Continuations and Polymorphic Lambek Calculus

Masaya Taniguchi
Japan Advanced Institute of Science and Technology

Abstract

We introduced the notion of continuation in lambda calculus for Lambek calculus and showed that the continuation-passing style transformation could be naturally derived from the rules of Lambek calculus. Furthermore, since the answer category of a continuation is given when the whole sentence is determined, we introduced a polymorphic category and generalized the continuation-passing style transformation in the polymorphic Lambek calculus.

1 Introduction

In formal grammars analysis of natural language, we use context-free grammar (CFG) and context-sensitive grammar (CSG). Especially, combinatory categorial grammar (CCG), which belongs to mildly CSG, have an advantage over other formal grammar theories. The categories and derivation trees correspond directly to their meanings. However, to explain more linguistic phenomena, it was necessary to add new grammatical rules, e.g., the Dutch forward cross composition rule. Taniguchi and Tojo showed that continuation-passing style (CPS) transformations could replace grammatical rules dealing with the following linguistic phenomena.

- We showed uncrossing a constituent tree over the context-free grammar (CFG) known as cross-serial dependencies in Dutch and Swiss-German [5]. For example, we cannot parse a Dutch sentence "dat ik Cecilia Henk de nijlpaarden zag helpen voeren." in CFG.

- We showed the transformation of the constituency tree to right-to-left constituent trees [4]. For example, the garden-path sentence "The old man the boat."

We, however, remained the two problems since the above works.

- It is hard to know what class of language contains CCG with CPS.
- The number of deduction rules is increasing, i.e., the computational complexity is increasing.

In the present paper, we employ the subset of the CPS rule into Lambek calculus. Moreover, we show the language class of Lambek calculus with the CPS rule. It is the same as the original language class CFG because we deduced the rule from the system.

2 Preliminaries

This section introduces two concepts, Lambek calculus and CPS transformation in λ calculus. First, the following is the product-free Lambek calculus, which has no production $\bullet$ operator compared to the original system. Someone shows the correspondence between the present system and CFG. It means the two systems are equivalent. Product-free Lambek calculus is one of the intuitionistic sequent calculi. There is only one formula on the right-hand side in sequents σ, and are two implications $/$ and $\backslash$ in formulae φ and no structural rule.

$$\sigma ::= \tau \Rightarrow \varphi \mid \Rightarrow \varphi \qquad \varphi ::= \varphi/\varphi \mid \varphi\backslash\varphi \mid \alpha \qquad \tau ::= \varphi \mid \varphi, \sigma$$

Definition 1 (Lambek Calculus [6])**.** *Let greek letters be sequences of formulae and roman letters be formulae. The calculus consists of six rules as follows.*

$$\frac{}{X \Rightarrow X}\,\text{Ax} \qquad \frac{\Gamma, X \Rightarrow Y}{\Gamma \Rightarrow Y/X}\,/R \qquad \frac{X, \Gamma \Rightarrow Y}{\Gamma \Rightarrow X\backslash Y}\,\backslash R$$

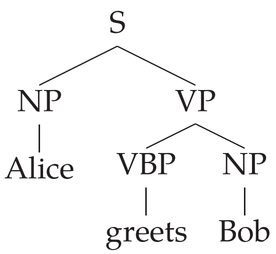
$$\frac{\Gamma, Y, \Delta \Rightarrow W \qquad \Sigma \Rightarrow X}{\Gamma, Y/X, \Sigma, \Delta \Rightarrow W}\,/L \qquad \frac{\Gamma \Rightarrow X \qquad \Delta, Y, \Sigma \Rightarrow W}{\Delta, \Gamma, X\backslash Y, \Sigma \Rightarrow W}\,\backslash L$$

We additionally define the following cut rule to the system for convenience. Independently, the two are also the same because of the cut-elimination theorem proved [2].

Definition 2 (Cut)**.** *Let capital letters be sequences of formulae and small letters be formulae. The rule is defined as follows.*

$$\frac{\Gamma, X, \Delta \Rightarrow W \qquad \Sigma \Rightarrow X}{\Gamma, \Sigma, \Delta \Rightarrow W}\,\text{Cut}$$

Lambek initially introduces this system inspired by categorial grammar (CG), which is equivalent to CFG. Thus, we can run on any sentence of context-language (CFL) by Lambek Calculus, e.g., we parse an English sentence, "Alice greets Bob.'' as follows.



$$\cfrac{\cfrac{NP \Rightarrow NP \qquad S \Rightarrow S}{NP, NP\backslash S \Rightarrow S} \qquad NP \Rightarrow NP}{NP, (NP\backslash S)/NP, NP \Rightarrow S}$$

The left-hand side of this figure is the CFG parsing tree, the the right-hand side is the Lambek calculus parsing tree, the category VBP corresponding to $(NP\backslash S)/NP$.

Lemma 1 (Type-raising rule)**.** *Related to combinatory categorial grammar* (*CCG*)*, Steedman* [3] *introduced rules lifting NP to* $X/(\text{NP}\backslash X)$ *and* $(X/\text{NP})\backslash X$ *for a giving category* X*. These rules are called type-raising rules, which is proved by Definition 1.*

$$\cfrac{\cfrac{X \Rightarrow X \qquad A \Rightarrow A}{X, X\backslash A \Rightarrow A}\,\backslash\text{I}}{X \Rightarrow A/(X\backslash A)}\,\text{I}/ \qquad \cfrac{\cfrac{A \Rightarrow A \qquad X \Rightarrow X}{A/X, X \Rightarrow A}\,\text{I}/}{X \Rightarrow (A/X)\backslash A}\,\text{I}\backslash$$

Lemma 1 shows the categories $A/(X\backslash A)$ and $(A/X)\backslash A$ hold for any categories A and X.

Second, the following is CPS transformation in λ-calculus, which we define the syntax as $\varphi ::= \lambda\psi.\varphi \mid \varphi\varphi \mid \psi$ and ψ atomic variables.

Definition 3 (CPS transformation)**.** *The transformation* $[\![\cdot]\!]$ *is recursively defined as;*

$$\begin{aligned}[\![x]\!] &\equiv \lambda k.kx \\ [\![\lambda x.M]\!] &\equiv \lambda k.k(\lambda x.[\![M]\!]) \\ [\![MN]\!] &\equiv \lambda k.[\![M]\!](\lambda m.[\![N]\!](\lambda n.mnk))\end{aligned}$$

We define the transformation on the untyped system. However, the Lambek calculus is on simply-typed λ-calculus. Thus, we next define the transformation of its type $\tau ::= \tau \to \tau \mid \sigma$, where σ is atomic types, e.g., the type of $\lambda x.y$ is $X \to Y$ if x and y is typed X and Y respectively.

Definition 4 (Type of CPS transformation)**.** *The transformation consists of the following twos,* $\langle \cdot \rangle$ *and* $\langle\!\langle \cdot \rangle\!\rangle$ *where* A *be the answer category of this calculation.*

$$\langle\!\langle T \rangle\!\rangle^A \equiv (\langle T \rangle^A \to A) \to A$$

$$\langle T \rangle^A \equiv \begin{cases} \langle X \rangle^A \to \langle\!\langle Y \rangle\!\rangle^A & \textit{if } T = X \to Y \\ T & \textit{otherwise} \end{cases}$$

We transform the $lambda$*-term* t *typed* T *to* $[\![t]\!]$ *typed* $\langle\!\langle T \rangle\!\rangle$*.*

3 CPS transformation in Lambek Calculus

In Section 2, we introduce CPS transformation only for simply-typed λ-calculus. We next define the transformation on Lambek calculus as a type in which there are two directed arrow / and \. Since there are several possible transformations in the calculus, we define the transformation as two relations $\to$ and $\hookrightarrow$ inductively. The relations are inductively defined as follows.

Definition 5 (CPS transformation in Lambek Calculus)**.** *Let* X *and* Y *be categories of Lambek calculus,* A *be an answer categories corresponding to the answer category of CPS transformation in* λ*-calculus, and two arrows* $\to$ *and* $\hookrightarrow$ *be relations corresponding to* $\langle\!\langle \cdot \rangle\!\rangle$ *and* $\langle \cdot \rangle$ *respectively.*

$$\frac{X \overset{A}{\hookrightarrow} X'}{X \overset{A}{\to} (A/X')\backslash A}\text{(i)} \qquad \frac{X \overset{A}{\hookrightarrow} X'}{X \overset{A}{\to} A/(X'\backslash A)}\text{(ii)} \qquad \frac{X \text{ is atomic}}{X \overset{A}{\hookrightarrow} X}\text{(iii)}$$

$$\frac{Y \overset{A}{\hookrightarrow} Y' \qquad X \text{ is atomic}}{Y/X \overset{A}{\hookrightarrow} Y'/X}\text{(iv)} \qquad \frac{X \text{ is atomic} \qquad Y \overset{A}{\hookrightarrow} Y'}{X\backslash Y \overset{A}{\hookrightarrow} X\backslash Y'}\text{(v)}$$

Compared to Definition 4, we note additional limitations to transform only the functional categories holding the atomic type as an argument type. Moreover, the transformation $\to$ is one more case than Definition 4 because there are two directed arrows / and \. Then, we show the transformation in Lambek calculus.

Theorem 1. *Let X be a category and A be an answer category. Then, we hold the following derivation.*

$$\frac{X \overset{A}{\hookrightarrow} X'}{X \Rightarrow X'}$$

Proof. We prove it by mathematical induction for the length of X. The possible forms of X are an atomic category and functional categories.

1. Assume X is atomic. Then, the possible transformation of X by $\hookrightarrow$ is only (iii), and the possible transformations of X by $\to$ are only (i) and (ii) in Definition 5.

   a. The first case is $X \overset{A}{\hookrightarrow} X$, $X \Rightarrow X$ by Ax in Definition 5.
   b. The second case is $X \overset{A}{\to} (A/X)\backslash A$, then $X \Rightarrow (A/X)\backslash A$ by Lemma 1.
   c. The last case is $X \overset{A}{\to} A/(X\backslash A)$, then $X \Rightarrow A/(X\backslash A)$ by Lemma 1..

2. Assume X is the left-functional category $B\backslash C$, where B be atomic and $C \overset{A}{\to} D$ for some category D. Then the possible transformation of X by $\hookrightarrow$ are only (v), and the possible transformations of X by $\to$ are only (i) and (ii) in Definition 5.

   a. The first case is $X \overset{A}{\hookrightarrow} B\backslash D$, $B \Rightarrow B$, and $C \Rightarrow D$ by the induction hypothesis, then $X \Rightarrow B\backslash D$ by Definition 1 as follows.

$$\cfrac{\cfrac{B \Rightarrow B \qquad C \Rightarrow D}{B, B\backslash C \Rightarrow D}\,\backslash\mathrm{L}}{B\backslash C \Rightarrow B\backslash D}\,\backslash\mathrm{R}$$

b. The second case is $X \overset{A}{\to} A/((B\backslash D)\backslash A)$, then $X \Rightarrow A/((B\backslash D)\backslash A)$ by Proof 2a and Lemma 1..

c. The last case is $X \overset{A}{\to} (A/(B\backslash D))\backslash A$, then $X \Rightarrow (A/(B\backslash D))\backslash A$ by Proof 2a and Lemma 1..

3. Assume X is the left-functional category C/B, where B be atomic and $C \overset{A}{\to} D$ for some category D. Then the possible transformation of X by $\hookrightarrow$ are only (iv), and the possible transformations of X by $\to$ are only (i) and (ii) in Definition 5..

a. The first case is $X \overset{A}{\hookrightarrow} D/B$, $B \Rightarrow B$, and $C \Rightarrow D$ by the induction hypothesis, then $X \Rightarrow D/B$ by Definition 1 as follows.

$$\cfrac{\cfrac{C \Rightarrow D \qquad B \Rightarrow B}{C/B, B \Rightarrow D}\,/\mathrm{L}}{C/B \Rightarrow D/B}\,/\mathrm{R}$$

b. The second case is $X \overset{A}{\to} A/((D/B)\backslash A)$, then $X \Rightarrow A/((D/B)\backslash A)$ by Proof 3a and Lemma 1..

c. The last case is $X \overset{A}{\to} (A/(D/B))\backslash A$, then $X \Rightarrow (A/(D/B))\backslash A$ by Proof 3a and Lemma 1..

Therefore, $X \Rightarrow X'$ if $X \overset{A}{\to} X'$ for any category X. □

4 Extension of CPS transformations

We need to specify the answer category of CPS transformation when we apply Theorem 1 according to Definition 5, but it is not always obvious to determine the answer category. For example, in incremental parsing, we cannot determine the answer category until retrieving the whole sentence. Hence, we introduce a polymorphic category as an answer category in polymorphic Lambek calculus [1].

Definition 6 (Polymorphic Lambek Calculus). *Polymorphic Lambek Calculus defines the following four extra rule into Lambek Calculus. The condition $Z!$ is that the category Z is not a free bellow line.*

$$\frac{\Gamma, X[Z := Y], \Delta \Rightarrow W}{U, \forall Z.X, \Delta \Rightarrow W}\ \forall\text{L} \qquad \frac{\Gamma \Rightarrow X}{T \Rightarrow \forall Y.X[Z := Y]}\ \forall\text{R}, Z!$$

$$\frac{\Gamma, X, \Delta \Rightarrow W}{\Gamma, \exists Y.X[Z := Y], V \Rightarrow W}\ \exists\text{L}, Z! \qquad \frac{\Gamma \Rightarrow X[Z := Y]}{\Gamma \Rightarrow \exists Z.X}\ \exists\text{R}$$

The original calculus has no quantified categories, which the polymorphic system added as new categories. Then, we generalize the CPS transformation defined in Definition 5 by universal quantifier because we mentioned above.

$$X \to \forall Z.X' \overset{def}{\Longleftrightarrow} X \overset{A}{\to} X'[Z := A] \text{ for all } A\text{, where Z is free in } X'$$

This generalization is consistent with the rule $\forall$L.

5 Conclusion

We defined the restricted version of CPS transformation in Lambek Calculus in Section 3. We also defined polymorphic CPS transformation excepting polymorphic categories in Section 4, i.e., we transformed only categories defined in Lambek calculus to polymorphic categories. The transformation from polymorphic categories to polymorphic categories remains as future work. The restriction is that the argument category of functional categories is only atomic, compared to that the original one transforms for all category. The proof of this restriction also remains as future work.

By giving proof of Theorem 1, we showed the grammar with the restricted CPS transformation is still in Lambek Calculus, i.e., such a grammar is CFG. We determined its language class and added only the grammar rule called the CPS rule.

References

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