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"Effectiveness of Bailout Policies for Asset Bubbles
in a Small Open Economy"

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Effectiveness of Bailout Policies for Asset Bubbles in a Small Open Economy

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Abstract

This study analyzes the effects of bailout policies on the growth rate and asset prices in a small open economy with asset bubbles. In our model, bubbles stimulate investment and economic activities (so-called "crowd-in effect" of bubbles). Thus, after bubble crushing occurs, recessions follow. Under this condition, we show that as long as bubbles persist, generous bailout policies raise the economic growth rate by enhancing the crowd-in effect. When bubbles burst, the bailout policy mitigates capital losses caused by the burst and accelerates economic growth and workers' wages compared to the no-bailout case. Since the bailout policy has growth and recovery enhancing effects, a generous bailout policy is a desirable one for governments from the perspective of taxpayers' welfare. It should be noted, however, that a U.S. monetary policy to reduce the interest rate enlarges the size of asset bubbles in a small open economy, and further reduction of the U.S. interest rate makes the size of asset bubbles too large to be sustainable without adequate policy intervention of the small open economy; the government needs to reduce the scale of bailouts to an appropriate level in response to the U.S. interest rate reduction.

Keywords: Asset Bubbles; U.S. Interest Rate Policy; Economic Growth; Collapse of Asset Bubbles; Asset Prices; Bailout Policy

JEL Classification: E32, E44, E61, F43

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1 Introduction

Although government bailouts have become more common in the wake of major economic dynamics, including the recent global financial crisis, there is not enough accumulated theoretical literature on their effectiveness. In addition, since the global financial crisis, the U.S. has adopted a policy of historically low interest rates, making it increasingly necessary to analyze the impact of the monetary policies of such a large country on other economies[2]. Our study provides a simple model to analyze the effectiveness of the bailout policy in small open economies and the impact of the U.S. policy changes on the nature of bailout policies in small countries.

In recent years, there has been an accumulating theoretical literature analyzing large economic movements and asset price fluctuations such as the occurrence and collapse of asset bubbles[3]. Based on the literature, the analysis of bailout policies has focused on their impact on economic growth before and after the bursting of bubbles. For example, Diamond and Rajan (2012) and Farhi and Tirole (2009, 2012a) show a negative impact on the ex-ante economic efficiency in terms of moral hazard. On the other hand, Hirano et al. (2015) show that, in an economy with incomplete financial market, the bailout policy can have both positive and negative impacts on the economy, depending on the size of bubbles. Their paper is unique in that they use an infinite-horizon model to analyze desirable bailout policies from a welfare perspective.

Most studies in this vein, however, consider closed economies, and they do not explain the effects of any external change in interest policies such as an interest rate reduction or hike by the U.S. central bank. In addition, although there are studies that deal with asset bubbles in open economies, they do not analyze the impact of the introduction of bailout policy on small open economies or the impact of the U.S.

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[2] For example, the Bank for International Settlements (BIS, 2019) notes that this low interest rate policy has strengthened economic activity, particularly in the short term, but has caused other problems in the long term, such as risk-taking and financial market sensitivity to policy tightening. The International Monetary Fund (IMF, 2020) reports that market valuations for equities appear to be stretched, and if risky assets are repriced, the next recession could be deeper and longer than before. These analyses underscore the importance of the relationship between the U.S. low interest rate policy, risk asset prices, and economic fluctuation risks.

[3] Examples include Martin and Ventura (2012), Farhi and Tirole (2012b), Hirano et al. (2015), Miao et al. (2015), Hirano and Yanagawa (2017), and Miao and Wang (2018). These studies focus on the incompleteness of financial markets, called "financial friction," and analyze large economic movements such as the occurrence and collapse of asset bubbles. Mitsui and Watanabe (1989) offer the first study in the literature regarding the relationship between the long-run economic growth rate and financial frictions.

monetary policy on the nature of the bailout policy in these countries[4]. Our study differs from these studies in that we analyze the effectiveness of the bailout policy in small open economies and the impact of the U.S. policy changes on the nature of bailout policies in small countries.

Our study makes three main contributions to the literature. First, we show that more generous bailout policies enhance asset bubble holdings, and as a result, asset prices and the economic growth rates rise as long as bubbles exist in small countries. This conclusion shows that bailout policies enhance the "crowd-in effects" of asset bubbles on the economic growth in small open economies as long as bubbles exist.

Second, we also show that more generous bailout policies are desirable from the perspective of taxpayers' welfare in small open economies. Before bubble bursting, generous bailout policies raise the economic growth rate by enhancing the crowd-in effect, which increases the rate of economic growth and, consequently, workers' wages and consumption. When bubbles burst, the capital losses caused by the bursting of bubbles are mitigated and economic growth is promoted, which in turn causes workers' wages to rise more rapidly than in the absence of bailout. That is, in a small open economy, the positive impacts of bailout policies outweigh the negative one of paying taxes.

Finally, we also indicated that a U.S. monetary policy to reduce the interest rate enlarges the size of asset bubbles in a small open economy, and further reduction of the U.S. interest rate makes the size of asset bubbles too large not to be sustainable without adequate policy intervention of the small open economy. In other words, the government needs to reduce the scale of bailouts to an appropriate level in response to the U.S. interest rate reduction. These conclusions provide one important perspective on the introduction of bailout policies in a small open economy. That is, they show that in an economy dominated by the crowd-in effect of bubbles, the introduction of bailout policies allows governments to accelerate economic growth and recovery before and after bubble bursting, but investors' expectations of bailout policies need to be flexibly adjusted to avoid the bursting of bubbles.

The remainder of this paper is organized as follows. In Section 2, we introduce the basic setup of the model. In Section 3, we define a competitive equilibrium based on the setup and derive the economic growth rate in a small open economy. In Section 4, we analyze the effects of bailout policies and the U.S. interest policy change on the

[4] Examples include Olivier (2000), Caballero and Krishnamurthy (2006), Martin and Ventura (2015), Shimizu (2018), and Motohashi (2016, 2020).

economic growth rate, respectively, before and after bubble bursting. In Section 5, we analyze the welfare of taxpayers. Finally, in Section 6, we summarize our main insights.

2 The Model

2.1 Background of the Model

We begin by constructing a model to analyze the effectiveness of asset bubbles and bailout policy in a small open economy by extending the models developed by Hirano et al. (2015) and Motohashi (2020). In this study, based on the literature, we specifically consider "partial bailouts," a policy that guarantees the bubbly assets of only a certain proportion of financial investments against losses. This is based on the fact that, during the recent global financial crisis, Lehman Brothers was not bailed out and AIG was rescued.

We also consider the effects of the U.S. interest rate policy on a small open economy's bailout policy. Since financial investors' portfolios usually include foreign assets as well as domestic ones in the global economy, most global portfolios contain government bonds from a large country such as the U.S. When the Fed adjusts the interest rates, not only asset holdings and prices but also bailout policies in such countries are affected. In our model, the international interest rate corresponds to the return on U.S. government bonds, which is exogenously introduced as "the U.S. interest rate" to analyze the effects. On the other hand, internally generated bubbly assets held by investors are defined as "bubbly assets." To analyze asset bubbles in a general equilibrium framework, we introduce the bubbly asset into the model as a type of security that exceeds its fundamental value.

2.2 Model Structure

A discrete-time economy model with financial friction is considered. There is no population growth, and the economy has one homogeneous good and a continuum of entrepreneurs and workers. A typical entrepreneur and worker have the following expected discounted utility function:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \log c_t^i\right], \quad (1)$$

where i is the index for each entrepreneur and c_t^i is his/her consumption at date t. The parameter $\beta \in (0,1)$ is the subjective discount factor and $E_0[x]$ is the expected value of x conditional on information at date 0[5].

Each entrepreneur encounters two types of investment projects in every period: highly productive investment projects (hereafter, H-projects) and nonproductive (low or negative return) investment projects (hereafter N-projects). Investments produce capital. At the beginning of every period, each entrepreneur encounters H-projects (N-projects) with probability p (probability $1-p$), which is exogenous and independent across entrepreneurs and constant over time. As a result, the productivity of each entrepreneur's investment portfolio changes over time. Throughout this discussion, an entrepreneur with H-projects (N-projects) is called an H-type (N-type) entrepreneur. The index i indicates the type of entrepreneur: $i=\{H,N\}$. Investments produce capital, and investment technologies are expressed as follows:

$$k_{t+1}^i=\alpha_t^i z_t^i, \tag{2}$$

where $z_t^i(\geq 0)$ is the investment level at date t and k_{t+1}^i is the capital at date $t+1$ produced by the investment at date t. Owing to the linearity of the production function, α_t^i corresponds to the marginal productivity of investments at date t. Since H-projects give high returns to H-types and N-projects give low or negative returns to N-types, α_t^i satisfies $\alpha_t^H>\alpha_t^N$. For simplicity, we assume $\alpha_t^i=\alpha^i$ and $\alpha^N\leq 1$[6]. Assuming the initial population measure of each type is p and $1-p$ at date 0, the population measure of each type after date 1 is p and $1-p$, respectively.

Each entrepreneur also faces borrowing constraints. They can pledge at most a fraction θ of future returns from investments to creditors due to financial friction, as stated by Kiyotaki and Moore (1997). Thus, the borrowing constraint is expressed as

$$r_t^{US} b_t^i \leq \theta q_{t+1}\alpha^i z_t^i, \tag{3}$$

where q_{t+1} is the price of capital relative to consumption goods at date $t+1$. r_t^{US} and b_t^i are the gross U.S. interest rate and the amount of borrowing, respectively, at date t. The parameter $\theta\in[0,1]$ corresponds to the degree of imperfection in the financial market and is assumed to be externally given.

Each entrepreneur faces the following flow of fund constraints in every period:

$$c_t^i+z_t^i+P_t x_t^i+g b_t^i=q_t\alpha^i z_{t-1}^i+P_t x_{t-1}^i+r_{t-1}^{US} g b_{t-1}^i- r_{t-1}^{US} b_{t-1}^i+b_t^i+m_t^i, \tag{4}$$

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[5] A log-linear utility function is adopted to analyze the effects of asset bubbles on countries where the ratio of consumption to income is stable. In other words, we focus mainly on countries that maintain modest growth, and third world countries are excluded.

[6] We consider the case where $\alpha^N>1$. In this setting, adjusting the assumption of the relationship between the return on N-projects and the gross U.S. interest rate, we can obtain the same results as in the present paper.

where x_t^i is the amount of bubbly assets purchased by type i entrepreneur and P_t is price. gb_t^i is the amount of the U.S. government bonds purchased by type i entrepreneurs and r_t^{US} is the return. m_t^i corresponds to bailout transfer. The left-hand side of (4) is, therefore, the gross expenditure, and the financing of this is expressed by the right-hand side, which is the return on investment and assets in the previous year plus net borrowing minus debt repayment and bailout transfer. Then, the net worth of the entrepreneur is defined as $e_t^i \equiv \alpha^i z_{t-1}^i + P_t x_{t-1}^i + r_{t-1}^{US} gb_{t-1}^i - r_{t-1}^{US} b_{t-1}^i + m_t^i$ to express the economic implications.

Bubbles survive with a probability of π and a collapse of $1-\pi$. A lower π value indicates riskier bubbles. When bubbles collapse at the beginning of date t, the price of bubbly assets becomes zero and the net worth of entrepreneurs investing in bubbly assets is wiped out. The government, however, bails out a certain proportion $\lambda \in (0,1)$ of entrepreneurs to mitigate these contractions (partial bailouts). When entrepreneur i is rescued, we assume that the government guarantees bubble investments against losses and that bailout is proportional to the entrepreneur's holdings of bubbly assets:

$$m_t^i = d_t^i x_{t-1}^i. \tag{5}$$

In this paper, we consider a bailout policy that fully guarantees bubble investments against losses. Hence, $d_t^i = P_t$ if entrepreneur i is rescued, and otherwise $d_t^i = 0$. The bailout scheme suggests that, from an ex-ante perspective, each entrepreneur anticipates government bailouts with probability λ. Thus, P_t is affected not only by the collapse risk but also by the probability of bailouts. We consider the case where the holding of bubbly assets cannot be negative:

$$x_t^i \geq 0. \tag{6}$$

We consider only the case where the U.S. interest rate is positive. Since the U.S. government bonds offer an opportunity for asset management in small countries, the interest rate in small countries converges to r_t^{US}. r_t^{US} is assumed to satisfy the following conditions[7]:

$$1 < r_t^{US} \leq \alpha^H. \tag{7}$$

We also assume that $r_t^{US} = r^{US}$ for simplicity.

Next, we consider workers' behavior. We assume that there are workers with a unit measure[8]. Each worker is endowed with one unit of labor in each period, which is

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[7] This assumption is introduced to exclude the case where entrepreneurs have all their assets in U.S. government bonds.

[8] Even if the case with N measure is considered, all the results in this paper hold.

supplied inelastically in labor markets, and earns a wage rate, w_t. We also assume that workers are hand-to-mouth all times.

$$c_t^u = w_t - T_t^u. \tag{8}$$

c_t^u is the equilibrium consumption level of a worker and T_t^u is the tax level at date t. When bubbles collapse, government levies a lump sum tax on workers and transfers those funds to entrepreneurs who suffer losses from bubble investments. This means that workers are taxpayers and incur the direct costs of bubbles' collapse.

Lastly, we explain the technology for producing final goods. There are competitive firms which produce final consumption goods by using capital and labor. The production technology of each firm is

$$y_t = \bar{z}_t k_t^{\sigma} n_t^{1-\sigma}, \tag{9}$$

where k_t and n_t are capital input and labor input at date t. $\overline{z_t}$ stands for Arrow-Romer type capital deepening externalities at date t[9]. Factors of production are paid for their marginal product:

$$q_t = \bar{z}_t \sigma K_t^{\sigma-1} \; and \; w_t = \bar{z}_t(1-\sigma)K_t^{\sigma}, \tag{10}$$

where K_t is the aggregate capital stock at date t. $\bar{z}_t = (1/\sigma)\bar{k}_t^{1-\sigma}$ and $\bar{k}_t$ is the average level of capital stock. As a result, equation (10) becomes

$$q_t = 1 \; and \; w_t = \frac{1-\sigma}{\sigma} K_t. \tag{11}$$

3 Equilibrium

3.1 Market Equilibrium

Next, we define competitive equilibrium and derive the economic growth rate of small countries. Competitive equilibrium is defined as a set of prices $\{r_t^{US} = r^{US},\ q_t = 1,\ w_t,\ P_t\}_{t=0}^{\infty}$ and other quantitative economic variables $\{C_t^H, C_t^N, C_t^u, Z_t^H, Z_t^N, B_t^H, B_t^N, P_t, X_t, GB_t, N_t, K_{t+1}, Y_t\}_{t=0}^{\infty}$ that satisfy the results of the optimal behavior of entrepreneurs $\left\{c_t^i, b_t^i, z_t^i, x_t^i, gb_t^i\right\}_{t=0}^{\infty}$ and workers $\{c_t^u = w_t\}_{t=0}^{\infty}$ and market clearing conditions as follows:

9 See Romer (1986) and Alogoskoufis and van der Ploeg (1991).

1. Each entrepreneur maximizes his/her utility under the following constraints:

$$\max_{c_t^i} E_0\left[\sum_{t=0}^{\infty} \beta^t \log c_t^i\right], \tag{12}$$

$$\begin{gathered} subject\ to\ c_t^i + z_t^i + P_t x_t^i + gb_t^i = e_t^i + b_t^i, \\ r^{US} b_t^i \leq \theta \alpha^i z_t^i\ and\ x_t^i \geq 0. \end{gathered} \tag{13}$$

2. The market-clearing conditions are

$$C_t^H + C_t^N + C_t^u + Z_t^H + Z_t^N + GB_t^H + GB_t^N = Y_t + r^{US} GB_{t-1}, \tag{14}$$

$$B_t^H + B_t^N = 0, \tag{15}$$

$$K_t = \sum_{i \in H_t \cup L_t} k_t^i, \tag{16}$$

$$N_t = 1, \tag{17}$$

$$X_t = X, \tag{18}$$

where the aggregate consumption, investment, purchasing of the U.S. government bonds, and borrowing and purchasing of bubbly assets of each type of entrepreneur at date t are, respectively, designated as $\sum_{i \in H_t} c_t^i \equiv C_t^H$, $\sum_{i \in N_t} c_t^i \equiv C_t^N$, $\sum_{i \in H_t} z_t^i \equiv Z_t^H$, $\sum_{i \in N_t} z_t^i \equiv Z_t^N$, $\sum_{i \in H_t} gb_t^i \equiv GB_t^H$, $\sum_{i \in N_t} gb_t^i \equiv GB_t^N$, $\sum_{i \in H_t} b_t^i \equiv B_t^H$, $\sum_{i \in N_t} b_t^i \equiv B_t^N$, $\sum_{i \in H_t} x_t^i \equiv X_t^H$, $\sum_{i \in N_t} x_t^i \equiv X_t^N$. C_t^u and N_t are the aggregate consumption and population of workers at date t.

It is well known that an entrepreneur with the log-linear utility function (1) consumes a fraction $1-\beta$ of net worth every period:

$$c_t^i = (1-\beta) e_t^i. \tag{19}$$

3.2 The Investment Function

We consider the investment function of each entrepreneur to derive the output level in equilibrium. N-types prioritize lending their assets to H-types rather than investing in N-projects because the lending interest rate and the expected return on bubbly assets (reflecting the bursting and bailout possibilities) exceed the marginal productivity of N-projects. N-types lend their assets to H-types up to the limit of the borrowing constraint and then buy bubbly assets or the U.S. government bonds using residual assets[10]. H-types borrow assets from N-types and invest all their assets in H-projects because the marginal productivity of H-projects exceeds the expected returns of the U.S. government bonds and bubbly assets. As a result, H-types are the only entrepreneurs who invest in internal production projects in small countries. Combining

[10] By introducing a negligible slight cost to buy bubbly assets and U.S. government bonds, N-types prioritize lending their assets to H-types.

the budget and borrowing constraints (13) and (19), the investment function of an H-type is

$$z_t^H = \frac{\beta e_t^H}{1-\frac{\theta\alpha^H}{r^{US}}} = \beta e_t^H \mu(r^{US}), \tag{20}$$

where $\mu(r^{US})$ is defined as $1/\left[1-\left(\frac{\theta\alpha^H}{r^{US}}\right)\right]$. Because βe_t^H represents the savings amount of H-types, the function $\mu(r^{US})$ corresponds to his/her multiple investments to owed capital. We call this the "leverage factor of investments." Since only H-types invest in internal projects, the investment function of the country is expressed as the aggregate investment of each H-type. Therefore, the investment function depends on the net worth of H-types at date t. As mentioned before, H-types at date t come from proportions p of N-types and H-types at date $t-1$. Thus, considering the market clearing condition (15), the net worth of H-types at date t is given by

$$E_t^H = p(\alpha^H Z_{t-1}^H - r^{US} B_{t-1}^H) + p(P_t X + r^{US} GB_{t-1} - r^{US} B_{t-1}^N) = p(\alpha^H Z_{t-1}^H + r^{US} GB_{t-1} + P_t X) = pE_t.$$

As a result, the investment function is expressed by

$$Z_t^H = \beta p E_t \mu(r^{US}). \tag{21}$$

3.3 The Demand Function for Bubbly Assets

Here, we consider the demand function for bubbly assets. N-types buy bubbly assets using their remaining savings after lending to H-types. An N-type chooses the optimal amount of x_t^N so that the marginal expected utilities from b_t^N, x_t^N, and gb_t^N are equalized. By solving the utility maximization problem explained in Appendix A, we can derive the demand function for bubbly assets of an N-type:

$$P_t x_t^N = \frac{\delta(\lambda)\frac{P_{t+1}}{P_t} - r^{US}}{\frac{P_{t+1}}{P_t} - r^{US}} \beta e_t^N, \tag{22}$$

where $\delta(\lambda) \equiv \pi + (1-\pi)\lambda$. Because only N-types hold bubbly assets, their aggregate demand is derived as

$$P_t X = \frac{\delta(\lambda)\frac{P_{t+1}}{P_t} - r^{US}}{\frac{P_{t+1}}{P_t} - r^{US}} \beta(1-p)E_t. \tag{23}$$

N-types' decisions regarding the amount of holdings of bubbly assets depend on the U.S. interest rate and the adjusted bursting risk of asset bubbles.

3.4 The Aggregate Capital Stock and the Economic Growth Rate

Finally, we consider the economic growth rate in small countries. From (9) and (21), the aggregate capital stock and the gross domestic products are expressed as follows:

$$Y_{t+1} = K_{t+1} = \alpha^H Z_t^H = \alpha^H \beta p E_t \mu(r^{US}). \tag{24}$$

The economic growth rate, therefore, is defined as the function of the growth rate of entrepreneurs' aggregate wealth as follows:

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{E_t}{E_{t-1}} \equiv a_{t-1}. \tag{25}$$

To characterize the economic growth rate, we express the aggregate wealth of entrepreneurs as follows:

$$E_{t+1} = \alpha^H Z_t^H + \left(\frac{P_{t+1}}{P_t}\right) P_t X + r^{US} G B_t. \tag{26}$$

In addition, we define the relative size of residual assets (φ_t), bubbly assets (η_t), and U.S. government bonds ($1-\eta_t$) as follows:

$$\varphi_t \equiv \frac{P_t X + GB_t}{\beta E_t} = 1 - \frac{Z_t}{\beta E_t} = 1 - p\mu(r^{US}), \tag{27}$$

$$\eta_t \varphi_t \equiv \frac{P_t X}{\beta E_t} \text{ and } (1-\eta_t)\varphi_t \equiv \frac{GB_t}{\beta E_t}, \tag{28}$$

where $\varphi_t = \varphi$ because the U.S. interest rate is externally given. From (21) and (23), and these definitions, the economic growth rate and the aggregate demand for bubbly assets can be expressed as follows:

$$g_{t+1} = a_t = \beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta_t \left[\frac{P_{t+1}}{P_t} - r^{US}\right], \tag{29}$$

$$\frac{P_{t+1}}{P_t} = r^{US} \frac{(1-p)-\eta_t\varphi}{\delta(\lambda)(1-p)-\eta_t\varphi}. \tag{30}$$

These equations point to important characteristics of bubbly assets with bailouts. In the first equation, the first term corresponds to the growth rate of entrepreneurs' aggregate wealth, which is realized when all assets become real investments, and the second term corresponds to the loss of investment opportunities due to the inability to invest in H-projects. The third term corresponds to improvement in the growth rate due to asset bubbles, which depend on the margin between the asset price and the U.S. interest rate. The second equation indicates that the price of bubbly assets depends on the bailout policy. It indicates that the price of a bubbly asset is higher than the U.S. interest rate due to the premium reflecting partial bailouts. The amount of premium is displayed as a coefficient of r^{US}, which reflects the risk of adjusted bubble bursting by bailouts.

4 Effects of Anticipated Bailouts on the Macro Economy

4.1 Stochastic Stationary Equilibrium with Asset Bubbles

We examine the dynamics and stochastic stationary equilibrium with asset bubbles. From the definition of $\eta_t\varphi$, we have

$$\eta_{t+1}\varphi = \frac{\frac{P_{t+1}}{P_t}}{\frac{E_{t+1}}{E_t}}\eta_t\varphi. \tag{31}$$

If an economy has a stable bubble equilibrium, the relative size of bubbly assets must be constant ($\eta_{t+1}\varphi/\eta_t\varphi = 1$). From equations (29), (30), and (31), we find the condition that $\eta_t\varphi$ should satisfy in a stationary equilibrium as follows:

$$\beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta_t r^{US}\left[\frac{(1-p)(1-\pi)}{\delta(\lambda)(1-p)-\varphi\eta_t}\right] = r^{US}\frac{(1-p)-\varphi\eta_t}{\delta(\lambda)(1-p)-\varphi\eta_t}. \tag{32}$$

Solving this equation for the ratio of holdings of bubbly assets, we obtain the ratio of the holdings of bubbly assets in the stochastic stable equilibrium as follows:

$$\varphi\eta^* = (1-p)\frac{\alpha^H[\theta-\beta\delta(\lambda)(\theta-p)]-r^{US}[1-\beta\delta(\lambda)(1-p)]}{\alpha^H[\theta-\beta\delta(\lambda)(\theta-p)+\beta p(1-\theta)(1-\delta(\lambda))]-r^{US}[1-\beta\delta(\lambda)(1-p)]}, \tag{33}$$

where * denotes the stochastic stable equilibrium.

Inserting equation (33) into (32), we find the economic growth rate in the stochastic steady state as follows:

$$g_{t+1}^* = a_t^* = \left(\frac{P_{t+1}}{P_t}\right)^* = \mu(r^{US})\frac{\beta\alpha^H p(1-\theta)}{1-\beta\delta(\lambda)(1-p)}. \tag{34}$$

Using (33) and the conditions for the U.S. interest rate, we can easily show that bubbly assets exist under relatively mild conditions, which means that the level of the maximum interest rate should be lower than the level of the equilibrium interest rate in the case of rational bubbles (see Appendix B).

$$Max\left[\theta\alpha^H\frac{\theta-\beta\delta(\lambda)(\theta-p)+\beta(1-\theta)\big(1-\delta(\lambda)\big)}{\theta-\beta\delta(\lambda)\theta(1-p)+(1-p)\beta(1-\theta)\big(1-\delta(\lambda)\big)},1\right] < r^{US}$$
$$< \alpha^H\frac{\big(1-\beta\delta(\lambda)\big)\theta+\beta p\delta(\lambda)}{1-\beta\delta(\lambda)(1-p)}\ and\ \theta < \beta\delta(\lambda)(1-p).$$

Using the existence conditions for bubbles for the U.S. interest rate, the level of bailouts that satisfy the existence conditions for bubbles is derived as follows:

$$\underline{\lambda} \equiv Max\left[\frac{r^{US}-\theta\alpha^H}{\beta(1-\pi)\{(1-p)r^{US}-\alpha^H(\theta-p)\}}-\frac{\pi}{1-\pi},0\right] < \lambda <$$
$$Min\left[\frac{r^{US}\{\theta+\beta(1-\theta)(1-p)\}-\theta\alpha^H\{\theta+\beta(1-\theta)\}}{\beta(1-\pi)(1-p)\big(r^{US}-\theta\alpha^H\big)}-\frac{\pi}{1-\pi},1\right] \equiv \overline{\lambda}. \tag{35}$$

4.2 Effects of Anticipated Bailouts on Boom-Bust

We consider the effects of bailout policies on small open economies: asset holding behavior, asset prices, and the economic growth rate. First, we consider the impact on asset holding behavior. Using (33), we have the following proposition.

Proposition 1. *More generous bailout policies enhance asset bubble holdings as long as bubbles exist in small open economies.*

Proof. From (33) and the existence conditions for bubbles ($\varphi\eta^* > 0$), we have $\frac{\partial \varphi\eta^*}{\partial \lambda} > 0$.

Proposition 1 means that the introduction of more generous bailout policies allows entrepreneurs to have more bubbly assets. This is because generous bailout policies turn bubbly assets into safer assets. As a result, entrepreneurs increase the ratio of bubbly asset holdings in the stochastic stable equilibrium.

Next, we analyze the effects on the economic growth rate. From (34), if the bailout policy is adjusted at date t, it affects the growth rate of the asset price and entrepreneurs' aggregate wealth at date t, and as a result, it changes the economic growth rate in the next period. As a result, we get the following proposition.

Proposition 2. *The introduction of more generous bailout policies at date* t *accelerates not only the growth rate of asset prices but also entrepreneurs' aggregate wealth in the same period, and as a result, it enhances the economic growth rate in the next period.*

Proof. From (34), we have $\frac{\partial g_{t+1}^*}{\partial \lambda} = \frac{\partial a_t^*}{\partial \lambda} = \frac{\partial \left(\frac{P_{t+1}}{P_t}\right)^*}{\partial \lambda} > 0$.

Here, looking at (29) and (30), it seems intuitive that the asset price and the economic growth rate are depressed by increasing the level of bailouts, if endogenous variables are constant. Proposition 2, however, shows contrasting results. This is because bailout policy changes affect not only the adjusted bursting risk of bubbly assets but also entrepreneurs' demand for them. As shown in Proposition 1, policy change enhances the holding of bubbly assets ratio, and the price of bubbly assets rises. Because entrepreneurs' aggregate wealth brings more capital investments, the amount of final consumption goods produced by using capital and labor increases. In other words, bailout policies have "crowd-in effects" on economic growth in small open economies as long as bubbles exist. This is one of the characteristic results of this paper compared to the literature.

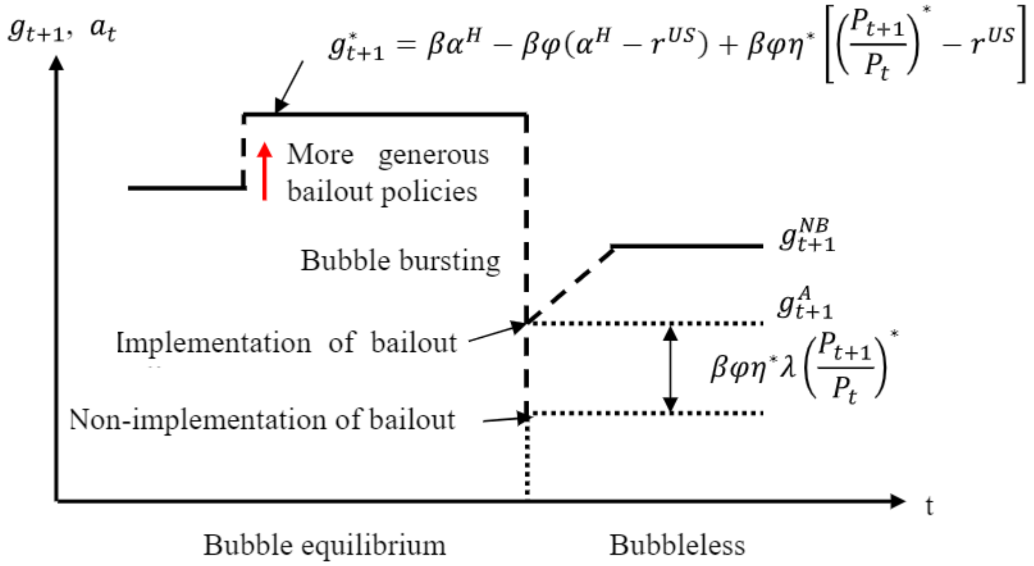
Finally, we consider the economic growth rate after bubble bursting. The value of bubbly assets is considered to be zero after the bubble bursts, and the proportion λ of entrepreneurs who suffer losses from bubble investments is rescued. Using equation (29) and considering P_{t+1} and η^* become zero in bubble equilibrium, the economic growth rate in the global economy after bubble bursting is as follows.

$$g_{t+1}^{A} = \beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta^*\left[\lambda\left(\frac{P_{t+1}}{P_t}\right)^* - r^{US}\right], \tag{36}$$

$$g_{t+1}^{NB} = \beta\alpha^H - \beta\varphi(\alpha^H - r^{US}), \tag{37}$$

where A is an index indicating an economy after bubble bursting and NB is an index indicating an economy with no bubbles. As a result, the economic growth rate before and after bubble bursting is illustrated in Figure 1.

Figure 1: Macro Effects of Bailouts



4.3 The U.S. Policy Change and Asset Holding Behavior

In this subsection, we consider the relationship between a U.S. interest rate reduction and bubbly assets holding behavior. According to the ratio of the holdings of bubbly assets and the economic growth rate in the stochastic stable equilibrium derived in

Section 4, we get the following lemma.

Lemma 1. *The U.S. interest rate reduction enhances the asset bubble holdings ratio and increases the growth rate of asset prices as long as bubbles exist in small open economies.*

Proof. From (33) and (27), we have $\partial\varphi\eta^*/\partial r^{US} < 0$ and $\partial\varphi/\partial r^{US} > 0$, respectively. As a result, $\partial\eta^*/\partial r^{US} < 0$. From (34), $\partial\left(\frac{P_{t+1}}{P_t}\right)^*/\partial r^{US} < 0$.

When we consider the U.S. interest rate reduction case, it eases the H-type borrowing constraint, and as a result, residual assets for financial investments decrease. N-types, however, rapidly pull up the holding ratio of bubbly assets and try to gain financial investment returns from limited residual assets and hold riskier assets. Such rapid changes in their holdings affect asset prices increase in these countries[11]. Thus, Lemma 1 shows that the U.S. interest rate reduction has the same effects as bailout policies with respect to asset bubbles holdings and price increases.

4.4 Effects of the U.S. Policy Change on the Existence Conditions for Bubbles

Next, we consider the relationship between the U.S. interest rate policy changes and the existence conditions for bubbles in a small open economy. Using (35), we get the following proposition.

Lemma 2. *The U.S. interest rate hike increases the maximum level of bailouts that satisfies the existence condition for bubbles.*

Proof. From (35), we have $\frac{\partial\bar{\lambda}}{\partial r^{US}} > 0$.

When we consider the U.S. interest rate reduction case, it enhances the asset bubble holdings ratio and increases the growth rate of asset prices as shown in Lemma 1. If the level of bailouts is at its maximum satisfying the existence conditions for bubbles,

[11] This result is similar to that of Motohashi (2020). Motohashi (2020) refers to the effects of a U.S. interest rate reduction as *asset holding change effect* and *borrowing constraint effect*.

further reduction of the U.S. interest rate makes the size of asset bubbles too large to be sustainable. Lemma 2, therefore, indicates that the level of bailouts needs to be reduced to an appropriate level in response to the U.S. interest rate reduction to avoid bubble bursting.

5 The Welfare Effect of Bailout on Taxpayers

5.1 Theoretical Analysis

In this section, we conduct a welfare analysis of bailouts to derive the optimal bailout level from workers' perspective. Before and after the bursting of bubbles, the effects of bailout policies have both ex-ante and ex-post effects. The ex-ante (i.e., before crash) effect is the positive effects; crowd-in effects of bailout policy. It enhances workers' wage income and consumption before bubble bursting. On the other hand, regarding ex-post effects, there are positive and negative effects. The negative one is tax payments. When bubbles collapse, workers have to pay taxes to rescue entrepreneurs, which lowers their consumption. On the other hand, the positive effect is wage recovery. Through bailouts, the net worth of the rescued entrepreneurs is increased and their investments are expanded compared to the no-bailout case. Therefore, the resulting increase in output increases wage income and workers' consumption. Thus, when considering the relationship between bailout policies and the welfare of taxpayers, it is necessary to compare the ex-ante and ex-post effects.

Here, let $V_t^{BB}(K_t)$ be the value function of taxpayers at date t in the bubble economy and $V_t^{BL}(K_t)$ be the value function of taxpayers at date t when bubbles collapse at date t. The Bellman equation can be written as

$$V_t^{BB}(K_t) = \log c_t + \beta[\pi V_{t+1}^{BB}(K_{t+1}) + (1-\pi)V_{t+1}^{BL}(K_{t+1})]. \tag{38}$$

The expected discounted welfare is weighted by the survival rate of bubbles, π. Solving it, we get taxpayers' value function as follows (see Appendix C for derivation):

$$V_t^{BB}(K_t) = \frac{1}{1-\beta\pi}\left[\frac{\beta}{1-\beta}\log H(\lambda) + \beta(1-\pi)M(\lambda) + \log\frac{1-\sigma}{\sigma}\right] + \frac{1}{1-\beta}\log K_t.$$

Here, $M(\lambda)$ and $H(\lambda)$ are defined as follows:

$$M(\lambda) \equiv \log\left(\frac{1-\sigma}{\sigma} - \lambda\varphi\eta^*\frac{H(\lambda)}{\alpha^H p\mu}\right) + \frac{\beta}{1-\beta}\log[\beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta^*\{\lambda H(\lambda) - r^{US}\}] + \frac{\beta}{1-\beta}\left[\log\frac{1-\sigma}{\sigma} + \frac{\beta}{1-\beta}\log[\beta\alpha^H - \beta\varphi(\alpha^H - r^{US})]\right] \text{ and } H(\lambda) \equiv \mu\frac{\beta\alpha^H p(1-\theta)}{1-\beta\delta(\lambda)(1-p)}.$$

The value function of taxpayers is composed of a weighted average of both effects, the ex-ante and ex-post effects, given a state variable K_t. The first term in the equation captures the expected ex-ante crowd-in effects of anticipated bailouts, which are influenced by change in $H(\lambda)$. $H(\lambda)$ corresponds to the economic growth rate in the stochastic steady state before bubble bursting; as shown in Proposition 2, it is the increasing function of λ. The second term captures the expected ex-post effects, which are influenced by the changes in $M(\lambda)$. $M(\lambda)$ consists of a positive and a negative effect as described above. The first term corresponds to tax burdens on workers, and the second term corresponds to wage recovery. Differentiating the value function with respect to λ, we have the following proposition.

Proposition 3. *The impact of bailout policies on taxpayers' welfare can be broken down into four categories: "crowd-in effect", "tax burden effect", "wage recovery effect", and "asset holding change effect." When the crowd-in and wage recovery effects are dominant, taxpayers' welfare improves.*

Proof. Differentiating V_t^{BB} with respect to λ, we have two effects:

$$\frac{dV_t^{BB}}{d\lambda} = \frac{\beta}{(1-\beta\pi)(1-\beta)}\left[\underbrace{\frac{d\log H(\lambda)}{d\lambda}}_{\text{crowd-in effect (ex-ante effect)}} + \underbrace{(1-\pi)(1-\beta)\frac{dM(\lambda)}{d\lambda}}_{\text{ex-post effect}}\right].$$

The latter effect can be further decomposed into two terms:

$$\frac{dM(\lambda)}{d\lambda} = \overbrace{-\varphi\eta^* H(\lambda)\frac{1+\lambda\left(\frac{\varphi\eta^{*\prime}(\lambda)}{\varphi\eta^*(\lambda)}+\frac{H'(\lambda)}{H(\lambda)}\right)}{\frac{1-\sigma}{\sigma}\alpha^H p\mu-\lambda\varphi\eta^* H(\lambda)}}^{\text{tax burden effect}} + \frac{\beta^2}{1-\beta}\varphi\eta^* H(\lambda)\frac{\overbrace{1+\lambda\frac{H'(\lambda)}{H(\lambda)}}^{\text{wage recovery effect}}+\overbrace{\left(\lambda-\frac{r}{H(\lambda)}\right)\frac{\varphi\eta^{*\prime}(\lambda)}{\varphi\eta^*(\lambda)}}^{\text{asset holding change effect}}}{\beta\alpha^H-\beta\varphi(\alpha^H-r^{US})+\beta\varphi\eta^*\{\lambda H(\lambda)-r^{US}\}}.$$

Among these effects, the crowd-in effect, the tax burden effect, and the wage recovery effect have already noted in Hirano et al. (2015). Since they analyze a closed economy model, crowd-out effect was also present. They pointed out that when the crowd-in and wage recovery effects are dominant, taxpayers' welfare increases. In contrast to Hirano et al. (2015), because of explicit introduction of U.S. government bonds our model has a new effect called the asset holding change effect (although the crowd-out effect is assumed away because of the small open economy setting). We find that when the bailout level is relatively small, the asset holding change effect has negative impact on taxpayers' welfare. This is because when a bailout policy increases the share of bubbly assets, the economic (wage) fluctuation before and after the bubble bursting increase. However, this effect may well be assumed to be smaller than the wage recovery effect because it is difficult to envision a situation where the rate of change in the share of bubbly assets exceeds 100 percent $(1 + \lambda H'(\lambda)/H(\lambda))$ caused by a marginal change in bailout policy. Thus, even in a small open economy, when the crowd-in effect exceeds the tax burden effect, taxpayers' welfare is improved. We provide a set of sufficient conditions for this case in Appendix D.

5.2 Policy Implication

What are the policy implications of this paper? Before bubble bursting, generous bailout policies raise the economic growth rate by enhancing the crowd-in effect, and as a result, workers' wages and consumption also increase. When bubbles burst, the bailout policy mitigates capital losses caused by the burst and accelerates recovery of the economy growth. As a result, workers' wages also quickly rise compared to the no-bailout case. This suggests that a more generous bailout policy, to the extent that the existence conditions for bubbles are satisfied, is desirable for workers.

When determining the level of bailout, however, it is important to note that the existence conditions for bubbles change depending on the U.S. monetary policy, as shown in Lemmas 1 and 2. That is, the U.S. monetary policy to reduce the interest rate enlarges the size of asset bubbles in a small open economy, and further reduction of the U.S. interest rate makes the size of asset bubbles too large to be sustainable without adequate policy intervention of the small open economy. In other words, the government, therefore, needs to reduce the scale of bailouts to an appropriate level in response to the U.S. interest rate reduction. If such changes cannot be made flexibly, the level of bailout would need to be set at a certain margin.

6 Concluding Remarks

This paper constructs a global model consisting of a large country (the U.S.) and a large number of small countries and analyzes the impact of bailout policies on the economic growth rate and asset prices of small countries. In addition, we identify the impact of changes in the U.S. interest policy on the bailout policies of small countries.

Our study makes several contributions. First, we show that more generous bailout policies enhance asset bubble holdings, and, as a result, asset prices and the economic growth rates rise as long as bubbles exist in small countries. This is because more generous bailout policies that turn bubbly assets into safer assets allow entrepreneurs to have more bubbly assets in the stochastic stable equilibrium. As a result, the price of bubbly assets rises, reflecting the increased demand for them, and entrepreneurs' aggregate wealth increases. Since entrepreneurs' aggregate wealth brings more capital investments, the amount of final consumption goods produced by using capital and labor increases. In other words, this conclusion indicates that bailout policies enhance the "crowd-in effects" of asset bubbles on the economic growth in small open economies as long as bubbles exist.

Second, we also indicate that more generous bailout policies are desirable from the perspective of taxpayers' welfare in small open economies. As already discussed, bailout policies have both ex-ante and ex-post effects. Before bubble bursting, generous bailout policies improve the economic growth rate by enhancing the crowd-in effect, and, as a result, workers' wages and consumption also increase. When bubbles burst, the bailout policy mitigates capital losses caused by the burst and accelerates recovery of the economy growth, and, as a result, workers' wages also quickly recover compared to the no-bailout case. That is, in our model, the crowd-in enhancement effect of the bailout policy lets these expected positive effects outweigh the expected negative ones of tax payments when the labor share satisfies the condition.

Finally, we also indicated that the U.S. monetary policy to reduce the interest rate enlarges the size of asset bubbles in a small open economy, and further reduction of the U.S. interest rate makes the size of asset bubbles too large to be sustainable without adequate policy intervention of the small open economy. In other words, the government needs to reduce the scale of bailouts to an appropriate level in response to the U.S. interest rate reduction. These conclusions provide one important perspective on the introduction of bailout policies in a small open economy. They show that in an economy dominated by the crowd-in effect of bubbles, the introduction of bailout policies allows governments to accelerate economic growth and recovery before and after bubble bursting, but they need to flexibly adjust investors' expectations of bailout policies to avoid bubble bursting.

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Appendix A

Each N-type chooses the optimal amounts of b_t^i, x_t^i, and gb_t^i so that the expected marginal utility from investing in the three assets is equalized. The first-order conditions with respect to x_t^i, gb_t^i, and b_t^i are

$$\left(x_t^i\right) : \qquad u'\left(c_t^{i,\pi}\right) = E_t\left[u'\left(c_{t+1}^i\right)\frac{P_{t+1}}{P_t}\right],$$

$$\left(gb_t^i\right)\, and\, \left(b_t^i\right) : \qquad u'\left(c_t^{i,\pi}\right) = E_t\left[u'\left(c_{t+1}^i\right) r^{US}\right].$$

$u(c) = \log c$ is the utility function of the entrepreneurs, and bubbles survive with a probability of π and collapse with that of $1-\pi$. Then, these equations are rewritten as

$$\left(x_t^i\right) : \qquad \frac{1}{c_t^{i,\pi}} = E_t\left[\frac{1}{c_{t+1}^i}\frac{P_{t+1}}{P_t}\right] = \pi\beta\frac{P_{t+1}}{P_t}\frac{1}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{d_{t+1}}{P_t},$$

$$\left(gb_t^i\right)\, and\, \left(b_t^i\right) : \qquad \frac{1}{c_t^{i,\pi}} = E_t\left[\frac{r^{US}}{c_{t+1}^i}\right] = \pi\beta\frac{r^{US}}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{r^{US}}{c_{t+1}^{i,(1-\pi)\lambda}} + (1-\pi)(1-\lambda)\beta\frac{r^{US}}{c_{t+1}^{i,(1-\pi)(1-\lambda)}},$$

where $c_{t+1}^{i,\pi} = (1-\beta)\left(-r^{US}b_t^i + P_{t+1}x_t^i + r^{US}gb_t^i\right)$ is the optimal consumption level at date $t+1$ when bubbles survive at date $t+1$, $c_{t+1}^{i,(1-\pi)\lambda} = (1-\beta)\left(-r^{US}b_t^i + r^{US}gb_t^i + d_{t+1}x_t^i\right)$ is the optimal consumption level at date $t+1$ when bubbles collapse at date $t+1$ and the entrepreneur is rescued, and $c_{t+1}^{i,(1-\pi)(1-\lambda)} = (1-\beta)\left(-r^{US}b_t^i + r^{US}gb_t^i\right)$. $d_{t+1} = P_{t+1}$ when the agent i is rescued after bubble bursting at date $t+1$. From these two equations and (13), we have equation (22).

Appendix B

We examined the existence conditions for bubbly assets. In the global economy, the following conditions need to be satisfied to sustain bubbly assets in a stochastic steady state:

$$0 < \varphi\eta^* \le \varphi \text{ and } 0 < \varphi < 1, \tag{39}$$

$$1 < r^{US} \le \alpha^H. \tag{40}$$

Condition (39) corresponds to the condition required based on the definition of the asset bubbly holding ratio. Condition (40) corresponds to the condition where the return from a safe asset becomes positive and does not exceed the marginal productivity of H-projects. As a result, we have the following existence condition for bubbly assets:

$$Max\left[\theta\alpha^H \frac{\theta-\beta\delta(\lambda)(\theta-p)+\beta(1-\theta)\big(1-\delta(\lambda)\big)}{\theta-\beta\delta(\lambda)\theta(1-p)+(1-p)\beta(1-\theta)\big(1-\delta(\lambda)\big)},1\right.$$
$$< r^{US} < \alpha^H \frac{\big(1-\beta\delta(\lambda)\big)\theta+\beta p\delta(\lambda)}{1-\beta\delta(\lambda)(1-p)} \; and \; \theta < \beta\delta(\lambda)(1-p). \tag{41}$$

The right-hand side of the inequality is equivalent to the equilibrium interest rate in a closed economy with stochastic asset bubbles. Compared with Hirano and Yanagawa (2017), who analyze the rational asset bubble case, the level of the maximum interest rate becomes lower than the level of the equilibrium interest rate in the case of rational asset bubbles. This is one of the well-known characteristics of stochastic bubbles. Solving this existence condition for λ, we have (35).

Appendix C

We derive taxpayers' value function. Suppose that at date t, bubbles collapse. After date t, the economy becomes a bubbleless economy. Let V_t^{BL} be the value function of taxpayers at date t when bubbles collapse and the government bails out entrepreneurs. First, we solve V_{t+1}^{BL}. Given the optimal decision rules, the Bellman equation after date $t+1$ can be written as

$$V_{t+1}^{BL}(K_{t+1}) = \log c_{t+1} + \beta V_{t+2}^{BL}(K_{t+2}), \tag{42}$$

with

$$\begin{cases} c_{t+1} = w_{t+1}, \\ K_{t+2} = [\beta\alpha^H - \beta\varphi(\alpha^H - r^{US})]K_{t+1}. \end{cases} \tag{43}$$

We guess that the value function is a linear function of $\log K$ after date $t+1$:

$$V_{t+1}^{BL}(K_{t+1}) = f + g\log K_{t+1}. \tag{44}$$

From (42)-(44), applying the method of undetermined coefficients yields

$$f = \frac{1}{1-\beta}\left[\log\frac{1-\sigma}{\sigma} + \frac{\beta}{1-\beta}\log[\beta\alpha^H - \beta\varphi(\alpha^H - r^{US})]\right], \tag{45}$$

$$g = \frac{1}{1-\beta}. \tag{46}$$

Thus, we have the following value function after date $t+1$:

$$V_{t+1}^{BL}(K_{t+1}) = \frac{1}{1-\beta}\left[\log\frac{1-\sigma}{\sigma} + \frac{\beta}{1-\beta}\log[\beta\alpha^H - \beta\varphi(\alpha^H - r^{US})]\right] + \frac{1}{1-\beta}\log K_{t+1}. \tag{47}$$

Next, we derive the value function of taxpayers at date t when bubbles collapse and the government bails out entrepreneurs by taking into account the effects of bailouts on the date t consumption and the date $t+1$ aggregate capital stock. The value function of taxpayers at date t satisfies

$$V_t^{BL}(K_t) = \log c_t + \beta V_{t+1}^{BL}(K_{t+1}), \tag{48}$$

with

$$\begin{cases} c_t = w_t - \lambda P_t X = \dfrac{1-\sigma}{\sigma}K_t - \lambda\varphi\eta^* \dfrac{\beta(1-\theta)}{1-\beta\delta(\lambda)(1-p)}K_t, \\ K_{t+1} = \left[\beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta^*\left\{\lambda\mu(r^{US})\dfrac{\beta\alpha^H p(1-\theta)}{1-\beta\delta(\lambda)(1-p)} - r^{US}\right\}\right]K_t. \end{cases} \tag{49}$$

From (47)-(49), we have

$$V_t^{BL}(K_t) = M(\lambda) + \frac{1}{1-\beta}\log K_t. \tag{50}$$

$M(\lambda) \equiv \log\left(\frac{1-\sigma}{\sigma} - \lambda\varphi\eta^*\frac{\beta(1-\theta)}{1-\beta\delta(\lambda)(1-p)}\right) + \frac{\beta}{1-\beta}\left[\log\frac{1-\sigma}{\sigma} + \frac{\beta}{1-\beta}\log[\beta\alpha^H - \beta\varphi(\alpha^H - r^{US})]\right] + \frac{\beta}{1-\beta}\log\left[\beta\alpha^H - \beta\varphi(\alpha^H - r^{US}) + \beta\varphi\eta^*\left\{\lambda\mu(r^{US})\frac{\beta\alpha^H p(1-\theta)}{1-\beta\delta(\lambda)(1-p)} - r^{US}\right\}\right].$

Now, we are in a position to derive the value function at any date t in the bubble economy. Let $V_t^{BB}(K_t)$ be the value function of taxpayers at date t in the bubble economy. Given optimal decision rules, the Bellman equation can be written as

$$V_t^{BB}(K_t) = \log c_t + \beta[\pi V_{t+1}^{BB}(K_{t+1}) + (1-\pi)V_{t+1}^{BL}(K_{t+1})], \tag{51}$$

with the optimal decision rule of aggregate capital stock until bubbles collapse:

$$K_{t+1} = H(\lambda)K_t, \tag{52}$$

where $H(\lambda) \equiv \mu(r^{US})\frac{\beta\alpha^H p(1-\theta)}{1-\beta\delta(\lambda)(1-p)}$.

We guess that the value function is a linear function of $\log K$:

$$V_t^{BB}(K_t) = s + Q\log K_t. \tag{53}$$

Applying the method of undetermined coefficients yields

$s = \frac{1}{1-\beta\pi}\left[\log\frac{1-\sigma}{\sigma} + \beta(1-\pi)\,M(\lambda) + \frac{\beta}{1-\beta}\log H(\lambda)\right],\ Q = \frac{1}{1-\beta}.$

Thus, we have the value function expressed in Section 5.

Appendix D

Differentiating the value function with respect to λ, we have

$$\frac{dV_t^{BB}}{d\lambda} = \frac{\beta}{(1-\beta\pi)(1-\beta)}\left[\frac{d\log H(\lambda)}{d\lambda} + (1-\pi)(1-\beta)\frac{dM(\lambda)}{d\lambda}\right].$$

As mentioned in Section 5, ex-post effect is expressed as follows:

$$\frac{dM(\lambda)}{d\lambda} = -\varphi\eta^* H(\lambda)\frac{1+\lambda\left(\frac{\varphi\eta^{*\prime}(\lambda)}{\varphi\eta^*(\lambda)}+\frac{H'(\lambda)}{H(\lambda)}\right)}{\frac{1-\sigma}{\sigma}\alpha^H p\mu-\lambda\varphi\eta^* H(\lambda)} + \frac{\beta^2}{1-\beta}\varphi\eta^* H(\lambda)\frac{1+\lambda\frac{H'(\lambda)}{H(\lambda)}+\left(\lambda-\frac{r}{H(\lambda)}\right)\frac{\varphi\eta^{*\prime}(\lambda)}{\varphi\eta^*(\lambda)}}{\beta\alpha^H-\beta\varphi\left(\alpha^H-r^{US}\right)+\beta\varphi\eta^*\{\lambda H(\lambda)-r^{US}\}}.$$

Considering a condition under which the second term is positive, we have

$$\varphi\eta^*(\lambda)H(\lambda) - \varphi\eta^{*\prime}(\lambda)r > 0. \tag{54}$$

Considering a condition under which the sum of the first term of $(1-\pi)(1-\beta)\frac{dM(\lambda)}{d\lambda}$ and $\frac{d\log H(\lambda)}{d\lambda}$ is positive, we have

$$\frac{1-\sigma}{\sigma} > \varphi\eta^*(1-\theta)\frac{1-\beta}{1-p}\left[1+\lambda\left\{\frac{\varphi\eta^{*\prime}(\lambda)}{\varphi\eta^*(\lambda)}+\frac{H'(\lambda)}{H(\lambda)}+\frac{\beta(1-p)}{(1-\beta)\left(1-\beta\delta(\lambda)(1-p)\right)}\right\}\right]. \tag{55}$$

Here, when we consider the above inequality condition, the left-hand side is a function of σ only and the right-hand side is a function independent of σ. Since the left-hand side can be taken from zero to infinity depending on the setting of σ, it is theoretically guaranteed that there exist parameters satisfying the above inequality condition. As a result, (54) and (55) constitute sufficient conditions.

Whether these conditions are satisfied in the real economy depends on countries. For example, in Japan, the conditions are likely to be satisfied because the labor share does not differ significantly from that of other developed countries, while households do not hold high percentage of risky assets.