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Abstract. We extend a market model of Kyle (1984) to a multi-period market model in which market makers are imperfectly competitive. When there are only a finite number of market makers who try to maximize their expected profits, Kyle's λ, price sensitivity to the order flow, is higher than that of Kyle (1985), who assumes perfect competition among risk-neutral market makers. We show that in a multi-period setting, the price movement has a negative serial correlation as Roll (1984) and Glosten and Milgrom (1985).

Keywords: Kyle's model, imperfectly competitive market makers, negative serial correlation.

1 Introduction

The problem of market making and insider trading has been studied extensively in the market microstructure literature. In his seminal paper, Kyle (1985) analyzed three types of market models; a one-shot model, a multi-period model, and a continuous-time model. He examined the market characteristics such as market liquidity and informational efficiency when a monopolistic informed trader behaves strategically. He showed that there exists a linear equilibrium in which the informed trader with long-lived private information trades in a gradual manner, and that market liquidity and informational efficiency are dependent on the variances of the liquidation value and of noise trades.

In Kyle (1985), the market maker is assumed to be risk-neutral and perfectly competitive, which implies that the conditional expected profit of the market maker is zero. Under this assumption, the price set by the market maker is given by

$$p = \mathrm{E}[v|\mathcal{F}],$$

where $\mathcal{F}$ is the information available to the market maker. Most of the subsequent literature building on Kyle's model follow the assumption of a risk-neutral and competitive market maker (e.g., see Subrahmanyam (1992), Foster and Viswanathan (1996), and Back et al. (2000)). This assumption leads to the result that the price in a multi-period setting satisfies a martingale property with respect to the information available to the market maker. In this case, the price change has no serial correlation.

In this paper, we present a Kyle-type market model where there are a finite number of imperfectly competitive market makers who are all risk-neutral. If a market maker sets the demand-price schedule more favorable for other traders, she gains less profit per unit trade, but can attract more trades. Market makers try to maximize their expected profit, while taking into account the effect of their own demand-price schedule on other traders' strategies.

An important issue arisen is how the characteristics of the market-making system affect the market equilibrium. On the so called "Black Monday" in 1987, most market makers in NASDAQ stopped dealing and the liquidity became scarce. On the other hand, specialists in New York Stock Exchange were compulsory for the market-making and so other traders can sell stocks to them. As a result, some of the specialists ran out of money.[1] This difference was said to be caused in part by the system of market-making. Another example to show the importance of market-making is the scandal of possible

[1] See, for example, Barro et al. (1989).

abuses by some specialists that match buyers and sellers on the floor of the New York Stock Exchange.

The investigation of competition among market makers is not new. Kyle (1984) analyzed a one-shot model with a finite number of market makers who are risk-neutral and aim to maximize their expected profits. He showed in his paper that the price sensitivity is decreasing with the number of market makers. We extend Kyle (1984) to a multi-period setting in which multiple market makers set their demand-price schedule, and clear orders by an informed trader and noise traders. Our major interest is how the imperfect competition among market makers affects the price behavior during the trading periods.

One of the major contributions in this paper is to show that in a multi-period setting, the time-series of the price change is negatively correlated, and the price movement has permanent and temporary impacts. This result echoes some of the existing works. For example, Roll (1984) showed in his simple model that the price has a negative serial correlation when the fundamental value of a risky asset is between the ask and bid prices, and the probability that the buy order arrives is equal to the probability of sell orders. Glosten and Milgrom (1985) obtained a similar result that when the market making needs some fixed cost for the market maker Holthausen et al. (1987) mentioned that there are three potential explanations for price changes: (i) liquidity costs, (ii) inelastic demand curves, and (iii) information effects. Thanks to the imperfect competition, each market maker can gain positive expected profit. When we regard the profit of market makers as some costs associated with market-making, the negative autocorrelation in our setting is consistent with previous theoretical papers.

There are some empirical papers which report some resilient effects in price, especially when price drops. Holthausen et al. (1987) and Holthausen et al. (1990) discovered that for seller-initiated transactions, there is evidence of negative correlations and temporary effects. Sentana and Wadhwani (1992) showed that when volatility is rather high, price returns exhibit negative autocorrelation. These empirical evidences can be partly explained by the imperfect competition among market makers.

We also extend the model to the cases of multiple informed traders in Holden and Subrahmanyam (1992), and of risk-averse informed traders in Holden and Subrahmanyam (1994). In the case of a monopolistic informed trader, the price change is negatively correlated in these two settings.

This paper is organized as follows. In Section 2, we review Kyle (1984), a one-shot model with imperfectly competitive market makers in order to make our discussion clear. We extend in Section 3 the one-shot model to a multi-period one, and show that the price change has permanent and temporary effects. We also demonstrate that our model converges to the result of the multi-period model in Kyle (1985) as the number of market makers goes to infinity. In Section 4, we discuss some extensions of our model. Section 5 concludes this paper.

2 A One-Shot Model

In this section, we review the model of Kyle (1984). The model consists of only two dates, time 0 and time 1. At time 0, market participants trade a risky asset based on their information, while at time 1, the payoff of the risky asset is realized and consumption occurs.

In the underlying market, one risk-free asset and one risky asset are traded by three types of market participants: a monopolistic informed trader who is risk-neutral and possesses private information about the payoff of the risky asset, noise traders who participate in the market for exogenous reasons, and risk-neutral market makers who set the price schedule based on the market orders by other traders. For simplicity, the risk-free interest rate is assumed to be zero. We denote by v the payoff of the risky asset. We assume that v follows a normal distribution with mean μ_0 and variance Σ_0. The ex ante distribution of the payoff is publicly known.

The monopolistic informed trader observes the liquidation value v before the market opens. However, the informed trader cannot observe the realized market price or the demand of other traders, and only submits a market order x based on v.

The assumption of noise traders is the same as other previous articles. We denote the cumulative order amount of noise traders by u, and assume that u follows a normal distribution with mean 0 and variance σ_u^2, and is independent of all other random variables.

There are M market makers in the market who are indexed by $m = 1, \ldots, M$. Market makers are all risk-neutral, i.e., each market maker maximizes her conditional expected profits. Contrary to the informed trader, market makers can submit a limit order which depends on the market price p. This means that market maker m provides a demand-price schedule $d_m = d_m(p)$. As we see below, observing the price p is equivalent to observing $x + u$, the total order amount by the informed trader and noise traders. We define $y := x + u$.

A *market equilibrium* is characterized by a trading strategy of the informed trader and demand-price schedules of market makers.

Definition 1 *An equilibrium is defined as a set* $(x, \{d_m\}_{m=1}^M)$ *satisfying the following three conditions:*

1. *Profit Maximization of the Informed Trader: The trading strategy* x *is determined so that the informed trader maximizes his conditional expected profit*

$$\mathrm{E}[(v-p)x|v].$$

2. *Profit Maximization of Market Makers: The demand-price schedule* d_m *is determined so that the market maker* m *maximizes her conditional expected profit*

$$\mathrm{E}[(v-p)d_m|p], \tag{2.1}$$

for $m = 1, \ldots, M$.

3. *Market Clearing: The equilibrium price satisfies the following equation*

$$y + \sum_{m=1}^{M} d_m = 0. \tag{2.2}$$

In this setting, Kyle (1984) obtained the following theorem.

Theorem 1 *When* $M \geq 3$, *there exists a linear equilibrium such that*

$$x = \sqrt{\frac{M-2}{M}}\sqrt{\frac{\sigma_u^2}{\Sigma_0}}(v-p) \tag{2.3}$$

and

$$d_m = \frac{2}{M}\sqrt{\frac{M-2}{M}}\sqrt{\frac{\Sigma_0}{\sigma_u^2}}(\mu_0 - p), \tag{2.4}$$

for $m = 1, \ldots, M$.

Proof See Kyle (1984).

Theorem 1 provides some properties of the market with imperfectly competitive market makers. For the informed trader, the market price is of the form

$$p = \mu_0 + \frac{1}{2}\sqrt{\frac{M}{M-2}}\sqrt{\frac{\Sigma_0}{\sigma_u^2}}y.$$

Hence, Kyle's λ, the price sensitivity to the order, is equal to $\frac{1}{2}\sqrt{\frac{M}{M-2}}\sqrt{\frac{\Sigma_0}{\sigma_u^2}}$. This result indicates Kyle's λ is monotonically decreasing with respect to the number of market makers. As M diverges to infinity, Kyle's λ converges to that $\frac{1}{2}\sqrt{\frac{\Sigma_0}{\sigma_u^2}}$, which is obtained in Kyle (1985), and the price satisfies

$$p = \mathrm{E}[v|y].$$

The aggregated expected profits of market makers are given by

$$M \times \mathrm{E}[(v-p)d_m|p] = \frac{2}{M-1}\sqrt{\frac{M-2}{M}\sigma_u^2\Sigma_0(\mu_0-p)^2}.$$

When $M \to \infty$, the total expected profit converges to zero.

The above analysis confirms that the market model actually converges to the one-shot model of Kyle (1985) when the number of market makers goes to infinity.

3 A Multi-Period Model

In this section, we extend the model of one-shot trading to a multi-period model. Trading of a risky asset takes place in N sequential auctions in a time interval which begins at $t = 0$ and ends at time $t = 1$. Let t_n be the time at which the nth auction takes place. As in Kyle (1985), we assume

$$0 = t_0 < t_1 < \cdots < t_N = 1,$$

and define $\Delta t_n := t_n - t_{n-1}$.

As in the previous section, the value of the risky asset is normally distributed with mean μ_0 and Σ_0. The ex ante distribution of v is public information, but the realization of v is only observed by the informed trader before the market opens.

Denote the total order quantity of noise traders by u_n, and define $\Delta u_n := u_n - u_{n-1}$. We assume that Δu_n is serially uncorrelated and is normally distributed with zero mean and variance of $\sigma_u^2\Delta t_n$. We also assume that the process $\{u_n\}$ is independent of all other random variables.

Let x_n be the position of the risky asset held by the informed trader at time t_n, and let Δx_n denote the quantity traded by the informed trader at the nth auction, i.e., $\Delta x_n := x_n - x_{n-1}$. At each auction, the informed trader chooses the optimal Δx_n based on the realization of v and the history of the market price. As in the previous section, the informed trader cannot observe the realization of the market price before submitting his order, i.e., he only submits a market order at each auction.

Denote by d_{mn} the position of the risky asset by the market maker m at time t_n, and Δd_{mn} the order quantity at time t_n, i.e., $\Delta d_{mn} := d_{mn} - d_{mn-1}$. Each market maker chooses her order quantity based on $\{y_1, \ldots, y_n\}$, the history of the total order flow from the informed trader and noise traders.

Let $\mathcal{F}_n$ be the sigma field generated by $\{y_1, \ldots, y_n\}$. As in the previous section, we easily see from the discussion below that observing $\{y_1, \ldots, y_n\}$ is equivalent to observing $\{p_1, \ldots, p_n\}$. The information $\mathcal{F}_n$ is public information just before each market maker submit her limit order at the nth auction.

Definition 2 *An equilibrium is defined as a set* $(\{x_n\}_{n=1}^N, \{\{d_{mn}\}_{n=1}^N\}_{m=1}^M)$ *such that the following conditions hold:*

1. *Profit Maximization of the Informed Trader: For all* $n = 1, \ldots, N$*, the trading strategy of the informed trader* $\{x_k\}_{k=n}^N$ *maximizes the expected profit*

$$\mathrm{E}[\pi_n | v, \mathcal{F}_{n-1}], \tag{3.1}$$

where $\pi_n := \sum_{k=n}^{N} (v - p_k)\Delta x_k$.

2. *Profit Maximization of Market Makers: For all* $m = 1, \ldots, M$ *and* $n = 1, \ldots, N$*, the trading strategy of market maker* m $\{d_{mk}\}_{k=n}^N$ *maximizes the expected profit*

$$\mathrm{E}\left[\sum_{k=n}^{N} (v - p_k)\Delta d_{mk} \middle| \mathcal{F}_n \right].$$

3. *Market Clearing: For* $n = 1, \ldots, N$*, the equilibrium price satisfies the following equation*

$$\Delta y_n + \sum_{m=1}^{M} \Delta d_{mn} = 0. \tag{3.2}$$

Figure 1 provides the time line of this market model. First, the informed trader submits a market order Δx_n based on v and $\{p_1, \ldots, p_{n-1}\}$. Second, noise traders submit Δu_n for exogenous reasons. Third, market maker m submits a limit order $\Delta d_{mn}(p_n)$ based on $\{y_1, \ldots, y_n\}$. Finally, the market clears such that (3.2) is satisfied.

Figure 1 is inserted here.

We now state a theorem which provides the difference equation system characterizing the equilibrium.

Theorem 2 *Define* $\mu_n := \mathrm{E}[v|\mathcal{F}_n]$*, and* $\kappa := \frac{M-2}{M-1}$*. For* $M \geq 3$*, there exists a linear equilibrium, in which there are constants* α_n*,* β_n*,* λ_n *and* Σ_n *such that for*

$$\begin{aligned}
\Delta x_n &= \beta_n (v - \mu_{n-1}) \Delta t_n \\
\Delta d_{mn} &= \frac{1}{M\lambda_n}(\mu_{n-1} - p_n), \quad m = 1, \ldots, M \\
\Sigma_n &= \mathrm{Var}[v|\mathcal{F}_n]
\end{aligned}$$

and

$$\mathrm{E}[\pi_n | v, \mathcal{F}_{n-1}] = \alpha_{n-1}(v - \mu_{n-1})^2 + \delta_{n-1}.$$

Given Σ_0*, the constants* α_n*,* β_n*,* λ_n *and* Σ_n *are the solution to the difference equation system*

$$\alpha_{n-1} = \frac{1 + 4(1-\kappa)\alpha_n \lambda_n}{4\lambda_n(1 - \kappa^2 \alpha_n \lambda_n)}, \tag{3.3}$$

$$\delta_{n-1} = \delta_n + \kappa^2 \alpha_n \lambda_n^2 \sigma_u^2 \Delta t_n, \tag{3.4}$$

$$\beta_n \Delta t_n = \frac{1 - 2\kappa \alpha_n \lambda_n}{2\lambda_n(1 - \kappa^2 \alpha_n \lambda_n)}, \tag{3.5}$$

$$\lambda_n = \frac{1}{\kappa} \frac{\beta_n \Sigma_n}{\sigma_u^2} \tag{3.6}$$

and

$$\Sigma_n = \left(1 - \kappa \beta_n \lambda_n \sigma_u^2 \Delta t_n\right) \Sigma_{n-1}, \tag{3.7}$$

subject to $\alpha_N = \delta_N = 0$ *and the second order condition*

$$\lambda_n(1 - \kappa^2 \alpha_n \lambda_n) > 0.$$

The conditional expectation μ_n *satisfies*

$$\mu_n = \mu_{n-1} + \kappa \lambda_n \Delta y_n.$$

Proof The proof has three steps. First, we derive optimality conditions of the informed trader given his value function and market makers' strategy. At the second step, the optimality condition for each market maker is obtained under the linear strategy of other players. Finally, we confirm that the conditions obtained in the first and second steps are consistent and lead to the statement in the proof.

Step 1: Optimality conditions for the informed trader Suppose that the informed trader conjectures that each market maker trades at the nth auction according to the following strategy:

$$\Delta d_{mn} = \xi_n(\mu_{n-1} - p_n), \quad m = 1, \dots, M, \tag{3.8}$$

where μ_n satisfies

$$\mu_n = \mu_{n-1} + \theta_n \Delta y_n \tag{3.9}$$

for some constant θ_n.[2] Now we proceed by backward induction. Assume for some constants α_n and δ_n, the value function of the informed trader is given by

$$\max \mathrm{E}[\pi_n | v, \mathcal{F}_{n-1}] = \alpha_{n-1}(v - \mu_{n-1})^2 + \delta_{n-1}. \tag{3.10}$$

Note that from (3.2) and (3.8), the price can be expressed as

$$p_n = \mu_{n-1} + \frac{1}{M\xi_n}\Delta y_n.$$

Then, we have

$$\begin{aligned} \max \mathrm{E}[\pi_n | v, \mathcal{F}_{n-1}] &= \max_{\Delta x} \mathrm{E}[(v - p_n)\Delta x + \alpha_n (v - \mu_n)^2 + \delta_n | v \mathcal{F}_{n-1}] \\ &= \max_{\Delta x} \Big\{ (v - \mu_{n-1} - \lambda_n \Delta x)\Delta x + \alpha_n (v - \mu_{n-1} - \theta_n \Delta x)^2 \\ &\quad + \alpha_n \theta_n^2 \sigma_u^2 \Delta t_n + \delta_n \Big\}. \end{aligned} \tag{3.11}$$

where $\lambda_n := \frac{1}{M\xi_n}$. The first order condition of (3.11) is given by

$$\Delta x_n = \frac{1 - 2\alpha_n \theta_n}{2(\lambda_n - \alpha_n \theta_n^2)}(v - \mu_{n-1}), \tag{3.12}$$

and the second order condition is

$$\lambda_n - \alpha \theta_n^2 > 0.$$

Plugging (3.12) into (3.11), we obtain

$$\alpha_{n-1} = \frac{1 + 4\alpha_n(\lambda_n - \theta_n)}{4(\lambda_n - \alpha_n \theta_n^2)}, \tag{3.13}$$

$$\lambda_n = \frac{1}{M\xi_n}, \tag{3.14}$$

$$\beta_n \Delta t_n = \frac{1 - 2\alpha_n \theta_n}{2(\lambda_n - \alpha \theta_n^2)} \tag{3.15}$$

and

$$\delta_{n-1} = \delta_n + \alpha_n \theta_n^2 \sigma_u^2 \Delta t_n. \tag{3.16}$$

[2]More formally, we should initially set $\mu_n = \mu_{n-1} + \theta_n \Delta y_n + h$ to derive a linear equilibrium, and then show that $h = 0$ as in Kyle (1985) or Holden and Subrahmanyam (1992). However, we skip the procedure since it seems tedious and not so important.

Step 2: Optimality conditions for market makers Suppose that the market maker m conjectures that for $n = 1, \dots, N$, the informed trader trades according to the following strategy

$$\Delta x_n = \beta_n(v - \bar{v}_{n-1})\Delta t_n, \tag{3.17}$$

and other market makers adopt the following strategy

$$\Delta d_{kn} = \xi_n(\bar{v}_{n-1} - p_n), \quad k \neq m, \tag{3.18}$$

where $\bar{v}_n$ is the expected value of v conditional on $\{y_1, \dots, y_n\}$, i.e. $\bar{v}_n := \mathrm{E}[v|\mathcal{F}_n]$. Let Σ_n be the conditional variance of v given $\mathcal{F}_n$. Then, from the projection theorem of normal distributions, $\bar{v}_n$ and Σ_n are given by

$$\bar{v}_v = \bar{v}_{n-1} + \frac{\beta_n \Sigma_{n-1}}{\beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_u^2} \Delta y_n$$

and

$$\Sigma_n = \frac{\sigma_u^2 \Sigma_{n-1}}{\beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_u^2},$$

respectively. By simple algebra, we have

$$\frac{\beta_n \Sigma_{n-1}}{\beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_u^2} = \frac{\beta_n \Sigma_n}{\sigma_u^2}$$

and

$$\Sigma_n = \left(1 - \frac{\beta_n^2 \Sigma_n}{\sigma_u^2} \Delta t_n\right) \Sigma_{n-1}. \tag{3.19}$$

Next, we prove the following lemma.

Lemma 1 *Assume* (3.17) *and* (3.18). *Then, the optimal strategy of each market maker is myopic in the sense that they only maximize the expected profit of each trading period.*

Proof of Lemma 1 From (3.2), the price at t_{n+1} is written as

$$p_{n+1} = \bar{v}_n + \frac{1}{(M-1)\xi_{n+1}}[\beta_n(v - \bar{v}_n)\Delta t_{n+1} + \Delta u_{n+1} + \Delta d_{mn+1}]. \tag{3.20}$$

Since $\bar{v}_n = \mathrm{E}[v|\Delta y_1, \dots, \Delta y_n]$ is $\mathcal{F}_n$-measurable and independent of Δd_{mn}, the strategies of the informed trader and other market makers at t_{n+1} are independent of Δd_{mn} from (3.17) and (3.18), respectively. Then, from (3.20), we see that p_{n+1} is independent of Δd_{mn}. We can show in a recursive way that p_{n+k} is independent of d_{mn} for $k = 1, \ldots, N-k$. The lemma follows readily. $\square$

From the market clearing condition (3.2), the market price for market maker m is written as

$$p_n = \bar{v}_{n-1} + \frac{1}{(M-1)\xi_n}(\Delta d_{mn} + \Delta y_n). \tag{3.21}$$

Then, the expected profit of the market maker m at time t_n is expressed as

$$\mathrm{E}[(v - p_n)\Delta d_{mn}|\mathcal{F}_n] = \left[\bar{v}_n - \left(\bar{v}_{n-1} + \frac{1}{(M-1)\xi_n}(\Delta d_{mn} + \Delta y_n\right)\right]\Delta d_{mn}. \tag{3.22}$$

The first order condition of (3.22) is

$$\Delta d_{mn} = (M-1)\xi_n\left(\bar{v}_n - p_n\right),$$

and the second order condition is $(M-1)\xi_n > 0$. Substituting (3.21) and $\bar{v}_n = \bar{v}_{n-1} + \frac{\beta_n \Sigma_n}{\sigma_n^2}\Delta y_n$ leads to

$$\Delta d_{mn} = (M-1)\xi_n\left(\bar{v}_{n-1} + \frac{\beta_n \Sigma_n}{\sigma_n^2}\Delta y_n - p_n\right). \tag{3.23}$$

Furthermore, the market clearing condition (3.2) yields

$$\Delta y_n = -\Delta d_m - (M-1)\xi_n(\mu_{n-1} - p_n). \tag{3.24}$$

Therefore, by substituting (3.24) into (3.23) and solving with respect to d_{mn}, we have

$$\Delta d_{mn} = \frac{1 - \frac{\beta_n \Sigma_n}{\sigma_u^2}(M-1)\xi_n}{\frac{\beta_n \Sigma_n}{\sigma_u^2} + \frac{1}{(M-1)\xi_n}}(\mu_{n-1} - p_n),$$

Step 3: Solution of the equilibrium Since the strategy of market makers is symmetric, we finally obtain

$$\xi_n = \frac{\kappa}{M}\frac{\sigma_u^2}{\beta_n \Sigma_n}$$

and

$$\lambda_n = \frac{1}{\kappa}\frac{\beta_n \Sigma_n}{\sigma_u^2}.$$

Now, suppose that $\theta_n = \kappa\lambda_n$ and $\mu_n = \bar{v}_n$. We can easily confirm that the value function of the informed trader is actually given by (3.10). Substituting $\theta_n = \kappa\lambda_n$ into

(3.13) - (3.16) and (3.19), we obtain the difference equations system. Now the proof is completed. □

We easily confirm that when M goes to infinity, the simultaneous differential equations converge to that of Kyle (1985). Hence, as in a one-shot model, our model in a multi-period setting is actually an extension of Kyle (1985).

From Theorem 1, κ can be thought of as a parameter representing how competitive market makers are. As κ approaches 1, the competition among market makers becomes fierce, and the market is close to Kyle (1985).

We provide the numerical example when $N = 5$ and $\Delta t_n = 0.2$ for $n = 1, \ldots, 5$. Other parameters are given by $\mu_0 = 0$, $\Sigma^{(0)} = 10$, and $\sigma_u = 1$, respectively.

Figure 2 depicts λ, and Figure 3 graphs Σ in the case of $M = 3$, 5, and ∞.

Figure 2 is inserted here.

Figure 3 is inserted here.

The figures illustrate how the competition affects the market liquidity and the informational efficiency. It is clear that both λ and Σ are decreasing with the number of market makers. When $M = 3$, λ is about 1.8 times as much as that in the case $M = \infty$. On the other hand, λ in the case $M = 10$ is around 1.1 times compared to λ in the case of $M = \infty$. Since the intensity of the competition among market makers is represented by $\kappa = \frac{M-2}{M}$, the market equilibrium is not so different from that in Kyle (1985) when M is large enough.

When the number of market makers is small, market makers set their price schedules to be more sensitive to the order amount, and earn more expected profits. In this case, the monopolistic informed trader trades less intensively. Therefore, the information owned by the informed trader is not incorporated into the price as in the case of the perfect competition, and the market becomes less efficient informationally. This result can be seen from Figure 3.

Theorem 2 leads to the following corollary.

Corollary 1 *The price has a negative serial correlation.*

Proof From Theorem 2,

$$\begin{aligned}
p_{n+1} =& \mu_n + \lambda_{n+1}\Delta y_{n+1} = \mu_{n-1} + \theta_n \Delta y_n + \lambda_{n+1}\Delta y_{n+1} \\
=& p_n - \frac{1}{M-2}\lambda_n \Delta y_n + \lambda_{n+1}\Delta y_{n+1} \\
=& p_n - \frac{1}{M-2}\lambda_n[\beta_n(v-\mu_{n-1})\Delta t_n + \Delta u_n] + \lambda_{n+1}[\beta_{n+1}(v-\mu_n)\Delta t_{n+1} + \Delta u_{n+1}].
\end{aligned}$$

Thus, we have

$$\mathrm{Cov}[\Delta p_n, \Delta p_{n+1}|\mathcal{F}_{n-1}] = -\frac{(\lambda_n\beta_n\Delta t)^2\Sigma_{n-1} + \lambda_n^2\sigma_n^2\Delta t_n}{(M-2)}$$

from the chain rule of conditional expectations. □

Holthausen et al. (1987) categorized price impact into three groups: (i) liquidity costs, (ii) inelastic demand curves, and (iii) information effects. The first two effects are temporary, whereas the last effect is permanent. In our setting, $\theta_n = \frac{\beta_n\Sigma_n}{\sigma_u^2}$ is an information effect, which is caused by updating the expectations by market makers. The temporary effect is given by $\lambda_n - \theta_n = \frac{1}{M-2}\frac{\beta_n\Sigma_n}{\sigma_u^2}$. By setting the price sensitivity to be higher, market makers can earn a profit from noise traders.

Some empirical studies have found that price movements have permanent and temporary impacts. Our model effectively explains how permanent and temporary effects are caused. To illustrate the permanent and temporary impacts, we provide a numerical example in Figure 4. The number of auctions is five, and the parameters are the same as in the previous numerical example. The aggregated order at each auction is given by 1, 0, -1, -1, and 0. The price processes in the case of $M = 3$, 5, and ∞ are as follows.

Figure 4 is inserted here.

Although the order at $t_2 = 0.4$ is zero, the price decreases when $M = 3$ and 5. This is because the price increases at t_1 has a temporary effect. The price increase at t_5 is also caused by the temporary effect of $\Delta p_4 = p_4 - p_3$. The figure illustrates the overshooting and the resilience of the price in a market with imperfectly competitive market makers.

4 Some Extensions

In this section, we discuss some extensions of our model.

4.1 Multiple informed traders

Holden and Subrahmanyam (1992) studied a Kyle-type market model with multiple informed traders. We extend our model to such a setting.

The market opens at $t = 0$ and ends at $t = 1$, and trading takes place at $t_1 < \cdots < t_N = 1$. We assume that there are I informed traders who privately observe the payoff of a risky asset before the market opens. We index each informed trader by $i = 1, \ldots, I$. Informed traders are risk-neutral in the sense that they maximize their conditional expected profit as (3.1).

As in the case of a monopolistic informed trader, we obtain the following theorem.

Theorem 3 *Let* $\mu_n = \mathrm{E}[v|\mathcal{F}_n]$ *and* $\kappa := \frac{M-1}{M-2}$. *When* $M \geq 3$, *there exists a linear equilibrium, in which there are constants* α_n, β_n, λ_n *and* Σ_n *such that for*

$$
\begin{aligned}
\Delta x_{in} &= \beta_n (v - \mu_{n-1}) \Delta t_n, \quad i = 1, \ldots, I, \\
\Delta d_{mn} &= \frac{1}{M\lambda_n}(\mu_{n-1} - p_n), \quad m = 1, \ldots, M, \\
\Sigma_n &= \mathrm{Var}[v|\mathcal{F}_n]
\end{aligned}
$$

and

$$
\mathrm{E}[\pi_{in}|v, \mathcal{F}_{n-1}] = \alpha_{n-1}(v - \mu_{n-1})^2 + \delta_{n-1}.
$$

Given Σ_0, *the constants* α_n, β_n, ξ_n *and* Σ_n *are the solution to the difference equation system*

$$
\begin{aligned}
\alpha_{n-1} &= \frac{1 + [1 - 2\kappa + 2I(1-\kappa^2) + I(1-\kappa)^2]\alpha_n \lambda_n}{\lambda_n [(I+1) - 2I\kappa^2 \alpha_n \lambda_n]^2}, \\
\delta_{n-1} &= \delta_n + \kappa^2 \alpha_n \sigma_u^2 \Delta t_n, \\
\beta_n \Delta t_n &= \frac{1 - 2\kappa \alpha_n \lambda_n}{\lambda_n [(I+1) - 2I\kappa^2 \alpha_n \lambda_n]}, \\
\lambda_n &= \frac{1}{\kappa} \frac{I \beta_n \Sigma_n}{\sigma_u^2}
\end{aligned}
$$

and

$$
\Sigma_n = \left(1 - I\kappa \beta_n \lambda_n \sigma_u^2 \Delta t_n\right) \Sigma_{n-1},
$$

subject to $\alpha_N = \delta_N = 0$ *and the second order condition*

$$
\lambda_n - \kappa^2 \alpha_n \lambda_n^2 > 0.
$$

The conditional expectation μ_n satisfies

$$\mu_n = \mu_{n-1} + \frac{I\beta_n\Sigma_n}{\sigma_u^2}\Delta y_n.$$

It is worth noting that when $M = \infty$, the simultaneous equations in Theorem 3 coincide with those obtained by Holden and Subrahmanyam (1992).

4.2 Risk-Averse Informed Traders

Holden and Subrahmanyam (1994) studied a multi-period model with multiple risk-averse informed traders. We here extend our model to the case of imperfectly competitive market makers.

Assume that I informed traders all have a exponential utility with risk-aversion coefficient A, i.e., the informed trader i maximizes

$$\mathrm{E}\left[\mathrm{e}^{-AW_{iN}}\,\middle|\, v\right], \quad i = 1, \dots, I,$$

where $W_{iN} := \sum_{n=1}^{N}(v - p_n)\Delta x_{in}$ denotes the final wealth of the informed trader i. As Proposition 1 in Holden and Subrahmanyam (1994), we have the following theorem.

Theorem 4 *When $M \geq 3$, there exists a unique linear equilibrium in which there are constants α_n, β_n, λ_n, Σ_n, and γ_n, characterized by the following:*

$$\begin{aligned}
\Delta x_{in} &= \beta_n(v - \mu_{n-1})\Delta t_n, \\
\Delta d_{mn} &= \frac{1}{M\lambda_n}(\mu_{n-1} - p), \\
\Sigma_n &= \mathrm{Var}[v|\mathcal{F}_n]
\end{aligned}$$

and

$$J_n(W_{n-1}) = -\gamma_n \mathrm{e}^{-A\left[W_{n-1} + \alpha_n(v-\mu_{n-1})^2\right]}$$

Given Σ_0, the constants α_n, β_n, λ_n, Σ_n, and γ_n are the solution of the simultaneous equation system

$$\begin{aligned}
\alpha_n =&(1 - I\lambda_n\beta_n\Delta t_n)\beta_n\Delta t_n + \alpha_n(1 - I\kappa\lambda_n\beta_n\Delta t_n)^2 \\
&- \frac{A\lambda_n^2[(1 - 2I\kappa^2\alpha_n\lambda_n)\beta_n\Delta t_n + 2\kappa\alpha_n]^2\sigma_u^2\Delta t_n}{2(1 + 2A\kappa^2\alpha_n\lambda_n^2\sigma_u^2\Delta t_n)}, \\
\beta_n\Delta t_n =&\frac{1 - 2\alpha_n\theta_n - 2A\alpha_n\theta_n(\lambda_n - \theta_n)\sigma_u^2\Delta t_n}{(I+1)\lambda_n - 2I\alpha_n\theta_n^2 + A\lambda_n^2\sigma_u^2\Delta t_n}, \\
\lambda_n =&\frac{1}{\kappa}\beta_n\Sigma_n\sigma_u^2, \\
\Sigma_n =&(1 - I\kappa\beta_n\lambda_n\sigma_u^2\Delta t_n)\Sigma_{n-1}
\end{aligned}$$

and

$$\gamma_{n-1} = \frac{\gamma_n}{\sqrt{1 + 2A\kappa^2\alpha_n\lambda_n^2\sigma_u^2\Delta t_n}}.$$

5 Conclusion

In this paper, we have studied a Kyle-type market model in which imperfectly competitive market makers set the price and clear the market. When market makers are imperfectly competitive, the price sensitivity is higher than that of Kyle (1985), the case of perfectly competitive market makers. In the multi-period setting, the price movement has permanent and temporary effects, and is negatively correlated in its time-series. This means that imperfect competition of market makers causes a overshooting and resilience of the market price.

Empirical papers such as Holthausen et al. (1987) discovered that the price change has permanent and temporary impacts, especially for seller-initiated transactions. The findings obtained in this paper indicate that the two types of price impacts can be partially explained by the imperfect competition among market makers.

We should note that this paper has not studied a continuous-time setting. When $\max_n \Delta t_n \to 0$ in the setting of Section 3, the price dynamics can be informally written as

$$\mathrm{d}p_t = -\frac{1}{M-2}\lambda_{t-}\mathrm{d}y_{t-} + \lambda_t \mathrm{d}y_t.$$

It is not clear whether the above stochastic process is well-defined. The continuous-time model will be investigated in future research.

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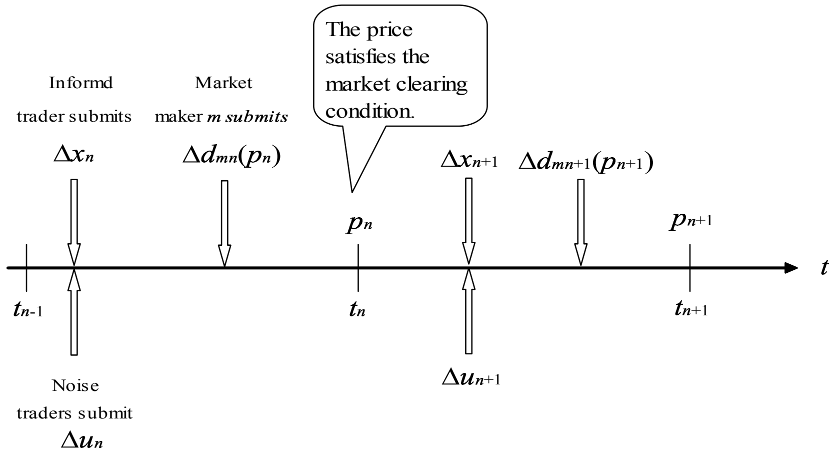
Figure 1: The time line of the model.



Notations	Order Type	Observables
Δx_n	Market order by Informed Trader	$\{v, p_1, \ldots, p_{n-1}\}$
Δu_n	Order by Noise Traders	Exogenous reasons
Δd_{mn}	Limit order by Market Maker m	$\{y_1, \ldots, y_n\}$

Figure 2: Price sensitivity to the order λ over the 5 trading periods. The market liquidity parameter λ is decreasing with M, the number of market makers.

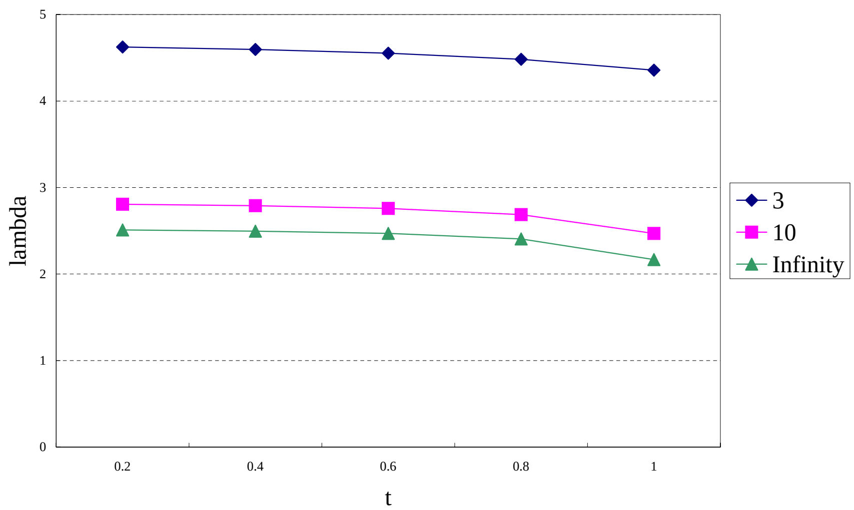


Figure 3: Informational efficiency Σ over the 5 trading periods. The figure shows that Σ is monotonically decreasing with respect to M, the number of market makers.

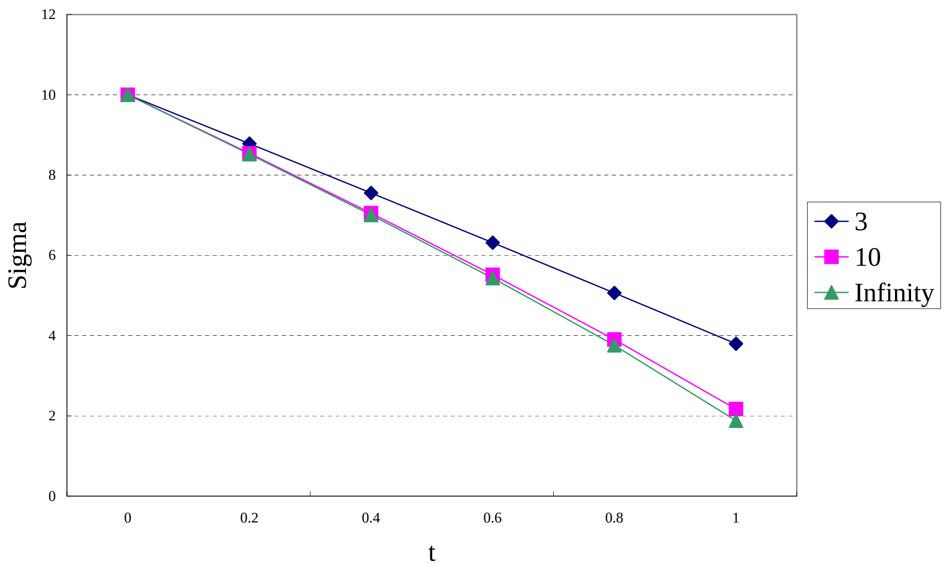


Figure 4: Price process over the 5 trading periods by a monopolistic informed trader. The figure effectively illustrates the permanent and temporary price impacts.

Δy_1	Δy_2	Δy_3	Δy_4	Δy_5
1	0	-1	-1	0