Linear Theory of Tearing Instability
with Some Types of Viscosity Effects

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研究目的 (Research Objective):

In past years, the linear perturbation equations of tearing instability derived by Loureiro (Loureiro, et.al., Phys. Plasmas 2007 (Loureiro,PoP2007)) has been deeply explored by numerically solving as the initial value problem (Shimizu, AAPPS-DPP2018, KDK Research Report2018). The Loureiro's linear perturbation equations are based on non-viscous case, but in this paper, two types of viscosity effect are introduced to the equations (Shimizu, AAPPS-DPP2021, KDK Research Report2020). Type 1 is a kind of non-uniform viscosity case and type 2 is uniform case, which respectively have different equilibriums. In addition, the WKB approximation introduced by Loureiro is extensively improved (Shimizu, SGEPSS2021). It is important that the improvement is established by the introduction of viscosity effect. In addition, the 4th order differential magnetic diffusion is applied to the theory (Shimizu & Fujimoto, AOGS2021), where usual resistivity is the 2nd order magnetic diffusion.

Introduction of Viscosity：

$$N\Phi'''' = ke^2\{(\lambda + 2Nk)\Phi'' - (\lambda + Nk)k^2e^2\Phi + \lambda k f(\xi)\Psi - kf(\xi)^2\Phi - f''(\xi)\Psi\}, \quad (1)$$

$$\Psi'' = k^2e^2\Psi + \lambda k\Psi - kf(\xi)\,\Phi, \quad (2)$$

$$f(\xi) = \xi_0 e^{-\xi^2/2}\int_0^{\xi} dz\, e^{z^2/2} \quad (3)$$

Eq.(1) is the perturbation equation of momentum for the tearing instability, in which viscosity effect is introduced with a constant viscosity coefficient N(=ν). Excepting the terms of N, eq.(1) is exactly the same as that of Loureiro's perturbation equations (Loureiro,PoP2007). Also, eqs.(2), (3) and every notation are exactly the same as those of the Loureiro's perturbation equations. Loureiro applied eq.(3) only to the inner region of the current sheet, i.e. ξ<1.307, and assumed f(ξ)=1.0 in the outer region, i.e. ξ>1.307. However, when viscosity effect is introduced in the Loureiro's equations, the change from eq.(3) to f(ξ)=1 breaks down the equilibrium of the plasma flow field. To correctly study the perturbation theory, the equilibrium must be rigorously satisfied. To rigorously keep the equilibrium, we take two strategies shown below, which have been proposed by last year (Shimizu, AAPPS-DPP2021, and Shimizu, KDK

Research Report2020). The latest definition of viscosity coefficient N(= ν) shown in eq.(1) has been slightly modified from that of AAPPS-DPP2021 and KDK Research Report 2020, but there is no essential difference of the numerical results originated from the slight modification.

Strategy 1 (Viscosity effect only in inner region):

In this case, eqs.(1)-(3) are solved with non-zero constant N value in the inner region, i.e. 0< ξ <1.307. Meanwhile, eq.(1) with N=0 and eq.(2) are solved in the outer region, i.e. ξ >1.307, where f(ξ)=1 is constantly set instead of eq.(3). Hence, this setup of f(ξ) is exactly the same as that of the original Loureiro theory, where the outer region becomes a uniform magnetic field region. However, to rigorously satisfy the equilibrium of plasma flow field, N(= ν)=0 is assumed in the outer region. Hence, this is a kind of non-uniform viscosity case.

How to solve the inner region as initial value problem is as follows. Firstly, when a set of λ , k, e(= ε), N(= ν), Φ(0), Φ'(0), Φ''(0), Φ'''(0), Ψ(0), and Ψ'(0) is given, we can uniquely obtain a perturbation solution of Φ(ξ) and Ψ(ξ). Finally, we must find any physically acceptable solutions. Let us assume the physical symmetry condition at the origin, i.e. Φ(0)=Φ''(0)=Ψ'(0)=0. This assumes that the tearing instability is symmetry for the current sheet plane. In addition, since eqs.(1) and (2) are homogeneous equations, Ψ(0)=1 can be set without the lack of generality of solutions. Then, k, e and N are uniquely set for a specified physical state of tearing instability and current sheet. The remaining λ , Φ'(0) and Φ'''(0) are adjustable control parameters to obtain a physically acceptable solution. Hence, as the initial value problem of eqs.(1)-(3), Φ and Ψ are numerically solved from ξ =0 until ξ =1.307. Then, Φ and Ψ in the outer region must be solved, smoothly connecting from Φ and Ψ obtained for the inner region. For the smooth connection, it is noted that eq.(1) must be simultaneously satisfied in both cases of non-zero N and N=0 at ξ =1.307. In other words, the next equation must be satisfied at ξ =1.307.

$$\Phi'''' - k^2e^2(2\Phi'' - k^2e^2\Phi) = 0 \tag{4}$$

This equation consists only of the N terms in eq.(1). Subsequently, Φ and Ψ beyond ξ=1.307 are numerically solved for eq.(1) of N=0, eq.(2), and f(ξ)=1. Then, we can obtain an unique zero-crossing solution as a physically acceptable solution, which satisfies Φ=Ψ=0 at a location defined as ξ c(>1.307). How to find such zero-crossing solutions is the same as that of non-viscous case (Shimizu, AAPPS-DPP2018). Finally, we can know how much is the linear growth rate λ of the zero-crossing solutions (Shimizu, AAPPS-DPP2021 and Fig.1(a) in KDK Research Report2020). In Figs.1(a) and (b), "Non-uniform-N" curves respectively show the k-dependence and N-dependence of the linear growth rate λ . As expected, Fig.1(b) shows that λ decreases for larger N. At this point, the slight increase of λ for large N(= ν) observed in Fig.2(a) of KDK Research Report2020, p.83, may be originated by numerical errors. In addition, λ in Fig.1(b) is plotted at N=0.0 and 0.004, which are extreme cases. The former is exactly obtained in the non-viscous (N=0) case. The latter almost coincides with what is exactly obtained for the large N limit of eq.(1). The latter suggests that, even in extreme large N, λ for k=1 cannot reach zero. Such a suggestion is unexpected, but it may be remembered that this strategy is a kind of non-uniform viscosity case. Rather, we may have to examine the uniform viscosity case, i.e., Strategy 2 shown below.

Strategy 2 (Uniform viscosity effect in all regions):

Unlike Strategy 1, eq.(3) is not switched to f(ξ)=1 at ξ =1.307. Hence, the magnetic field intensity of the equilibrium, i.e, f(ξ), is monotonically weakened to zero for large ξ . In this case, eqs.(1)-(3) are seamlessly solved beyond ξ=1.307 from the origin ξ=0 until ξ=ξc. This strategy is much simpler than Strategy 1, where the uniform viscosity is assumed not only in the inner region and also in the outer region with a constant N. The equilibrium of plasma flow field is rigorously satisfied. However, since the equilibrium inflow speed toward the current sheet is infinite at infinite ξ , this strategy may be non-realistic as ξ c is large. In order to uniquely find a physically acceptable solution, let us additionally require Φ'(ξ c)=0 at the crossing point. Let us call it zero-contact solution (e.g., Fig.1(b) in KDK Research Report2020).

Finally, we can know how much is the linear growth rate λ of the zero-contact solutions. In Figs.1(a) and (b), "Uniform-N" curves respectively show the k-dependence and N-dependence of the linear growth rate λ. In Fig.1(b), λ plotted for N=0 is exactly obtained in the non-viscous (N=0) case. Meanwhile, λ reaches zero beyond N=0.003, which is exactly obtained for the λ =0 limit of eqs.(1)-(2). It means that, in contrast to Strategy 1, Strategy2 has a critical N value for the tearing instability, beyond which it is stabilized.

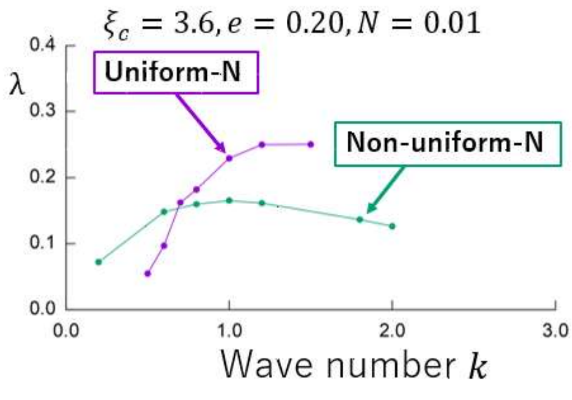

Fig.1(a): Growth rate λ vs Wave number k

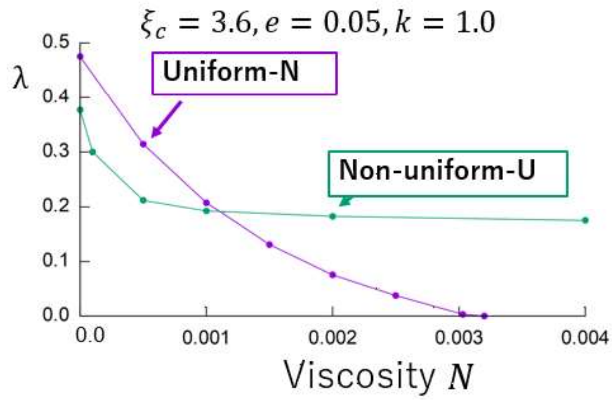

Fig.1(b): Growth rate λ vs Viscosity N

Improvement of WKB approximation (for Strategy 2):

Exactly, we may have to say that the original Loureiro's perturbation equation is applicable only for k>1 because the lowest level of WKB approximation is applied there (i.e., see the translation from eqs.(6) and (7) to eqs.(8) and (9) in Loureiro, PoP2007). Hence, eqs.(1) and (2) based on eqs.(8) and (9) in his paper may be also inapplicable for k<1. Meanwhile, eqs.(6) and (7) in his paper will be applicable for k<1, but unfortunately cannot be solved at the origin, i.e. ξ =0, until viscosity is introduced. In other words, eqs.(1) and (2) shown above can be extended to the higher level of WKB approximation. The resulting perturbation equations to be solved are as follows.

$$N\Phi'''' = ke^2\{(\lambda + 2Nk)\Phi'' - (\lambda + Nk)k^2e^2\Phi - (\xi/k)\Phi''' + ke^2\ (2\Phi + \xi\,\Phi') + \lambda k f(\xi)\Psi - kf(\xi)^2\Phi\ - f(\xi)\xi\Psi'\ - f''(\xi)\Psi\}, \tag{5}$$

$$\Psi'' = k^2e^2\Psi + \lambda k\Psi - \xi\Psi' - kf(\xi)\ \Phi \tag{6}$$

It may be noted that these eqs.(5) and (6) are obtained by the combination of eqs.(6) and (7) in Loureiro,PoP2007 with eqs.(1) and (2) shown above. Basically, eqs.(5) and (6) can be solved in the same manner as Strategy 2, but unfortunately, any zero-contact solution of (5) and (6) is not found at present. Alternately, the behaviors of the zero-crossing solutions for the variations of $\phi'(\xi c)$ value are being explored (Shimizu, SGEPSS2021).

Introduction of Hyper-viscosity effect:

In general MHD studies, the resistivity is the 2nd order differential magnetic diffusion. In many numerical MHD studies, the resistivity is assumed to be uniform in time and space but some types of non-uniform resistivity are also often studied, which may be called anomalous resistivity (e.g., Ugai, JPP1977). Anyway, unfortunately, MHD cannot answer about the identity of the resistivity, which must be studied in kinetic plasma physics. It suggests that the 2nd order differential magnetic diffusion, itself, must be reconsidered to efficiently promote the fast magnetic reconnection process. Recently, the 4th order differential magnetic diffusion is predicted in some kinetic plasma particle simulations (Fujimoto, Astrophys.J.Lett.2021), which may be called “hyper viscosity”. When the 4th order diffusion is introduced in MHD simulations of tearing instability, the reconnection rate tends to be higher in contrast to the case of usual resistivity, i.e. the 2nd order diffusion (Shimizu & Fujimoto, AOGS2021).

Hence, it will be worth to extend the linear perturbation theory of the tearing instability to the 4th order diffusion. To do so, firstly, the equilibrium must be found because eq.(3) is inapplicable in that case. Alternately, a new equilibrium has been found by numerically solving an initial value problem (Shimizu & Fujimoto, AOGS2021). Based on the new equilibrium, the perturbation equations are being numerically studied as another initial value problem, at present.

公表状況（Publications and Presentations）：

1. Tohru Shimizu., A new viewpoint for linear theory of tearing instability, 2nd Asia-Pacific Conf. on Plasma Phys (AAPPS-DPP2018). Kanazawa, Japan, SPG-04, 2018.
2. 清水徹、テアリング不安定性の磁気流体線形理論における粘性効果、SGEPSS2021, 2021,Nov. (Remote, Japanese domestic meeting).
3. Tohru Shimizu, Linear Theory of Viscous-Resistive Tearing Instability, Invited Presentation of AAPPS-DPP2021, SG-I40, Fukuoka (Remote, 2021, July)
4. Tohru Shimizu and K. Fujimoto, Higher-Order Differential Magnetic Diffusion Effect in MHD Simulations of Petschek Reconnection Model, Proc. of AOGS2021, Singapore (Remote, Aug.,1-6)