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Comment on 'Piezoelectricity as a mechanism on generation of electromagnetic precursors before earthquakes' by J.H. Wang

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SUMMARY

Wang (2021, hereafter JHW) recently investigated elastic–electric coupling (EEC) in terms of the piezoelectric effect to assess both the plausibility of and necessary conditions for generating pre-earthquake electromagnetic phenomena, including variations in the total electron content (TEC). The study considered a 1-D model to simulate the piezoelectric effect, and derived a quantitative relationship between the dislocation and intensity of the electric field. One of JHW's conclusions was that the piezoelectric effect is a potential mechanism for generating previously reported pre-earthquake TEC anomalies. However, the quantitative discussion in JHW contains a serious error. JHW had chosen an incorrect mode between the two solutions during the derivation of a quantitative relationship between the displacement and the electric field, which subsequently led to an incorrect estimation of the ratio of the generated electric field to the displacement. The opposite conclusion to that drawn by JHW is attained when a correct mode is used.

Key words: Electromagnetic theory; Earthquake hazards; Mechanics, theory and modelling.

INTRODUCTION

Wang (2021, hereafter JHW) recently investigated elastic–electric coupling (EEC) in terms of the piezoelectric effect to assess both the plausibility of and necessary conditions for generating pre-earthquake electromagnetic (EM) phenomena, such as variations in the total electron content (TEC; e.g. Heki 2011). JHW considered a one-dimensional model to simulate the piezoelectric effect, and derived the following quantitative relationship between the dislocation and intensity of the electric field:

$$|E| \sim (c/v)^2 \zeta^{-1} |ku| , \tag{1}$$

where u represents the dislocation, E is the electric field, k is the spatial wavenumber, c is the speed of light, v is the speed of the elastic wave and ζ is the piezoelectric coupling coefficient, which is assumed to be 10^{-12} CN^{-1}. (Note that eq. 1 corresponds to eq. 16 in JHW. Hereafter, the equations in this comment and JHW are identified as 'Equation' and 'JHW equation,' respectively.) JHW also used simulation results of ionospheric dynamics (Kuo *et al.* 2011, 2014) to define the critical intensity of the electric field, $E_c = 5 \times 10^5$ Vm^{-1}, which is the minimum intensity of the electric field required to generate the pre-earthquake TEC anomaly. JHW discussed the necessary conditions for potential earthquake nucleation processes to generate an electric field that is sufficiently large to induce pre-earthquake TEC anomalies based on eq. (1) and this E_c value. An important implication of JHW's conclusion is that the piezoelectric effect is a possible mechanism for generating previously reported pre-earthquake TEC anomalies.

However, the discussion in JHW contains a serious error in using eq. (1) to estimate the intensity of the electric field generated by deformation processes, as this derives an incorrect conclusion. Eq. (1) leads to a paradoxical consequence whereby a given strain will generate an infinitely large electric field in the absence of piezoelectric coupling (i.e. $\zeta \to 0$). Here I review the mathematical arguments presented in the derivation of eq. (1) in section 3 of JHW, which was used to discuss the electric field generated by deformation processes, and highlight a major error by discussing the physical meaning of eq. (1) (JHW eq. 16) and a related equation (JHW eq. 17). Readers can recognize the error easily if they follow the discussion carefully. However, it is assumed that this type of misunderstanding is commonly shared, even amongst specialists in fields related to the study, since both the author and the reviewers of JHW overlooked this error. This issue is summarized here to clarify the misunderstanding of JHW and to convey a positive implication of JHW's work after the error has been corrected, in the hope that the correct treatise on elastic–electromagnetic (EM) coupling will be disseminated to the scientific community.

DERIVATION OF THE RATIO OF THE ELECTRIC FIELD INTENSITY TO THE SIZE OF THE DISLOCATION: A REVIEW

There are several issues that need to be reconciled in JHW's discussion, all of which influence the main conclusion of the study. For example, both the conduction current due to the generated electric field and the relaxation process of the charges in the conductive earth's crust are ignored, although the reasoning for not including these parameters in the analysis is questionable. However, these issues are not discussed in this comment, which focuses only on the error in JHW's modelling framework. Some unclear points in JHW, such as confusion between the electric field ($\boldsymbol{E}$) and one of its components (E), are also ignored to focus on this major issue.

The derivation of eq. (1) is summarized here for the purpose of the following explanation. The presented derivation is already described by JHW, and the following derivation is essentially the same as that described by JHW. However, in section 3 of JHW, the phenomena with and without EEC are discussed together, such that it is difficult to follow the essence of JHW's modelling approach or which formulae apply to each case. In addition, equations in JHW contain numerous typos and minor errors. Therefore, the derivations of the formulae with and without EEC are presented here in a systematic manner and with the correction, since this information will provide clarity for the later discussion. The variables and physical quantities presented here are denoted using the same symbols as in JHW, unless mentioned otherwise, with minor revisions applied to the presented equations.

The starting point of the discussion in JHW is the derivation of eq. (1) from the following two equations that express EEC via the piezoelectric effect:

$$\epsilon = \mu^{-1}\sigma + \zeta \hat{E} \tag{2}$$

and

$$D = \zeta\sigma + \chi\hat{E}, \tag{3}$$

where ϵ is the strain, μ is the rigidity, σ is the stress, $\hat{E}$ is the electric field, D is the electric displacement or electric induction and χ is the electric permeability in a vacuum ($\sim 10^{-11}$ Fm^{-1}). Eqs (2) and (3) correspond to JHW eqs (3) and (4), respectively. Note that the electric displacement, which is denoted by I in JHW, is denoted by D in this comment to follow the convention used in electromagnetism and to avoid confusion regarding the electric current. This comment follows the 1-D problem that is considered in JHW, and defines the physical quantities such that they depend only on x. The strain ϵ is then defined as follows:

$$\epsilon = \partial\hat{u}/\partial x, \tag{4}$$

where $\hat{u}$ is the displacement. This equation is not numbered in JHW. The momentum equation is given as follows:

$$\rho\partial^2\hat{u}/\partial t^2 = \partial\sigma/\partial x, \tag{5}$$

where ρ is the density. Eq. (5) corresponds to JHW eq. (5). Maxwell's equations for describing the electric field ($\boldsymbol{E}$) and magnetic field ($\boldsymbol{B}$) are given as follows:

$$\nabla\times\boldsymbol{E} = -\partial\boldsymbol{B}/\partial t \tag{6}$$

and

$$\nabla\times\xi^{-1}\boldsymbol{B} = \partial\boldsymbol{D}/\partial t, \tag{7}$$

where ξ is the magnetic permeability of a material. Following JHW, the electric current, which should appear on the right-hand side of eq. (7), is ignored, although the absence of this term is disputable. Eqs (6) and (7) correspond to JHW eqs (7) and (8), respectively. Maxwell's equations are then reduced to a single equation:

$$\nabla^2\boldsymbol{E} = \xi\partial^2\boldsymbol{D}/\partial t^2, \tag{8}$$

which is a generalized form of JHW eq. (9). Note that $\nabla\times\hat{\boldsymbol{E}}$ in the right-hand side of JHW eq. (9) should be replaced with $\nabla^2\hat{\boldsymbol{E}}$. Eqs (2)–(5) and (8) are summarized in the following equations for $\hat{u}$ and $\hat{E}$:

$$\partial^2\hat{u}/\partial x^2 = v^{-2}\partial^2\hat{u}/\partial t^2 + \zeta\partial\hat{E}/\partial x \tag{9}$$

and

$$\partial^2\hat{E}/\partial x^2 = c^{-2}\left(1-\zeta^2\mu\chi^{-1}\right)\partial^2\hat{E}/\partial t^2 + \zeta\mu\xi\partial^3\hat{u}/\partial x\partial t^2, \tag{10}$$

where v and c are equal to $(\mu/\rho)^{1/2}$ and $(\xi\chi)^{-1/2}$, respectively. Eqs (9) and (10) are the same as JHW eqs (10) and (11), respectively. Note that c^2 appears in the first term of the right-hand side of JHW eq. (11) should be replaced with c^{-2}, as in eq. (10). Eqs (9) and (10) form the basis for deriving the quantitative relationship between $\hat{u}$ and $\hat{E}$ via EEC.

Eqs (9) and (10) are reduced to the following algebraic equations by assuming $\hat{E}$ and $\hat{u}$ take the forms $\hat{E} = Ee^{i(\omega t-kx)}$ and $\hat{u} = ue^{i(\omega t-kx)}$, respectively, and replacing $\partial/\partial t$ and $\partial/\partial x$ with $i\omega$ and $-ik$, respectively:

$$-k^2u = -\omega^2v^{-2}u - ik\zeta E \tag{11}$$

and

$$-k^2E = -\omega^2c^{-2}\left(1-\zeta^2\mu\chi^{-1}\right)E + ik\omega^2\zeta\mu\xi u. \tag{12}$$

From eqs (11) and (12), the following quadratic equation for ω^2 is derived:

$$\left(1-\zeta^2\mu\chi^{-1}\right)\left(\omega^2\right)^2 - \left(c^2+v^2\right)k^2\omega^2 + c^2v^2k^4 = 0, \tag{13}$$

which is equivalent to JHW eq. (12). The solutions are expressed as a power series of v^2/c^2 when $v^2/c^2 \ll 1$ and $\zeta^2\mu\chi^{-1} \ll 1$. The first solution, which JHW referred to as the 'fast mode,' is

$$\omega_+^2 = c^2k^2\cdot\left[\frac{1}{1-\zeta^2\mu\chi^{-1}} + \frac{\zeta^2\mu\chi^{-1}}{1-\zeta^2\mu\chi^{-1}}\left(v^2/c^2\right)+\ldots\right], \tag{14}$$

which corresponds to JHW eq. (14). By substituting eq. (14) into 11 (or into 12), a good approximation of the relation between E and u corresponding to the 'fast mode', denoted by E_+ and u_+ respectively, is derived as

$$E_+/u_+ \cong ik\zeta^{-1}\left(c^2/v^2\right), \tag{15}$$

which corresponds to JHW eq. (16). The second solution, which JHW referred to as the 'slow mode,' is

$$\omega_-^2 = v^2k^2\cdot\left[1-\zeta^2\mu\chi^{-1}\left(v^2/c^2\right)+\ldots\right], \tag{16}$$

which corresponds to JHW eq. (16). By substituting eq. (16) into 11 (or into 12), a good approximation of the relation between E and u corresponding to the 'slow mode', denoted by E_- and u_- respectively, is derived as

$$E_-/u_- \cong -ik\zeta\mu\chi^{-1}\left(v^2/c^2\right), \tag{17}$$

which corresponds to JHW eq. (17).

Note that JHW eq. (15), which represents ω_-^2/k^2, is incorrect due to the inclusion of a factor $(1-\zeta^2\mu\chi^{-1})^{-1} \cong (1+\zeta^2\mu\chi^{-1})$, which yields an incorrect estimation of E_-/c_- via JHW eq. (17). JHW eq. (17) is obtained by substituting JHW eq. (15) into either eq. (11) or JHW eq. (10). When only the dominant term of ω_-^2/k^2

(i.e. the zeroth-order term of u^2/c^2 in eq. 16), instead of JHW eq. (15), is substituted into eq. (11) (or JHW eq. 10), we obtain an estimation of $E_-/u_- = 0$; however, this is illogical because the first- and higher-order terms of u^2/c^2 have been ignored. E_-/u_- can only be estimated via eq. (12) (or JHW eq. 11) if we only use the dominant term of ω_-^2/k^2. Similarly, E_+/u_+ can only be estimated via eq. (11) (or JHW eq. 10) if we only use the dominant term of ω_+^2/k^2. When both the zeroth- and first-order terms of u^2/c^2 are considered for $\omega_\pm^2/k^2$, eqs (11) and (12) (or JHW eqs 10 and 11) yield the consistent estimations for E_+/u_+ and E_-/u_-, as presented in eqs (15) and (17), respectively. The difference between eq. (17) and JHW eq. (17) is also noteworthy, as the difference is a factor of v^2/c^2, which is on the order of 10^{-9} or 10^{-10}.

MEANING OF THE FAST AND SLOW MODES

Eq. (1), which appeared at the beginning of this comment, is based on eq. (15). This means JHW used only the 'fast mode' during the discussion. However, there are not only the 'fast mode' but also the 'slow mode'. Both modes provide quite different E estimates for the same u value, as noted by JHW. The issue that arises is which mode should be used to evaluate EEC. This issue can be assessed by clarifying the physical meanings of the 'fast mode' and 'slow mode'.

We clarify the meaning of the 'fast mode' and 'slow mode' by first evaluating the intensity of the coupling between the strain (stress) and electric field (electric displacement) by considering the derivation of the quantity $\zeta^2\mu\chi^{-1}$ as follows. The second term on the right-hand side of eq. (2) expresses the EEC, whereas the first term expresses the purely elastic effect. Therefore, the contribution of the electric field to the strain through EEC relative to that of the stress to the strain is quantified by the ratio of the second term to the first term in eq. (2), $|\zeta E|/|\mu^{-1}\sigma|$. Similarly, the contribution of the stress to the electric displacement through EEC relative to that of the electric field to the electric displacement is quantified by the ratio of the first term to the second term in eq. (3), $|\zeta\sigma|/|\chi E|$. The dimensionless quantity is defined as the product of these two ratios, $\zeta^2\mu\chi^{-1}$, and therefore expresses the intensity of the coupling. For further details, see eq. (2.41) and the related explanation in Ikeda (1990).

We recognize that the fully coupled problem described by eqs (2) and (3) (JHW eqs 3 and 4) can be approximately decomposed into two semi-coupled problems with a sufficient accuracy when $\zeta^2\mu\chi^{-1}$ is quite small. If both types of EEC (i.e. the influence of the electric field to the strain and the influence of the stress to the electric displacement) cannot be ignored, then $\zeta^2\mu\chi^{-1}$ should not be $\ll 1$. However, this is not the case. We obtain $\zeta^2\mu\chi^{-1} \sim 3 \times 10^{-3} \ll 1$ by applying the set of values adopted in JHW, as presented at the beginning of section 5.1 in JHW. We note that the piezoelectric constant of 2×10^{-12} CN^{-1} adopted in Wang (2021) is too large. The piezoelectric constant for quartz-rich rocks is on the order of 10^{-15} CN^{-1} (e.g. Bishop 1981), which makes $\zeta^2\mu\chi^{-1}$ even smaller. The smallness of $\zeta^2\mu\chi^{-1}$ means that we can ignore at least one of the two coupling terms that appear in eqs (2) and (3) (or JHW eqs 3 and 4).

The meaning of the 'fast mode' is clarified by considering a semi-coupled problem whereby the influence of the electric field to the strain is included but that of the stress to the electric displacement is ignored. This semi-coupling is expressed as follows:

$$\epsilon = \mu^{-1}\sigma + \zeta\hat{E}$$

and

$$D = \chi\hat{E}. \tag{18}$$

The former is the same as eq. (2), but the latter ignores the first term on the right-hand side of eq. (3). Eq. (9) is unchanged in this situation, whereas eq. (10) is reduced to

$$\partial^2\hat{E}/\partial x^2 = c^{-2}\partial^2\hat{E}/\partial t^2. \tag{19}$$

Note that this equation is self-contained to determine $\hat{E}$ independent of $\hat{u}$. When $\hat{E}$ is expressed in the form $\hat{E} = E_1\, e^{i(\omega_1 t - kx)}$, the angular frequency ω_1 satisfies

$$\omega_1 = kc, \tag{20}$$

which is identical to ω_+ given by eq. (14) when $v^2/c^2 \ll 1$ and $\zeta^2\mu\chi^{-1} \ll 1$. The displacement $\hat{u} = u_1\, e^{i(\omega_1 t - kx)}$ that corresponds to this electric field is determined from eq. (9), which yields

$$-u_1 = -c^2 v^{-2} u_1 - ik^{-1}\zeta E_1. \tag{21}$$

The relationship between u_1 and E_1 is identical to that between u_+ and E_+ in eq. (15) when $v^2/c^2 \ll 1$. We have thus far demonstrated that almost the same estimation of E/u is obtained for the 'fast mode', even when the influence of the stress to the electric displacement is ignored. This means that the stress (strain) cannot be considered the primary cause of the 'fast mode' (eqs 14 and 15 or JHW eqs 14 and 16). The fast mode should basically be considered a mode that describes the electric field as a cause and the strain as an effect, although the electric field may experience weak modulation due to the coupling effect that should be expressed by the ignored terms in eq. (15).

The meaning of the 'slow mode' is also clarified by considering another semi-coupled problem whereby the influence of the stress to the electric displacement is ignored but that of the electric field to the strain is ignored. This semi-coupling is expressed as follows:

$$\epsilon = \mu^{-1}\sigma \tag{22}$$

and

$$D = \zeta\sigma + \chi\hat{E}.$$

The former ignores the second term on the right-hand side of eq. (2), whereas the latter is the same as eq. (3). This set of equations also represents semi-coupling: the stress influences the electric displacement, but the electric field does not influence the strain. In this situation, eq. (9) is reduced to

$$\partial_x^2\hat{u} = v^{-2}\partial^2\hat{u}/\partial t^2, \tag{23}$$

whereas eq. (10) is unchanged. Eq. (23) is self-contained to determine $\hat{u}$ independent of $\hat{E}$. When $\hat{u}$ is expressed in the form $\hat{u} = u_2\, e^{i(\omega_2 t - kx)}$, the angular frequency ω_2 satisfies

$$\omega_2 = kv, \tag{24}$$

which is identical to ω_- given by eq. (16) when $\zeta^2\mu\chi^{-1}(v^2/c^2) \ll 1$. The electric field $\hat{E} = E_2\, e^{i(\omega_2 t - kx)}$ is determined from eq. (10) after substituting $\hat{u} = u_2\, e^{i(\omega_2 t - kx)}$, which yields

$$-E_2 = -v^2c^{-2}\left(1 - \zeta^2\mu\chi^{-1}\right)E_2 + ikv^2c^{-2}\zeta\mu\chi^{-1}u_2. \tag{25}$$

The relationship between u_2 and E_2 is identical to that between u_- and E_- in eq. (17) when $v^2/c^2 \ll 1$. We have thus far demonstrated that almost the same estimation of E/u is obtained for the 'slow mode', even when the influence of the electric field to the strain is ignored. This means that the electric field cannot be considered the primary cause of the 'slow mode' (eqs 16 and 17 or JHW eqs 15 and 17). The 'slow mode' should basically be considered a mode that describes the strain as a cause and the electric field as an effect, although the strain experiences weak modulation due to the coupling effect that should be expressed by the ignored terms in eq. (17).

DISCUSSION AND CONCLUSION

It is important to reiterate that each of the 'fast' and 'slow' modes does not represent a mutual interaction, but rather a one-sided action: the 'fast mode' represents a conversion from EM waves to elastic waves, and the 'slow mode' represents a conversion from the elastic waves to the electric field.

The assumption by JHW that only the mechanical deformations are converted to the electric field means that the 'slow mode' formula, as opposed to the 'fast mode' formula, must be used. Nevertheless, the discussions in JHW are based on the 'fast mode' formula (JHW eq. 16). Therefore, most of the quantitative discussion of JHW concerning the generation process of the anomalous electric field is not justified and should be reconsidered.

The opposite conclusions to those presented by JHW are attained when the 'slow mode' formula (eq. 17, which is a corrected version of JHW 1) is applied. We use the 'slow mode' formula to estimate the amplitude of the strain, ϵ_c, required to generate an electric field as large as the critical intensity of the electric field, $E_c = 5 \times 10^5$ Vm^{-1}, as follows:

$$|\epsilon_c| = |ku| \cong E_c \times \zeta^{-1}\mu^{-1}\chi\left(c^2/v^2\right). \quad (26)$$

This equation highlights that the strain ϵ must be on the order of 10^5, even if we assume a large value of $\zeta \sim 10^{-12}$ CN^{-1}, which is obviously impossible. This means that when using the JHW modelling framework, the piezoelectric effect is rejected as a potential generation mechanism for the pre-earthquake EM phenomena. Additional mechanisms, which were ignored in the JHW modelling framework, must therefore exist for the piezoelectric effect to generate pre-earthquake EM phenomena.

A scenario based on the 'fast mode' formula may only be justified when the variations in the EM field are the cause, and the strain changes and subsequent earthquake are the effect. The discussion in JHW may have merit and potentially advance our understanding of pre-earthquake EM phenomena if this scenario is adopted. However, if this scenario is adopted, it will be necessary to investigate an alternative generation mechanism for the pre-earthquake EM anomalies that are not caused by earthquake nucleation processes.

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DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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