ON THE DENSITY THEOREM RELATED TO THE SPACE OF NON-SPLIT TRI-HERMITIAN FORMS I

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The purpose of this paper is to announce the result in [10], [8]. The result in [10], [8] is over an arbitrary number field k. However, we assume here that the ground field is $\mathbb{Q}$ for simplicity.

Let k be a non-normal cubic field unramified at $2, 3$ for simplicity. (In [10], [8], k can be cyclic cubic, and can be ramified at 2. k can also be ramified at 3 unless $k \otimes \mathbb{Q}_3$ is not a field.) For quadratic fields F, let Δ_F be the absolute discriminant of F. If L is a number field, $h_L, R_L, \zeta_L(s)$ are the class number, the regulator and the Dedekind zeta function of L respectively.

For any prime number p or $p = \infty$, we define the type of k as follows. If $k \otimes \mathbb{Q}_p \cong \mathbb{Q}_p^3$ then the type of k at p is (triv). If $k \otimes \mathbb{Q}_p \cong k_p \oplus \mathbb{Q}_p$, $[k_p : \mathbb{Q}_p] = 2$ and $k_p/\mathbb{Q}_p$ is ramified then the type of k at p is (2 rm). The type (2 ur) is defined similarly. If $k \otimes \mathbb{Q}_p = k_p$ is a field and $k_p/\mathbb{Q}_p$ is ramified then the type of k at p is (3 rm). The type (3 ur) is defined similarly.

For $p < \infty$ as above we define a constant E_p as follows.

Case	E_p
(triv)	$(1 - p^{-2})(1 - 3p^{-3} + 2p^{-4} + p^{-5} - p^{-6})$
(2 ur)	$(1 - p^{-4})(1 - p^{-2} - p^{-3} + p^{-4})$
(2 rm)	$(1 - p^{-1})(1 - p^{-4})$
(3 ur)	$1 - p^{-2} - p^{-4} + p^{-5} - p^{-7} + p^{-8}$
(3 rm)	$(1 - p^{-1})(1 - p^{-4})$

We define a constant E_∞ as follows.

Case	E_∞
(triv)	$\frac{1}{8} + \frac{1}{2\pi^2}$
(2 rm)	$\frac{1}{2\pi}$

The following theorem is the main theorem of [10], [8].

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Theorem

$$\lim_{X\to\infty} X^{-2} \sum_{[F:\mathbb{Q}]=2, |\Delta_F|<X} \frac{h_{k\cdot F} R_{k\cdot F}}{h_F R_F} = |\Delta_k|^{\frac{1}{2}} h_k R_k \zeta_k(2) \prod_p E_p.$$

The above result is based on a consideration of the non-split prehomogeneous vector space in Section3 [1]. The poles of its zeta function was determined in [9]. The local theory and the proof of the density theorem of another non-split case was carried out in [2], [3], [4]. The constants in the above table for the cases (triv), (2 rm), (2 ur) are due to the results in these papers. In the present case, the correspondence between rational orbits of the prehomogeneous vector space and field extensions is not one-to-one. It makes us to deal with the formula on Tamagawa numbers of tori proved by T. Ono. For this the reader should see [7], [5], [6].

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