Cat Bond for Extreme events*

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Abstract

This paper investigates the applicability of Catastrophe Bonds to natural disaster such as Nankai trough tsuname, which might be more than 20 meters hight of waves to west half of Japan in near future. In Japan there were serious disasters as Kobe earthquake, East-Japan earthquake which caused not only human casualty but also economic destruction. The economic burden to recover from disaster is depending on Government bonds which are to be paid by the future generation. The Cat bonds is a kind of insurance which the present generation can pay in the occasion of catastrophe.

1 Introduction

The Catastrophe bond is a security that pays the issuer when a predefined disaster risk is realized. It has been started since 1997 due to Hurricane Andrew disaster(1992) according to [10]. In Japan serious disasters had occurred and predicted by the government as Table 1. Two catastrophic earthquakes has increased significantly government debt and recent COVID-

Table 1: Recent catastrophe in Japan

Year	Disaster	Scale	Casualties	Damaged res.	Gov.Expense
1995	Earthquake Kobe	M7.9	6,439	640,000	16 tril.¥
2011	Earthquake East-Japan	M9.0	20,960	1,138,000	37 tril.¥
2020-1	COVID-19	-	18,360*	-	91 tril.¥
?	Nankai trough tsunami	M7	320,000	2,380,000	215 tril.¥

(* 2021.12.6)

19 has worsen. The Cabinet office has alerted the Government expense for coming Nankai trough tsunami may reach 215 trillion yen [12]. This financial burden cannot be sustained any more by traditional economic policies. In this paper is discussed the possibility of financial risk management feasibility against the catastrophe tail risk using Catastrophe bonds.

In Figure 1 the Cabinet office reports the prediction of wave hight in the Tunami which might be more than 20 meters heigh at full tide. The Cabinet alerts the occurrence within 10 years is at 70% probability. The damaged area stars from Shizuoka to Miyazaki prefecture where it includes major industrial areas. The direct economic loss by Nankai trough tsunami is estimated from 100 to 172 trillion yen according to the report. The estimate is almost twice amount of government annual budget.

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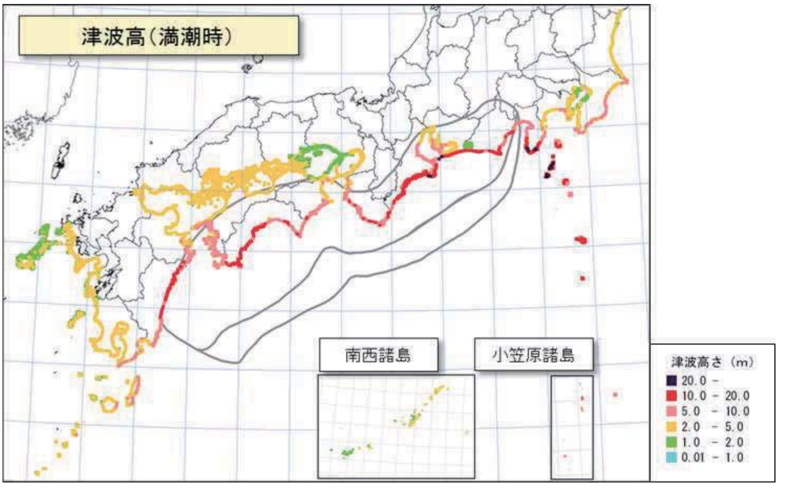
This article investigates the possibility of Catastrophe bond (Cat bond) to cover the damage as an insurance. The origin of Cat bond is design to covered the huge tail risk caused the catastrophe hurricane through the utilization of capital market, see [10].

In next section 2 describes the structure of Cat bond money flow and the contract scheme. In section 3, the pricing for cat bond is applied by the option pricing theory of Jump difffusion Merton model [3] which can describe the tail risk of catastrophe events. In numerical example assuming the distribution from Trottier [5], the prices of cat bonds are calculate for different maturities and insurance benefits. Finally we discuss the possibility of Cat bond for Nankai Trough Tunami and summalize a conclusion.

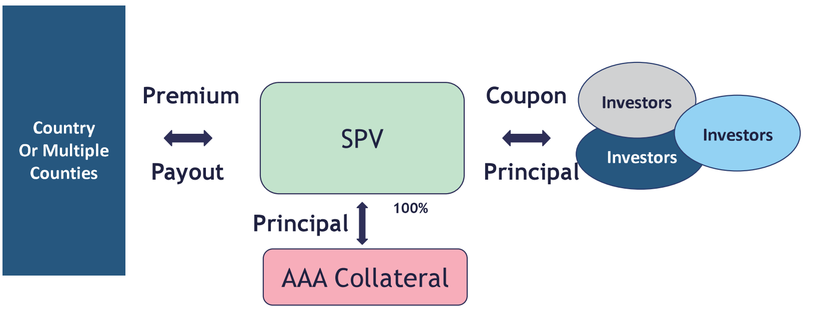
Figure 1: **Nankai trough tsunami**



2 Catastrophe bonds and the cash flow

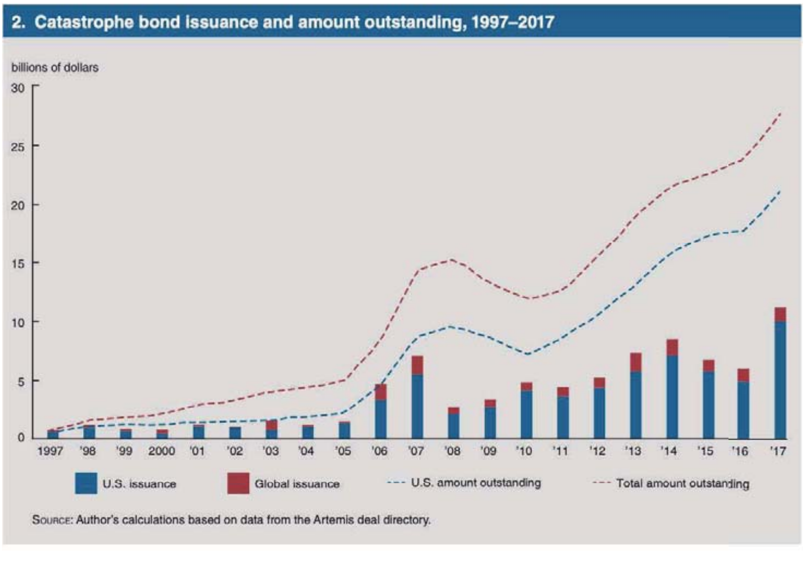
Cat bonds are risk-linked securities which transfer a predefined risks from a sponsor to investors. The sponsor is a kind of insurance vehicle which receives the premium from those who wish to receive insurance to catastrophe incident. The sponsor issues the bond which pays the higher coupon than normal bonds. The principal that Cat bond investors invest is reserved the safe deposit called SPV(Special Purpose Vehicle). The SPV is the safe deposit witch is AAA collateral. If no incident happen, all deposits will return to Cat bond investors. If incident occurs, the insurers will receive the payout which is the insurance money. This is shown in Figure 2.

Figure 2: **Cat bond and Capital market**



The historical development of Cat bond is seen in Figure 3 from [10]. The most of contracts are in US market and has been increased rapidly. Recently the total amount of Cat bonds has reached to around 30 billion dollars. Partly because bonds market has been in low interest rate for these periods. The market information are summarized in Web site ARTEMIS [6].

Figure 3: **Historical increase of Cat bond contracts values**



3 Mathematical model for catastrophe bonds

The extreme event can be described by Jump diffusion model as seen [5], which is simple and whose analytical solution is well known for European options [3].

Let assume the loss distribution as a Jump diffusion model;

$$\frac{dL_I(t)}{L_I(t)} = \mu_I dt + \sigma_I dW_I(t) + (Y_i - 1)dN_t \tag{3.1}$$

where $W_I(t)$ is a Brownian motion and N_t is Poisson process whose intensity is λ, The jump size is assumes to be a log -normal distributed as,

$$Z_i := \log(Y_i) \sim N(a, b^2)$$

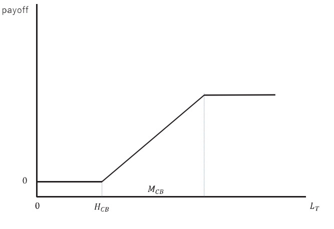
$N(\cdot, \cdot)$ is Normal distribution.

The solution for time T is for the initial value $L_I(0)$;

$$L_I(T) = L_I(0)\exp\{(\mu_I - \sigma_I^2/2)T + \sigma_I W_I(T) + \sum_{j=1}^{N_T} Z_j\} \tag{3.2}$$

The simplified payoff for Cat bond seller which means the insurance holder is as follows; The insurer will receive the insurance money for the damage which is bigger than the attachment point value H_C until the damage exceed the detachment value $H_C + M_C$, when the damage exceeds the point the money is a constant as the limit (detachment point), seen as in Figure 4.

Figure 4: **Attachment $H_C(H_{CB})$ and detachment point $H_C + M_C$ of Cat Bond Insurer**



The value of insurer B_I of Cat bond will be at maturity T

$$B_I = \min\{(L_I(T) - H_C)^+, M_C\}.$$

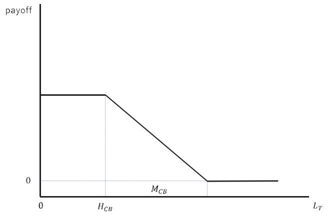
The value of Cat Bond issuer B_C will be at T

$$B_C = \min\{M_C, (H_C + M_C - L_I(T))^+\}.$$

The value of Cat bond investor at the maturity is seen in Figure 5;
Clearly we see the sum of payoffs is constant as,

$$B_I + B_C = M_C. \tag{3.3}$$

Figure 5: **Payoff of CAT bond investor** $M_{CB} = M_C$



3.1 Prices of Cat bond

The insurer's payoff of Cat bond at T is considered by two call options as seen in Figure 6,

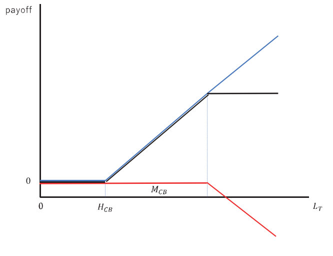
$$\begin{aligned} B_I &= \min\{(L_I(T) - H_C)^+, M_C\} \\ &= (L_I(T) - H_C)^+ - (L_I(T) - (H_C + M_C))^+ \end{aligned}$$

By Risk neutral probability Q, the price of insurance is the difference of two call options,

$$\begin{aligned} \pi_C(H_C, M_C) &= E^Q[e^{-rT} B_I] \\ &= E^Q[e^{-rT}\{(L_I(T) - H_C)^+ - (L_I(T) - (H_C + M_C))^+\}] \\ &= Call(L_I, H_C, T) - Call(L_I, H_C + M_C, T) \end{aligned} \tag{3.4}$$

Where $Call(L_I, H_C, T)$ is the call option price of strike price of H_C and maturity T.

Figure 6: **Payoff of Cat Bond as two options difference**



By Merton Jump diffusion model of options [3], we can get the value of Cat bond;

$$\begin{aligned} Call(L_I, H_C, T) &= \sum_{n=0}^{\infty} \frac{(\lambda' T)^n}{n!} \exp(-\lambda' T) C_{BS}(L_I, H_C, T, \sigma_n, r_n) \\ Call(L_I, H_C + M_C, T) &= \sum_{n=0}^{\infty} \frac{(\lambda' T)^n}{n!} \exp(-\lambda' T) \\ &\quad \times C_{BS}(L_I, H_C + M_C, T, \sigma_n, r_n) \end{aligned}$$

$$C_{BS}(L_I, H_C, T, \sigma_n, r_n) = L_I(0)N(d_n^1(H_C)) - H_C e^{-r_n T} N(d_n^2(H_C))$$

$$d_n^1(K) = \frac{\log\{L_I/K\} + (r_n + \sigma_n^2/2)T}{\sigma_n\sqrt{T}}, d_n^2(K) = d_n^1(K) - \sigma_n\sqrt{T}$$

$$\lambda' = \lambda m, m = E(Y) = e^{a+b^2/2}$$

$$\sigma_n^2 = \sigma_I^2 + \frac{nb^2}{T}, r_n = r - \lambda(m-1) + nlog(m)/T.$$

3.2 Numerical examples

The parameters of catastrophic ross L_I are adopted from Trottier [5],

$$\frac{dL_I(t)}{L_I(t)^-} = \mu_I dt + \sigma_I dW_I(t) + (Y_i - 1)dN_t(\lambda),$$

Let maturities of Cat bonds be $T = 1, \cdots, 6$ years, where parameters of the loss are $\mu_I = 0.025$, $\sigma_I = 0.5$, $Z_i = \log Y_i$, $Z_i \sim N(a = 0.179, b^2 = 0.083^2)$, $\lambda = 0.1$.
The loss size is assumed $L_I(0) = 120M$, and the interest rate is $r = 0.02$, Let Attachment point be $H_{CB} = 0.7L_I(0) = 84M$ and for Detachment points let $M_{CB} = 100M, 200M, \cdots, 600M$, where $M = 1,000,000$ Dollars.

3.2.1 Premium of Insurance

Prices of insurance by Cat bond π_C are shown in Table 2 for maturities from 1 year to 6 years and for the detachment point values M_C from 100 million dollars to 600 million dollars. The detachment value is the maximum covered value by the Cat bond. In Table 2 the premium increases as guarantee values M_C increases. The graph of price of Cat bond insurance is in Figure 7, where the premium increases as the guarantee increases. In Table 3 for each Guarantee M_C, premium rates for insurance are shown for maturities. The annual premium is smaller for the bigger guarantee, it is seen in Figure 8.

The premium has the characteristics that the longer and bigger guarantee decreases the cost, seen in Figure 7 and 8 as the ordinary insurance contracts.

Table 2: Price of insurance by CAT bond π_C (million dollar) for maturities 1 to 6 years

M_C	100	200	300	400	500	600
1Y	4.49	5.79	6.02	6.11	6.14	6.15
2 Y	7.40	10.28	11.51	12.09	12.38	12.54
3Y	9.45	13.74	15.94	17.18	17.93	18.4
4 Y	10.9	16.3	19.4	21.2	22.5	23.4
5Y	11.9	18.1	21.9	24.4	26.1	27.3
6Y	12.5	19.4	23.7	26.7	28.8	30.4

(million dollar)

Table 3: CAT Bond premium rate （%）

Gurantee M_C	100	200	300	400	500	600
premium 1year	2.44	2.01	1.57	1.26	1.05	0.9
2 years	1.97	1.77	1.47	1.23	1.05	0.91
3years	1.67	1.57	1.36	1.15	1.01	0.88
4 years	1.44	1.38	1.23	1.07	0.94	0.84
5years	1.25	1.24	1.11	0.98	0.87	0.78
6years	1.1	1.1	1.0	0.89	0.80	0.72

Figure 7: Premium for Guarantee M

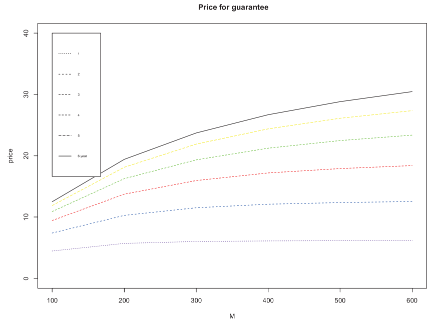
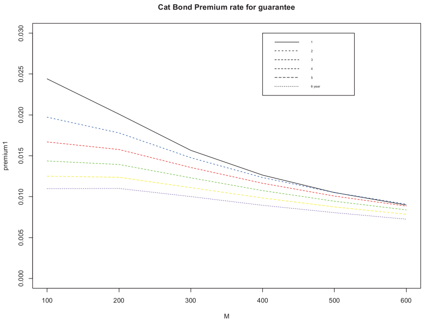


Figure 8: Premium par year of Cat Bond



3.2.2 Cat Bond prices and rates of return

The investors could calculate the Cat bond price by Put-Call parity condition. The price of Cat bond for investor π_B is obtained from payoff of maturity in Figure 5,

$$\pi_B = Put(L_I, H_C, T) - Put(L_I, H_C + M_C, T).$$

Cat bond prices are in Table 4 for guarantee from 100 to 600 million dollars and for maturities from 1 to 6 years.

Table 4: Cat Bond price π_B for Guarantee M_C

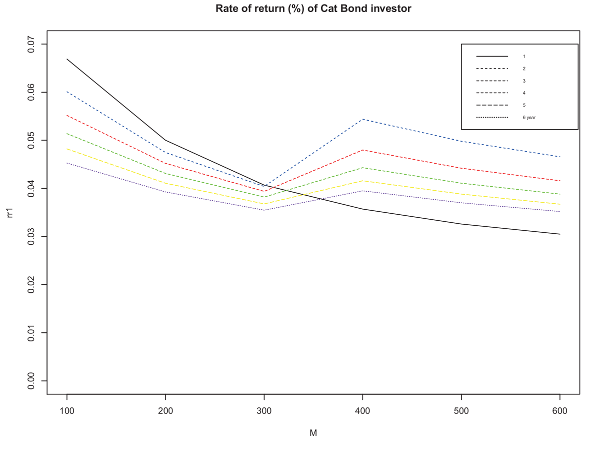
GuaranteeM_C	100	200	300	400	500	600
1Y(maturity)	93.53	190.25	288.04	385.97	483.95	581.97
2 Y	88.68	181.88	276.73	358.75	452.60	546.61
3Y	84.73	174.61	266.59	346.31	437.84	529.68
4Y	81.41	168.32	257.53	335.09	424.25	513.83
5Y	78.58	162.87	249.55	324.84	411.80	499.29
6Y	76.19	157.98	242.38	315.62	400.43	485.76

When we calculate the annual expected rate of return from Table 4, the best investment performance is the case of 1 year maturity and the least guarantee 100 million \$. In Figure 9 the expected returns are compared for maturities and guarantees. Up to 300 Million \$ rates of return are decreasing but at 400 million \$ they jump and decreasing. This kinked problem may cause from what we calculate the rate of returns for options. See the discussion in [9].

Table 5: Rate of returns in Cat Bond Investors

Guarant M_C	100	200	300	400	500	600
1 Y	6.69	5.00	4.07	3.57	3.26	3.05
2 Y	6.01	4.75	4.04	5.44	4.98	4.66
3 Y	5.52	4.52	3.94	4.80	4.42	4.16
4 Y	5.14	4.31	3.82	4.43	4.11	3.88
5 Y	4.82	4.11	3.68	4.16	3.88	3.67
6 Y	4.53	3.93	3.55	3.95	3.70	3.52

Figure 9: Rate of return for Cat Bond investor for Guaranty M



4 Conclusion

The Cat bonds with longer maturities are empirically known as an effective source for natural disaster. The Cat bond is the risk transfer from private bond market to disaster affected countries, and it will bring win-win relation to suffering countries government and the investors. The success example is Pacific Alliance countries joint Cat bond for earthquake which started since 2018. Feb., where Chili, Columbia, Mexico and Peru participate with 1.36 billion US dollar contract [7]. In this paper we propose a simple option pricing model for Cat bond. The further research is expected to develop financing scheme for coming catastrophe risk.

The economic damage by Nankai trough tsunami estimated by the cabinet office of government is over 200 trillion yen which is about twice bigger than annual government budget, which might be impossible to depend the future tax revenue as East Japan earthquake. The Cat bond is different from national bond, and the principal of Cat bond will be extinct once the catastrophe occurs. The burden of bond is payed by the investor of the present generation. We can conclude that the Cat bond is very important financial instrument for these disasters.

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