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<td>Title</td>
<td>斜面侵食モデリング</td>
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<tr>
<td>Author(s)</td>
<td>沈大勇 宝馨 金争平</td>
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<tr>
<td>Citation</td>
<td>京都大学防災研究所年報. B = Disaster Prevention Research Institute annuals. B (2006), 49(B): 45-53</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2006-04-01</td>
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<tr>
<td>URL</td>
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<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
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京都大学
**Hillslope Erosion Modeling**

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**Synopsis**

This paper discusses mathematically dialectical logic (MDL)-based modelling idea. Selecting one watershed in China Loess Plateau region as research object, the authors calculated erosion from the watershed by using Remote Sensing Information Model of Water Erosion on Hillslopes (RSIMWEH), a MDL-based erosion model using measured data. The simulation result shows that RSIMWEH can serve the practice of soil and water conservation.

**Keywords:** mathematically dialectical logic, Loess Plateau, soil erosion

1. Introduction

1.1 Mathematical modelling of soil erosion by water on hillslopes

Existing models of water erosion can be mainly divided into three types—empirically based model (Musgrave, 1947; Wischmeier and Smith, 1978; Singh et al., 1982), partially conceptually and partially empirically based model (Johnson, 1943; Rendon-Herrero, 1978; Sharma and Dickinson, 1979), and partially physically and partially empirically based model (Foster, 1982; Prasad and Singh, 1982; Duan et al., 1998).

The advantages and disadvantages of each type are listed below:

[1] Empirically based model: is based on fieldwork, modelling by means of random statistics, and not necessary to simplify any factor or process. This type of model is comparatively reliable when used in areas with similar natural conditions. But many model dimensions are inconsistent, and many conceptions and causal relationships are not clear.

[2] Partially conceptually based and partially empirically based model: this approach attempts to model logically and testify with measured data, based on the analysis of causal relationships. Although absorbed the advantages of equations and statistics, it belongs to lumped model.

[3] Partially physically based and partially empirically based model: this approach characterizes the quantitative relations between the process of soil erosion and the factors that affect it in the form of equation. But it is often impossible to find the analytical solutions to the complicated equations. As a result, model simplification is necessary.

Ma (1997) put forward Remote Sensing Information Model of Water Erosion on Hillslopes (RSIMWEH). Via imaging independent variables of both remote sensing information and geoscientific information, the model is computed pixel-by-pixel and easy to display in 3-D space (Ma, 1997). Also in 1997, the Ministry of Water Resources of P. R. China listed RSIMWEH as a standard for classification of soil erosion. Ma said that the model has reached dialectical logic computation (Ma, 2001). However, how to theoretically prove its dialectics and how to make the model practical are still very challenging, because in the original model some parameter value can hardly be acquired. From 1998 till now, Shen has been exploring scientific approach to solve the model (Shen, 2002; Shen et al., 2002; Shen, 2003). This paper tries to prove its dialectics from different point of view and solve the model.

1.2 MDL

MDL coincides with the three main laws of dialectics as presented by Engels (1940) in The Dialectics of Nature. These laws are the transformation of quantity into quality, the unity of opposites, and the negation of the negation. Actually, dialectics are important in every domain where knowledge is not certain; that is, everywhere assumptions must be made (St-Vincent et al., 1995). In mathematics, nonlinear dynamical models e.g. those capable of generating catastrophic discontinuities (Trotman and Zeeman, 1976), chaotic dynamics (Lorenz, 1963), and a variety of other complex dynamics such as self-organization (Turing, 1952) can be interpreted as manifesting these laws (Sabelli, 1995; Dobronravova, 1997; Rosser, 2000). In computer science, Gordon (1994) proposed a new subfield, computational dialectics, which investigates computational models of the processes by...
which groups of natural or artificial agents construct judgement, agreement, or other forms of social choices (Brewka et al., 2003). Based on computational dialectics, Zeno, a mediating system was developed for supporting discussion, argumentation and decision-making in groups (Karacapilidis and Gordon, 1995).

1.3 Challenges

Traditional erosion models can be written in a general form shown in Equation (1):

$$ E = b_0 f_1^{b_1} \cdots f_n^{b_n} $$

(1)

where $E$ represents erosion, $f_0$, ..., $f_n$ represent dimensionless or dimensional affecting factors of erosion, and $b_0$, ..., $b_n$ are pending constants. Previous researches have found that $b_0$, ..., $b_n$ actually change over space and time, but there is not any ideal explanation.

1.4 Main contents of this paper

To address above challenges, this paper will mainly discuss MDL based erosion modelling idea and model implementation.

2. MDL based erosion modelling

Suppose that $q_1$, ..., $q_n$ are all affecting factors of erosion that have already been known, considering the challenge in traditional erosion models, we build a new model shown in Equation (2):

$$ E = x_0 \pi_1^{\pi_1} \cdots \pi_m^{\pi_m} $$

(2)

where $E$ represents erosion, $\pi_1$, ..., $\pi_m$ are independent dimensionless products of $q_1$, ..., $q_n$ and $x_0$, ..., $x_m$ are pending variables. Based on the Buckingham Pi Theorem (Randall 2003), if $k$ is the minimum number of primary quantities necessary to express the dimensions of $q_1$, ..., $q_n$, then

$$ m = n - k $$

(3)

where $k > 0$ and $m < n$. Note that in Equation (1) $b_0$, ..., $b_n$ are constants, but in Equation (2) $x_0$, ..., $x_m$ are variables that change over space and time. Setting $x_0$, ..., $x_m$ as variables can well explain the pending problem in traditional erosion models. Actually, Equation (2) integrates MDL:

[1] Fig. 1 shows the MDL based modelling to a unity of opposites. In Equation (2), all determinate factors are represented by $\pi_1$, ..., $\pi_m$ and both determinate factors and unknown factors are represented by $x_0$. Therefore, the model theoretically represents all affecting factors of $E$.

[2] Fig. 2 shows the law of the negation of the negation. In Equation (2), when a new determinate factor $\pi_{m+1}$ is discovered, $x_0$ will be factorised as $x_0 \pi_{m+1}$ and then Equation (2) will be written as:

$$ E = x_0 \pi_1^{\pi_1} \cdots \pi_m^{\pi_m} \pi_{m+1}^{x_{m+1}} $$

(4)

where $x_{0m}$ is similar to $x_0$, containing both indeterminate factors and unknown factors. So Equation (4) is inherited from Equation (2) but Equation (4) is better than Equation (2) due to containing more determinate factors.

Fig. 1 MDL based modeling to a unity of opposites (Shen and Takara, in press)

Fig. 2 The law of the negation of the negation

(Sawamura et al., 2000; Sawamura, 2002)
Fig. 3 The law of transformation of quantity into quality: leap system evolution and envelope curves (Petrov, 2002)

[3] Fig. 3 shows that quantitative changes in a system take place continuously accordingly the S-curve of evolution. When a certain limit of quantitative evolution is reached, a system experiences qualitative changes. During this process, quantitative changes take place continuously whereas qualitative changes take place in discrete steps. The duration and characteristics of the discrete step can differ: long and short, impetuous and relatively calm and so on. In Equation (2), values of \( x_1, \ldots, x_m \) in each pixel in the research area are relatively stable in the short term. But \( x_1, \ldots, x_m \) will become unstable if one or more factors among \( \pi_1, \ldots, \pi_m \) change greatly or a new determinate factor is discovered and factorised out of \( x_0 \). Then it is necessary to re-compute \( x_1, \ldots, x_m \) for better reflecting their changes over space and time.

Take remote sensing information model of water erosion on hillslopes (RSIMWEH) as an example, which was put forward by Ma (1997,2001) and adjusted by Shen (2002), it is written as:

\[
E_{sl} = x_0 \left( \frac{I - I_0}{I} \right)^{h_d \phi} S_{ve} \exp(-x_i S_{ve})
\]

(5)

where \( E_{sl} \) (mm) is the erosion depth on hillslopes; \( I \) is the intensity of rainfall (mm min\(^{-1}\)); \( I_0 \) is the critical value of the intensity of rainfall causing erosion; \( h_d \) (mm) is the depth of overland flow; \( \phi \) (\%) is the percentage of the main soil particle size; \( \alpha \) (radian) is the gradient; \( S_{ve} \) (\%) is vegetation coverage; \( x_0 \) is geoscientific coefficient and \( x_1, x_2, x_3, x_4 \) are geoscientific indices. Note that RSIMWEH integrates MDL based modelling idea because \( x_0, x_1, x_2, x_3, x_4 \) in the model are all variables.

Main characteristics of MDL based erosion modeling are listed as follows:

1. Coefficients and exponents are variables.
2. Theoretically represents all affecting factors.
3. Coefficients and exponents can represent complicated spatio-temporal changes of a geoscientific phenomenon or process.

To solve Equation (2), simultaneous nonlinear equations are created and then solved based on sample data of \( E \) and \( \pi_1, \ldots, \pi_m \) to get values of \( x_0, \ldots, x_m \) at sample points. After that, the model is calibrated and verified. Finally \( E \) is computed pixel by pixel. Section 3 will describe the algorithms in detail.

3. Case study

To test the model performance and to support decision making for conserving water and soil loss, we select Wufendigou small watershed as a research object. Wufendigou watershed, which has an area of 3.85 km\(^2\), is on the east bank of the middle reaches of Changchuan, one branch of Huangfuchuan in China Loess Plateau region. The watershed belongs to Wufendigou experimental area as shown in Fig. 4. Because of severe water and soil loss, research on this area will absolutely improve understanding the dynamic change of water erosion in China Loess Plateau region (Jin et al., 1992, 1993). In this case study, considering different levels of erosion affecting factors \( I_{30}, h_d, \phi, \alpha \) and \( S_{ve} \), we obtained data of 26 rainfall events occurring from June 1987 to August 1994, which was measured from 24 small experimental plots on the hillslopes as shown in Fig. 4 and Table 1. The area of each plot is equal to or less than 100 m\(^2\). The edges of each plot were surrounded by concrete plates. At the down slope of the plot there was a collecting tank. After a rainfall event, water level in the collecting tank was measured and water and sediment sample was taken from the tank. Then runoff volume and sediment yield were calculated by filtering, drying and weighting the sample. The main data sources include a 1:5000 topographic map, a 1:10000 colour infrared aerial photograph, a soil type map and measured rainfall data. For calculation and 3D visualization, colour infrared aerial photograph has been registered according to DEM. One pixel in this research is 5m \( \times \) 5m. As it is very difficult to measure \( I_0 \), we use maximum 30-minute rainfall intensity \( I_{30} \) (mm min\(^{-1}\)) during one rainfall event as an independent model factor, which has been applied by Chen et al. (1988) and Wang and Jiao (1996) and proved quite effective, to replace \( \frac{I - I_0}{I} \). And then we get:

\[
E_{sl} = x_0 I_{30}^h h_d \phi S_{ve} \exp(-x_i S_{ve})
\]

(6)

In Equation (6), determinate factors coincide well with other research achievements e.g.:

1. \( I_{30} \) has been found having good linear correlation with hillslope erosion during one rainfall event (Chen et al., 1988; Wang and Jiao, 1996; Li et al., 1999).

2. Hjelmfelt et al. (1975) and Hjelmfelt (1976) derived \( E \approx h_d^b \), where \( b \) is an exponent.

3. Liu and Singh (2004) found that the effect of slope angle on the flow velocity and shear stress...
of overland flow can be both positive and negative (Fig. 5). The flow velocity and shear stress initially increase and then begin to decrease when the slope angle reaches a critical value within the range of 40 degree-50 degree. This range was also found by Liu et al. (2001) in their theoretical analysis. In Equation (6) if \( \sigma = 45 \) degree, \( E_{sl} \) will reach maximum value if other factors remain unchanging.

[4] Rainfall energy at the ground surface decreases exponentially with increasing vegetation coverage (Elwell, 1981; Laflen and Colvin, 1981; Morgan et al., 1984; Wang et al., 1992) i.e. \( E \propto \exp(-cS_{vc}) \), where \( c \) is an exponent and \( c > 0 \). Note that human impact on erosion takes effects via natural factors. For example, irrigation changes \( h_{sl} \); step-like paddy field changes \( \alpha \); planting and grass growing change \( S_{vc} \).

The methodology of a MDL integrated erosion model is shown in Fig. 6.
Table 1 Characteristics of experimental plots

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<td>7</td>
<td>Loess soil</td>
</tr>
<tr>
<td>②</td>
<td>6</td>
<td>Soft rock soil</td>
</tr>
<tr>
<td>③</td>
<td>3</td>
<td>Loess soil</td>
</tr>
<tr>
<td>④</td>
<td>8</td>
<td>Sandy soil</td>
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Fig. 4 Slope map and the location of experimental plots ① to ④

Fig. 5 The effect of slope angle on the flow velocity and shear stress of overland flow (Liu and Singh, 2004)

![Graph showing relationship between slope gradient and flow velocity/shear stress](image)

Fig. 6 Methodology of RSIMWEH

1. Creation of factor data. There are mainly three types of soil in Wufendigou watershed: sandy soil, loess soil and soft rock soil. Measured hillslope erosion $E_m$ (mm) is calculated using Equation (7):

$$E_m = \frac{E_{mo}}{10000 \rho}$$

where $\rho$ is soil density (g · cm$^{-3}$) and $E_{mo}$ is erosion module (kg · hm$^{-2}$). Supposing that erosion affecting factors are homogeneous within each plot ($\leq$100m$^2$), $h_{sl}$ (mm) is calculated using Equation (8):

$$h_{sl} = \frac{V}{A} \times 1000$$

where $V$ is the volume of overland flow from the plot during one rainfall event (m$^3$), and $A$ is the area of the plot (m$^2$). Slope angle was calculated from relief map, and vegetation coverage was calculated based on field investigation and aerial photograph.

2. Model calibration. We got 140 measured data of 26 rainfall events and divided the data into two groups (112 for model calibration and 28 for model verification) in view of different levels in $I_{sl}$, $h_{sl}$, $\phi$, $\alpha$ and $S_{ve}$. Note that in this case study we neglect indeterminate factors and unknown factors and set $x_0 = 1.0$. The average relative deviation $ARD$ during model calibration is 28%, i.e. the accuracy is 72% as shown in Figs. 7(a-c). Fig. 8 shows the zoomin effect of Fig. 7a.

3. Model verification. The average relative deviation $ARD$ during model verification is 25%, i.e. the accuracy reaches 75% as shown in Fig. 9.

4. Conclusion and discussion

RSIMWEH was created based on physical experiments and Buckingham’s $\pi$ theorem, which makes the model obey physical principles. In terms of simulation of measured data, maximum $ARD$ is 71.5%, minimum $ARD$ is 0.04%. Accuracy for the whole set of data is 73.01%. Especially for non-bare land plots, the accuracy reaches 77.12%. Compared to USLE, RSIMWEH obeys physical principles and model parameters are easy to measure. Moreover, RSIMWEH can represent complicated spatio-temporal changes of soil erosion as the model is MDL-based. In this case study, we found that the ac-
Accuracy of RSIMWEH for bare land plots is 71.25% and bare land data occupies 86% among all sample data with $ARD > 30\%$. Meanwhile, as slope of experimental plots ranges from $3^\circ$-$70^\circ$, it is meaningful to pay more attention to hyper-concentrated flow, which causes debris (Vandine et al., 2005) for steep slope in the erosion model. Erosion simulation of bare land and steep slope will be deeply studied in next work.

Generally, RSIMWEH is a MDL based hillslope erosion model, where coefficient and index are all defined as variables, which will change over large space or long duration. But inside a small space and short duration, they can be set as constants. MDL based modelling idea can well address the challenge in traditional models. Moreover, RSIMWEH is pixel based. Supported by advanced technology such as remote sensing, which can provide detailed information inside a watershed, MDL and pixel based erosion modelling can provide detailed information for the practices of water and soil conservation and land use planning.

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**Fig. 7a RSIMWEH modeling (Part 1)**

**Fig. 7b RSIMWEH modeling (Part 2)**
Acknowledgements

This work is supported by JSPS (Japan Society for the Promotion of Science) Fellowship and the MEXT Grant-in-Aid for Scientific Research (No. 16004360). We are also grateful to Ma, A.N., Lin, H., Chen X.W., Liu, Y.L., Mao, S.J. and Wu, L. for their help and advice.

References


斜面侵食モデリング

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要旨

本論文は、数理弁証論理（MDL）ベースのモデリングを行う。中国の黄土高原地域の流域を対象に、測定データを使用して斜面侵食リモート・センシング情報モデル（RSIMWEH）－MDLベースの侵食モデルにより土壌侵食量を計算する。シミュレーション結果はRSIMWEHが土壌と水保険の実務に有用であることが示される。

キーワード：数理弁証論理、黄土高原、土壌侵食