RIMS 2020 - KYOTO - JAPAN

Title of the presentation:

Stability for a first order coefficient in a non self adjoint wave equation from Dirichlet to Neumann map

Joint work with Pr. Mourad Bellassoued

Presented by:

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In this presentation we focus on the study of an inverse problem for a non-self-adjoint hyperbolic equation. More precisely, we attempt to stably recover a first order coefficient appearing in a wave equation from the knowledge of Neumann boundary data. We show in dimension n greater than two, a stability estimate of Hölder type for the inverse problem under consideration. The proof involves the reduction to an auxiliary inverse problem for an electro-magnetic wave equation and the use of an appropriate Carleman estimate.

Keywords Inverse problem, Stability result, Dirichlet-to-Neumann map, Carleman estimate.

1. Summary of the presentation

The main purpose of this work is the study of an inverse problem of determining a coefficient of order one on space appearing in a non-self-adjoint wave equation. Let $\Omega \subset \mathbb{R}^n$ with $n \geqslant 2$, be an open bounded set with a sufficiently smooth boundary $\Gamma = \partial\Omega$. For $T > 0$, we denote by $Q = \Omega \times (0,T)$ and $\Sigma = \Gamma \times (0,T)$. We introduce the following initial boundary value problem for the wave equation with a velocity field V,

$$\begin{cases} \mathscr{L}_V u := (\partial_t^2 - \Delta + V \cdot \nabla) u = 0 & \text{in } Q, \\ u_{|t=0} = \partial_t u_{|t=0} = 0 & \text{in } \Omega, \\ u = f & \text{on } \Sigma, \end{cases} \tag{1.1}$$

where $V \in W^{1,\infty}(\Omega, \mathbb{R}^n)$ is a real vector field and $f \in \mathcal{H}_0^1(\Sigma) := \{f \in H^1(\Sigma),\ f_{|t=0} = 0\}$ is the Dirichlet data that is used to probe the system. We may define the so-called Dirichlet-to-Neumann (DN) map associated with the wave operator $\mathscr{L}_V$ as follows

$$\begin{array}{rcl} \Lambda_V : \mathcal{H}_0^1(\Sigma) & \longrightarrow & L^2(\Sigma) \\ f & \longmapsto & \partial_\nu u, \end{array}$$

where ν denotes the unit outward normal to Γ at x and $\partial_\nu u$ stands for $\nabla u \cdot \nu$.

The inverse problem we address is to determine the velocity field V appearing in (1.1) from the knowlegde of the DN map Λ_V and we aim to derive a stability result for this problem. To our knowledge this work is the first that treats the recovery of a coefficient of order one on space appearing in a wave equation.

The problem of recovering coefficients appearing in hyperbolic equations gained increasing popularity among mathematicians within the last few decades and there are many works related to this topic. But they are mostly concerned with coefficients of order zero on space.

The stability for problems associated with non self-adjoint operators is never treated before. In this work, we consider this challenging problem and we establish a stability estimate of Hölder type for the recovery of the first order coefficient V appearing in the wave operator $\mathscr{L}_V$ from the knowledge of the DN map Λ_V. The proof of the stability estimate requires the use of an L^2-weighted inequality called a Carleman estimate. Let us introduce the admissible set of the coefficients V. Given $M>0$ and $V_0 \in W^{1,\infty}(\Omega,\mathbb{R}^n)$, we define

$$\mathcal{V}(M,V_0) := \{V \in W^{1,\infty}(\Omega,\mathbb{R}^n),\ \|V\|_{W^{1,\infty}(\Omega)} \leqslant M,\ V = V_0 \text{ on } \Gamma\}.$$

Then our main result can be stated as follows

Theorem 1.1. *Let $V_1,\ V_2 \in \mathcal{V}$ such that $V_1 - V_2 \in W^{2,\infty}(\Omega)$. Then, there exist positive constants $\kappa \in (0,1)$ and $C > 0$ such that*

$$\|V_1 - V_2\|_{L^2(\Omega)} \leqslant C\|\Lambda_{V_1} - \Lambda_{V_2}\|^{\kappa}.$$

Here the constant C is depending only on Ω and M and $\|\cdot\|$ denotes the norm in $\mathcal{L}(\mathcal{H}_0^1(\Sigma); L^2(\Sigma))$.

The above statement claims stable determination of the velocity field V from the knowledge of the DN map Λ_V, where both the Dirichlet and Neumann data are performed on the whole boundary Σ. By Theorem 1.1, we can readily derive the following

Corollary 1.2. *Let $V_1,\ V_2 \in \mathcal{V}$. Then, we have that $\Lambda_{V_1} = \Lambda_{V_2}$ implies $V_1 = V_2$ everywhere in Ω.*

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