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AUTHOR(S):
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CITATION:

ISSUE DATE:
2020-06

URL:
http://hdl.handle.net/2433/262418

RIGHT:
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Statistical double $\Lambda$ hypernuclear formation from $\Xi^-$ absorption at rest in light nuclei

Akira Ohnishi$^{1,*}$, Chikako Ishizuka$^2$, Kohsuke Tsubakihara$^{2,3}$, and Yuichi Hirata$^4$

$^1$Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
$^2$Laboratory for Advanced Nuclear Energy, Institute of Innovative Research, Tokyo Institute of Technology, Tokyo 152-8550, Japan
$^3$National Institute of Technology, Asahikawa College, Asahikawa 071-8142, Japan
$^4$Central Institute of Isotope Science, Hokkaido University, Sapporo 060-0815, Japan

*E-mail: ohnishi@yukawa.kyoto-u.ac.jp

Received November 28, 2019; Revised March 16, 2020; Accepted March 17, 2020; Published June 18, 2020

We investigate double $\Lambda$ hyperfragment formation from the statistical decay of double $\Lambda$ compound nuclei produced in the $\Xi^-$ absorption at rest in the light nuclei $^{12}$C, $^{14}$N, and $^{16}$O. We examine the target and the $\Lambda\Lambda$ bond energy dependence of the double $\Lambda$ hyperfragment formation probabilities, especially of those double hypernuclei observed in experiments. For the $^{12}$C ($^{14}$N) target, the formation probabilities of $^6\Lambda\Lambda$He and $^{10}\Lambda\Lambda$Be ($^{13}\Lambda\Lambda$B) are found to be reasonably large as they are observed in the KEK-E373 (KEK-E176) experiment. By comparison, for the $^{16}$O target, the formation probability of $^{11}\Lambda\Lambda$Be is calculated to be small with $\Delta B_{\Lambda\Lambda}$ consistent with the Nagara event. We also evaluate the formation probability of $^5\Lambda\Lambda$H from a $\Xi^-$–$^6$He bound state, $^7\Xi$H.

Subject Index D00, D25, D29, D33

1. Introduction

Formation of double $\Lambda$ hypernuclei (DAHN) from $\Xi^-$ absorption in nuclei is of importance for several reasons. $\Xi^-$ absorption at rest in nuclei is the most efficient way to produce DAHN, and uniquely identified DAHN [1–6] provide strong constraints on the $\Lambda\Lambda$ interaction [7,8]. The strength and density dependence of the $\Lambda\Lambda$ interaction are the keys to solving the hyperon puzzle in neutron star physics. Until now, four DAHN formation events have been uniquely identified, and more will be found in the J-PARC-E07 experiment, where $10^4$ $\Xi^-$ absorption events in nuclei are expected to be observed. Let us comment on these points in order.

The baryon–baryon interaction has been one of the central subjects in nuclear physics. Compared with nucleon–nucleon interactions, hyperon–nucleon scattering data are much more scarce and single $\Lambda$ hypernuclear data are also used to constrain the $\Lambda N$ interaction. For the $\Lambda\Lambda$ interaction, there are theoretical predictions in the meson exchange model [9], the quark cluster model [10], and lattice QCD calculations [11]. Experimentally, by comparison, it is not possible to perform scattering experiments, and hence the binding energies of DAHN [1–6] and the correlation function data from high-energy nuclear collisions [12–23] have been utilized to experimentally constrain the $\Lambda\Lambda$ interaction. While the correlation function technique has recently been applied to investigate several hadron–hadron interactions [12–15], we need further theoretical and experimental studies to constrain the interactions precisely [15–23]. At present, the strongest constraint on the $\Lambda\Lambda$ interaction...
While the analysis is still ongoing, a new D/Lambda1 reaction is provided by the Lambda Lambda bond energy of DΛΛN, 6/Lambda1Lambda1 He, observed in the Nagara event [4]. The Lambda Lambda bond energy represents the strength of the Lambda Lambda interaction, and is defined as 

\[ \Delta B_{\Lambda\Lambda} = S_{\Lambda\Lambda}(A_{\Lambda\Lambda}Z) - 2S_{\Lambda\Lambda}(A_{\Lambda\Lambda} - 1)Z, \]

where \( S_{\Lambda\Lambda} \) and \( S_{\Lambda} \) are the separation (binding) energies of Lambda Lambda and Lambda. The bond energy of 6/Lambda1Lambda1 He is found to be \( \Delta B_{\Lambda\Lambda}(6/Lambda1Lambda1 He) = 0.67 \pm 0.12 \text{ MeV} \) [5]. This bond energy can be fitted by a Lambda Lambda interaction with the low-energy scattering parameters of \( (a_0, r_{\text{eff}}) = (-0.44 \text{ fm}, 10.1 \text{ fm}) \) [7], where \( a_0 \) and \( r_{\text{eff}} \) are the scattering length and the effective range, respectively.

The Lambda Lambda interaction also plays a crucial role in neutron star physics. With most of the well-known attractive Lambda Lambda two-body interactions, Lambda hyperon mixing is calculated to take place in neutron star matter at (2–4)\( \rho_0 \) and the equation of state (EOS) is softened [24,25]. Consequently, it is hard to support two-solar-mass neutron stars [26–28]. In order to solve this problem, known as the hyperon puzzle, several mechanisms have been proposed so far. One of the natural ways is to introduce repulsive three-baryon interactions [29–34]. For example, it is possible to support massive neutron stars by introducing repulsive three-body contact couplings in relativistic mean field models [34]. It should be noted that the three-baryon repulsion also needs to operate among Lambda Lambda and Lambda Lambda, otherwise Lambda matter becomes more stable than nuclear matter with hyperon mixing at high densities. The Lambda Lambda three-baryon interaction will cause effective density-dependent Lambda Lambda and Lambda Lambda interactions in nuclear matter. If we can observe and uniquely identify many DΛΛN in a wide mass region, it would be possible to deduce the density dependence of the Lambda Lambda interaction and the underlying Lambda Lambda three-baryon interaction.

The most efficient reaction to form DΛΛN is \( \Xi^- \) absorption at rest in nuclei. DΛΛN formation proceeds in the following steps. First, \( \Xi^- \) particles are produced in the \( (K^-, K^+) \) reactions on nuclei or protons [1–6] or \( \bar{p} \)–nucleus collisions [35]. The produced \( \Xi^- \) particle is absorbed in a nucleus, and converted to two Lambda particles via the \( \Xi^- p \to \Lambda\Lambda \) reaction in the nucleus. If two LAs are trapped in the nucleus in the pre-equilibrium stage, a double Lambda compound nucleus (DAC) is formed [36]. The compound nucleus de-excites by emitting nucleons, LAs, \( \alpha \)s, and other clusters. When these two Lambda particles occasionally stay in the same fragment with an excitation energy below the particle emission threshold, a DΛΛN is formed. When the sequential weak decay of the DΛΛN is observed, one can identify that an \( S = -2 \) nucleus is formed. The production and detection of DΛΛN in heavy-ion collisions [37] are also expected, while there is no clear evidence of DΛΛN formation in these reactions yet.

Until now, several experiments have been performed to find DΛΛN, and four of them have been uniquely identified from \( \Xi^- \) absorption in nuclei, as summarized in Table 1 [1–5]: 10/Lambda1Lambda1 Be from the \( \Xi^- + 12\text{C} \) reaction at CERN [1,2], 13/Lambda1Lambda1 B from \( \Xi^- + 14\text{N} \) in the KEK-E176 experiment [3], 6/Lambda1Lambda1 He from \( \Xi^- + 12\text{C} \) (Nagara event) [4], and 6/Lambda1Lambda1 Be* from \( \Xi^- + 12\text{C} \) (Demachi–Yanagi event) [5] in the KEK-E373 experiment. Another report on 6/Lambda1Lambda1 He [38] was questioned [39] and found to be not consistent with the Nagara event [4,5]. It should be noted that the identifications of the above four DΛΛN rely on consistency between the events. For example, \( B_{\Lambda\Lambda} \) values of 10/Lambda1Lambda1 Be in Refs. [1,2,5] can be made consistent by assuming the channel of \( 10/Lambda1Lambda1 Be \to 9/Lambda1Lambda1 Be^* + p + \pi^- \) in the weak decay in the event of Refs. [1,2] and the formation of the excited state \( 10/Lambda1Lambda1 Be^* \) in the Demachi–Yanagi event [5]. In order to further observe DΛΛN, the J-PARC-E07 experiment has been carried out. While the analysis is still ongoing, a new DΛΛN formation event was discovered recently, 10/Lambda1Lambda1 Be from \( \Xi^- + 16\text{O} \) (Mino event) [6]. We summarize the uniquely identified DΛΛN events in Table 1. We also show the candidate fragmentation reactions in the Mino event.

Since further events are expected to be observed from the J-PARC-E07 and future experiments, it would be possible to perform statistical analysis of fragment formation events from \( \Xi^- \) absorption.
Table 1. Uniquely identified double $\Lambda$ hypernuclear formation events [1–5]. We also show the possible formation channels in the Mino event, a new event observed in the J-PARC-E07 experiment [6].

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reaction</th>
<th>$\Delta B_{\Lambda\Lambda} (\text{MeV})$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN</td>
<td>$\Xi^- + ^{12}\text{C} \rightarrow ^{10}_{\Lambda\Lambda}\text{Be} + d + n$</td>
<td>1.3 ± 0.4</td>
<td>[1,2]</td>
</tr>
<tr>
<td>KEK-E176</td>
<td>$\Xi^- + ^{14}\text{N} \rightarrow ^{\alpha}_{\Lambda\Lambda}\text{Br} + p + n$</td>
<td>0.6 ± 0.8</td>
<td>[3]</td>
</tr>
<tr>
<td>Nagara</td>
<td>$\Xi^- + ^{12}\text{C} \rightarrow ^{4}_{\Lambda\Lambda}\text{He} + \alpha + t$</td>
<td>0.67 ± 0.17</td>
<td>[4,5]</td>
</tr>
<tr>
<td>Demachi–Yanagi</td>
<td>$\Xi^- + ^{12}\text{C} \rightarrow ^{10}_{\Lambda\Lambda}\text{Be}^* + d + n$</td>
<td>$-1.52 \pm 0.15$</td>
<td>[5]</td>
</tr>
<tr>
<td>Mino</td>
<td>$\Xi^- + ^{16}\text{O} \rightarrow ^{10}_{\Lambda\Lambda}\text{Be} + \alpha + t$</td>
<td>1.63 ± 0.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Xi^- + ^{16}\text{O} \rightarrow ^{11}_{\Lambda\Lambda}\text{Be} + \alpha + d$</td>
<td>1.87 ± 0.37</td>
<td>[6]</td>
</tr>
<tr>
<td></td>
<td>$\Xi^- + ^{16}\text{O} \rightarrow ^{12}_{\Lambda\Lambda}\text{Be}^* + \alpha + p$</td>
<td>$-2.7 \pm 1.0$</td>
<td></td>
</tr>
</tbody>
</table>

at rest in light nuclei. Statistical decay of DAC was studied by using a canonical fragmentation model [40], a sequential binary statistical decay model [41], and a microcanonical fragmentation model [42]. We also note that the $\Xi^-$ absorption reaction in $^{12}\text{C}$ was analyzed by using the direct reaction model in Ref. [43]. In Ref. [41], the $\Xi^-$ absorption reaction in $^{12}\text{C}$ was analyzed in a combined framework of a transport model and a statistical decay model of hypernuclei. The formation probability of the DAC ($^{13}_{\Lambda\Lambda}\text{B}^*$) was evaluated to be around $P_{\text{DAC}} \simeq 30\%$ in the pre-equilibrium stage by using the antisymmetrized molecular dynamics (AMD) transport model calculation [41], the sum of branching ratios to form DÅHN from DAC in the statistical decay was found to be around $P_{\text{DAC}}^{\text{tot}} = 60\%$, and then the total DÅHN formation probability was found to be around $P_{\text{DÅHN}}^{\text{tot}} = P_{\text{DAC}}^{\text{tot}} \times P_{\text{Br}}^{\text{tot}} \simeq 18\%$ after the statistical decay. This analysis was performed before the discovery of the Nagara event [4], and a strongly attractive $\Lambda\Lambda$ interaction was adopted, $\Delta B_{\Lambda\Lambda} (^{13}_{\Lambda\Lambda}\text{B}) \simeq 4.9 \text{ MeV}$, as suggested by the KEK-E176 experiment [3]. After the Nagara event, the KEK-E176 event was interpreted and the $\Lambda\Lambda$ bond energy is now considered to be $\Delta B_{\Lambda\Lambda} (^{13}_{\Lambda\Lambda}\text{B}) = 0.6 \pm 0.8 \text{ MeV}$. With these updated, less attractive, $\Lambda\Lambda$ interactions, it would be possible to predict the DÅHN formation probabilities in a more reliable manner.

It should be noted that the formation probability of DAC ($P_{\text{DAC}}$) depends on the definition of compound nucleus formation, and in practice it depends on the transport model adopted in describing the dynamical stage. For example, the DÅC formation probability in stopped $\Xi^-$ absorption in $^{12}\text{C}$ is calculated to be smaller, $P_{\text{DAC}} (\text{AMD-QL}) = 16\%$, in AMD with additional quantum fluctuations (AMD-QL) [41]. In addition to the difference in $P_{\text{DAC}}$, emission of nucleons and light clusters in the dynamical stage would modify the mass dependence of the formation probability of DÅC. In Sect. 3.1 we compare the statistical decay model results from DAC ($^{13}_{\Lambda\Lambda}\text{B}^*$) and AMD-QL results with the statistical decay in Ref. [41].

In this article we discuss the formation of DÅHN from the statistical decay of DAC formed via the $\Xi^-$ absorption at rest in nuclei. Specifically, we concentrate on the target nuclei $^{12}\text{C}$, $^{14}\text{N}$, and $^{16}\text{O}$. These are the main light components of emulsion, and some DÅHN have been reported to be formed. We mainly discuss the formation probabilities $^{6}_{\Lambda\Lambda}\text{He}$ and $^{10}_{\Lambda\Lambda}\text{Be}$ from $\Xi^- + ^{12}\text{C}$, $^{13}_{\Lambda\Lambda}\text{B}$ from $\Xi^- + ^{14}\text{N}$, and $^{11}_{\Lambda\Lambda}\text{Be}$ from $\Xi^- + ^{16}\text{O}$. We also discuss the dependence of the formation probabilities on the $\Lambda\Lambda$ bond energy, and the decay of the $\Xi$ hypernucleus formed in the $^7\text{Li}(K^-, K^+)$ reaction.

The paper is organized as follows. In Sect. 2 we provide a general idea of statistical decay of compound hypernuclei. In Sect. 3, we evaluate DÅHN formation probabilities from the $\Xi$ absorption reaction at rest in $^{12}\text{C}$, $^{14}\text{N}$, and $^{16}\text{O}$. In Sect. 4 we discuss $^{5}_{\Lambda\Lambda}\text{H}$ formation probability from $\Xi^- + ^{6}\text{He}$ bound state at various values of the $\Xi^-$ binding energy. A summary and discussion are given in Sect. 5.
2. Statistical decay of double Λ compound nuclei and hypernuclear binding energies

In this paper, the statistical decay of DAC is calculated by using a sequential binary statistical decay model (SDM) [44]. In the SDM, an excited nucleus 1 is assumed to decay into nuclei 2 and 3 with a decay rate of

\[ \Gamma_{1 \rightarrow 23}dE_2dE_3 = \frac{\rho_2(E_2, J_2) \rho_3(E_3, J_3)}{2\pi \rho_1(E_1, J_1)} \sum_{L_j_{23}} T_L dE_2 dE_3, \]  

where \( E_i \) and \( J_i \) denote the excitation energy and the angular momentum of the \( i \)th nucleus. We adopt the back-shifted Fermi gas model to evaluate the level density \( \rho_j(E, J) \) [44,45]. Since we are interested in decays of compound nuclei in equilibrium, the density of narrow excited levels needs to be considered and the level density is reduced at excitation energies above the threshold for charge-neutral particle emission and above the Coulomb barrier for charged particle emission [45]. The factor \( T_L = \Theta(L_c - L) \) is the transmission coefficient of the partial wave \( L \) in the fusion reaction \( 2 + 3 \rightarrow 1 \) with the incident energy corresponding to the excitation energies and the \( Q \)-value. We assume strong absorption in the inverse fusion process \( 2 + 3 \rightarrow 1 \), and take the form \( T_L = \Theta(L_c - L) \) with \( L_c \) being the maximum orbital angular momentum of the fusion. The statistical decays are assumed to be binary and to proceed until all the fragments are in their ground states. When combined with the transport models describing the pre-equilibrium stage, the SDM has been found to work well for intermediate energy heavy-ion collisions [46–48] and for hypernuclear formation reactions [41,49].

We would also like to mention here that the combined framework of the transport model and the SDM has been successfully applied to the light-ion induced reaction, \( p + ^{12}\text{C} \) at 45 MeV, where the excitation energy of the compound nucleus is similar to the \( \Xi^- \) absorption reaction at rest in nuclei and essentially the same SDM program is used [41].

The essential inputs of the SDM are the binding energies of nuclei that would be formed during the sequential decay processes. While the binding energies of normal nuclei are well known, information on single and double hypernuclear binding energies is limited. We have constructed the mass table based on the following assumptions. First, all existing normal nuclei plus \( \Lambda \) and \( \Lambda \Lambda \) are assumed to form single and double hypernuclei, respectively. Normal nuclei include those whose ground states are resonance and unstable to particle emission. We also consider dineutron (\( nn \)) and \( ^2\text{He} \) (\( pp \)) as resonance nuclei, and their energies are assumed to be 100 keV above the threshold, in order to mimic three-body decays. Second, for single hypernuclei we adopt the separation energy if measured. We use the separation energy data summarized by Bando et al. (BMZ) [50] and by Hashimoto and Tamura (HT) [51]. We add 0.5 MeV to the separation energies measured in \( (\pi^+, K^+) \) experiments [51] to take account of the recalibration as shown in Ref. [52]. If not measured, we adopt the \( \Lambda \) separation energy \( S_\Lambda \), parameterized as a function of the mass number and fitted to the observed separation energies,

\[ S_\Lambda = \frac{c_0(A - 1)}{A} \frac{1 - c_1 x}{1 + c_2 x + c_3 x^2}, \]  

where \( A \) denotes the hypernuclear mass number and \( x = 1/(A - 1)^{2/3} \). By fitting to the BMZ and HT data, we determined the parameters of \( c_0 = 30.18 \text{ MeV}, c_1 = 1.52, c_2 = 3.17, \) and \( c_3 = 4.38 \). The results of the fit are shown in Fig. 1. Third, for DAHN, the \( \Lambda \Lambda \) bond energy is given as a linear function of the mass number. We compare the results of the three models, \( \Delta B_{\Lambda \Lambda} = 4.9 \text{ MeV} \) (model A), \( \Delta B_{\Lambda \Lambda} = 0.67 \text{ MeV} \) (model B), and \( \Delta B_{\Lambda \Lambda}(A = 6) = 0.67 \text{ MeV} \) and \( \Delta B_{\Lambda \Lambda}(A = 11) = 1.87 \text{ MeV} \).
Fig. 1. Left: Separation energy of single hypernuclei. Data summarized in Refs. [50] (open circles) and [51] (filled circles) are shown by symbols, and the results of the fit are shown by the solid curve. Right: ΛΛ bond energy of DAHN. The filled circles show the results of the uniquely identified ground state ΛΛ bond energy ΔB_{ΛΛ}. The dotted, dashed, and solid lines show ΔB_{ΛΛ} in the models A, B, and C, respectively. The open circles and squares show experimental data of ΔB_{ΛΛ} without unique identifications and rejected values, respectively.

(model C). These model parameters are determined by the old rejected value of ΔB_{ΛΛ} from the E176 experiment [53] (model A), the Nagara event (model B), and the Nagara and Mino events (model C).

3. Double hypernuclear formation from Ξ⁻ absorption in nuclei

We now discuss DAHN formation from Ξ⁻ absorption reactions at rest in ¹²C, ¹⁴N, and ¹⁶O. We assume that Ξ⁻ is absorbed from the 3D atomic orbit, so the Ξ⁻ binding energy is \( B = \mu \alpha^2 Z^2 / 2n^2 = 0.126, 0.174, \) and 0.230 MeV for ¹²C, ¹⁴N, and ¹⁶O targets, respectively, where \( \mu \) is the reduced mass, \( Z \) is the atomic number of the target, and \( n = 3 \) for the 3D orbit. Part of the released energy in the conversion process, \( M_{Ξ⁻} + M_p - 2M_Λ = 28.62 \) MeV, is used by the separation energy of the proton, 15.96, 7.55, and 12.13 MeV, and thus the energies from the threshold, ¹¹B + ΛΛ, ¹³C + ΛΛ, and ¹⁵N + ΛΛ, are obtained as 12.66, 21.07, and 16.49 MeV, for ¹²C, ¹⁴N, and ¹⁶O targets, respectively. These energies are summarized in Table 2.

The open channels including DAHN from the Ξ⁻ absorption reactions are shown in Fig. 2. Many channels, including two-, three-, and four-body, can be populated in the final state. We have performed the statistical decay model calculation from the DAC until all the fragments are de-excited to the ground state. In order to handle the variety of decay paths, the decay channel, excitation energies, and angular momenta of daughter fragments are chosen with a Monte Carlo method.

Table 2. Relevant energies in the stopped Ξ⁻ absorption reaction at rest in ¹²C, ¹⁴N, and ¹⁶O, showing the target binding energies \( B_T \), the proton separation energies \( S_p \), and the energy from the ΛΛ emission threshold \( E_{th} \). We also show the case of the ⁶He target, a core nucleus of ²²H, which may be formed in the ⁷Li(K⁻, K⁺) reaction.

<table>
<thead>
<tr>
<th>Target</th>
<th>( B_T ) (MeV)</th>
<th>( S_p ) (MeV)</th>
<th>( E_{th} ) (MeV)</th>
<th>( B_{Ξ} ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>¹²C</td>
<td>92.165</td>
<td>15.957</td>
<td>12.569 - ( B_{Ξ} )</td>
<td>0.126</td>
</tr>
<tr>
<td>¹⁴N</td>
<td>76.047</td>
<td>7.551</td>
<td>21.065 - ( B_{Ξ} )</td>
<td>0.174</td>
</tr>
<tr>
<td>¹⁶O</td>
<td>127.624</td>
<td>12.127</td>
<td>16.489 - ( B_{Ξ} )</td>
<td>0.230</td>
</tr>
<tr>
<td>⁶He</td>
<td>29.268</td>
<td>23.482</td>
<td>5.137 - ( B_{Ξ} )</td>
<td>0 - 4</td>
</tr>
</tbody>
</table>
First, we discuss the $\Xi^-$ absorption in $^{12}$C, where $^{12}$B$^*(\Xi^-+^{12}$C$)$, $^{15}$C$^*(\Xi^-+^{14}$N$)$, and $^{17}$N$^*(\Xi^-+^{16}$O$)$. Table 3 summarizes the formation probabilities of DAHN in the SDM calculation from the DAC, $^{13}$B$^*(\Xi^-+^{12}$C), $^{15}$C$^*(\Xi^-+^{14}$N), and $^{17}$N$^*(\Xi^-+^{16}$O). The total DAHN formation probability in the SDM is calculated to be $P_{\text{DAHN}} = (25–80\%)$. If we assume that the DAC formation probability is common in the three targets and set to be $P_{\text{DAC}} = 30\%$ [41], the total DAHN formation probability is $P_{\text{DAHN}} = P_{\text{DAC}} \times P_{\text{Br}} = (7.5–24\%)$, which is larger than the lower limit of the $2\Lambda$ trapping probability (double and twin hypernuclear formation), 4.8% for light nuclei, evaluated by the KEK-E176 collaboration [53,54]. By comparison, the calculated DAHN formation probability is larger than the $2\Lambda$ trapping probability captured by light nuclei, $5.0 \pm 1.7\%$ [55]. The small $2\Lambda$ trapping probability data may suggest small bond energies as in model B.

3.1 $\Xi^-$ absorption in $^{12}$C

First, we discuss the $\Xi^-$ absorption in $^{12}$C, where $^{6}$He and $^{10}$Be [1,2,5] were observed. In Fig. 3, we show the DAHN mass distribution obtained in the SDM calculation of the DAC, $^{13}$B$^*$, assumed to be formed in the $\Xi^-$ absorption in $^{12}$C. We find that DAHN with $A = 11$ is most frequently formed; $P_{\text{Br}} = (18.9–27.1\%)$ for $^{11}$Be and $P_{\text{Br}} = (4.3–8.7\%)$ for $^{11}$B. The next most frequently formed DAHN is $^{10}$Be with a probability of $P_{\text{Br}} = (5.5–18.7\%)$. While the $\Delta B_{\Lambda\Lambda}$ model dependence is not significant for heavier ($A \geq 11$) hypernuclei, small $\Delta B_{\Lambda\Lambda}$ in models B and C suppresses the formation probabilities of lighter hypernuclei. For example, the formation probability of $^{6}$He is calculated to be $P_{\text{Br}} = 4.3\%$ in model A, while it becomes $P_{\text{Br}} = 1.6\%$ and 1.3% in models B and C, respectively.

In the SDM, the formation probability strongly depends on the $Q$-value and the number of fragments in the final state, as shown in Fig. 4. Since the statistical binary decay favors decays into the excited state having large level densities, two-body decays with large $Q$-values do not necessarily exhaust the probabilities. The formation and decay of $^{6}$He in the Nagara event [4] have been uniquely
Table 3. Formation probabilities of DAHN in the statistical decay model from the DAC, $^{13}$ΛΛ $B^*(\Xi^- + 12C)$, $\Lambda^*\Lambda C^*(\Xi^- + 14N)$, and $\Lambda^*\Lambda N^*(\Xi^- + 16O)$. The probabilities ($P_{Br}$) are given in %. The — symbols show that the DAHN are either unbound, energetically not accessible, or not populated in the Monte Carlo sampling of the decay paths in the SDM calculation.

<table>
<thead>
<tr>
<th>$\Lambda^*\Lambda Z$</th>
<th>$^{12}C$</th>
<th>$^{14}N$</th>
<th>$^{16}O$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Sum</td>
<td>71.2</td>
<td>41.0</td>
<td>50.6</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda H$</td>
<td>0.03</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda H$</td>
<td>0.6</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda He$</td>
<td>4.3</td>
<td>1.6</td>
<td>1.3</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda He$</td>
<td>1.6</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda He$</td>
<td>0.6</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Li$</td>
<td>0.4</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Li$</td>
<td>3.9</td>
<td>1.6</td>
<td>1.7</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Li$</td>
<td>0.6</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Li$</td>
<td>0.2</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Be$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Be$</td>
<td>18.7</td>
<td>5.5</td>
<td>8.0</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Be$</td>
<td>27.1</td>
<td>18.9</td>
<td>24.3</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Be$</td>
<td>0.8</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda Be$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda B$</td>
<td>8.7</td>
<td>4.3</td>
<td>6.3</td>
</tr>
<tr>
<td>$\Lambda^*\Lambda B$</td>
<td>3.6</td>
<td>6.0</td>
<td>5.9</td>
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<tr>
<td>$\Lambda^*\Lambda N$</td>
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identified to take place as

$$\Xi^- + ^{12}C \rightarrow ^6\Lambda\Lambda \text{He} + \alpha + \tau, \quad ^6\Lambda\Lambda \text{He} \rightarrow ^5\Lambda \text{He} + p + \pi^-.$$  \hspace{1cm} (3)

In the formation of $^{10}\Lambda\Lambda \text{Be}$ [1,2], the following sequence was found not to be inconsistent with the Nagara event [56,57]:

$$\Xi^- + ^{12}C \rightarrow ^{10}\Lambda\Lambda \text{Be} + d + n, \quad ^{10}\Lambda\Lambda \text{Be} \rightarrow ^9\Lambda \text{Be}^* + p + \pi^-.$$  \hspace{1cm} (4)

These fragmentation branches have relatively large $Q$-values, 8.4 and 9.8 MeV in the $\Delta B_{\Lambda\Lambda}$ model C, for the channels in Eqs. (3) and (4), respectively. As a result, the event probability of the decay channel in Eq. (3) (the decay channel in Eq. (4)) in the SDM is calculated to be $P_{Br} = 2.6\%$, 1.0\%, and 0.9\% ($P_{Br} = 8.8\%$, 3.5\%, and 5.0\%) in models A, B, and C, respectively. The results are summarized in Table 4.
Fig. 3. DAHN mass distribution ($P_{Br}$) obtained in the statistical decay model starting from the DΛC ($^{13}_{\Lambda\Lambda}B^*$) assumed to be formed from $\Xi^-$ absorption at rest from the 3D atomic orbit in $^{12}$C. The results of models A (triangles), B (squares), and C (circles) for the $\Lambda\Lambda$ bond energy $\Delta B_{\Lambda\Lambda}$ are compared. The diamonds show the results of AMD-QL with strong fluctuations on the DAHN formation probability [41] divided by 0.3.

Fig. 4. Threshold energy dependence of the event probabilities ($P_{Br}$) obtained in the SDM from $^{13}_{\Lambda\Lambda}B^*$ ($\Xi^- + ^{12}$C). The filled circles, squares, and triangle show the event probability in the SDM as a function of the threshold energy in two-, three-, and four-body channels, respectively. The open circles and squares show the channels including $^6\Lambda\Lambda$He and $^{10}_{\Lambda\Lambda}$Be, respectively.

Table 4. Event probabilities of hypernuclear formation events including $^6\Lambda\Lambda$He and $^{10}_{\Lambda\Lambda}$Be from the DΛC ($^{13}_{\Lambda\Lambda}B^*$) assumed to be formed in the $\Xi^-$ absorption reaction in $^{12}$C. The probabilities ($P_{Br}$) are given in %.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Prob. (%)</th>
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<tr>
<td></td>
<td>A</td>
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<tr>
<td>$^6\Lambda\Lambda$He + $\alpha + t$</td>
<td>2.6</td>
</tr>
<tr>
<td>$^6\Lambda\Lambda$He + $^7$Li</td>
<td>0.7</td>
</tr>
<tr>
<td>$^6\Lambda\Lambda$He + $^6$Li + $n$</td>
<td>0.4</td>
</tr>
<tr>
<td>$^6\Lambda\Lambda$He + $\alpha + d + n$</td>
<td>0.7</td>
</tr>
<tr>
<td>$^{10}_{\Lambda\Lambda}$Be + $d + n$</td>
<td>8.8</td>
</tr>
<tr>
<td>$^{10}_{\Lambda\Lambda}$Be + $t$</td>
<td>0.7</td>
</tr>
<tr>
<td>$^{10}_{\Lambda\Lambda}$Be + $p + n + n$</td>
<td>9.2</td>
</tr>
</tbody>
</table>

We shall now evaluate the expected number of events in experiments. In the KEK-E373 experiment, about $10^3$ events of $\Xi^-$ absorption in emulsion nuclei were analyzed [5]. About half of the $\Xi^-$ absorption events are those with light nuclei, $^{12}$C, $^{14}$N, and $^{16}$O; it would therefore be reasonable to assume that about 160 events of $\Xi^-$ absorption in $^{12}$C were analyzed. The formation probability of DΛC, $^{13}_{\Lambda\Lambda}B^*$, is about $P_{DAC} \simeq 30\%$ in AMD [41]. Thus the number of decays in a given channel may
be given as $N_{ev} = 160 \times P_{DAC} \times P_{Br}$, with $P_{Br}$ being the branching ratio (event probability) in the statistical decay. For the fragmentation channel in Eq. (3) (for the formation of $^6\Lambda\Lambda$He), the estimated number of events in the E373 experiment is $N_{ev} = 1.2, 0.5$, and 0.4 ($N_{ev} = 2.1, 0.8$, and 0.6) in models A, B, and C, respectively. Since these numbers are close to unity, it is not unreasonable that the KEK-E373 experiment observed one event in the channel of Eq. (3). Since the $\Delta Br_{\Lambda\Lambda}$ models B and C are more realistic, we can judge that the KEK-E373 experiment was reasonably lucky provided that the detection efficiency of $^6\Lambda\Lambda$Be in the E373 experiment was reasonable, as long as the detection efficiency of the sequential weak decay is not small.

Now let us discuss the transport model dependence. In Fig. 3, we also show the $D\Lambda\Lambda$HN formation probabilities obtained by using AMD with additional fluctuations (AMD-QL) with the statistical decay effects [41] normalized by $P_{DAC} = 30\%$. We show the results with strong fluctuations at $g_0 = 0.5$, with $g_0$ being the fluctuation strength parameter, and a strongly attractive $\Lambda\Lambda$ interaction similar to model A was adopted. Additional fluctuations promote nucleon and $\Lambda$ emissions in the dynamical stage, then the total DAC formation probability is found to be around half, $P_{DAC}(AMD-QL) = 16\%$, compared with the AMD results, and the total $D\Lambda\Lambda$HN formation probability is also reduced. By comparison, lighter DAC with smaller excitation energies are found to be formed in AMD-QL, and the formation probabilities of some $D\Lambda\Lambda$HN are enhanced. For example, $^6\Lambda\Lambda$Be is formed at $P_{DAC}(^6\Lambda\Lambda$Be) = 2.0\%, which corresponds to $P_{Br}(^6\Lambda\Lambda$Be) $\simeq$ 6.7\% and is larger than the SDM calculation results. When normalized by $P_{DAC} = 30\%$, the differences of the results in AMD-QL with statistical decays from the SDM results using model A are within a factor of two. If we assume similar differences in the SDM with models B and C, which are more realistic, the expected numbers of events of $^{10}\Lambda\Lambda$Be and $^6\Lambda\Lambda$Be in KEK-E373 become closer to unity with additional quantum fluctuation effects in the dynamical stage. Thus these differences are not negligible, but the conclusion in the previous paragraph, that the observations of $^{10}\Lambda\Lambda$Be and $^6\Lambda\Lambda$Be in KEK-E373 are reasonable, does not change.

### 3.2. $\Xi^-$ absorption in $^{14}\Lambda\Lambda$N

Next, we proceed to discuss the $\Xi^-$ absorption in $^{14}\Lambda\Lambda$N, where $^{13}\Lambda\Lambda$B was observed [3]. Figure 5 shows the $D\Lambda\Lambda$HN mass distribution in the SDM calculation of $^{15}\Lambda\Lambda$C* from $\Xi^- + ^{14}\Lambda\Lambda$N. As in the case of $\Xi^- + ^{12}\Lambda\Lambda$C, two-nucleon evaporation processes are favored, and $D\Lambda\Lambda$HN with $A = 13$ is most frequently formed: $P_{Br} = (11.2–18.5)\%$ for $^{13}\Lambda\Lambda$B and $P_{Br} = (5.4–10.0)\%$ for $^{13}\Lambda\Lambda$C. We also find that $^{12}\Lambda\Lambda$B ($P_{Br} = (3.4–16.4)\%$), $^{10}\Lambda\Lambda$Be ($P_{Br} = (1.4–2.6)\%$), and $^6\Lambda\Lambda$He ($P_{Br} = (1.1–3.6)\%$) are also frequently formed. Since the proton separation energy is smaller and the initial $^{15}\Lambda\Lambda$C* energy is larger for $^{14}\Lambda\Lambda$N, the $\Delta Br_{\Lambda\Lambda}$ model dependence is smaller than other target nuclei.

In the $D\Lambda\Lambda$HN formation event in E176 [3], the following sequence is found to be consistent with the Nagara event [57],

$$\Xi^- + ^{14}\Lambda\Lambda N \rightarrow ^{13}\Lambda\Lambda B + p + n, \quad ^{13}\Lambda\Lambda B \rightarrow ^{13}C^* + \pi^-.$$  

(5)

As shown in Fig. 6, this decay channel is found to have the largest event probability, $P_{Br} = 17.9\%$, $10.4\%$, and $14.6\%$ ($P_{Br} = 18.5\%$, $11.2\%$, and $15.5\%$) for the event probability of this channel.
Fig. 5. Double $\Lambda$ hyperfragment mass distribution ($P_{Br}$) in the SDM from $^{15}_{\Lambda\Lambda}C^+ (\Xi^- + ^{14}N)$.

Fig. 6. Threshold energy dependence of the event probabilities ($P_{Br}$) in the SDM from $^{15}_{\Lambda\Lambda}C^+ (\Xi^- + ^{14}N)$. The open circles show the channels including $^{13}_{\Lambda\Lambda}B$.

(formation probability of $^{13}_{\Lambda\Lambda}B$) in models A, B, and C, respectively. In the KEK-E176 experiment, the number of events was estimated to be $77.6 \pm 5.1$ for $\Xi^-\Lambda$ absorption at rest in emulsion [54], and $31.1 \pm 4.8$ for absorption events on light nuclei [53]. Then we may roughly estimate the number of $\Xi^-\Lambda$ absorption events for $^{14}N$ at around 10. With the D$\Lambda$C formation probability of around $P_{D\Lambda C} \simeq 30\%$, the expected number of events in the fragmentation channel in Eq. (5) (formation of $^{13}_{\Lambda\Lambda}B$) is $N_{ev} = 0.5$, 0.3, and 0.4 ($N_{ev} = 0.6$, 0.3, and 0.5) in models A, B, and C, respectively. We can judge that the KEK-E176 experiment was also reasonably lucky provided that the detection efficiency of the sequential weak decay of $^{13}_{\Lambda\Lambda}B$ is high.

3.3. $\Xi^-\Lambda$ absorption in $^{16}O$

We now discuss the $\Xi^-\Lambda$ absorption in $^{16}O$, where $\Lambda\Lambda Be$ was reported to be observed [6]. In Fig. 7, we show the D$\Lambda$HN mass distribution in the SDM calculation of $^{17}_{\Lambda\Lambda}N^+$ from $\Xi^- + ^{16}O$. The most frequently formed D$\Lambda$HN is $^{14}_{\Lambda\Lambda}C$, which is a bound state of $^{12}C + \Lambda + \Lambda$, and the formation probability in the SDM is $P_{Br} = 59.3\%$, 38.5\%, and 52.8\% in models A, B, and C, respectively. The next most frequently formed one is $^{13}_{\Lambda\Lambda}C$, which is formed via two-nucleon evaporation from $^{17}_{\Lambda\Lambda}N^+$. We also find that formation of D$\Lambda$HN with $A = 7$ and 8 is strongly suppressed in D$\Lambda$C models B and C.

The J-PARC E07 experiment is expected to detect 100 D$\Lambda$HN formation events among $10^4$ stopped $\Xi^-\Lambda$ events. As of November 2018, 920 $\Xi^-\Lambda$ absorption events were analyzed [58], and 8 and 6($+2$) double and twin hypernuclear formation events were detected. Among them, there is one event (the Mino event) where the D$\Lambda$HN formed was identified to be $^{13}_{\Lambda\Lambda}Be$ [6]. The event was interpreted as
one of three candidates:

\[
\Xi^- + ^{16}\text{O} \rightarrow ^{10}_{\Lambda\Lambda}\text{Be} + \alpha + t \quad (\Delta B_{\Lambda\Lambda} = 1.63 \pm 0.14 \text{MeV}),
\]

(6)

\[
\Xi^- + ^{16}\text{O} \rightarrow ^{11}_{\Lambda\Lambda}\text{Be} + \alpha + d \quad (\Delta B_{\Lambda\Lambda} = 1.87 \pm 0.37 \text{MeV}),
\]

(7)

\[
\Xi^- + ^{16}\text{O} \rightarrow ^{12}_{\Lambda\Lambda}\text{Be}^* + \alpha + p \quad (\Delta B_{\Lambda\Lambda} = -2.7 \pm 1.0 \text{MeV}).
\]

(8)

Experimentally, the second candidate (\(^{11}_{\Lambda\Lambda}\text{Be}\) formation) is considered to be the most probable, and the \(^{11}_{\Lambda\Lambda}\text{Be}\) formation probability is reasonably large (\(P_{\text{Br}} = 1.1\%\)) in the SDM with model A, as shown in Table 3. By comparison, \(^{11}_{\Lambda\Lambda}\text{Be}\) formation is less probable, \(P_{\text{Br}} = 0.2\%\) and 0.3\% in models B and C, respectively. By assuming the number of \(\Xi^-\) absorption events in \(^{16}\text{O}\) to be around 150 and a DAC formation probability of \(P_{\text{DAC}} \simeq 30\%\), the expected numbers for the second candidate are \(N_{\text{ev}} = 0.14, 0.04,\) and 0.06 for models A, B, and C, respectively. In the SDM, the third candidate (\(^{12}_{\Lambda\Lambda}\text{Be}^*\) formation) is more probable: the expected numbers are \(N_{\text{ev}} = 0.25, 0.15,\) and 0.17 for models A, B, and C, respectively. The probabilities are summarized in Fig. 8 and Table 5.

4. Double hypernuclear formation from \(\Xi^-\) nucleus

In addition to \(\Xi^-\) absorption in nuclei, conversion of \(\Xi\) nuclei would also make DAHN. (\(K^-\), \(K^+\)) reactions on nuclear targets populate certain \(\Xi\) hypernuclear states, which become doorway states to DAHN. In particular, the \(^7\text{Li}(K^-\), \(K^+)\) reaction may produce the neutron-rich \(\Xi\) hypernucleus \(^7_2\text{H} = \Xi^- + ^6\text{He}\), if it exists. Kumagai-Fuse and Akaishi proposed that the branching ratio of \(^7_2\text{H}\) conversion to form \(^5_2\text{H}\) is surprisingly large, at around \(P_{\text{Br}} = 90\%\) [59]. The first reason for this
large branching ratio is the limited decay channels,

\[ ^7_8H \rightarrow ^5_6^7H + n + n, \quad ^4_2^7H + ^3_2^7H + n + n, \quad ^4_2^7H + ^3_2^7H + ^2_1^7H + n \]

if the separation energy of \( \Xi \) in \( ^7_8H \) is around 2 MeV or more. In the left panel of Fig. 9 we show the decay channels from \( ^7_8H \). There are two channels, \( ^3_2^7H + ^3_2^7H + ^2_1^7H + n \) and \( ^2_1^7H \). The second reason for the large branching ratio in the small \( Q \)-values. Since a large part of the released energy in \( p \Xi^- \rightarrow \Lambda \Lambda (\sim 28.6 \text{ MeV}) \) is exhausted in breaking the \( \alpha \) cluster in \( ^7_8H \), the threshold energy of \( \Xi^- + ^6\text{He} \) is only 5.1 MeV above the \( \Lambda \Lambda \) emitting threshold (5 \( ^3H \) + \( \Lambda \Lambda + \Lambda \) threshold). This energy difference is much smaller than in the case of \( \Xi^- \) absorption in \( ^{12}\text{C} \) (12.7 MeV), \( ^{14}\text{N} \) (21.1 MeV), and \( ^{16}\text{O} \) (16.5 MeV). Furthermore, the binding energy of \( \Xi^- \) suppresses the \( Q \)-values. The \( Q \)-values in the channels shown in Eq. (9) are estimated as \( \sim 11 \). The \( \Xi^- \) and the corresponding \( \Lambda \Lambda \) are small. Hence, three-body decays would be favored over four- or five-body decays. As a result, the decay process will be dominated by the \( ^5_6^7H + 2n \) channel having the largest \( Q \)-value and three bodies in the final state. In Ref. [59], approximate but explicit calculations were performed for the decay process of \( ^7_8H \), and a large branching ratio decaying to \( ^5_6^7H \) is obtained. Based on this estimate of a high branching ratio to form \( ^5_6^7H \), Fujioka et al. proposed a new experiment at J-PARC (P75) [60]. Since the branching ratio is crucial to determining the feasibility of the experiment, it is valuable to evaluate the branching ratio in different approaches.

We shall now discuss the branching ratio of the \( ^7_8H \) decay to \( ^5_6^7H + 2n \) in the SDM described in Sect. 2 and used in Sect. 3; \( DB_{\Lambda \Lambda} \) model C is applied as an example. There are several differences from Ref. [59]. First, the matrix elements are evaluated statistically as given in Eq. (1), and excited
The binding energy differences. We adopt the measured and fitted sequences such as $^7\Lambda\Lambda\Lambda\Lambda H \rightarrow ^5\Lambda\Lambda\Lambda\Lambda H + n + n$ (circles) and $^7\Lambda\Lambda\Lambda\Lambda H \rightarrow ^4\Lambda\Lambda\Lambda\Lambda H + \Lambda + n + n$ (triangles) in the SDM as functions of the binding energy of $\Xi^-$ in $^8\Lambda\Lambda H$. The solid (dotted) lines show the results in $\Delta B_{\Lambda\Lambda\Lambda\Lambda}$ model C with (without) dineutron emission. The dashed lines show the results without considering particle unstable nuclei in the decay processes and with the channels and binding energies used in Ref. [59].

Figure 10 shows the branching ratios of $^7\Lambda\Lambda\Lambda\Lambda H \rightarrow ^5\Lambda\Lambda\Lambda\Lambda H + n + n$ ($P_{\text{Br}}(^5\Lambda\Lambda\Lambda\Lambda H))$ and $^7\Lambda\Lambda\Lambda\Lambda H \rightarrow ^4\Lambda\Lambda\Lambda\Lambda H + \Lambda + n + n$ ($P_{\text{Br}}(^4\Lambda\Lambda\Lambda\Lambda H))$ obtained in the SDM calculation using $\Delta B_{\Lambda\Lambda\Lambda\Lambda}$ model C as functions of the binding energy of $\Xi^-$ in $^8\Lambda\Lambda H$ (solid curves). The $\Xi^-$ binding energy in $^8\Lambda\Lambda H (\Lambda H)$ was evaluated from the twin hypernuclear formation in the Kiso event, $\Xi^- + ^{14}\Lambda + ^{15}C \rightarrow ^{10}\Lambda\Lambda\Lambda Be^{(*)} + ^5\Lambda\Lambda He$, as $B_{\Xi} = 3.87 \pm 0.21$ MeV or $1.03 \pm 0.18$ MeV [57,61]. These two values are for decays to the ground state and the excited state of $^{10}\Lambda\Lambda Be$. The binding energy would be smaller in $^8\Lambda\Lambda H$, so the binding energy region of $0 \leq B_{\Xi} \leq 4$ MeV would be enough.

The branching ratio $P_{\text{Br}}(^5\Lambda\Lambda\Lambda\Lambda H)$ increases with increasing $B_{\Xi}$, as expected, and takes values between $P_{\text{Br}}(^5\Lambda\Lambda\Lambda\Lambda H) = 43.6\% (B_{\Xi} = 0$ MeV) and $P_{\text{Br}}(^4\Lambda\Lambda\Lambda\Lambda H) = 57.8\% (B_{\Xi} = 4$ MeV) in the $B_{\Xi}$ region of interest. These values are smaller than those in Ref. [59], but still a large branching ratio of around $P_{\text{Br}} = 50\%$ seems to be probable in the SDM.

Let us discuss the difference of the branching ratio in the present study and in Ref. [59]. We show the SDM results without dineutron emission by dotted lines in Fig. 10. Model C is used for $\Delta B_{\Lambda\Lambda\Lambda\Lambda}$. In this case, $^5\Lambda\Lambda\Lambda\Lambda H(B(\Lambda\Lambda\Lambda\Lambda H) = 10.44$ MeV) is assumed to decay into $^4\Lambda\Lambda\Lambda\Lambda H(B(\Lambda\Lambda\Lambda\Lambda H) = 10.52$ MeV) in the final state, while the one-neutron emission into $^3\Lambda\Lambda\Lambda\Lambda H(B(\Lambda\Lambda\Lambda\Lambda H) = 8.83$ MeV) is not allowed energetically. The branching ratio to form $^5\Lambda\Lambda\Lambda\Lambda H$ takes $P_{\text{Br}}(^5\Lambda\Lambda\Lambda\Lambda H) = 42.3\% (B_{\Xi} = 0$ MeV) and $52.0\% (B_{\Xi} = 4$ MeV). These values are slightly smaller than those with dineutron emission. This is because the direct three-body decay of $^7\Lambda\Lambda\Lambda\Lambda H^* \rightarrow ^5\Lambda\Lambda\Lambda\Lambda H + 2n$ simulated by dineutron emission is suppressed, while the effect of dineutron emission is not significant and hypernuclear formation is dominated by binary decay sequences such as $^7\Lambda\Lambda\Lambda\Lambda H^* \rightarrow ^6\Lambda\Lambda\Lambda\Lambda H^{(*)} + n \rightarrow ^5\Lambda\Lambda\Lambda\Lambda H + 2n$ and $^7\Lambda\Lambda\Lambda\Lambda H^* \rightarrow ^6\Lambda\Lambda\Lambda\Lambda H^{(*)} + \Lambda \rightarrow ^5\Lambda\Lambda\Lambda\Lambda H + \Lambda + n$. Since the $Q$-values are small, it is reasonable to expect that two-body (binary) decays are expected to be favored over three- or four-body decays. We also show the branching ratios in the SDM with the $\Delta B_{\Lambda\Lambda\Lambda\Lambda}$ values and channels used in Ref. [59] by dashed lines in Fig. 10. Here we adopt.
We have investigated double $\Lambda\Lambda$ formation with respect to particle decay, $^5\Lambda\Lambda H$, $^6\Lambda\Lambda H$, and $^7\Lambda\Lambda H$, in the decay processes. The branching ratio to $^5\Lambda\Lambda H$ takes larger values than those with particle unstable nuclei, and takes values of $P_{\text{Br}}(^5\Lambda\Lambda H) = 46.7\%$ ($B_\Sigma = 0$ MeV) and $67.8\%$ ($B_\Sigma = 4$ MeV). These values are still smaller than those in Ref. [59]. Therefore, the larger branching ratio around $P_{\text{Br}} = 90\%$ in Ref. [59] is found to be a result of an explicit evaluation of the transition width in a specific model treatment in addition to kinematical reasons such as the limited number of decay channels and ignoring particle unstable states during the decay processes. For a more serious estimate, we need calculations with updated $\Sigma^-N$ and $\Lambda\Lambda$ interactions, and experimental confirmation of the three-body decay width in the same theoretical treatment.

It should be noted that the formation of $\Delta\Lambda\Lambda$ and its statistical decay may be a less reliable picture of hypernuclear formation for lighter nuclear targets. For example, while AMD+SDM roughly explains the $^4\Lambda H$ formation probability from the $K^-$ absorption reaction at rest in $^{12}\text{C}$ and $^{16}\text{O}$, the combined framework underestimates [49] the probability for $^7\text{Li}$ and $^9\text{Be}$ targets [62]. These light target nuclei, $^7\text{Li}$ and $^9\text{Be}$, have a cluster structure in the ground states and easily dissociate to fragments after the conversion process of $K^-N \rightarrow \Lambda\Lambda$. Then, $\Delta\Lambda\Lambda$ formation may take place without going through $\Delta\Lambda\Lambda$.

5. Summary and discussion

We have investigated double $\Lambda$ hypernuclear (DAHN) formation from the double $\Lambda$ compound nuclei (DAC), $^{13}\Lambda\Lambda B^*$, $^{15}\Lambda\Lambda C^*$, $^{17}\Lambda\Lambda N^*$, and $^7\Lambda\Lambda H^*$ in the statistical decay model. The first three compound nuclei would be formed in $\Sigma^-$ absorption at rest in light nuclear targets in emulsion, $\Sigma^- + ^{12}\text{C} \rightarrow ^{13}\Lambda\Lambda B^*$, $\Sigma^- + ^{14}\text{N} \rightarrow ^{15}\Lambda\Lambda C^*$, and $\Sigma^- + ^{16}\text{O} \rightarrow ^{17}\Lambda\Lambda N^*$, and the last one would be formed from $\Sigma$-hypernuclei, $^7\text{Li}(K^-, K^+)^7\text{H}_{\text{gs}}$ followed by $^7\text{H}_{\text{gs}} \rightarrow ^7\Lambda\Lambda H^*$. The SDM has been demonstrated to work well in describing fragment formation processes, especially when combined with the transport model calculation to populate the compound nuclei and the excitation energies. In the antisymmetrized molecular dynamics calculations of $\Sigma^-$ absorption in $^{12}\text{C}$, the DAC are dominated by $^{13}\Lambda\Lambda B^*$ and its formation probability is around $P_{\text{DAC}} \simeq 30\%$. Assuming that this mechanism also applies to the $\Sigma^-$ absorption in other target nuclei, we have evaluated the formation probabilities of DAHN. We have also applied the same SDM to evaluate the branching ratio of the $\Sigma$ hypernucleus $^7\text{H}$ to form $^5\Lambda\Lambda H$.

We have examined the target and the $\Lambda\Lambda$ bond energy ($\Delta B_{\Lambda\Lambda}$) dependence of the DAHN formation probabilities, and the event probabilities are also examined for the channels in which DAHN have been observed in experiments. The bond energy is given as $\Delta B_{\Lambda\Lambda} = 4.9$ MeV (model A), $\Delta B_{\Lambda\Lambda} = 0.67$ MeV (model B), and $\Delta B_{\Lambda\Lambda}(A = 6) = 0.67$ MeV and $\Delta B_{\Lambda\Lambda}(A = 11) = 1.87$ MeV (model C). In models A and B $\Delta B_{\Lambda\Lambda}$ is assumed to be independent of the hypernuclei, and in the model C $\Delta B_{\Lambda\Lambda}$ is assumed to be a linear function of the DAHN mass number. Models B and C are consistent with the Nagarasa event result.

In the $\Sigma^-$ absorption in $^{12}\text{C}$, $^6\Lambda\Lambda\text{He}$ and $^{10}\Lambda\Lambda\text{Be}$ are observed in the fragmentation channel of $\Sigma^- + ^{12}\text{C} \rightarrow ^6\Lambda\Lambda\text{He} + \alpha + \gamma$ and $\Sigma^- + ^{12}\text{C} \rightarrow ^{10}\Lambda\Lambda\text{Be} + d + n$, respectively. In the SDM, these channels are found to have relatively large probabilities, and the expected numbers of events are consistent with unity within a factor of three. For a $^{14}\text{N}$ target, $^{13}\Lambda\Lambda\text{B}$ has been observed in the channel $\Sigma^- + ^{14}\text{N} \rightarrow ^{13}\Lambda\Lambda\text{B} + p + n$. The SDM calculation shows that this channel has the largest event probability among other fragmentation channels, and the expected number of events is $(0.3–0.4)$ in $\Delta B_{\Lambda\Lambda}$ models B and C. For a $^{16}\text{O}$ target, $^{15}\Lambda\Lambda\text{Be}$ has been observed. The mass number and channels are not uniquely identified, but the experimentally most probable channel is $\Sigma^- + ^{16}\text{O} \rightarrow ^{11}\Lambda\Lambda\text{Be} + \alpha + d$. 

\[ \Delta B_{\Lambda\Lambda} = 1.92 \text{ MeV} \]
This channel is found to have a small event probability of $P_{\text{Br}} = (0.2–0.3)\%$ and the expected number of events is $(0.04–0.06)$ in $\Delta B_{\Lambda\Lambda}$ models B and C. If the observation was not accidental, we may need other mechanisms than the statistical decay of DAC.

The $\Delta B_{\Lambda\Lambda}$ model dependence of the DAHN formation probabilities is not significant in the main decay channels, where a few nucleons are evaporated: two-nucleon emission for $^{12}\text{C}$ and $^{14}\text{N}$ targets and three-nucleon emission for $^{16}\text{O}$, as shown in Figs. 3, 5, and 7. One-nucleon emission to excited levels is favored in each step of the statistical binary decay because of the large level densities in the daughter nuclei, $\rho \propto A^{-5/3} e^{2\sqrt{aE}}$, with $a$ being the level density parameter, $a \simeq A/8 \text{ MeV}^{-1}$ [44,45]. Since the mass number dependence of $\Delta B_{\Lambda\Lambda}$ is assumed to be small, the $\Delta B_{\Lambda\Lambda}$ difference results in a shift of the excitation energies of the parent and daughter nuclei simultaneously and does not strongly affect the decay width of one-nucleon emission. By comparison, lighter DAHN formation depends more strongly on $\Delta B_{\Lambda\Lambda}$. Since the initial energy is fixed in $\Xi^{-}$ absorption in nuclei, the $Q$-value difference of around $4 \text{ MeV}$ in emitting light DAHN is significant in the total released energy of $(10–30) \text{ MeV}$ in the fragmentation. Transport model dependence is also discussed via comparison with the results in AMD with quantum fluctuation effects [41], and a factor of two difference may appear in the DAHN formation probabilities.

We also discussed the branching ratio to form $^{5}_{\Lambda\Lambda}\text{H}$ from a $\Xi$ hypernucleus $^{2}_{\Lambda}\text{H}$, which can be formed in the $^{7}\text{Li}(K^{-}, K^{+})$ reaction and is assumed to be converted to a DAC $^{7}_{\Lambda\Lambda}\text{H}^{+}$. It was proposed that the branching ratio would be around $P_{\text{Br}} = 90\%$ [59]. In SDM calculations, the branching ratio is found to be $P_{\text{Br}} = (40–60)\%$ in a $\Xi^{-}$ binding energy range of $(0–4) \text{ MeV}$. This branching ratio is still large, but lower than that in Ref. [59]. The difference seems to come from the method used to evaluate the decay width: explicit few-body calculations in Ref. [59] and the statistical assumptions in the present work.

The theoretical framework adopted in the present work may be too simplified to describe realistic DAHN formation processes, and we need improvements in the theoretical frameworks for more quantitative estimates of the formation probabilities of DAHN from $\Xi^{-}$ absorption reactions at rest and $\Xi$ hypernuclei. First, the initial DAC formation probabilities need to be evaluated with updated $\Delta B_{\Lambda\Lambda}$ values consistent with the Nagara event and for $^{14}\text{C}$, $^{14}\text{N}$, and $^{16}\text{O}$ target nuclei in a consistent way. We have adopted a formation probability of DAC in the initial pre-equilibrium stage of around $P_{\text{DAC}} \simeq 30\%$, based on the AMD calculation of $\Xi^{-}$ absorption at rest in $^{12}\text{C}$ [41]. This probability is obtained with a strongly attractive $\Lambda\Lambda$ interaction, which gives the $\Lambda\Lambda$ bond energy $\Delta B_{\Lambda\Lambda} = 4.9 \text{ MeV}$ in $^{13}_{\Lambda\Lambda}\text{B}$, and $^{14}\text{N}$ and $^{16}\text{O}$ targets were not considered. In heavier targets ($^{14}\text{N}$ and $^{16}\text{O}$), the $\Lambda$ trapping probabilities are found to be larger in the case of $K^{-}$ absorption at rest [49], and then similar DAC formation probabilities or more are also expected, while this expectation may be too optimistic and is premature. Second, we need more care in discussing hypernuclear formation in reactions on light nuclear targets such as $^{7}\text{Li}$ and $^{9}\text{Be}$. When the ground state of the target has a developed cluster structure, direct reaction processes are found to be more important than in reactions on C, N, and O targets [49]. This may also apply to the decay of light $\Xi^{-}$ nuclei such as $^{2}_{\Lambda}\text{H}$, as discussed in Sect. 4. Thirdly, we may need other quantum effects or multifragmentation mechanisms to understand DAHN formation consistently with twin hypernuclear formation, where two single $\Lambda$ hypernuclei are formed. The twin hypernuclear formation probability is known to be comparable to that of DAHN, but the SDM predicts a much lower probability of twin hypernuclear formation. In Ref. [41], quantum fluctuation effects are considered and are found to promote twin hypernuclear formation, but the twin hypernuclear formation probability is still underestimated. One of the possible mechanisms for producing twin single hypernuclei efficiently is to go through the
resonance states of two single hypernuclei around the threshold energy of $\Xi^-$ and target nuclei [43].

This mechanism seems plausible, since resonance states tend to appear around the threshold, and there are several twin hypernuclear channels around the $\Xi^-$ target threshold. Another candidate mechanism is multifragmentation, simultaneous decay to three or more fragments. An excitation energy of $E^* = (30–50)$ MeV corresponds to a temperature of $T \simeq \sqrt{E^*/a} \sim 5$ MeV, and multifragmentation may be relevant at this temperature. Actually, canonical and microcanonical statistical fragmentation models are applied to hypernuclear formation in Refs. [40,42]. Thus it is desirable to examine $\Delta$HN formation probabilities in multifragmentation models.

Acknowledgments

The authors would like to thank Hiroyuki Fujioka, Tomokazu Fukuda, and Emiko Hiyama for giving us motivation for this work; Hirokazu Tamura and Toshiyuki Gogami for telling us the recalibration results in SKS data; and Jørgen Randrup for useful discussions and careful reading of our manuscript. This work is supported in part by Grants-in-Aid for Scientific Research from JSPS (nos. 19H05151 and 19H01898), and by the Yukawa International Program for Quark-hadron Sciences (YIPQS).

References

    100, 069902 (2019) [erratum].