opposite effect on economic fluctuations. Although interest hikes can slow the global economy before a bubble bursts, it has the effect of limiting potential economic fluctuation risk. This conclusion implies that small countries need to be prepared for overheated asset prices associated with U.S. interest policies.

References

BIS, 2019. Annual Economic Report. https://www.bis.org/publ/arpdf/ar2019e.htm

Caballero, R., Krishnamurthy, A., 2006. Bubbles and capital flow volatility: Causes and risk management. J. Monetary Econ. 53, 35-53.

Farhi, E., Tirole, J., 2012. Bubbly Liquidity. Rev. Econ. Studies, 79, 678-706.

Hirano, T., Yanagawa, N., 2017. Asset Bubbles, Endogenous Growth, and Financial Friction. Rev. Econ. Studies, 1, 406-443.

Hirano, T., Inaba, M., Yanagawa, N., 2015. Asset Bubbles and bailouts. J. Monetary Econ. 76, 71-89.

IMF, (2020). Global Financial Stability Report Update: Financial Conditions Have Eased, but Insolvencies Loom Large. https://www.imf.org/en/Publications/GFSR/Issues/2020/06/25/global-financial-stability-report-june-2020-update

Kiyotaki, N., Moore, J., 1997. Credit Cycles. J. Political Econ. 105, 211-248.

Martin, A., Ventura, J., 2012. Economic Growth with Bubbles. Amer. Econ. Rev. 102, 3033-3058.

Mitsui, T., Watanabe, S., 1989 Monetary growth in a turnpike environment. J. Monetary Econ. 24, 1, 123-137

Motohashi, A., 2016. Economic Growth with Asset Bubbles in a Small Open Economy. Theor. Econ. Lett. 6, 942-961.

Olivier, J., 2000. Growth-Enhancing Bubbles. Int. Econ. Rev. 41, 133-151.

Shimizu, R., 2018. Bubbles, growth and imperfection of credit market in a two-country model. Ann. Finance, 14, 353-377.

Tirole, J., 2005. The Theory of Corporate Finance. Princeton University Press Princeton, NJ.

Appendix A

To clarify the meaning of financial friction, we provide a micro-foundation for θ. We define the revenue that an entrepreneur gains by working conscientiously as R ($\equiv \alpha^i z_t^i$), and the lucre he/she gains by embezzling company funds as L. Since banking corporations (lenders) want to avoid the entrepreneur (borrower) embezzling funds, the borrowing condition should satisfy $L \leq R - r^{US} b_t^i$. From an elementary calculation, this inequality can be rewritten as

$$r^w b_t^i \leq (1 - L/R)R. \tag{30}$$

Here, we can redefine the parameter θ as $(1 - \mathrm{L/R})$. The degree of θ depends on the amount of L. Thus, the degree of financial friction depends on the level of monitoring technology in banking corporations. In a country with a relatively undeveloped financial sector, the entrepreneur finds it easy to embezzle company funds, and banking corporations limit their lending. To simplify the discussion, the ratio of L/R is assumed to be constant and exogenously given in this paper.

Appendix B

Each N-type chooses the optimal amounts of b_t^i, x_t^i, and gb_t^i so that the expected marginal utility from investing in three assets is equalized. The first-order conditions with respect to x_t^i, gb_t^i and b_t^i are

$$(x_t^i)\ :\qquad u'(c_t^i) = E_t\left[u'(c_{t+1}^i)\frac{P_{t+1}}{P_t}\right],$$

$$(gb_t^i)\ and\ (b_t^i)\ :\qquad u'(c_t^i) = E_t\left[u'(c_{t+1}^i)r^{US}\right]. \tag{31}$$

As mentioned in Section 2.2, $u(c) = \log c$ is the utility function of the entrepreneurs, and bubbles survive with a probability of π and collapse with that of $1 - \pi$. Then, these equations are rewritten as

$$(x_t^i)\ :\qquad \frac{1}{c_t^i} = E_t\left[\frac{1}{c_{t+1}^i}\frac{P_{t+1}}{P_t}\right] = \pi\beta\frac{P_{t+1}}{P_t}\frac{1}{c_{t+1}^{i,\pi}},$$

$$(gb_t^i)\ and\ (b_t^i)\ :\qquad \frac{1}{c_t^i} = E_t\left[\frac{r^{US}}{c_{t+1}^i}\right] = \pi\beta\frac{r^{US}}{c_{t+1}^{i,\pi}} + (1-\pi)\beta\frac{r^{US}}{c_{t+1}^{i,1-\pi}}, \tag{32}$$

where $c_{t+1}^{i,\pi} = (1-\beta)\left(-r^{US}b_t^i + P_{t+1}x_t^i + r^{US}gb_t^i\right)$ is the optimal consumption level at date $t+1$ when bubbles survive at date $t+1$, and $c_{t+1}^{i,1-\pi} = (1-\beta)\left(-r^{US}b_t^i + r^{US}gb_t^i\right)$ is the optimal consumption level at date $t+1$ when bubbles collapse at date $t+1$. From these two equations and the level of consumption in each state, we have equation (17).

Appendix C

We examine the existence conditions for bubbly assets. In the global economy, the following conditions need to be satisfied to sustain bubbly assets in a stochastic steady state:

$$0 < \varphi\eta^* \leq \varphi \text{ and } 0 < \varphi < 1, \tag{33}$$

$$1 < r^{US} \leq \alpha^H. \tag{34}$$

Condition (33) corresponds to the condition required based on the definition of the asset bubbly holding ratio. The second condition (34) corresponds to the condition that the return from a safe asset becomes positive and does not exceed the marginal productivity of H-projects. As a result, we have the following existence condition for bubbly assets:

$$Max\left[\theta\alpha^H \frac{\theta - \beta\pi(\theta - p) + \beta(1-\theta)(1-\pi)}{\theta - \beta\pi\theta(1-p) + (1-p)\beta(1-\theta)(1-\pi)}, 1\right] < r^{US} < \alpha^H \frac{(1-\beta\pi)\theta + \beta p\pi}{1 - \beta\pi(1-p)} \; and \; \theta < \beta\pi(1-p). \tag{35}$$

The right side of the inequality is equivalent to the equilibrium interest rate in a closed economy with stochastic asset bubbles. Compared with Hirano and Yanagawa (2017), who analyze the rational asset bubble case, the level of the maximum interest rate becomes lower than the level of the equilibrium interest rate in the case of rational asset bubbles. This is one of the well-known characteristics of stochastic bubbles.