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Three-dimensional Model for Hydrodynamic and Cohesive Sediment Transport

Fitri Riandini[*] and Takao YAMASHITA

[*] Graduate School of Engineering, Kyoto University

Synopsis

A three dimensional finite difference model system for hydrodynamics and cohesive sediment transport is described. The hydrodynamics model is based on the hydrostatic and Boussinesq approximation. The simulation of cohesive sediment transport process is performed solving the 3D-conservative advection-diffusion equation of concentration.

From fractal description of mud flocs, a new formulation for settling velocity is provided as a function of floc size, which is consistent with Stoke's law. The evolution of floc size, hence settling velocity in a turbulent environment, is described through a new flocculation model of Winterwerp (1999) that includes the effects of turbulence induced aggregation and floc breakup. The flocculation model is introduced and implemented in ECOMSED, a three-dimensional hydrodynamics and sediment transport model, and applied to simulation of evolution of settling velocity of cohesive sediment.

Keywords: cohesive sediment, settling velocity, flocculation, Stoke's law

1. Introduction

The prediction of concentrations of suspended cohesive sediments and their transport is vital importance to their management with respect to cope with various problems (e.g. wetland protection and restoration, maintenance of navigation channels, dredging and dredged material relocation, effects of construction works on siltation and turbidity levels, dispersion of pollutans, nutrient transport, etc). Detailed mathematical models including three-dimensional codes, are necessary tools for the development and application of this knowledge.

It is well known that the transport and fate of fine-grained sediment in dynamic environment such as estuaries and coastal waters is a function of the effective settling velocity of the sediment, which in turn is affected by flocculation. The term "flocculation" is used to described the combined processes of aggregation and floc breakup and the term "flocs" for aggregates of fine-grained cohesive sediment.

Due to the cohesion of fine sediment, collision of particles may result in aggregation. From studies by O'Melia (1980), McCave (1984), Van Leussen (1994) and Stolzenbach & Elimenich (1994), Winterwerp (1999) concludes that collision due to Brownian motion and differential settling is negligible in estuaries and that collision of particles can be totally attributed to turbulent motions. For small shear stresses, aggregation is the dominant process, whereas at higher shear stresses breakup of flocs becomes dominant.

As the result of gravity, flocs of cohesive sediment settle in stagnant water. In flowing water, turbulent motions result in mixing processes, opposing settling. Under calm flow conditions (low flow velocity) turbulence levels are low and flocs settle. Concentration in shear flow are highest near the bed because of settling. At high floc concentrations, the settling velocity of individual flocs decreases, because of the influence neighbouring flocs (return flow). This hindered settling process leads to a further increase of sediment concentration near the bed.

From the description of mud flocs, which is treated as self-similar fractal entities, a new formulation for the settling velocity as a function of floc size is derived. The evolution of floc size in turbulent environment is described through flocculation model (Winterwerp, 1999) in a Eulerean framework that includes the effects of turbulence induced aggregation and floc breakup.

This paper focuses on the research to introduce flocculation process on the cohesive sediment transport. The flocculation model is implemented in a three-dimensional hydrodynamic and sediment transport model, ECOMSED, an applied to simulate the evolution of settling velocity of cohesive sediments.

2. Model Outline

A three-dimensional finite difference model system for hydrodynamics and cohesive sediment transport, ECOMSED solves the Navier-Stokes equations with a free surface boundary condition and the advection-diffusion equations of the temperature, the salinity and any other variable. Density effects, wind stress on the free surface, heat exchange with the atmosphere and the coriolis force are included in the model. Variations of the density are taken into account in the momentum equation via the Boussinesq approximation. The physical problems considered allow assuming that the pressure is hydrostatic.

To model sediment transport, two specific modules were developed:

2.1 Suspended Sediment Module

The transport of suspended sediment is described by the following advection-diffusion equation:

$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial x_i}\left(\left(u_i-\delta_{i3}w_s\right)c\right)= \frac{\partial}{\partial x_1}\left(A_H\frac{\partial c}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(A_H\frac{\partial c}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(K_H\frac{\partial c}{\partial x_3}\right) \quad (1)$$

in which c: concentration of the suspended sediment, and u, v, w: velocity components of ($i=1,2,3$). A_H is horizontal diffusivity and K_H is vertical eddy diffusivity.

At the water surface, z_s, the net sediment flux is zero. At the sediment-water interface, z_b , the flux is estimated by the rates of erosion and deposition, $F_{erosion}$ and $F_{dipposition}$. The bottom boundary condition can therefore be formulated as follows:

$$\left.\left\{\left(u_i-w_s\right)c-K_H\frac{\partial c}{\partial x_3}\right\}\right|_{x_3=Z_s}=0 \quad (2a)$$

and

$$\left.\left\{\left(u_i-w_s\right)c-K_H\frac{\partial c}{\partial x_3}\right\}\right|_{x_3=Z_b}=E_{b,c} \quad (2b)$$

with $E_{b,c}=F_{erosion}+F_{deposition}$.

The erosion rate is represented by Partheniades' formulation as:

$$F_{erosion}=M\left(\frac{\tau_b}{\tau_e}-1\right) \quad (3)$$

where, M is a positive empirical erosion parameter, τ_b is the bed shear stress, τ_e is the critical shear stress for erosion, is a function of the concentration of the top bed layer, which itself is given by the state of consolidation. The erosion coefficient M may also be function of the concentration. The deposition rate is calculated according to Krone's formula:

$$F_{deposition}=P_d w_s c \quad (4)$$

where P_d is the probability for deposition described by,

$$P_d=1-\left(\frac{\tau_b}{\tau_d}\right). \quad (5)$$

where τ_d is the critical shear stress for deposition

2.2 A model for flocculation and evolutional settling velocity

(1) Relation between settling velocity and floc size

Winterwerp (1998) developed three-dimensional Eulerian model of the evolution of the settling velocity of fine-grained cohesive sediment in turbulent open channel flow:

$$w_{sr}=\frac{\alpha}{18\beta}\frac{\left(\rho_s-\rho_w\right)}{\mu}D_p^{3-n_f}\frac{D^{n_f-1}}{1+0.15\,\mathrm{Re}^{0.687}} \quad (6)$$

in which D is the actual floc size, D_p is the diameter of the primary particle and n_f is fractal dimension for sediment particles. α and β are coefficients depending on the spherity of the particles, and $R_e=w_{s,r}D/\nu$ is the particle Reynolds.

Mud flocs seldom settle as individual particles. When their concentration becomes high enough, the settling flocs start to hinder each other in their movement, generally known as hindered settling. The effective settling velocity w_s in suspension of cohesive sediment affected by the process of hindered settling defined as:

$$w_s = w_{sr} \frac{(1-\phi_*)(1-\phi_p)}{1+2.5 \quad \phi} \tag{7}$$

In which the factor $(1-\phi_*)$ accounts for the return-flow effect. The volumetric concentration $\phi_* = \min(1,\phi)$ to account for the fact that c/c_{gel} can exceed unity in a consolidating fluid mud layer.

(2) A model for turbulence-induced flocculation

The differential equation for the flocculation of cohesive sediment under the influence of turbulent shear, then becomes:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x_i}\left(\left(u_i - \delta_{i,3}\frac{(1-\phi_*)(1-\phi_p)}{(1+2.5\phi)} \quad w_{sr}\right)N\right) - \frac{\partial}{\partial x_i}\left((D_s + \Gamma_T)\frac{\partial N}{\partial x_i}\right)$$

$$= -k'_A (1-\phi_*) G D^3 N^2 + k_B G^{q+1} (D - D_p)^p \; D^{2q} N \tag{8}$$

in which the parameter k'_A and k_B are defined as follows (e.g. Winterwerp, 1998):

$k'_A = \frac{3}{2} e_c \pi e_d$ and

$$k_B = a e_b D_p^{-p}\left(\frac{\mu}{F_y}\right)^q$$

where e_c, e_d, and e_b are efficiency parameters for collision, diffusion and breakup respectively. μ is the dynamic viscosity of the suspension, F_y is the strength of the mud flocs, and numbers of power, p and q are to be established empirically. The definition and relation of the number concentration, N, and the mass concentration, c, and the volumetric concentration ϕ are:

$$\phi = f_s N D^3 = \frac{c}{\rho_s}\left(\frac{D}{D_p}\right)^{3-n_f} \tag{9}$$

where f_s is the shape factor.

The aggregation and floc breakup terms are set to zero at the water surface and at the horizontal water-bed interface. Hence the boundary conditions lead:

$$\left\{(u_i - w_s)N - (D_s + \Gamma_T)\frac{\partial N}{\partial x_3}\right\}\Bigg|_{x_3 = Z_s} = 0 \tag{10a}$$

and

$$\left\{(u_i - w_s)N - (D_s + \Gamma_T)\frac{\partial N}{\partial x_3}\right\}\Bigg|_{x_3 = Z_b} = E_{b,N} \tag{10b}$$

The source-sink term $E_{b,N}$ represents the exchange with the bed and is modeled with classical formulae of Partheniades and Krone, so that the water-bed exchange formulation is consistent with the one for the mass balance.

Those set of equation, forming the flocculation model are introduced and simultaneously implemented in ECOMSED. The turbulent shear flow and sediment concentration govern the flocculation process, hence the settling velocity. The settling velocity affects the vertical concentration profile, hence the eddy viscosity and thus the settling velocity profile

3. Numerical Experiments

As no data are presently available on the evolution of the floc size and settling velocity, a sensitivity analysis of the flocculation model was conducted by assuming open channel flow shown in Fig.1. The flow direction is from left to right, and the constant sediment concentration with uniform profile is imposed at the left-side boundary. The right boundary is an open boundary for unidirectional steady flow tests. On the other hand, the right boundary is a sea side boundary at which tidal elevation is imposed and the left boundary is river side boundary which supplies a constant sediment concentration profile, for tidal flow tests. The number of grid mesh 52 x 7 grids with grid size is 100 m.

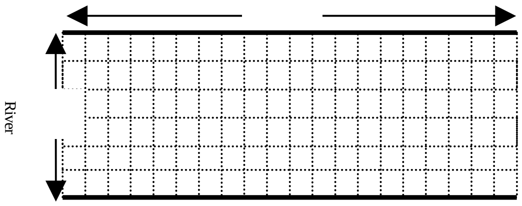

Fig.1 Computational set-up.

3.1 Unidirectional steady flow tests

First, a series of simulation for unidirectional steady flow is carried out with constant flow velocity and zero settling velocity of floc (w_s=0 mm/s). The various parameters used in the numerical experiment are listed in Table 1.

Table 1 Parameter settings in numerical experiment of flocculation tests.

Parameter	Value	Remarks
Still water depth	8 m	Constant; also for tidal flow
Flow velocity	0.2 – 1.0 m/s	Steady state
Tidal flow amplitude	0.5 m/s	
Bed roughness	1 mm	
Water density	1000 kg/m^3	
Sediment density	2650 kg/m^3	
Initial sediment concentration for steady flow tests	0.5 and 1.0 kg/m^3	Homogeneous profile for unidirectional steady flow tests
Initial sediment concentration for tidal flow tests	Linear distribution 0.5-0.59 kg/m^3	surface concentration: 0.5 kg/m^3 bottom concentration: 0.59 kg/m^3
Fractal dimension	2.0	
Initial particle size	4.0 μm	

The computed logarithmic velocity profile and related turbulent dissipation parameter $G = \sqrt{\varepsilon / \nu}$, which represents the turbulent dissipation of $k-\varepsilon$ simulation, are presented in Figs. 2 and 3. In the definition of G , ν is molecular dynamic viscosity. Weaker the flow velocity, smaller the dissipation parameter is shown in the figure.

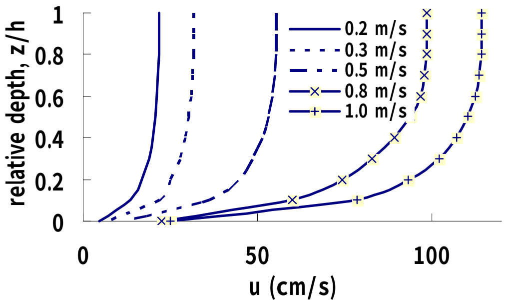

Fig. 2 Vertical profiles of flow velocity.

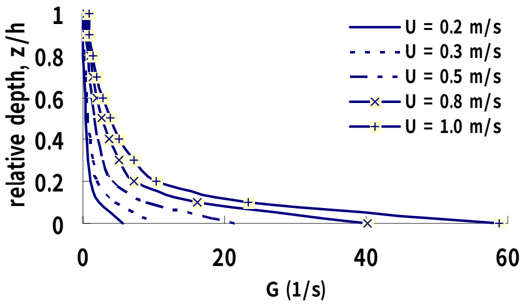

Fig. 3 Dissipation parameter for steady flow condition.

The settling velocity is computed by using equation (6). The vertical distribution of suspended sediment concentration at the steady state (final distribution after one day simulation) is presented in Fig.4. The corresponding floc size diameter, settling velocity and concentration are shown in Fig.5. The floc size and settling velocity show an increase near the surface and very small decrease in water column, then a large decrease near the bed. This large decrease is caused by the rapid increase of dissipation parameter, G near the bed.

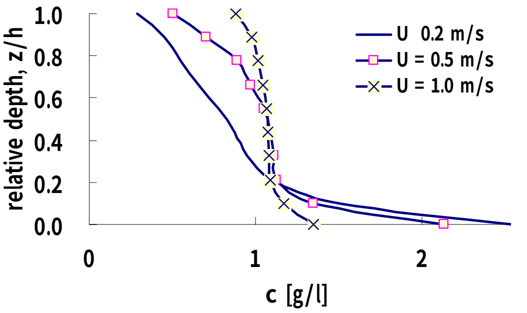

Fig. 4 Final distributions of suspended sediment concentration.

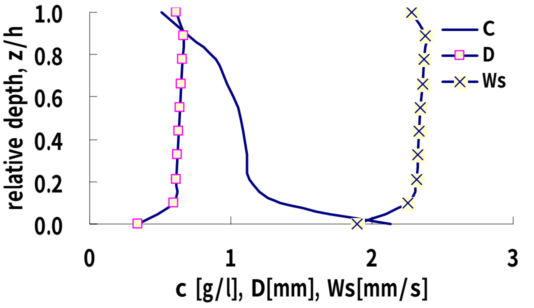

Fig.5 Relation between concentration, floc size and settling velocity.

3.2 Tidal flow conditions

The behaviour of the floc model for tidal flow condition is computed with two series. In the first series, the depth is kept constant at $h = 8$ m, and the amplitude of the tidal velocity at $U_m = 0.5$ m/s with tidal period of $T = 12.5$ hrs. The computed vertical distributions of the flow velocity, *u*, and dissipation parameter, *G*, are presented in Figs.6 and 7.

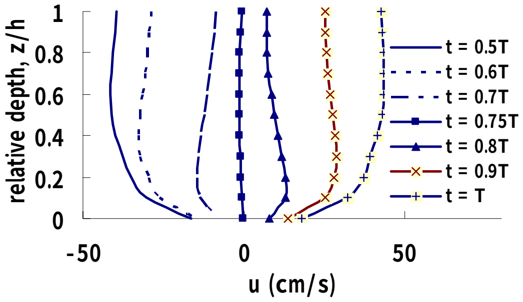

Fig. 6 Vertical profiles of flow velocity.

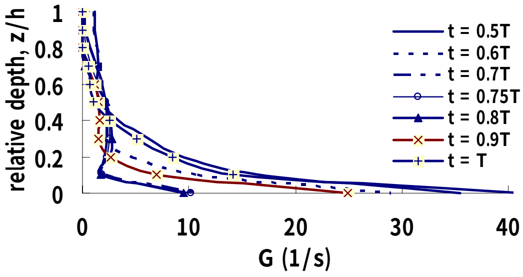

Fig. 7 Dissipation parameter for tidal conditions.

In the second series, we used six major tidal constituents as the boundary condition of open sea boundary (left) and a mean constant inflow of 100 m^3/s was imposed at the river boundary (right). For the sediment boundary conditions, constant sediment concentration with a constant value 500 mg/l at surface, increase gradually to 590 mg/l at the bottom was specified at the river side boundary (left).

In Figs.8 and 9, the computed variation of concentrations and settling velocities at three depths of surface, middle and near bottom are presented. It is shown that during the flood period ($13 < t < 20$ hrs), when the sediment well mixed over the water depth, the settling velocity higher in the water column is larger than near the bed. This is the result of the smaller values of the dissipation parameter *G* higher in the water column.

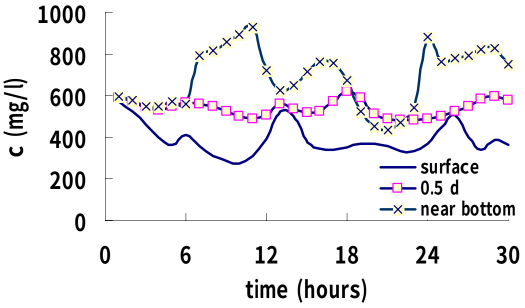

Fig. 8 Computed variations in concentration.

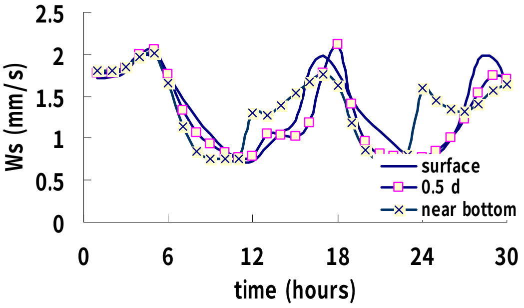

Fig. 9 Computed variations in settling velocity.

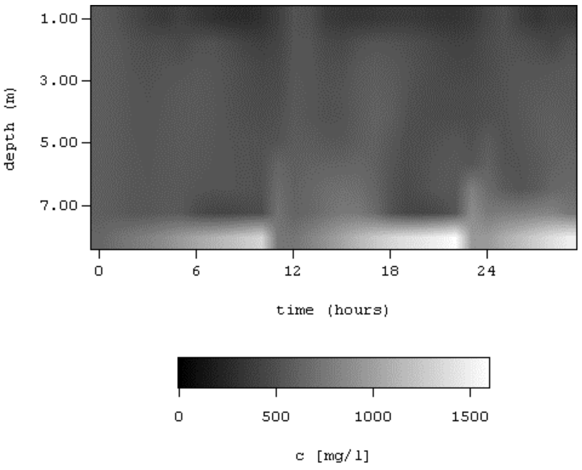

Fig. 10 "Isolutals" for the simulation with flocculation model.

On the other hand, during the ebb period ($20 < t < 27$), the greater settling velocities are found near the bed because the effect of the higher sediment concentration dominates the dissipation parameter effect. This is the stable distribution of sediment giving rise to considerable vertical gradients in concentration, as predicted by the model.

The computed vertical concentration profile at the fixed point is also plotted as a function of time in Fig.10

in the form lines of "isolutals" that is the sediment concentration contour in the time-depth space. This figure showing almost homogeneous conditions around *t*=12 and 24 hrs, i.e. around mean velocity, and large vertical gradients around 10 and 22 hrs, i.e. around slack water.

4. Conclusions

In this paper, a model for flocculation is introduced and implemented in ECOMSED, a hydrodynamics and sediment transport model, together with the mass balance, momentum equation and turbulence closure model, and a number of algebraic relations to relate floc size, settling velocity, number mass and volumetric concentration. The model is derived to describe the evolution of the settling velocity of flocs as a function of various parameters, i.e. the dissipation parameter and suspended sediment concentration

The flocculation equation has the form of an advection-diffusion equation, which can be solved in conjunction with the commonly applied mass-balance equation for suspended sediment.

Simulation results show that there is relation between flow velocity and dissipation parameter, weaker flow velocity resulting smaller dissipation parameter.

Fluid mud concentration varies with hydrodynamics conditions (ebb-flood tide) and the sediment concentration.

It is observed that the computed sediment concentration is almost homogeneous over the depth, except around slack water, when large vertical concentrations are found. The model predicts large variation in settling velocity over depth and strongly with time, with the higher values around slack water.

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３次元流体運動・底泥輸送モデル

ﾌｨﾄﾘ　ﾘｱﾝﾃﾞｨﾆ*・山下隆男

*京都大学工学研究科

要　旨

本研究では，３次元の流体運動（乱流，平均流）と粘着性底泥の挙動および輸送を再現するモデルとそのテスト計算結果を示す。流体運動のモデルはBoussinesq近似と静水圧近似を仮定した準3次元のモデルである。一方，粘着性底泥のモデルは，3次元の底質濃度に関する移流・拡散方程式の数値計算に基づいている。

フロックの沈降速度は，底泥のフラクタル次元を用いたフロックの大きさを関数とした新しい定式化を行っており，均一サイズの場合はストークス則に一致する。これはWinterwerp (1999)によって提案されたもので，流体の乱流運動によってフロックが凝集したり破壊されたりしてフロックの大きさが時間的に変化する効果が含まれている。本研究では，このフロックの沈降速度の定式化を，３次元の流体運動を含む底質輸送の公開モデル，ECOMSEDに導入し，エスチャリーでの底泥の挙動を解析するテスト計算を行った。

キーワード: 粘着性底泥，沈降速度，凝集，ストークス則