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"Dynamically Consistent Menu Preferences"

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Abstract

We provide a unified analysis of dynamically consistent menu preferences that may exhibit preference for flexibility, preference for commitment, or both. Our work generalizes prior results, which investigated this problem for an agent who always exhibits preference for flexibility. By using two types of consistency conditions, we characterize an agent with a subjective state space who reacts to information about her subjective states in a dynamically consistent way. We apply our reuslts to the multiple temptations and the anticipating regret models.

JEL classification: D11; D81; D91

Keywords: Dynamic consistency; Preference for flexibility; Preference for commitment; Bayesian updating; Subjective state space

1 Introduction

Uncertainty and information are basic concerns in economics. Preference shocks, or uncertainty about future preferences, are prevalent in many decision-making models and applications. This paper theoretically studies how people react to new information about her future preferences.

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Since Kreps [14] and subsequently Dekel, Lipman and Rustichini [3, 4] (henceforth DLR and DLR2, respectively), the decision theory literature on models with uncertain future preferences has been growing. The set of possible future preferences is often referred to as a *subjective state space*, because they are unobservable by the modeler. To model a subjective state space, Kreps [14], DLR [3], and DLR2 [4] analyze preferences over menus of alternatives. Suppose that the decision maker (DM) is unsure whether she will prefer chicken or fish for dinner. She may prefer to keep both options available than to commit to either option. Thus preference uncertainty may lead to a *preference for flexibility*. At the same time, it is well-known that many people have self-control problems and exhibit a *preference for commitment*, which is a desire to reduce options available for future choice. Preference for commitment has been extensively studied theoretically and empirically.[1] Moreover, several papers address the importance of combining preference for flexibility and preference for commitment, for example, in behavioral contract theory.[2]

In this paper, we provide a unified analysis of dynamically consistent menu preferences that may show preference for flexibility, preference for commitment, or both. Then we apply our analysis to the multiple temptations model of Stovall [20] (see also DLR2) and the anticipating regret model of Sarver [18], in both of which a DM may exhibit preference for commitment. Some might associate preference for commitment with *dynamic inconsisntency*. Gul and Pesendorfer [11] and Noor [16] characterize a DM who suffers from temptation by implicitly assuming that self-control problems arise only when an alternative is chosen from the menu, but it does not arise when a menu is chosen. As in those papers, we consider preferences over menus instead of preferences over alternatives. This is why we investigate dynamically consistent menu preferences even when we allow for preference for commitment.

We follow the classical approach where consistency conditions between preferences before and after receiving information are investigated. When the state space is objective, dynamic consistency can be directly imposed and its consequences can be investigated, as in Epstein and Le Breton [8] and Ghirardato [10]. With a subjective state space, a difficulty arises. Although we are often able to observe that the DM has received relevant information, we cannot observe how she interprets this information in terms of her subjective state space. Riella [17] investigated this problem for the most specialized model in DLR. He characterized, for an agent with a positive additive expected utility (EU) representation, when an objective signal is interpreted as a measurable event in the DM's subjective state space and, upon learning this event, the DM acts in a dynamically consistent way.

Because Riella [17] works with positive additive EU representations, the condition in that

[1] See, for example, Bryan, Karlan, and Nelson [2] for a survey.
[2] See Amador, Werning, and Angeletos [1] and Galperti [9].

paper is related to how the DM's preference for flexibility changes after the signal. We generalize the analysis in [17] by working with additive EU representations and, therefore, allowing the DM to also display preference for commitment in some situations. The main insight we obtain is that, altough information may change the DM's preferences, a dynamically consistent behavior disciplines how preference reversals may occur. Precisely, our main condition imposes that whenever the signal causes a preference reversal, this may be attributed to the fact that the DM values flexibility or commitment more before the signal than after. We also work with a weaker condition where we impose a similar restriction, but only after one of the reasons, flexibility or commitment, for the preference reversal is ruled out.

We then apply our results to the multiple temptations model of Stovall [20] (see also DLR2) and the anticipating regret model of Sarver [18]. To apply it to the multiple temptations representation, we need to assume that the DM's "normative" preference is independent of information and remains the same before and after the signal. In some situations, however, it might be the case that the normative preference is affected by the new information as well. We use our weaker condition, then, to allow for situations where information affects both the DM's normative preference and beliefs. We also apply our weaker condition to the anticipating regret representation of Sarver [18].

The papers most closely related to our work are those on subjective learning: Dillenberger, Lleras, Sadowski, and Takeoka [6] (henceforth DLST) and Dillenberger and Sadowski [7]. In both papers, a DM has a preference over menus of Anscombe and Aumann acts and expects to receive a subjective signal that is a probability over objective states. There are two differences between these papers and our work. The first is the timing of arrival of information. In DLST [6] and Dillenberger and Sadowski [7], the DM receives information after the choice of a menu and before the choice of an act out of the menu. In our work, the DM receives information before the choice of a menu. The second is the structure of the signal. In these papers, the signal is subjective and appears only as part of the representation of the DM's preferences over menus. In contrast, we assume the existence of objective signals observed by the modeler. The difficulty in our case is that, since the state space is subjective, one cannot know how the DM interprets the signal in terms of her subjective state space.

2 Setup

Let Z be a finite set of prizes and let Δ denote the set of probability measures on Z. We see Δ as a metric subspace of $\mathbb{R}^Z$. Let $\mathcal{K}$ be the collection of all closed and nonempty subsets of Δ. A typical element of $\mathcal{K}$ is called a *menu* and denoted by A, B, C, *etc.*, while typical elements of Δ are denoted by p, q, r, *etc.* We write $int(\mathcal{K})$ to represent the subset of all

menus located entirely in the relative interior of Δ. That is, $int(\mathcal{K})$ is the set of all menus that include only lotteries with full support. We need the following definition:

Definition 1. Given a binary relation $\succsim \subseteq \mathcal{K} \times \mathcal{K}$, a *finite additive EU representation* of $\succsim$ is a tuple (S, μ, U), where S is a finite set, μ is a signed measure on S, and $U : S \times \Delta \to \mathbb{R}$ is a state dependent utility function such that

1. for each $s \in S$, $U(s, \cdot)$ is nonconstant and of the expected utility type;[3]
2. the function $V : \mathcal{K} \to \mathbb{R}$ defined by

$$V(A) := \sum_{s \in S} \mu(s) \max_{p \in A} U(s, p),$$

   for each $A \in \mathcal{K}$, represents $\succsim$;
3. μ has full support and, for each distinct s and s' in S, $U(s, \cdot)$ and $U(s', \cdot)$ are not positive affine transformations of each other.

If $\succsim$ admits a finite additive EU representation, we say that $\succsim$ is a finite additive EU preference. Relations that admit an additive EU representation were axiomatized by DLR [3] and DLRS [5]. DLR2 [4] characterized when such relations can be represented with a finite state space.

We note that the set S in a finite additive EU representation is just an index set and is not directly relevant. What is important is the set of *ex post* preferences induced by $\{U(s, \cdot) : s \in S\}$. Following DLR [3], we refer to the set of expected-utility preferences induced by $\{U(s, \cdot) : s \in S\}$ as the *subjective state space*. We define a state s to be positive (negative) if $\mu(s) > 0$ ($\mu(s) < 0$). DLR [3] showed that finite additive EU preferences have unique subjective state spaces, in the sense that if (S, μ, U) and $(\hat{S}, \hat{\mu}, \hat{U})$ are two finite additive EU representations of the same relation, then they both induce the same subjective state space and, for every pair of states $s \in S$ and $\hat{s} \in \hat{S}$ that induce the same subjective state we have $\mu(s)\hat{\mu}(\hat{s}) > 0$. That is, two different finite additive EU representations of the same relation induce exactly the same sets of positive and negative states. Given these observations, we can assume, without loss of generality, that the index set S in a given finite additive EU representation coincides with its subjective state space. From now on, whenever we write a state space S, it is understood that each $s \in S$ corresponds to the expected-utility preference relation represented by $U(s, \cdot)$. In addition, given a finite additive EU

[3] That is, there exists a nonconstant function $u : Z \to \mathbb{R}$ such that, for each $p \in \Delta$, $U(s, p) = \sum_{z \in Z} p(z)u(z)$.

representation (S, μ, U), we will write $\mathcal{P}(S)$ to denote the set of positive states and $\mathcal{N}(S)$ the set of negative ones. That is, $\mathcal{P}(S) := \{s \in S : \mu(s) > 0\}$ and $\mathcal{N}(S) := \{s \in S : \mu(s) < 0\}$.

When an additive EU representation (S, μ, U) satisfies that $\mu(s) > 0$ for every $s \in S$, or $\mu(s) < 0$ for every $s \in S$, we say that it is a positive or a negative finite additive EU representation, respectively. In such cases, we usually assume, without loss of generality, that μ or $-\mu$ is a probability measure, respectively.

3 Dynamically Consistent Menu Preferences

We will work with a pair of relations $\succsim$ and $\succsim^*$ on $\mathcal{K}$. The interpretation is that both $\succsim$ and $\succsim^*$ represent the preferences of the same individual. The difference is that $\succsim^*$ represents the individual's preferences after she has received an objective signal. We assume that $\succsim$ and $\succsim^*$ are finite additive EU preferences. Both $\succsim$ and $\succsim^*$ may exhibit preferences for flexibility and comittment. Hence, we will consider the following postulate linking $\succsim$ and $\succsim^*$ by comparing attitudes toward flexibility and commitment.

When talking about "consistency," most papers consider the parts where the two preferences coincide. Instead, following Riella [17], we focus on preference reversals and investigate a consistency condition about how the two preferences differ.

Axiom 1 (Menu Preference Consistency). *For any pair of menus A and B in $int(\mathcal{K})$, $B \succsim A$ and $A \succ^* B$ implies that there exists a menu C such that (i) $A \cup B \cup C \succ A \cup C$, but $A \cup B \cup D \sim^* A \cup D$ for every $D \supseteq C$ or (ii) $A \cup B \cup C \prec B \cup C$, but $A \cup B \cup D \sim^* B \cup D$ for every $D \supseteq C$.*

When $\succsim$ and $\succsim^*$ are finite positive additive EU representations and, consequently, value only flexibility, Menu Preference Consistency reduces to the Flexibility Consistency postulate of Riella [17].

Axiom 2 (Flexibility Consistency). *For any pair of menus A and B in $int(\mathcal{K})$, $B \succsim A$ and $A \succ^* B$ implies that there exists a menu C such that $A \cup B \cup C \succ A \cup C$, but $A \cup B \cup C \sim^* A \cup C$.*

Similarly, if $\succsim$ and $\succsim^*$ are negative additive EU preferences and, therefore, value only commitment, Menu Preference Consistency becomes equivalent to the following postulate:

Axiom 3 (Commitment Consistency). *For any pair of menus A and B in $int(\mathcal{K})$, $B \succsim A$ and $A \succ^* B$ implies that there exists a menu C such that $A \cup B \cup C \prec B \cup C$, but $A \cup B \cup C \sim^* B \cup C$.*

Following Riella [17], we note that Flexibility Consistency admits the following interpretation. Whenever the relations $\succsim$ and $\succsim^*$ disagree in the comparison of two menus A and

B as in the statement of the postulate, there must exist at least one situation where the relation $\succsim$ strictly values the flexibility provided by the alternatives in B, in the presence of A, but the relation $\succsim^*$ sees no value on that. In a way, the postulate demands that all the disagreement between the two relations can be blamed on the fact that $\succsim$ values flexibility more than $\succsim^*$. Commitment consistency has a similar interpretation, with the difference that it demands the existence of a situation where $\succsim$ values the commitment provided by B and $\succsim^*$ does not.

In a nutshell, Menu Preference Consistency imposes that when the two relations disagree, either the condition in Flexibility Consistency or the one in Commitment Consistency must hold. We note, however, that it requires a stronger condition on $\succsim^*$; This is due to the fact that when $\succsim^*$ is an arbitrary additive EU preference, we can no longer interpret the conditions $A \cup B \cup C \sim^* A \cup C$ and $A \cup B \cup C \sim^* B \cup C$ as situations where $\succsim^*$ does not see any flexibility or commitment value in B, respectively. In the first case, for example, there may be two alternatives $p, q \in B$ such that $A \cup \{p\} \cup C \succ^* A \cup C$ and $A \cup \{q\} \cup C \prec^* A \cup C$. The indifference between $A \cup B \cup C$ and $A \cup C$ may be due to the fact that the flexibility value $\succsim^*$ sees in B is exactly offset by the commitment value it sees in A. To overcome this problem, we impose the stronger condition implying that $\succsim^*$ never reveals any flexibility value in B nor any commitment value in A once the options in C are available. We can now state the following result:

Theorem 1. *Let $\succsim$ and $\succsim^*$ be two finite additive EU preferences. The following statements are equivalent:*

1. *$\succsim$ and $\succsim^*$ satisfy Menu Preference Consistency;*

2. *Let S and S^* be the unique subjective state spaces of $\succsim$ and $\succsim^*$, respectively, and let $U : (S \cup S^*) \times \Delta \to \mathbb{R}$ be such that, for each $s \in S \cup S^*$, $U(s, \cdot)$ is an expected utility function that represents s. For any two menus A and B with*

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p) \text{ for all } s \in S \setminus S^*,$$

   $A \succsim B \iff A \succsim^* B$;

3. *For every finite additive EU representation (S, μ, U) of $\succsim$, there exists $T \subseteq S$ such that $(T, \mu|_T, U|_T)$ represents $\succsim^*$.*[4]

The interpretation of the above theorem is similar to Riella's main result [17]. We interpret Part 2 as a subjective state space version of Dynamic Consistency under finite additive EU

[4]By $\mu|_T$ and $U|_T$ we mean the restrictions of μ and U to the set T.

preferences. The relation $\succsim^*$ represents the DM's preferences after she observes the event S^*. Then, similarly to the objective state space version of Dynamic Consistency, $\succsim$ and $\succsim^*$ have the same ranking over any two menus that achieve the same utility levels outside S^*. Part 3 says that the agent identifies the subset T of the subjective state space S, and updates her preference in a Bayesian-like way.

With respect to Part 3, some might be concerned about the significance of a Bayesian updating result in an environment where the relative weights of the states are not uniquely identified. Indeed, for finite additive EU representations one cannot disentangle the utility and the measure of each state. That is, for a given state s, we can always change the magnitude (but not the sign) of the measure of s as long as we renormalize the state dependent utility function accordingly. We note, however, that Part 3 guarantees that for any representation of the relation $\succsim$ it is possible to find an event T such that $\succsim^*$ can be seen as a version of $\succsim$ that learns T, ignores the states outside T after that, and keeps aggregating the weights and utilities inside T in the same way as $\succsim$.

Another important observation is that some approaches to derive the uniqueness of the measure μ in a finite additive EU representation have appeared in the literature. The first approach is to introduce state-independence into the model by considering some form of separability in the ex post preferences as discussed in DLR [3, p. 912]: Krishna and Sadowski [15], and Schenone [19]. The second approach is to introduce state-independence into the model by using a natural restriction of ex post preferences: Dillenberger, Lleras, Sadowski, and Takeoka [6], Higashi, Hyogo, and Takeoka [12], Hyogo [13], and Takeoka [22]. The third is Stovall [20].[5] In Stovall [20], there is a state-independent component in the relationship between subjective states related to preference for flexibility and those related to preference for commitment. All these approaches end up implying some specific normalization for the function U. Part 3 above shows that irrespectively of the specific normalization of the function U in the representation of $\succsim$, if $\succsim$ and $\succsim^*$ satisfy Menu Preference Consistency, then $\succsim^*$ can be represented by the same measure and utility, but ignoring the states outside some event T.

In turn, we will be interested in situations where only the positive or negative part of the representation of $\succsim^*$ may be seen as an update of $\succsim$. For that, we will need to weaken Menu Preference Consistency so that it applies only to situations where commitment or flexibility reasons for the two preferences to disagree have been ruled out. Consider the following weakening of Menu Preference Consistency:

Axiom 4 (Weak Flexibility Consistency)**.** *For any pair of menus A and B in int($\mathcal{K}$), $B \succsim A$ and $A \succ^* B$ implies that there exists a menu C such that (i) $A \cup C \succ^* A \cup B \cup C$, (ii)*

[5] We show uniqueness of a belief over subjective states in Stovall [20]. See Theorem 4 for more details.

$B \cup C \succ A \cup B \cup C$, or (iii) $A \cup B \cup C \succ A \cup C$, but $A \cup B \cup C \sim^* A \cup C$.

In the postulate above, the relations $\succsim$ and $\succsim^*$ disagree in the comparison of the menus A and B. The postulate then demands one of three conditions. The first two say that $\succsim^*$ sees some commitment value in A in the presence of B, or $\succsim$ sees some commitment value in B in the presence of A. Suppose those conditions are not satisfied. That is, suppose we cannot blame the disagreement between $\succsim$ and $\succsim^*$ on commitment reasons. In this case, the postulate demands that the condition in the flexibility consistency postulate be satisfied. Note that we no longer need the stronger condition in Menu Preference Consistency. In fact, $\succsim^*$ does not see any commitment value in A in the presence of the options in C because (i) is not satisfied. Thus the indifference in condition (iii) is enough to guarantee that $\succsim^*$ does not see any flexibility value in B once the options in C are available.

Note that both Flexibility Consistency and Menu Preference Consistency imply Weak Flexibility Consistency, while Flexibility Consistency and Menu Preference Consistency do not have a logical relationship in general. When $\succsim$ and $\succsim^*$ are finite positive additive EU representations, all three consistency conditions are equivalent.

We can now state the following result:

Theorem 2. *Let $\succsim$ and $\succsim^*$ be two finite additive EU preferences. The following statements are equivalent:*

1. *$\succsim$ and $\succsim^*$ satisfy Weak Flexibility Consistency;*

2. *Let S and S^* be the unique subjective state spaces of $\succsim$ and $\succsim^*$, respectively, and let $U : (S \cup S^*) \times \Delta \to \mathbb{R}$ be such that for each $s \in S \cup S^*$, $U(s, \cdot)$ is an expected utility function that represents s. For any $A, B \in \mathcal{K}$ with*

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p)$$

   for all $s \in (S \cup S^*) \setminus \mathcal{P}(S^*)$, we have $A \succsim B \iff A \succsim^* B$;

3. *For every finite additive EU representation (S, μ, U) of $\succsim$, there exists a representation (T, μ^*, U^*) of $\succsim^*$ such that $\mathcal{P}(T) \subseteq \mathcal{P}(S)$, and, for every $s \in \mathcal{P}(T)$, $\mu(s) = \mu^*(s)$ and $U(s, \cdot) = U^*(s, \cdot)$.*

The interpretation of Theorem 2 is similar to that of Theorem 1. Part 2 can be viewed as a version of Dynamic Consistency that applies only to the positive states of both relations. It says that both relations must agree on the ranking of any pair of menus that attain the same utility levels on all states outside the set of positive states of $\succsim^*$. In Part 3, it

is as if the positive part of $\succsim^*$'s representation is a Bayesian update of the positive part of $\succsim$'s representation. Note that it does not restrict in any way the negative parts of the representations of $\succsim$ and $\succsim^*$.

The analysis above focuses on flexibility. We can perform a symmetric exercise focusing on commitment.

Axiom 5 (Weak Commitment Consistency)**.** *For any pair of menus A and B in int($\mathcal{K}$), $B \succsim A$ and $A \succ^* B$ implies that there exists a menu C such that (i) $A \cup B \cup C \succ^* B \cup C$, (ii) $A \cup B \cup C \succ A \cup C$, or (iii) $A \cup B \cup C \prec B \cup C$, but $A \cup B \cup C \sim^* B \cup C$.*

The interpretation of the above postulate is symmetric to the one of Weak Flexibility Consistency. That is, it imposes that whenever we cannot blame the disagreement between $\succsim$ and $\succsim^*$ on flexibility reasons, the condition in the commitment consistency postulate must be satisfied.

We have the following result analogous to Theorem 2 above:

Theorem 3. *Let $\succsim$ and $\succsim^*$ be two finite additive EU preferences. The following statements are equivalent:*

1. *$\succsim$ and $\succsim^*$ satisfy Weak Commitment Consistency;*
2. *Let S and S^* be the unique subjective state spaces of $\succsim$ and $\succsim^*$, respectively, and let $U : (S \cup S^*) \times \Delta \to \mathbb{R}$ be such that for each $s \in S \cup S^*$, $U(s, \cdot)$ is an expected utility function that represents s. For any $A, B \in \mathcal{K}$ with*

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p)$$

   for all $s \in (S \cup S^*) \setminus \mathcal{N}(S^*)$, we have $A \succsim B \iff A \succsim^* B$;
3. *For every finite additive EU representation (S, μ, U) of $\succsim$, there exists a representation (T, μ^*, U^*) of $\succsim^*$ such that $\mathcal{N}(T) \subseteq \mathcal{N}(S)$, and, for every $s \in \mathcal{N}(T)$, $\mu(s) = \mu^*(s)$ and $U(s, \cdot) = U^*(s, \cdot)$.*

In the next section, we apply the results above to some special cases of finite additive EU representations that have appeared in the literature.

4 Applications

4.1 Multiple Temptations

In this subsection, we apply the results of Section 3 to the multiple temptations model introduced by Stovall [20]. Stovall's model is a generalization of the temptation and self-

control model of Gul and Pesendorfer [11]. In Stovall's model, the individual chooses a menu knowing that in the future she may be susceptible to multiple temptation states and in each state she will make a choice that is a compromise between her normative preference and her corresponding temptation. In addition, for each temptation state, she pays a self-control cost which is a function of the most tempting alternative in the menu. Formally, we will work with the following model:

Definition 2. Given a binary relation $\succsim \subseteq \mathcal{K} \times \mathcal{K}$, a *finite multiple temptations representation* of $\succsim$ is a tuple (S, μ, U, u), where S is a finite set, μ is a probability measure on S, $U : S \times \Delta \to \mathbb{R}$ is a state dependent utility function, and u is a nonconstant expected utility function such that

1. For each $s \in S$, $U(s, \cdot)$ is nonconstant and of the expected utility type;
2. the function $V_T : \mathcal{K} \to \mathbb{R}$ defined by

$$V_T(A) := \sum_{s \in S} \mu(s) \left\{ \max_{p \in A} [u(p) + U(s,p)] - \max_{p \in A} U(s,p) \right\},$$

   for each $A \in \mathcal{K}$, represents $\succsim$;
3. μ has full support and, for each distinct s and s' in S, $U(s, \cdot)$ and $U(s', \cdot)$ are not positive affine transformations of each other.

In the definition above, u represents the DM's normative preference. This is the preference the DM would like to use to make her choice from a given menu, but she knows that in the future she will suffer from temptation. Temptation is random and any of the temptation states in S may happen with some positive probability. Each temptation state $s \in S$ induces a temptation utility function $U(s, \cdot)$. Given a temptation state $s \in S$, the DM maximizes a compromise between her normative utility u and her temptation utility $U(s, \cdot)$, which takes the form of the simple addition of the two functions. Finally, the DM suffers an additional cost that is given by the maximum temptation utility she could obtain from the menu.

As it is the case with finite additive EU representations, the state space S in a finite multiple temptations representation is just an index set whose relevant aspects are only the preference relations represented by the functions in $\{U(s, \cdot) : s \in S\}$. Again, we may assume, without loss of generality, that the elements of S are exactly these preference relations. Moreover, finite multiple temptations representations are a special case of finite additive EU representations and it turns out that the uniqueness of the state space of the latter implies the uniqueness of the state space of the former. In fact, we can show that finite multiple

temptations representations have even stronger uniqueness properties, as the probability measures that are part of this type of representation are also unique.

Theorem 4. *Let (S, μ, U, u) be a finite multiple temptations representation. Then, $(\hat{S}, \hat{\mu}, \hat{U}, \hat{u})$ is another multiple temptations representation of the same relation if and only if (i) $S = \hat{S}$, (ii) $\mu = \hat{\mu}$, (iii) there exist $(\alpha, \beta) \in \mathbb{R}_{++} \times \mathbb{R}$ such that $\hat{u} = \alpha u + \beta$, and (iv) for each $s \in S$ such that $U(s, \cdot)$ is not a positive affine transformation of u, there exists $\beta_s \in \mathbb{R}$ such that $\hat{U}(s, \cdot) = \alpha U(s, .) + \beta_s$.*

In Theorem 4, we can prove the uniqueness of the probability measure over uncertain temptation states in multiple temptations representations because the normative preference is not uncertain. Stovall [21] considers a preference over menus of menus of lotteries setting and characterizes a DM who is represented by a corresponding finite multiple temptations representation. In a richer setting than Stovall [20] and ours, Stovall [21] shows that if the normative preference is not uncertain, the probability measure over uncertain temptation states is uniquely identified.

Given a relation $\succsim$ with a finite multiple temptations representation (S, μ, U, u), define a relation $\tilde{\succsim} \subseteq \Delta \times \Delta$ by $p \tilde{\succsim} q \iff \{p\} \succsim \{q\}$. The relation $\tilde{\succsim}$ can be interpreted as the agent's normative preference and the function u is an expected utility representation of it. We may now state the following result:

Theorem 5. *Let $\succsim$ and $\succsim^*$ be two finite multiple temptations preferences. The following statements are equivalent:*

1. *$\succsim$ and $\succsim^*$ satisfy Menu Preference Consistency, and $\tilde{\succsim} = \tilde{\succsim}^*$;*
2. *For every finite multiple temptations representation (S, μ, U, u) of $\succsim$, there exists $T \subseteq S$ such that $(T, \mu_T, U|_T, u)$ is a finite multiple temptations representation of $\succsim^*$, where μ_T is the Bayesian update of μ after the observation of T.*

A statement like 2 in Theorem 1 is not readily available for finite multiple temptations representations. We note, however, that statement 2 in Theorem 5 is more meaningful than statement 3 in Theorem 1. Since the prior in a finite additive EU representation is not unique, Bayesian updating has a more limited meaning for that type of representation. It only captures the fact that the DM acts in a dynamically consistent way. In contrast, the prior in a finite multiple temptations representation is unique, which makes the priors and Bayesian updating fully meaningful concepts for this type of representation.

A finite multiple temptations representation is silent about the relationship between the normative utility function u and the temptation states. It is conceivable, thus, that the

change in the state space induced by the reception of the signal might affect it. This is captured by the following result:

Theorem 6. *Let $\succsim$ and $\succsim^*$ be two finite multiple temptations preferences. The following statements are equivalent:*

1. *$\succsim$ and $\succsim^*$ satisfy Weak Commitment Consistency;*

2. *For every finite multiple temptations representation (S, μ, U, u) of $\succsim$, there exists $T \subseteq S$ and a nonconstant expected utility function $u^* : \Delta \to \mathbb{R}$ such that $(T, \mu_T, U|_T, u^*)$ is a finite multiple temptations representation of $\succsim^*$, where μ_T is the Bayesian update of μ after the observation of T.*

4.2 Anticipating Regret

Another interesting special case of additive EU representations that has appeared in the literature is the anticipated regret model of Sarver [18]. In Sarver's regret model, the DM chooses a menu and an alternative from the menu, both before some subjective uncertainty is resolved. Because of that, the DM anticipates that when uncertainty is finally resolved she may regret her choice. Formally:

Definition 3. Given a binary relation $\succsim \subseteq \mathcal{K} \times \mathcal{K}$, a *finite regret representation* of $\succsim$ is a tuple (S, μ, U, K), where $K \in \mathbb{R}_{++}$, S is a finite set, μ is a probability measure on S, and $U : S \times \Delta \to \mathbb{R}$ is a state dependent utility function such that

1. For each $s \in S$, $U(s, \cdot)$ is nonconstant and of the expected utility type;

2. the function $V_R : \mathcal{K} \to \mathbb{R}$ defined by

$$V_R(A) := \max_{p \in A} \left\{ \sum_{s \in S} \mu(s) \left[U(s,p) - K \left(\max_{q \in A} U(s,q) - U(s,p) \right) \right] \right\},$$

   for each $A \in \mathcal{K}$, represents $\succsim$;

3. μ has full support and, for each distinct s and s' in S, $U(s, \cdot)$ and $U(s', \cdot)$ are not positive affine transformations of each other;

4. the real valued function $\sum_{s \in S} \mu(s) U(s, \cdot)$ defined on Δ is not a positive affine transformation of any of the functions in $\{U(s, \cdot) : s \in S\}$.

As it happens with finite additive EU representations, the state space S in a finite regret representation is just an index set whose only relevant aspect is the set of expected utility preferences induced by $\{U(s,\cdot\,) : s \in S\}$. Again, this collection of expected utility preferences is unique and we can assume, without loss of generality, that the elements of S are exactly the expected utility relations represented by the functions in $\{U(s,\cdot\,) : s \in S\}$.

As we have discussed above, in the regret representation the individual anticipates that she will make a choice from the menu A before the subjective state is revealed. Therefore, when choosing a menu she takes into account the psychological cost she will have once she realizes she did not make the best choice for the realized subjective state. This psychological cost is proportional to the utility difference between the best alternative in the corresponding state and the individual's choice.

The model can be equivalently written as

$$V_R(A) = (1+K)\max_{p\in A}\sum_{s\in S}\mu(s)U(s,p) - K\sum_{s\in S}\mu(s)\max_{q\in A}U(s,q),$$

which makes it clear that it is a special case of a finite additive EU representation with one positive state (represented by $\sum_{s\in S}\mu(s)U(s,\cdot\,)$) and $|S|$ negative states. We may now state the following result:

Theorem 7. *Let $\succsim$ and $\succsim^*$ be two relations that admit finite regret representations. The following statements are equivalent:*

1. *$\succsim$ and $\succsim^*$ satisfy Weak Commitment Consistency;*

2. *Let S and S^* be the unique state spaces (in the regret representation sense) of $\succsim$ and $\succsim^*$, respectively. Let $U : (S \cup S^*) \times \Delta \to \mathbb{R}$ be such that, for each $s \in S \cup S^*$, $U(s,\cdot)$ is an expected utility function that represents s. Then, for any $A, B \in \mathcal{K}$ with*

$$\begin{aligned}
\max_{p\in A} V_R(\{p\}) &= \max_{p\in B} V_R(\{p\}),\\
\max_{p\in A} V_R^*(\{p\}) &= \max_{p\in B} V_R^*(\{p\}),\ \textit{and}\\
\max_{p\in A} U(s,p) &= \max_{p\in B} U(s,p)\ \textit{for all}\ s \in S\setminus S^*,
\end{aligned}$$

   we have $A \succsim B \iff A \succsim^* B$;

3. *For every finite regret representation (S,μ,U,K) of $\succsim$, there exists $T \subseteq S$ and $\hat{K} > 0$ such that $(T,\mu_T,U|_T,\hat{K})$ is a finite regret representation of $\succsim^*$, where μ_T is the Bayesian update of μ after the observation of T.*

For a finite regret representation, $\max_{p\in A} V_R(\{p\})$ gives us the expected utility the DM would derive from menu A in the absence of the anticipated regret cost. We may call it the regret-free utility of menu A. So, Part 2 of Theorem 7 says that for menus A and B that have the same regret-free utility for both $\succsim$ and $\succsim^*$, and have the same maximal attainable utility for all states in $S \setminus S^*$, the two relations must agree on how to compare them. Again, it can be seen as a subjective state space version of Dynamic Consistency, but that comes into play only for pairs of menus that have the same regret-free utility for both relations. Similarly to Part 2 of Theorem 5, Part 3 above says that $\succsim^*$ may be interpreted as a version of $\succsim$ that learned that some event T occurred and updated its prior using Baye's rule.

Appendix

A Proof of Theorem 1

Lemma 1. *Let (S^*, μ^*, U) be a Finite Additive EU representation of $\succsim^*$. Suppose that $F \in \text{int}\mathcal{K}$. If $E \cup F \cup D \sim^* E \cup D$ for all $D \supseteq C$, we have*

$$\max_{p\in E\cup F\cup C} U(s,p) = \max_{p\in E\cup C} U(s,p)$$

for all $s \in S^*$.

Proof. We show the result by contraposition. Suppose that there exists $s^* \in S^*$ such that $\max_{p\in E\cup F\cup C} U(s^*,p) \neq \max_{p\in E\cup C} U(s^*,p)$. Then we have that $\max_{p\in F} U(s^*,p) > \max_{p\in E} U(s^*,p)$.

Now we construct a menu $D \supseteq C$ such that

$$\max_{l\in E\cup F\cup D} U(s,p) = \max_{p\in E\cup D} U(s,p) \text{ for all } s \in S^* \setminus \{s^*\},$$
$$\max_{l\in E\cup F\cup D} U(s^*,p) > \max_{l\in E\cup D} U(s^*,p).$$

And hence, $E \cup F \cup D \nsim^* E \cup D$

The following construction is the same as in Theorem 1 in Riella [17]. In order to have

the desired D, we consider the following three cases for $s \in S^* \setminus \{s^*\}$:

$$(i)\ \max_{p\in E} U(s,p) \geq \max_{p\in F} U(s,p);$$

$$(ii)\ \max_{p\in F} U(s,p) > \max_{p\in E} U(s,p) \text{ and } \operatorname*{argmax}_{p\in F} U(s,p) \not\subset \operatorname*{argmax}_{p\in F} U(s^*,p);$$

$$(iii)\ \max_{p\in F} U(s,p) > \max_{p\in E} U(s,p) \text{ and } \operatorname*{argmax}_{p\in F} U(s,p) \subset \operatorname*{argmax}_{p\in F} U(s^*,p).$$

For the first case, let p_s be any lottery in $\operatorname*{argmax}_{p\in E} U(s,p)$. For the second case, p_s be any lottery in $\operatorname*{argmax}_{p\in F} U(s,p) \setminus \operatorname*{argmax}_{p\in F} U(s^*,p)$. It is clear that $\max_{p\in F} U(s^*,p) > U(s^*,p_s)$ for both the first and second cases. Consider the third case. Since $s \neq s^*$, we can pick lotteries p and p' with $U(s,p) = U(s,p')$ and $U(s^*,p) < U(s^*,p')$ such that p and p' are arbitrary close to each other. Pick p, p' and $q \in \operatorname*{argmax}_{p\in F} U(s,p)$ so that $r := q+(p-p')$ is a lottery. Note that for any $\lambda \in (0,1)$, we have that $U(s, \lambda q+(1-\lambda)r) = U(s,q)$ and $U(s^*, \lambda q+(1-\lambda)r) < U(s^*,q)$. Let $p_s := \lambda q+(1-\lambda)r$. Then it follows from the construction that $U(s,p_s) = \max_{p\in F} U(s,p)$ and $\max_{p\in F} U(s^*,p) > U(s^*,p_s)$. Define $D := C \cup \{p_s : s \in S^* \setminus \{s^*\}\}$. Then we have the desired D.

□

In the following, we denote the set of maximizers over E of ex-post utility functions $U(s,p)$ for $s \in O$ by

$$M_E(O) = \{q : q = \operatorname*{argmax}_{p\in E} U(s,p)\}.$$

Note that, if O is a finite set and E is a sphere, then $M_E(O)$ is a finite set. Moreover, if $E \in \text{int}\mathcal{K}$, then $M_E(O) \in \text{int}\mathcal{K}$

[1 $\implies$ 2] Suppose 1 is true. To begin with, we show that $S^* \subseteq S$. Suppose in negation that there exists $s^* \in S^* \setminus S$. First, consider a case where $s^* \in \mathcal{P}(S^*)$. Fix a sphere $E \in \text{int}\mathcal{K}$. Define $B := M_E(S \cup S^*) \in \text{int}\mathcal{K}$. Note that B is finite because $S \cup S^*$ is finite. For small enough $\varepsilon > 0$, we can take $p^* \in \Delta(X)$ with full support such that $U(s^*,p^*) = \max_{p\in B} U(s^*,p) + \varepsilon$ and $U(s,p^*) < \max_{p\in B} U(s,p)$ for every $s \in S \cup S^* \setminus \{s^*\}$. Let $A := B \cup \{p^*\} \in \text{int}\mathcal{K}$.

It follows from the construction that $A \sim B$ and $A \succ^* B$. It is clear that $A \cup B \cup C \sim A \cup C$ for any $C \in \mathcal{K}$ because $A \supseteq B$. By construction, for any menu C, $\max_{p\in A\cup B\cup C} U(s,p) = \max_{p\in B\cup C} U(s,p)$ for all $s \in S$. Thus we have $A \cup B \cup C \sim B \cup C$ for any $C \in \mathcal{K}$. This contradicts to Menu Preference Consistency.

Next, consider a case where $s^* \in \mathcal{N}(S^*)$. We rename B in the above to A and A in the

above to B. Then by the same argument, we have $A \sim B$ and $A \succ^* B$, but $A \cup B \cup C \sim A \cup C$ and $A \cup B \cup C \sim B \cup C$ for any $C \in \mathcal{K}$. Again, this contradicts to Menu Preference Consistency. Summarizing the above, we conclude that $S^* \subseteq S$.

Now we turn to show the second part of statement 2. In negation, suppose that the second part of statement 2 fails: there exist menus $A, B \in \mathcal{K}$ such that

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p)$$

for all $s \in S \setminus S^*$, but either (i) $A \succsim B$ and $B \succ^* A$ or (ii) $A \succsim^* B$ and $B \succ A$.

We argue that the negation of the second part of statement 2 implies that there exist menus A and B with

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p) \tag{1}$$

for all $s \in S \setminus S^*$, but either (i') $A \succ^* B$, but $B \succsim A$, or (ii') $A \sim^* B$, but $B \succ A$. If the condition (i) of the negation of the second part of statement 2 holds, rename A to B and B to A. Then we have such menus A and B satisfying the equation (1) and the condition (i'). If the condition (ii) of the negation of second part of statement 2 holds, (i') or (ii') holds.

Next, we show that the condition (ii') reduces to the condition (i'). Suppose that we have the condition (ii') and let $s^* \in S^*$. Let $E \in \text{int}\mathcal{K}$ be a sphere. First consider a case where $s^* \in \mathcal{P}(S^*)$. For each $\lambda \in (0,1)$, define $A_\lambda := \lambda A + (1-\lambda) M_E(S)$ and $B_\lambda := \lambda B + (1-\lambda) M_E(S \setminus \{s^*\})$. It can be checked that, for sufficiently large λ,

$$\max_{p \in A_\lambda} U(s,p) = \max_{p \in B_\lambda} U(s,p)$$

for all $s \in S \setminus S^*$, but $A_\lambda \succ^* B_\lambda$ and $B_\lambda \succ A_\lambda$. Rename A_λ to A and B_λ to B. Then we have the condition (i').

Second, consider a case where there exits $s^* \in \mathcal{N}(S^*)$. For each $\lambda \in (0,1)$, define $A_\lambda := \lambda A + (1-\lambda) M_E(S \setminus \{s^*\})$ and $B_\lambda := \lambda B + (1-\lambda) M_E(S)$. Then the same argument as above holds, and we have the condition (i'). Summarizing the above, the negation of the second part of statement 2 implies that there exist menus A and B with

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p) \tag{2}$$

for all $s \in S \setminus S^*$, and $A \succ^* B$ but $B \succsim A$.

Finally, we show that Menu Preference Consistency fails for such A and B.[6] First, we

[6] If $A \notin \text{int}\mathcal{K}$ or $B \notin \text{int}\mathcal{K}$, combine both menus with a lottery with full support. "Independence axiom" guarantees that all of the rankings remain the same.

argue that the condition (i) in Menu Preference Consistency does not hold. Suppose $C \in \mathcal{K}$ is a menu such that $A \cup B \cup D \sim^* A \cup D$ for all $D \supseteq C$. By Lemma 1, we have that $\max_{p \in A \cup B \cup C} U(s,p) = \max_{p \in A \cup C} U(s,p)$ for all $s \in S^*$. This equality and the equation (2) together imply that $A \cup B \cup C \sim A \cup C$. We conclude that the condition (i) in Menu Preference Consistency does not hold. Similarly, it can be shown that the condition (ii) in Menu Preference Consistency does not hold, either.

[2 $\implies$ 3] Suppose 2 is true. Take any two menus A and B, and let E be a sphere in int$\mathcal{K}$. Define $C := M_E(S \setminus S^*)$ and $D := M_E(S^*)$. For each $\lambda \in (0,1)$, define $A_\lambda := C \cup (\lambda D + (1-\lambda)A)$ and $B_\lambda := C \cup (\lambda D + (1-\lambda)B)$. For sufficiently large λ, we have that $\max_{p \in A_\lambda} U(s,p) = \max_{p \in B_\lambda} U(s,p)$ for every $s \in S \setminus S^*$, and $\max_{p \in A_\lambda} U(s,p) - \max_{p \in B_\lambda} U(s,p) = (1-\lambda)(\max_{p \in A} U(s,p) - \max_{p \in B} U(s,p))$ for every $s \in S^*$. By statement 2 and the representations of $\succsim$ and $\succsim^*$, we obtain that

$$
\begin{aligned}
& A \succsim^* B \\
\iff & A_\lambda \succsim^* B_\lambda \\
\iff & A_\lambda \succsim B_\lambda \\
\iff & \sum_{s \in S} \mu(s) \max_{p \in A_\lambda} U(s,p) \geq \sum_{s \in S} \mu(s) \max_{p \in B_\lambda} U(s,p) \\
\iff & \sum_{s \in S \cap S^*} \mu(s) \max_{p \in A_\lambda} U(s,p) \geq \sum_{s \in S \cap S^*} \mu(s) \max_{p \in B_\lambda} U(s,p) \\
\iff & \sum_{s \in S \cap S^*} \mu(s) \max_{p \in A} U(s,p) \geq \sum_{s \in S \cap S^*} \mu(s) \max_{p \in B} U(s,p),
\end{aligned}
$$

where the second equivalence is by the second part of statement 2, and the fourth equivalence is by $S^* \subseteq S$. By defining $T := S \cap S^*$, we obtain that 2 $\implies$ 3.

[3 $\implies$ 1] Suppose 3 is true. Fix $A, B \in$ int$\mathcal{K}$ such that $B \succsim A$ and $A \succ^* B$. Then there exists $\hat{s} \in S \setminus T$ satifying (i)$\mu(\hat{s}) > 0$ and $\max_{p \in B} U(\hat{s},p) > \max_{p \in A} U(\hat{s},p)$, or (ii)$\mu(\hat{s}) < 0$ and $\max_{p \in A} U(\hat{s},p) > \max_{p \in B} U(\hat{s},p)$.

Consider the first case. Then, following the same argument as in the proof of Lemma 1,

we can construct a menu C such that[7]

$$\max_{p\in A\cup B\cup C} \max U(s,p) = \max_{p\in A\cup C} \max U(s,p) \text{ for all } s \in S \setminus \{\hat{s}\},$$
$$\max_{p\in A\cup B\cup C} \max U(\hat{s},p) > \max_{p\in A\cup C} \max U(\hat{s},p).$$

By the representations of $\succsim$ and $\succsim^*$, we have $A \cup B \cup C \sim^* A \cup C$, but $A \cup B \cup C \succ A \cup C$. Moreover, it holds that $A \cup B \cup D \sim^* A \cup D$ for all $D \supseteq C$. Hence, part (i) of Menu Preference Consistency holds.

Consider the second case. Again, following the same argument as in the proof of Lemma 1, we can construct a menu C such that[8]

$$\max_{p\in A\cup B\cup C} \max U(s,p) = \max_{p\in B\cup C} \max U(s,p) \text{ for all } s \in S \setminus \{\hat{s}\},$$
$$\max_{p\in A\cup B\cup C} \max U(\hat{s},p) > \max_{p\in B\cup C} \max U(\hat{s},p).$$

By the representations of $\succsim$ and $\succsim^*$, we have $A \cup B \cup C \sim^* B \cup C$ but $A \cup B \cup C \prec B \cup C$. Moreover, it holds that $A \cup B \cup D \sim^* A \cup D$ for all $D \supseteq C$. Hence, Commitment Consistency at (A, B) holds. Hence, part (ii) of Menu Preference Consistency holds.

B Proof of Theorem 2

[1 $\implies$ 2] Suppose 1 is true. To begin with, we show $\mathcal{P}(S^*) \subseteq \mathcal{P}(S)$. Suppose contrary that there exists $s^* \in \mathcal{P}(S^*) \setminus \mathcal{P}(S)$. Fix a sphere $E \in \text{int}\mathcal{K}$. Define $A := M_E(S \cup S^*) \in \text{int}\mathcal{K}$. Note that A is finite because $S \cup S^*$ is finite. For small enough $\varepsilon > 0$, we can take $p^* \in \Delta(X)$ with full support such that $U(s^*, p^*) = \max_{p\in A} U(s^*, p) + \varepsilon$ and $U(s, p^*) < \max_{p\in A} U(s, p)$ for every $s \in S \cup S^* \setminus \{s^*\}$. Let $B := A \cup \{p^*\} \in \text{int}\mathcal{K}$. Then menus A and B in $int\mathcal{K}$ satisfy

$$\max_{p\in A} U(s,p) = \max_{p\in B} U(s,p)$$

for all $s \in (S \cup S^*) \setminus \{s^*\}$ and

$$\max_{p\in A} U(s^*,p) < \max_{p\in B} U(s^*,p).$$

By the representations of $\succsim$ and $\succsim^*$, such menus satisfy that $A \succsim B$, but $B \succ^* A$. However, by the representations of $\succsim$ and $\succsim^*$, we have $A \cup B \cup C \succsim^* A \cup C$, $A \cup B \cup C \sim B \cup C$, and

[7] We can think a menu A (B) as a menu E (F) in Lemma 1 and define $C := \{p_s : s \in S \setminus \{s^*\}\}$.
[8] We can think a menu A (B) as a menu F (E) in Lemma 1 and define $C := \{p_s : s \in S \setminus \{s^*\}\}$.

$A \cup B \cup C \sim A \cup C$ for every menu C. Thus $\succsim$ and $\succsim^*$ violate Weak Flexibility Consistency. We conclude that $\mathcal{P}(S^*) \subseteq \mathcal{P}(S)$.

We turn to show statement 2. By the same argument in Theorem 1, negation of statement 2 reduces to the following: there exist such menus A and B with

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p)$$

for all $s \in (S \cup S^*) \setminus \mathcal{P}(S^*)$, $A \succ^* B$, but $B \succsim A$, or $A \sim^* B$, but $B \succ A$. Suppose we have the second case and let $s^* \in \mathcal{P}(S^*)$.[9] Fix any sphere E in $int\mathcal{K}$. Define $\{p^*\} := M_E(\{s^*\})$. Let q^* be any lottery such that $U(s^*, q^*) > U(s^*, p^*)$. For $\lambda, \alpha \in (0,1)$, define $A_{\lambda,\alpha} := ((\lambda(M_E((S \cup S^*) \setminus \{s^*\}) \cup \{\alpha p^* + (1-\alpha) q^*\}) + (1-\lambda)A)$ and $B_\lambda := (\lambda(M_E((S \cup S^*) \setminus \{s^*\}) \cup \{p^*\}) + (1-\lambda)B)$. It can be checked that, for λ and α large enough,

$$\max_{p \in A_{\lambda,\alpha}} U(s,p) = \max_{p \in B_\lambda} U(s,p)$$

for all $s \in (S \cup S^*) \setminus \mathcal{P}(S^*)$, $A_{\lambda,\alpha} \succ^* B_\lambda$ and $B_\lambda \succ A_{\lambda,\alpha}$. This shows that whenever 2 is not satisfied we can find menus A and B such that

$$\max_{p \in A} U(s,p) = \max_{p \in B} U(s,p)$$

for all $s \in (S \cup S^*) \setminus \mathcal{P}(S^*)$, $B \succsim A$ and $A \succ^* B$. Without loss of generality, we can assume that $A, B \in int\mathcal{K}$.[10]

By the representations of $\succsim$ and $\succsim^*$, and $\mathcal{P}(S^*) \subseteq \mathcal{P}(S)$, the condition above implies that $A \cup B \cup C \succsim^* A \cup C$ and $A \cup B \cup C \succsim B \cup C$ for every menu C. These rankings contradict (i) and (ii) in Weak Flexibility Consistency. Now suppose that the menu C is such that $A \cup C \sim^* A \cup B \cup C$. Since, by assumption,

$$\max_{p \in A \cup C} U(s,p) = \max_{p \in A \cup B \cup C} U(s,p)$$

for all $s \in (S \cup S^*) \setminus \mathcal{P}(S^*)$ and $\mathcal{P}(S^*) \subseteq \mathcal{P}(S)$, the representation of $\succsim^*$ implies that

$$\max_{p \in A \cup C} U(s,p) = \max_{p \in A \cup B \cup C} U(s,p)$$

for all $s \in \mathcal{P}(S^*)$. Now the representation of $\succsim$, implies that $A \cup C \sim A \cup B \cup C$. This contradicts (iii) in Weak Flexibility Consistency. We conclude that $1 \implies 2$.

[9] Notice that $B \succ A$ implies that $\mathcal{P}(S^*) \neq \emptyset$.

[10] If this is not the case, just mix both A and B with a lottery p with full support.

[2 $\implies$ 3] Now suppose statement 2 is true and let (S, μ, U) and (S^*, μ^*, U^*) be finite additive EU representations of $\succsim$ and $\succsim^*$, respectively. Note that statement 2 implies that $\mathcal{P}(S^*) \subseteq \mathcal{P}(S)$. Suppose contrary that there exists $s^* \in \mathcal{P}(S^*) \setminus \mathcal{P}(S)$. Let E be any sphere in $int\mathcal{K}$. Define $A := M_E(S \cup S^*)$ and $B := M_E((S \cup S^*) \setminus \{s^*\})$. Then such A and B satisfy

$$\max_{p \in A} U(s,p) = \max_{p \in N} U(s,p)$$

for all $s \in S \setminus S^*$, but $A \sim B$ and $A \succ^* B$. This contradicts statement 2.

Without loss of generality, we may assume that $U(.,s) = U^*(.,s)$ for every $s \in \mathcal{P}(S^*)$ and $\mu(\mathcal{P}(S^*)) = \mu^*(\mathcal{P}(S^*))$. Fix any two menus A and B and let E be any sphere in $int\mathcal{K}$. Define $C := M_E(S \cup S^*)$ and $D := M_E(\mathcal{P}(S^*))$. For each $\lambda \in (0,1)$, define $A_\lambda := C \cup (\lambda D + (1-\lambda)A)$ and $B_\lambda := C \cup (\lambda D + (1-\lambda)B)$. For λ large enough, we have that $\max_{p \in A_\lambda} U(s,p) = \max_{p \in B_\lambda} U(s,p)$ for every $s \in (S \cup S^*) \setminus \mathcal{P}(S^*)$ and $\max_{p \in A_\lambda} U(s,p) - \max_{p \in B_\lambda} U(s,p) = (1-\lambda)\left(\max_{p \in A} U(s,p) - \max_{p \in B} U(s,p)\right)$ for every $s \in \mathcal{P}(S^*)$. By statement 2 and the representations of $\succsim$ and $\succsim^*$, we get that

$$\begin{aligned}
&\sum_{s \in \mathcal{P}(S^*)} \mu(s) \max_{p \in A} U(s,p) \leq \sum_{s \in \mathcal{P}(S^*)} \mu(s) \max_{p \in B} U(s,p) \\
\iff &\sum_{s \in \mathcal{P}(S^*)} \mu(s) \max_{p \in A_\lambda} U(s,p) \leq \sum_{s \in \mathcal{P}(S^*)} \mu(s) \max_{p \in B_\lambda} U(s,p) \\
\iff &A_\lambda \succsim B_\lambda \\
\iff &A_\lambda \succsim^* B_\lambda \\
\iff &\sum_{s \in \mathcal{P}(S^*)} \mu^*(s) \max_{p \in A_\lambda} U(s,p) \leq \sum_{s \in \mathcal{P}(S^*)} \mu^*(s) \max_{p \in B_\lambda} U(s,p) \\
\iff &\sum_{s \in \mathcal{P}(S^*)} \mu^*(s) \max_{p \in A} U(s,p) \leq \sum_{s \in \mathcal{P}(S^*)} \mu^*(s) \max_{p \in B} U(s,p).
\end{aligned}$$

Since $\mathcal{P}(S^*) \subseteq \mathcal{P}(S)$, this shows that $(\mathcal{P}(S^*), \mu, U)$ and $(\mathcal{P}(S^*), \mu^*, U)$ represent the same Positive Additive EU preferences. Since $\mu(\mathcal{P}(S^*)) = \mu^*(\mathcal{P}(S^*))$, the uniqueness properties of such representations imply that $\mu(s) = \mu^*(s)$ for every $s \in \mathcal{P}(s^*)$. This shows that 2 $\implies$ 3.

[3 $\implies$ 1] Finally, suppose 3 is satisfied and let (S, μ, U) and (T, μ^*, U^*) be Finite additive EU representations of $\succsim$ and $\succsim^*$, respectively, that satisfy the conditions in 3. Pick menus A and B in $int\mathcal{K}$ such that $A \succ^* B$ and $B \succsim A$. There are three cases. First, if there is $s^* \in \mathcal{N}(T)$ with

$$\max_{p \in B} U^*(s^*,p) > \max_{p \in A} U^*(s^*,p),$$

we can follow the steps in Lemma 1 to find a menu C such that $\max_{p\in A\cup B\cup C} U^*(s,p) = \max_{p\in A\cup C} U^*(s,p)$ for every $s \in T \setminus \{s^*\}$, but $\max_{p\in A\cup B\cup C} U^*(s^*,p) > \max_{p\in A\cup C} U^*(s^*,p)$.[11] For such a menu, we have $A \cup C \succ^* A \cup B \cup C$. Hence (i) in Weak Flexibility Consistency holds.

Second, if there is $\hat{s} \in \mathcal{N}(S)$ with

$$\max_{p\in A} U(\hat{s},p) > \max_{p\in B} U(\hat{s},p),$$

we can similarly find a menu C such that $B\cup C \succ A\cup B\cup C$.[12] Hence (ii) in Weak Flexibility Consistency holds.

Finally, suppose that $\max_{p\in A} U^*(s,p) \geq \max_{p\in B} U^*(s,p)$ for every $s \in \mathcal{N}(T)$ and $\max_{p\in B} U(s,p) \geq \max_{p\in A} U(s,p)$ for every $s \in \mathcal{N}(S)$. Since the representations of $\succsim$ and $\succsim^*$ satisfy the conditions in 3, this can happen only if there exists $\tilde{s} \in \mathcal{P}(S) \setminus \mathcal{P}(T)$ with $\max_{p\in B} U(\tilde{s},p) > \max_{p\in A} U(\tilde{s},p)$. Note that $\tilde{s} \in \mathcal{P}(S) \setminus T$ because $\max_{p\in A} U^*(s,p) \geq \max_{p\in B} U^*(s,p)$ for every $s \in \mathcal{N}(T)$. As in Lemma 1, we can find a menu C such that $\max_{p\in A\cup B\cup C} U(s,p) = \max_{p\in A\cup C} U(s,p)$ for every $s \in (S\cup T) \setminus \{\tilde{s}\}$, but $\max_{p\in A\cup B\cup C} U(\tilde{s},p) > \max_{p\in A\cup C} U(\tilde{s},p)$.[13] For such a menu C, we have $A \cup B \cup C \sim^* A \cup C$, but $A \cup B \cup C \succ A \cup C$. This shows that $\succsim$ and $\succsim^*$ satisfy (iii) in Weak Flexibility Consistency. That is, $3 \implies 1$.

C Proof of Theorem 4

Suppose (S,μ,U,u) and $(\hat{S},\hat{\mu},\hat{U},\hat{u})$ are multiple temptations representations of the same relation. Note that S and $\hat{S}$ are sets of negative states of the same finite additive EU relation, so we must have $S = \hat{S}$. Let V_T and $\hat{V}_T$ be the respective utility functions over menus induced by the two representations. By the uniqueness properties of affine representations, we know that there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that, for any menu A, $\hat{V}_T(A) = \alpha V_T(A) + \beta$. This implies that $\hat{u}(p) = \hat{V}_T(\{p\}) = \alpha V_T(\{p\}) + \beta = \alpha u(p) + \beta$ for every $p \in \Delta$. Note that $(S,\hat{\mu},\frac{1}{\alpha}\hat{U} - \beta, u)$ is another finite multiple temptations representation of the same relation. By the uniqueness properties of finite additive EU representations, for each $s \in S$ there exist (α_s,β_s) and (α_s',β_s') in $(\mathbb{R}_{++},\mathbb{R})$ such that

$$\frac{1}{\alpha}\hat{U}(s,.) - \beta = \alpha_s U(s,.) + \beta_s \tag{3}$$

[11] We can think a menu A (B) as a menu E (F) in Lemma 1 and define $C := \{p_s : s \in T \setminus \{s^*\}\}$.
[12] We can think a menu A (B) as a menu F (E) in Lemma 1 and define $C := \{p_s : s \in S \setminus \{\hat{s}\}\}$.
[13] We can think a menu A (B) as a menu E (F) in Lemma 1 and define $C := \{p_s : s \in (S\cup T) \setminus \{\tilde{s}\}\}$.

and

$$u + \frac{1}{\alpha}\hat{U}(s,.) - \beta = \alpha_s^{'}(u + U(s,.)) + \beta_s^{'} \tag{4}$$

for every $s \in S$. Suppose first that $s \in S$ is such that $U(s,.)$ and u represent different relations. In this case, we can find two lotteries p and q such that $U(s,p) = U(s,q)$ (consequently, $\hat{U}(s,p) = \hat{U}(s,q)$), but $u(p) \neq u(q)$. Then, (4) implies that

$$\begin{aligned} u(p) - u(q) &= u(p) + \frac{1}{\alpha}\hat{U}(s,p) - u(q) - \frac{1}{\alpha}\hat{U}(s,q) \\ &= \alpha_s^{'}(u(p) + U(s,p)) - \alpha_s^{'}(u(q) + U(s,q)) \\ &= \alpha_s^{'}(u(p) - u(q)), \end{aligned}$$

which implies that $\alpha_s^{'} = 1$. We can also find lotteries p and q such that $u(p) = u(q)$, but $U(s,p) \neq U(s,q)$. Now (4) implies that

$$\begin{aligned} \frac{1}{\alpha}(\hat{U}(s,p) - \hat{U}(s,q)) &= u(p) + \frac{1}{\alpha}\hat{U}(s,p) - u(q) - \frac{1}{\alpha}\hat{U}(s,q) \\ &= u(p) + U(s,p) - u(q) - U(s,q) \\ &= U(s,p) - U(s,q). \end{aligned}$$

Using (3), we now obtain that

$$\begin{aligned} U(s,p) - U(s,q) &= \frac{1}{\alpha}(\hat{U}(s,p) - \hat{U}(s,q)) \\ &= \alpha_s(U(s,p) - U(s,q)), \end{aligned}$$

which implies that $\alpha_s = 1$. That is,

$$\frac{1}{\alpha}\hat{U}(s,.) - \beta = U(s,.) + \beta_s.$$

If $s \in S$ is such that u and $U(s,.)$ represent the same relation, it is clear from the formula of a finite multiple temptations representations that $\hat{U}(s,.)$ can be replaced by any positive affine transformation of it in the representation $(S, \hat{\mu}, \frac{1}{\alpha}\hat{U} - \beta, u)$ and continue representing the same preference over menus.[14] The observations above imply that $(S, \hat{\mu}, U, u)$ is another multiple temptations representation of the same preference over menus. Now the uniqueness properties of additive EU representations give us that $\mu = \hat{\mu}$.

[14] Because in this case $\max_{p \in A}[u(p) + U(s,p)] - \max_{p \in A} U(s,p) = \max_{p \in A} u(p)$ for any $U(s,.)$ that represents the same relation as u.

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