Domination for quantum Markov semigroups

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Abstract: A well-known theorem of W. Arveson states that a completely positive (CP) map **dominated** (difference is CP) by a given CP map is described through a positive contraction in the commutant of the homomorphism of Stinespring representation. This contraction plays the role of Radon-Nikodym derivative of measure theory. We look at implications of this to one parameter semigroups of unital completely positive maps, known as quantum Markov semigroups.

We focus our attention to quantum Markov semigroups of $\mathcal{B}(\mathcal{H})$, the algebra of all bounded operators on a Hilbert space. These semigroups can be dilated to semigroups of unital endomorphisms (E_0-semigroups) of $\mathcal{B}(\mathcal{K})$ for some other Hilbert space $\mathcal{K}$. Here we see that dominated CP semigroups dilate to dominated semigroups of the dilation. Moreover at the level of E_0-semigroups the dominated CP semigroups are described through positive contractive local cocycles, which are really families of contractions of 'Radon-Nikodym derivatives' mentioned above. We analyze the structure for quantum semigroups with bounded generators.