Long-Run Consequences of Population Decline in an Economy with Exhaustible Natural Resources

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Abstract

This study explores how population decline affects the long-run performance of an economy in which exhaustible natural resources are indispensable in the process production . Using a one-sector neoclassical growth model with external increasing returns, we inspect the conditions under which the per capita income and consumption persistently expand in the long-run equilibrium. We find that it is population decline, rather than exhaustible resources, that might terminate persistent growth in per capita income and consumption.

Keywords:exhaustible natural resources, population decline, long-run growth, external increasing returns

JEL Classification: O13; O44; Q32; Q43

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1 Introduction

Exhaustible natural resources and population decline are two major obstacles to persistent economic growth. First, triggered by the Club of Rome's 1972 report, *The Limit to the Growth*, the effects of exhaustible natural resources on long-run economic growth have been widely discussed since the 1970s. The Club of Rome's gloomy prediction was critically evaluated by many economists. Stiglitz (1974a, 1974b) revealed that the presence of exhaustible natural resources does not necessarily terminate economic growth, since the prices of exhaustible natural resources continue to rise, which makes firms substitute capital and labor for exhaustible resources[1]. Therefore, although exhaustible natural resources are indispensable inputs in production, persistent income growth can still be sustained by technical progress. Second, population decline is a relatively new issue. The recent decline in fertility rates observed in many high-income countries rekindled research interest in the relationship between population scale and economic growth. In particular, a small number of authors such as Christiaans (2011), Jones (2020), Sasaki (2015), and Sasaki and Hoshida (2017) examined growth models in which the growth rate of the labor force population takes negative values. They found that negative population growth may yield outcomes that cannot be observed in the case of positive population growth.

In this study, we consider the two aforementioned issues in a single setting. We analyze a one-sector growth model in which exhaustible natural resources are necessary for production and the population growth rate may take negative values. We first characterize the steady-state growth equilibrium and then investigate the conditions under which the per capita income and consumption continue to expand in the long run. Our baseline analytical framework is close to Groth and Schou's (2002) growth model with exhaustible natural resources, where the production technology exhibits increasing returns to scale[2]. They showed that the per capita income and consumption grow in the long-run equilibrium without exogenous technical progress if the production technology exhibits a certain degree of increasing returns. Our study departs from Groth and Schou (2002) in two respects: first, we treat a decentralized, competitive economy with external increasing returns, while Groth and Schou

[1] See also Solow (1974), Dasgupta and Heal (1974), and Suzuki (1978).

[2] Groth and Schou (2002) generalized the optimal growth model with exhaustible natural resources explored by Stigliz (1974b), who assumed that the aggregate production function exhibits constant returns to scale.

(2002) studied an optimal growth model in which the central planner realizes efficient resource allocation. Second, Groth and Schou (2002) assumed that the growth rate of the labor force is non-negative, but we focus on the case where the population shrinks at a fixed rate[3]. Additionally, Sasaki and Mino (2021) investigated a semi-endogenous growth model with exhaustible resources and population decline. They assumed that the household saving rate is fixed over time. In contrast, this study explores a fully micro-founded model that explicitly formulates the optimizing behaviors of firms and households.

Specifically, our model considers final-goods-producing firms, resource-extracting firms, and households. The final goods firms employ capital, labor, and exhaustible natural resources. The private technology of each firm satisfies constant returns to scale, whereas the social technology that involves external effects associated with the aggregate capital exhibits increasing returns. The resource-extracting firms extract natural resources without paying any cost to maximize the present value of their revenues. The households select the optimal sequence of consumption to maximize a discounted sum of utilities. For simplicity of exposition, we assume that the households own the firms; thus, the sales of natural resources are distributed to the households. Given these assumptions, we first define the perfect-foresight-competitive equilibrium in our economy We confirm that, given a set of plausible restrictions on magnitudes of the model parameters, the perfect-foresight competitive equilibrium path converges to a unique steady-state growth equilibrium. We then explore the conditions under which per capita income and consumption continue to increase in the steady-state growth equilibrium. We find that in our baseline setting, the per capita income and consumption cannot increase in the steady-state growth equilibrium, as long as the population growth rate is strictly negative; hence, population expansion is a necessary condition for the persistent growth of per capita income and consumption.

We then examine alternative situations in which both the per capita income and consumption may expand in the steady-state growth equilibrium under negative population growth. First, we investigate the case in which the degree of external effects is sufficiently high in our baseline model. In this case, we find that the per capita income and consumption can grow at a positive rate in the long-run equilibrium under negative population; however, the

[3]Althouth Groth and Shou (2002) mostly focused on the case of positive population growth, they also referred to the case of constant population.

degree of increasing returns should be implausibly high. We also examine the models that explicitly consider technical progress. We first reconfirm that in the neoclassical setting in which the aggregate production technology satisfies constant returns to scale and there is exogenous labor augmenting technical progress, the steady-state growth rate of per capita income and consumption can be positive under negative population growth, if the rate of technical progress exceeds a certain level. However, since the endogenous growth theory was developed to generalize the traditional neoclassical growth models with constant returns and exogenous technical progress, we also consider the prototype R&D-based endogenous growth model. In this case, we find that the technology of knowledge production should exhibit strong increasing returns, which is not consistent with existing empirical observations. Consequently, our study suggests that it is population decline, rather than exhaustible natural resources, that might terminate persistent economic growth.

The reminder of this paper is organized as follows. Section 2 sets up the baseline model. Section 3 analyzes the existence and stability of the steady-state growth equilibrium. Section 4 explores the conditions under which the per capita income and consumption increase in the steady-state growth equilibrium. Section 5 examines the role of technical progress that may sustain persistent expansion of the per capita income and consumption under negative population growth. Section 6 concludes.

2 Baseline Model

2.1 Setup

We consider a competitive economy in which there are final-goods-producing firms, resource-extracting firms, and households. The behavior of each agent is specified ass follows.

Final goods firms

There is a continuum of identical firms with a unit mass that produce homogeneous final goods. The production function of each firm is given by

$$Y_t = AK_t^{\alpha}\bar{K}_t^{\gamma}L_t^{\beta}\hat{R}_t^{1-\alpha-\beta}, \quad A > 0, \;\; \alpha > 0, \;\; \beta > 0, \;\; \alpha + \beta < 1, \;\; \gamma > 0, \tag{1}$$

where Y_t K_t, L_t, and $\hat{R}_t$ denote output, capital stock, labor, and input of an executable resource use by the firm, respectively. Since the mass of firms is normalized to unity, each variable denotes its aggregate value as well. Here, $\bar{K}_t^{\gamma}$ represents Romer's (1986) type of positive external effects associated with capital stock in the economy at large. If there are no external effects (i.e. $\gamma = 0$) and A grows at a positive, constant rate, then the production technology reduces to Stiglitz's (1974a, 1974b) original setting. The firm maximizes a discounted sum of net cash flow given by

$$\Pi_{Y,0} = \int_o^{\infty} \exp\left(-\int_0^t r_s ds\right)\left(AK_t^{\alpha}\bar{K}_t^{\gamma}\hat{L}_t^{\beta}\hat{R}_t^{1-\alpha-\beta} - w_t\hat{L}_t - I_t - p_t\hat{R}_t\right) dt$$

subject

$$\dot{K}_t = I_t - \delta K_t, \quad K_0 = \text{given } > 0.$$

Here, r_t is the real interest rate, w_t is the real wage, I_t is gross investment, and p_t denotes the price of resources in terms of the final good. The firm selects $\left\{I_t, L_t, \hat{R}_t\right\}_0^{\infty}$ under given sequences of prices, $\{r_t, w_t, p_t\}_{t=0}^{\infty}$, and the external effects, $\{\bar{K}_t\}_{t=0}^{\infty}$.

Owing to the assumption of the representative firm, in equilibrium it holds that $\bar{K}_t = K_t$ for all $t \geq 0$, implying that the social production function that internalizes the external effects can be written as

$$Y_t = AK_t^{\alpha+\gamma}\hat{L}_t^{\beta}\hat{R}_t^{1-\alpha-\beta}. \tag{2}$$

In what follows, we assume that the marginal product of capital in the social production function is diminishing:

Assumption 1 $\alpha + \gamma < 1$.

Resource extracting firms

Assuming a continuum of identical firms with a unit measure that extract an exhaustible resource without any cost, each firm maximizes a discounted sum of its revenue

$$\Pi_{S,0} = \int_0^{\infty} \exp\left(-\int_0^t r_s ds\right) p_t R_t dt$$

subject to

$$\dot{S}_t = -R_t, \text{ and } \int_0^\infty R_t dt \leq S_0 = \text{given } > 0, \tag{3}$$

where S_t is the stock of the exhaustible resource, and R_t denotes the level of resource extraction in each moment.

Households

We assume that households are identical and constitute a continuum whose mass is denoted by L_t, which changes at a rate of n so that $L_t = L_0 e^{nt}$. We set $L_0 = 1$. In this paper, the members of the dynastic family may die, meaning that n may take a negative value[4]. For simplicity of exposition, we assume that households directly own the final good firms and the resource extraction firms. We employ a dynastic family setting, so that the representative household maximizes a discounted sum of utilities for all members of the dynasty that is given by

$$U_0 = \int_0^\infty e^{-(\rho-n)t} \frac{c_t^{1-\sigma}}{1-\sigma} dt$$

subject to the flow budget constraint

$$\dot{b}_t = r_t b_t + w_t + p_t \frac{R_t}{L_t} - c_t - n b_t, \quad b_0 = \text{given}, \tag{4}$$

together with the no-Ponzi-game condition: $\lim_{t\to\infty} \exp\left(-\int_0^t r_s ds\right) b_t \geq 0$. Here, b_t is the per-capita stock of financial asset (IOU) and c_t is the per capita consumption.

Market equilibrium conditions

The market equilibrium conditions for final goods, labor, and resource markets are respectively given by

$$Y_t = I_t + c_t L_t, \quad \hat{L}_t = L_t, \quad \text{and } \hat{R}_t = R_t. \tag{5}$$

[4] As usual, we assume that the event of death follows a Poisson processes with a constant density μ. Thus the probability of survival at time t of an agent born at time 0 is

$$\pi_t = e^{-\mu t}$$

It is also assumed that the mas of newly born agents at time t is given by $dN_t = \beta N_t dt$, where β denotes the birth rate. Sincet $N_t = N_0 e^{\beta t}$, the labor force population at time t is given by

$$L_t = \pi_t N_t = e^{(\beta-\mu)t}$$

Then we denote $\beta - \mu = n$.

We assume that financial assets represent claims to capital stock, and, hence, the equilibrium condition for the bond market is

$$b_t L_t = K_t. \tag{6}$$

Definition of the competitive equilibrium

We define the perfect-foresight competitive equilibrium in our economy as follows:

Definition 1 *The perfect-foresight competitive equilibrium (PFCE) is established if the following conditions are met:*
(i) The final good firms maximize a discounted sum of cash flow under given sequences of $\{r_t, w_t, p_t, \bar{K}_t\}_{t=0}^{\infty}$.
(ii) The resource extracting firms maximize a sum of discounted revenues under given sequences of $\{r_t, p_t, R_t\}_{t=0}^{\infty}$.
(iii) The households maximize a discounted sum of utilities under given sequences of $\{r_t, w_t, p_t\}_{t=0}^{\infty}$
.
(iv) The markets for final goods, labor, resources, and bonds clear in each moment.
(v) The consistency condition, $\bar{K}_t = K_t$, *holds in each moment.*

2.2 Optimization Conditions

To characterize the PFCE of our economy, we first derive the optimization conditions for the firms and households. The Hamiltonian function for the final-goods firm's optimization problem is set as

$$H_t^f = D_t \left(A K_t^{\alpha} \bar{K}_t^{\gamma} \hat{L}_t^{\beta} \hat{R}_t^{1-\alpha-\beta} - w_t \hat{L}_t - I_t - p_t \hat{R}_t \right) + q_t^f \left(I_t - \delta K_t \right),$$

where $D_t = \exp\left(-\int_0^t r_s ds\right)$ and q_t^f denotes the implicit price of capital. The first-order conditions for an optimum include the following:

$$
\begin{aligned}
\max_{\hat{L}_t} H_t^f \quad &\Longrightarrow \quad \beta \frac{Y_t}{L_t} = w_t, &(7)\\
\max_{\hat{R}_t} H_t^f \quad &\Longrightarrow \quad (1-\alpha-\beta) \frac{Y_t}{\hat{R}_t} = p_t, &(8)\\
\max_{I_t} H_t^f \quad &\Longrightarrow \quad D_t = q_t^f, &(9)\\
\dot{q}_t^f \quad &= \quad -\frac{\partial H_t^f}{\partial K_t} = \delta q_t^f - D_t \alpha \frac{Y_t}{K_t}. &(10)
\end{aligned}
$$

together with the transversality condition: $\lim_{t\to\infty} q_t^f K_y = 0$.

Since (9) leads to $\dot{q}_t^f / q_t^f = \dot{D}_t / D_t = -r_t$, from (10) we obtain

$$\alpha \frac{Y_t}{K_t} = r_t + \delta. \tag{11}$$

Similarly, the optimization conditions for resource-extracting-firms can be obtained by setting the following Hamiltonian function:

$$H_t^e = D_t p_t R_t - q_t^e R_t,$$

where q_t^e is the implicit price of the stock of resource. The first-order conditions give:

$$
\begin{aligned}
\max_{R_t} H_t^e \quad &\Longrightarrow \quad D_t p_t = q_t^e, &(12)\\
\dot{q}_t^e \quad &= \quad -\frac{\partial H_t^e}{\partial S_t} = 0,, &(13)
\end{aligned}
$$

and the transversality condition: $\lim_{t\to\infty} q_t^e S_t = 0$. Thus, (12), (13) and $\dot{D}_t / D_t = -r_t$ yield the following relation (Hotteling's rule):

$$\frac{\dot{p}_t}{p_t} = r_t. \tag{14}$$

Finally, the Hamiltonian function for the household's optimization problem is given by

$$H_t^h = e^{-(\rho-n)t} \frac{c_t^{1-\sigma}}{1-\sigma} + q_t^h \left(r_t b_t + w_t + \frac{p_t R_t}{L_t} - c_t - n b_t \right),$$

where q_t^h is the utility price of bond. The first-order conditions are:

$$\max_{c_t} H_t^h \implies e^{-(\rho-n)t} c_t^{-\sigma} = q_t^h, \tag{15}$$

$$\dot{q}_t^h = -\frac{\partial H_t^h}{\partial b_t} = q_t^h (n - r_t). \tag{16}$$

These conditions lead to the Euler equation:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_t - \rho), \tag{17}$$

together with the transversality condition: $\lim_{t\to\infty} q_t^h b_t = 0$.

3 Model Analysis

3.1 Definition of the Steady-State Growth

Before analyzing our model, we define the steady-state growth of our economy.

Definition 2 *The steady-state growth of our economy is established, if each of Y_t, K_t, L_t, R_t, C_t, I_t, and S_t changes at a constant rate over time, and each variable takes a strictly positive value for all $t \geq 0$.*

Denote the steady rate of change in X_t by g_X, where $X_t = Y_t, K_t, L_t, R_t, C_t, I_t$, and S_t. Then, the market equilibrium condition for the final goods, $Y_t = C_t + I_t$, gives, in the steady-sate growth,

$$g_Y = \frac{I_t}{Y_t} g_I + \frac{C_t}{Y_t} g_C.$$

Additionally, the capital accumulation equation, $\dot{K}_t = I_t - \delta K_t$, gives

$$g_K = \frac{I_t}{K_t} - \delta. \tag{18}$$

Because we restrict our attention to the case where Y_t, K_t, I_t and C_t are strictly positive, the above two equations mean that $g_Y = g_K = g_I = g_C$ Otherwise, at least one of I_t/K_t, I_t/Y_t and C_t/Y_t continues to change, so that g_Y and/or g_K cannot stay constant over time[5]. Similarly, (3) shows that $g_s = g_R$. Consequently, to find a meaningful long-run equilibrium of our economy, we should derive a complete dynamic system whose stationary solution hods that $g_Y = g_K = g_I = g_C$ as well as $g_S = g_R$.

3.2 Dynamic System

Denoting the aggregate consumption as $C_t = c_t L_t$ and combining (11) , (17) and $L_t = e^{nt}$, we obtain the Euler equation of the aggregate consumption:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma}\left(\alpha\frac{Y_t}{K_t} - -\delta - \rho\right) + n. \tag{19}$$

The equilibrium condition of the final good market given in (5) yields

$$\frac{\dot{K}_t}{K_t} = \frac{Y_t}{K_t} - \frac{C_t}{K_t} - \delta. \tag{20}$$

From (8) , (14) , and $\hat{R}_t = R_t$ given in (5) , it holds that

$$\frac{\dot{R}_t}{R_t} = \frac{\dot{Y}_t}{Y_t} - r_t. \tag{21}$$

In view of (2) , (11) , and (21), we obtain

$$\frac{\dot{Y}_t}{Y_t} = (\alpha+\gamma)\frac{\dot{K}_t}{K_t} + \beta n + (1-\alpha-\beta)\left.\quad \frac{\dot{Y}_t}{Y_t} - \alpha\frac{Y_t}{K_t} + \delta\right).$$

Hence, from (20) the rate of change in the final goods is expressed as

$$\frac{\dot{Y}_t}{Y_t} = \frac{\alpha+\gamma}{\alpha+\beta}\left(\frac{Y_t}{K_t} - \frac{C_t}{K_t} - \delta\right) + \frac{\beta}{\alpha+\beta}n + \left(\frac{1-\alpha-\beta}{\alpha+\beta}\right)\left(\delta - \alpha\frac{Y_t}{K_t}\right). \tag{22}$$

Letting $x_t = Y_t/K_t$ and $z_t = C_t/K_t$, the dynamic behaviors of x_t and z_t are respectively given by

$$\frac{\dot{x}_t}{x_t} = \left[\frac{\gamma-\beta-\alpha(1-\alpha-\beta)}{\alpha+\beta}\right]x_t + \left(\frac{\gamma-\beta}{\alpha+\beta}\right)z_t + \frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta}, \tag{23}$$

[5] The above discussion follows Shlict (2006) and Jones and Scrimgeour (2008) who presented alternative, simple proofs of Uzawa's (1961) theorem that gives the necessary and sufficient conditions for the existence of the steady-state growth in a one-sector neoclassical growth model with technical change.

$$\frac{\dot{z}_t}{z_t} = \left(\frac{\alpha}{\sigma} - 1\right) x_t + z_t + n + \left(1 - \frac{1}{\sigma}\right) \delta - \frac{\rho}{\sigma}. \tag{24}$$

Additionally, denoting $R_t/S_t = v_t$, from (3) and (21) , the dynamic equation of v_t is expressed as

$$\frac{\dot{v}_t}{v_t} = \frac{\gamma}{\alpha+\beta} x_t - \frac{\alpha+\gamma}{\alpha+\beta} z_t + v_t + \frac{\beta n - (1-\alpha-\gamma)\,\delta}{\alpha+\beta}. \tag{25}$$

Differential equations (23) , (24) and (25) constitute a complete dynamic system. Evidently, when all of x_t, z_t and v_t are constant over time, it holds that $g_Y = g_K = g_I = g_C$ and $g_S = g_R$. Hence, the stationary solution of the dynamics system derived above satisfies our definition of the steady-state growth.

3.3 Existence and Stability of the Steady-State Growth Path

In our baseline model, we impose the following conditions:

Assumption 2 $\beta > \gamma$ and $\sigma > \alpha$.

The first assumption states that the external effects associated with the aggregate capital do not exceed the labor share of income. Since in reality the value of β is around 2/3, this is a natural assumption. The second assumption is also plausible because the conventional value of the intertemporal elasticity of consumption ($= 1/\sigma$) is less than one so that σ is higher than 1.0.

Although the presence of externalities makes our model differ from the optimal growth model examined by Groth and Schu (2002), the complete dynamic system of our model has the same structure as theirs. Letting the steady-state values of x_t z_t and v_t be x, z and v,respectively, we see that x, z and v fulfill the following conditions:

$$\left[\frac{\gamma - \beta - \alpha\,(1-\alpha-\beta)}{\alpha+\beta}\right] x - \left(\frac{\gamma-\beta}{\alpha+\beta}\right) z + \frac{\beta n + (1-\alpha-\gamma)\,\delta}{\alpha+\beta} = 0, \tag{26}$$

$$\left(\frac{\alpha}{\sigma} - 1\right) x + z + n + \left(1 - \frac{1}{\sigma}\right) \delta - \frac{\rho}{\sigma} = 0, \tag{27}$$

$$\frac{\gamma}{\alpha+\beta} x - \frac{\alpha+\gamma}{\alpha+\beta} z + v + \frac{\beta n + (1-\alpha-\gamma)\,\delta}{\alpha+\beta} = 0. \tag{28}$$

Since (23) and (24) constitute a complete dynamic system with respect to x_t and z_t, we first focus on this subsystem. The $\acute{x}_t = 0$ and $\dot{z}_t = 0$ loci in (23) and (24) are depicted by Figure

1. The $\dot{x}_t = 0$ and $\dot{z}_t = 0$ loci are respectively given by

$$\begin{aligned}\dot{x}_t &= 0 \text{ locus: } z_t = \left[1 + \frac{\alpha\,(1-\alpha-\beta)}{\beta-\gamma}\right] x_t + \frac{\beta n + (1-\alpha-\gamma)\,\delta}{\gamma-\beta}, \\ \dot{z}_t &= 0 \text{ locus: } z_t = \left(1 - \frac{\alpha}{\sigma}\right) x_t + \frac{\rho}{\sigma} - n - \left(1 - \frac{1}{\sigma}\right)\delta.\end{aligned}$$

Under Assumptions 1 and 2, both loci are positively sloped, and the $\dot{x}_t = 0$ locus is steeper than the $\dot{z}_t = 0$ locus. If we allow $n < 0$, the sings of their intercepts on the vertical axis are not predetermined. Figure 1 assumes the intercept of the $\dot{z}_t$ locus to be positive, while that of the $\dot{x}_t = 0$ locus to be negative. As the figure indicates, regardless of the signs of the intercepts, the necessary condition to hold $x > 0$ and $z > 0$ is

$$\frac{\rho}{\sigma} - n - \left(1 - \frac{1}{\sigma}\right)\delta > \frac{\beta n + (1-\alpha-\gamma)\,\delta}{\gamma-\beta}, \tag{29}$$

under which the intercept of the $\dot{z}_t = 0$ locus is higher that of the $\dot{x}_t = 0$ locus, so that there may exist an interior steady state in which there exist positive x and z that fulfill (26) and (27) . Then, given x and z, (28) determines v, which is strictly positive under appropriate restrictions on the parameter values[6].

As to the stability of the subsystem, the coefficient matrix of the linearized system of (23) and (24) evaluated at the steady state is given by

$$J_1 = \begin{bmatrix} x & 0 \\ 0 & z \end{bmatrix} \begin{bmatrix} \frac{\gamma-\beta-\alpha(1-\alpha-\beta)}{\alpha+\beta} & -\frac{\gamma-\beta}{\alpha+\beta} \\ \frac{\alpha}{\sigma} - 1 & 1 \end{bmatrix},$$

which leads to

$$\det J_1 = \frac{xz}{\alpha+\beta}\left[(\gamma-\beta)\frac{\alpha}{\sigma} - \alpha\,(1-\alpha-\beta)\right]. \tag{30}$$

Because $0 < \alpha + \beta < 1$, under Assumption 2 the determinant of J is strictly negative, so that the steady-state solution of (23) and (24) exhibits a regular saddle point property. As Figure 1 shows that the stable saddle paths are positively sloped, and it is less steep than the $\dot{z}_t = 0$

[6] In this studyr, we restrict our attention to the interior steady state. We can confirm that depending on the parameter values, we may have a boundary steady state in which $x = 0$ and $z > 0$. Sasaki and Mino (2021) showed that the boundary steady state also satisfies stability.

locus at least around the steady state.

[Figure 1]

Let us denote the saddle paths in Figure1 as

$$z_t = \zeta(x_t), \quad \zeta'(x_t) > 0, \qquad \lim_{x_t \to x} \zeta(x_t) = z. \tag{31}$$

Then, using (24), (25) and (31), we express a complete dynamic system as follows:

$$\frac{\dot{x}_t}{x_t} = \left[\frac{\gamma - \beta - \alpha(1-\alpha-\beta)}{\alpha+\beta}\right] x_t - \left(\frac{\gamma-\beta}{\alpha+\beta}\right)\zeta(x_t) + \frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta}, \tag{32}$$

$$\frac{\dot{v}_t}{v_t} = \frac{\gamma}{\alpha+\beta} x_t - \frac{\alpha+\gamma}{\alpha+\beta}\zeta(x_t) + v_t + \frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta}. \tag{33}$$

Hence, the $\dot{x}_t = 0$ and $\dot{v}v_t = 0$ loci in the (x_t, v_t) space are respectively given by

$$\begin{aligned} \dot{x}_t &= 0 \text{ locus: } \left[\frac{\gamma - \beta - \alpha(1-\alpha-\beta)}{\alpha+\beta}\right] x_t - \left(\frac{\gamma-\beta}{\alpha+\beta}\right)\zeta(x_t) + \frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta} = 0, \\ \dot{v}_t &= 0 \text{ locus: } v_t = -\frac{\alpha+\gamma}{\alpha+\beta}\zeta(x_t) - \frac{\gamma}{\alpha+\beta}x_t - \frac{\beta n + (1-\alpha-\gamma)\delta}{\alpha+\beta}. \end{aligned}$$

The above shows that $\dot{x}_t = 0$ locus is the vertical, while the sign of the slope of $\dot{v}_t = 0$ locus is not predetermined under our restrictions of parameter values. The coefficient matrix of (32) and (33) linearized at the stationary point is

$$J_2 = \begin{bmatrix} x & 0 \\ 0 & v \end{bmatrix} \begin{bmatrix} \frac{\gamma-\beta-\alpha(1-\alpha-\beta)}{\alpha+\beta} - \left(\frac{\gamma-\beta}{\alpha+\beta}\right)\zeta'(x), & 0 \\ \frac{1}{\alpha+\beta} - \frac{\alpha+\gamma}{\alpha+\beta}\zeta'(x_t), & 1 \end{bmatrix},$$

which leads to

$$\det J_2 = \frac{xv}{\alpha+\beta}\left\{-\alpha(1-\alpha-\beta) + (\gamma-\beta)\left[1-\zeta'(x)\right]\right\}. \tag{34}$$

Note that the saddle paths in Figure 1 is less steep than the $\dot{z}_t = 0$ locus with slope of $1 - \alpha/\sigma$ in $(x_t,\, v_t)$ space, which means that $0 < \zeta'(x_t) < 1$ at least around the steady state. Hence, (34) shows that $\det J_2 < 0$ under Assumption 2. The phase diagram of (32) and (33) is illustrated in Figure 2. In this figure, we assume that $\dot{v}_t = 0$ locus has a positive slope. We see that if the slope of $\dot{v}_t = 0$ locus is negative, the stable saddle paths are negatively sloped as well. In addition, if the slope of the $\dot{v}_t = 0$ locus is negative, the intercept of that locus on the vertical axis, which is given by $\frac{\alpha+\gamma}{\alpha+\beta}\zeta(0) - \frac{\beta n+(1-\alpha-\gamma)\delta}{\alpha+\beta}$ should be strictly positive. (In the case of Figure 2, $\frac{\alpha+\gamma}{\alpha+\beta}\zeta(0) - \frac{\beta n+(1-\alpha-\gamma)\delta}{\alpha+\beta}$ can be negative, but it must be higher than a certain level.)

[Figure 2]

In our dynamic system, K_t, S_t and L_t are predetermined endogenous variables, while c_t and R_t are jump variables. From the social production function (2) and the equilibrium condition, $\hat{R}_t = R_t$, it holds that

$$v_t = R_t/S_t = \frac{1}{S_{0t}}\left(x_t A K_t^{\alpha+\gamma-1} L_t^{\beta}\right)^{\frac{1}{1-\alpha-\beta}}. \tag{35}$$

This means that the initial levels of x_t and v_t must satisfy

$$v_0 = \frac{1}{S_0}\left(x_0 A K_0^{\alpha+\gamma} L_0^{\beta}\right)^{\frac{1}{1-\alpha-\beta}}$$

Figure 2 also shows the graph of (35) . In the figure, Point B is a unique initial position of the economy that can converge to the steady state: the initial levels of R_0 and c_0 are selected to make the initial position of the economy is Point B. Moreover, once x_0 and z_0 are selected, the initial levels of c_0 and R_0 (so Y_0) are respectively given by

$$R_0 = \left(x_0 A K_0^{\alpha+\gamma-1} L_0^{\beta}\right)^{\frac{1}{1-\alpha-\beta}},$$

$$c_0 = \xi(x_0)\frac{K_0}{L_0}.$$

Consequently, if there is a feasible, interior steady state, the PFCE of our economy is uniquely determined. To sum up, we have confirmed the following:

Proposition 1 *Provided that the dynamic system involves an interior steady state, then there exists a unique, stable PFCE path that converges to the steady-state growth equilibrium.*

4 Persistent Growth of Per Capita Income and Consumption

4.1 Growth Rates of Income and Consumption

From (2), the steady-state rate of change in Y_t, K_t, L_t and R_t satisfy

$$g_Y = (\alpha + \gamma)\, g_K + \beta n + (1 - \alpha - \beta)\, g_R.$$

Equation (21) shows that $g_R = g_Y - r$, where $r \left(= \alpha \frac{Y_t}{K_t} - \delta \right)$ is the steady state rate of net rate of return to capital. Thus, using the steady-state growth condition, $g_Y = g_K$, we find that

$$g_Y = \frac{\beta}{\beta - \gamma} n - \frac{1 - \alpha - \beta}{\beta - \gamma} r.$$

Hence, the steady-state growth rate of per capita output is given by

$$g_y = g_Y - n = \frac{\gamma}{\beta - \gamma} n - \frac{1 - \alpha - \beta}{\beta - \gamma} r. \tag{36}$$

Further, the steady-state rate of change in per capita consumption, c_t, is given by

$$g_c = \frac{1}{\sigma} r - \frac{\rho}{\sigma}. \tag{37}$$

The steady-state growth conditions include $g_Y = g_C$, so that $g_y = g_c$ should hold on the steady-state growth path. Figure 3 illustrates the graphs of (36) and (37). Panel (a) assumes that the population growth rate, n, is strictly positive and the balanced growth rate, $g^* = g_y = g_c$, is also strictly positive. As the figure shows, under our assumption of $\beta > \gamma$, the necessary and sufficient condition for positive growth of per capita income and consumption in the steady-state growth equilibrium is $\frac{\gamma n}{1-\alpha-\beta} > \rho$ or

$$\gamma > \frac{(1 - \alpha - \beta)}{n}. \tag{38}$$

However, if the population growth rate is strictly negative (or $n = 0$), the graphs of (36) and (37) are given as in Figure 3-(b). In this case, it is impossible to hold that both g_y and g_c have positive values.

[Figure 3]

Consequently, given Assumption 2, to hold positive growth of per capita income and consumption in the long-run equilibrium, the population growth rate, n, must be positive and the degree of external effect, γ, should be higher than a certain level shown by (38).

Proposition 2 *Under $\beta > \gamma$, the necessary and sufficient conditions for the presence of the steady-state growth equilibrium on which the balanced growth rate of per capita income and consumption is strictly positive are:*

$$n > 0 \text{ and } \gamma > \frac{(1-\alpha-\beta)}{n}. \tag{39}$$

4.2 The Case of Strong Increasing Returns

We have confirmed that given Assumption 2, it is impossible to hold that both the per capita income and consumption expand on the steady-growth path in the presence of exhaustible resources and negative population growth. In this subsection, we assume that the external effect of aggregate capital is large enough to hold $\gamma > \beta$. Figure 4 shows the graphs of (36) and (37) under $\gamma > \beta$,. All panels in this figure show the cases that hold $g^* = g_y = g_c > 0$. Panels (a) and (b) assume that $n > 0$. Based on these graphs, we find that if one of the following conditions are met

$$\frac{\beta n}{1-\alpha-\beta} > \rho \text{ and } \frac{1-\alpha-\beta}{\gamma-\beta} > \frac{1}{\sigma}, \tag{40}$$

$$\rho > \frac{\beta n}{1-\alpha-\beta} \text{ and } \frac{1}{\sigma} > \frac{1-\alpha-\beta}{\gamma-\beta}, \tag{41}$$

then it holds that $g^* = g_y = g_c > 0$. (Conditions (40) and (41) are assumed in panels (a) and (b), respectively.) Conversely, if $n < 0$, then as shown by panel (c), $g^* = g_y = g_c > 0$,holds under the following condition:

$$\frac{1}{\sigma} > \frac{1-\alpha-\beta}{\gamma-\beta}. \tag{42}$$

[Figure 4]

Note that if $\gamma - \beta$ is sufficiently small, the saddle-point conditions, $\det J_1 < 0$ and $\det J_2 < 0$ given in (30) and (34), may hold even if $\gamma > \beta$. The following proposition summarizes the findings in the case of strong increasing returns:

Proposition 3 *Suppose that* $\gamma > \beta$. (i) *If the population growth rate is positive, then the positive growth in the per capita income and consumption is realized in the steady-state growth equilibrium, if*

$$\frac{\beta n}{1-\alpha-\beta} > \rho \ \text{ and } \frac{1-\alpha-\beta}{\gamma-\beta} > \frac{1}{\sigma},$$

or if

$$\rho > \frac{\beta n}{1-\alpha-\beta} \text{ and } \frac{1}{\sigma} > \frac{1-\alpha-\beta}{\gamma-\beta}.$$

(ii) *If* $n \leq 0$, *then the per capita income and consumption continue to increase in the steady-state growth equilibrium, if and only if*

$$\frac{1}{\sigma} > \frac{1-\alpha-\beta}{\gamma-\beta}.$$

In reality, the income share of natural resources, $1-\alpha-\beta$, is relatively small, and it is usually set at around 0.05. Hence, if we set $\sigma = 1.5$, $\alpha = 0.345$, and $\beta = 0.65$, then (42) can be established if $\gamma > 0.725$. In this case, the degree of returns to scale of aggregate social production function (2) is higher than 1.725. This value is much higher than the degree of returns to scale suggested by previous empirical studies such as Basu and Fernald (1997). Therefore, although it is theoretically possible to establish positive growth in the per capita income and consumption in the long run under negative population growth, in our model economy, we should assume an empirically implausible degree of increasing returns.

5 The Role of Technical Progress

So far, we have not explicitly considered the presence of technical progress as a source of productivity growth of the aggregate production technology. Instead, following Romer (1986), we have assumed that productivity growth is generated by the spillover of knowledge embodied with the capital stock in the economy at large. In this section, we briefly discuss the role of technical progress in the model with exhaustible natural resource.

5.1 Exogenous Technical Progress

Let us re-express the aggregate production function as

$$Y_t = Q_t K_t^{\alpha} L_t^{\beta} R_t^{1-1\alpha-\beta}, \quad 0 < \alpha < 1, \quad 0 < \beta < 1, \quad \alpha + \beta < 1.$$

This expression means that under a given level of the total factor productivity (TFP) denoted as Q_t, the aggregate production technology exhibits constant returns to scale with respect to capital, labor and resource input, and that a rise in Q_t represents technical progress. In the baseline model, we set $Q_t = K_t^{\gamma}$, meaning that TFP growth stems from the external effects associated with the aggregate capital. Following the tradition of the neoclassical growth theory, Stiglitz (1974a, 1974b) assumed that the aggregate production function involves exogenous TFP growth, that is, $Q_t = e^{\lambda t}$, where $\lambda > 0$. In this case, the production is written as

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{\beta} R_t^{1-\alpha-\beta},$$

where $A_t = e^{at}$ and $a = \lambda/\beta$. Then we find that the steady-state growth rate of the aggregate income is given by

$$g_Y = a + n - \frac{1-\alpha-\beta}{\beta} r, \tag{43}$$

meaning that the steady-state growth rate of the per capita income is given by

$$g_y = a - \frac{1-\alpha-\beta}{\beta} r \tag{44}$$

The steady-state growth rate of per capita consumption is given by (37). By inspecting the graphs of (37) and (44), we confirm that as long as $a > 0$, the balanced growth rate, $g^* = g_y = g_c$, is strictly positive if and only if

$$\rho < \frac{\beta a}{1-\alpha-\beta}, \tag{45}$$

or $a > \frac{\rho(1-\alpha-\beta)}{\beta}$. Therefore, if we set $\sigma = 1.5$, $\rho = 0.02$, $\alpha = 0.345$ and $\beta = 0.65$, then if $a > 0.0016$, the per capita income and consumption can grow at a positive rate in the steady-state growth equilibrium. Notice that in the traditional neoclassical environment with constant returns and exogenous growth of labor efficiency, the conditions for positive growth of the per capita income and consumption are independent of the population growth rate. Moreover, under plausible parameter values, a very small growth in labor efficiency may sustain long-run expansion of the per capita income and consumption even in the presence of exhaustible natural resources. However, considering that the endogenous growth theory has been developed in response to the neoclassical growth models with exogenous technical progress, we should examine the case in which technical progress evolves endogenously.

5.2 Endogenous Technical Progress

If Q_t evolves endogenously through the purposeful activities of firms, we should consider R&D investments of firms. As an example, consider a simplified version of Romer's (1990) model. The production function of the final goods is

$$Y_t = K_t^{\alpha} (A_t \theta_t L_t)^{\beta} R_t^{1-\alpha-\beta}, \tag{46}$$

where $\theta_t \in [0, 1]$ denotes the rate of labor allocation to the final good production. The production function of knowledge is given by

$$\dot{A}_t = \chi \left(1 - \theta_t\right) L_t A_t^{\eta}, \quad \chi > 0 \text{ and } \eta > 0. \tag{47}$$

Equation (47) means that

$$g_{A,t} = \chi \left(1 - \theta_t\right) L_t A_t^{\eta - 1},$$

so that

$$\frac{\dot{g}_{A,t}}{g_{A,t}} = -\frac{\dot{\theta}_t}{1 - \theta} + n + (\eta - 1)\, g_{A,t}.$$

First, suppose that $\eta \neq 1$. In the steady-state growth equilibrium, both θ_t and $g_{A,,t}$ remain constant over time, it holds that

$$g_A = \frac{n}{1 - \eta}.$$

Hence, the steady-state growth rate of the aggregate income is given by

$$g_Y = \frac{n}{1-\eta} + n - \frac{1}{\beta}(1-\alpha-\beta)\, r, \tag{48}$$

and the steady growth rate of per capita income is

$$g_y = \frac{n}{1-\eta} - \frac{1}{\beta}(1-\alpha-\beta)\, r. \tag{49}$$

As a result, (37), (49) and $g_y = g_c$ determine the balanced growth rate of the per capita income and consumption.

If $0 < \eta < 1$, then the graphs of (49) and (37) are given as panel (a) in Figure 5, which shows the case in which $g^* = g_y = g_c > 0$. As figure reveals, it holds that $g^* > 0$, if the following condition is met:

$$\frac{\beta n}{(1-\eta)(1-\alpha-\beta)} > \rho. \tag{50}$$

Panel (b) assumes that $n < 0$. In this case, there is no steady-state growth path that satisfies $g^* = g_c = g_y > 0$. Therefore, the steady-state characterization in the case of semi-endogenous growth ($0 < \eta < 1$) is essentially the same as the case of $\beta > \gamma$ discussed in Section 4.2.

[Figure 5]

Next, assume that $\eta > 1$. In this case, if $n > 0$, the graphs of (37) and (49) are the same as panel (b) so that it is impossible to hold $g^* = g_y = g_c > 0$. If $\eta > 1$ and $n < 0$, then the graphs are given in panel (a). Hence, the necessary condition for the presence of a positive growth in per capita income and consumption under $n < 0$ is the same as (50) . The following proposition summarizes our finding:

Proposition 4 *In our R&D-based endogenous growth model, persistent growth of per capita income and consumption is possible, if* $0 < \eta < 1$ *and* $n > 0$ *or if* $\eta > 1$ *and* $n < 0$. *In both cases, the necessary and sufficient condition for* $g^* = g_y = g_c > 0$ *is*

$$\frac{\sigma n}{(1-\eta)(1-\alpha-\beta)} > \rho.$$

Finally, consider the hairline case in which $\eta = 1$. Given this condition, we have

$$g_{A,t} = \chi \left(1 - \theta_t\right) L_t.$$

Since $\theta_t \in [0, 1]$, the above expression means that when L_t continues to decease, $g_{A,t}$ ultimately converges to zero and technical progress is terminated. Consequently, A_t and θ_t stay constant in the steady-state growth equilibrium, and from (49) the steady-state growth rate of per capita income is given by

$$g_y = -\frac{1 - \alpha - \beta}{\beta} r. \tag{51}$$

Hence, from (37) and (51), we see that regardless of the sign of n, it is impossible to hold $g^* = g_y = g_c > 0$.

In summary, in the case of semi-endogenous growth setting $(0 < \eta < 1)$ as well as in the case of $\eta = 1$, the per capita income and consumption cannot expand in the steady-state growth equilibrium under negative population growth. Persistent expansion in the per capita income and consumption under negative population growth is attained only when $\eta > 1$ so that the stock of existing knowledge in the firms' R&D activities exhibits increasing returns[7]. However, as was seen, if $\eta > 1$ and $n > 0$, there is no steady-state growth path on which the per capita income continues to increase. This obviously contradicts Jones' (1995) finding that the labor share of the R&D sector in the US economy was tripled during 50 years after World War II, while both the per capita income and the labor force population persistently increased during that period.

6 Conclusion

After almost 50 years since the publication of the Club of Rome's report, it seems that the negative impact of exhaustible natural resources on economic growth has not been so serious as the report predicted. This is because energy-saving technologies and alternative sources of energy have been developed. In this sense, the theoretical investigations by Stiglitz (1974a,

[7] Agihon and Howitt (2009, Section 3 in Chapter 16) and Bretschger (2013) analyzed R&D-based endogenous growth models with exhaustible resources. Aghion and Howitt used a quality ladder model, while Bretschger (2013) employed a variety expansion model of growth. Both studies explored the conditions for persistent growth of income; however, they did not consider the effects of population decline. See also Cabo et al. (2016).

1974b) and others conducted in the 1970s did not miss the point. However, following the neoclassical tradition, these studies assumed constant returns to scale and exogenous technical progress, under which the balanced growth rate of per capita income and consumption is independent of the population growth rate. We have shown that once we depart from the traditional neoclassical setting, it is hard to sustain persistent growth of per capita income and consumption in an economy with exhaustible natural resources and population decline: we should assume an implausibly high degree of increasing returns either in final goods production or in R&D activities of firms. Consequently, our study suggests that it is population decline, rather than exhaustible natural resources, that might terminate persistent growth in per capita income and consumption.

Finally, we can state that a permanent decline in population is not a realistic assumption but a thought experiment. In reality, population changes endogenously. Therefore, as conducted by Jones (2020), we should examine an alternative model in which the population growth rate is a choice variable of the households[8]. Such a model allows us to discuss the linkage between exhaustible natural resources, population change, and long-run economic growth in a micro-founded, unified framework. Nevertheless, this topic needs further investigation by future studies[9].

[8] It is to be noted that Cigno, (1981) endogenized the population growth rate in Stiglitz's (1974a) model. The author used an ad-hoc model of population change and did not consider the case of negative population growth.

[9] Naso et al. (2020) empirically confirmed that the negative effect of population decline on economic growth would be much larger than that of exhaustible resources. See also Ashral et al. (2012).

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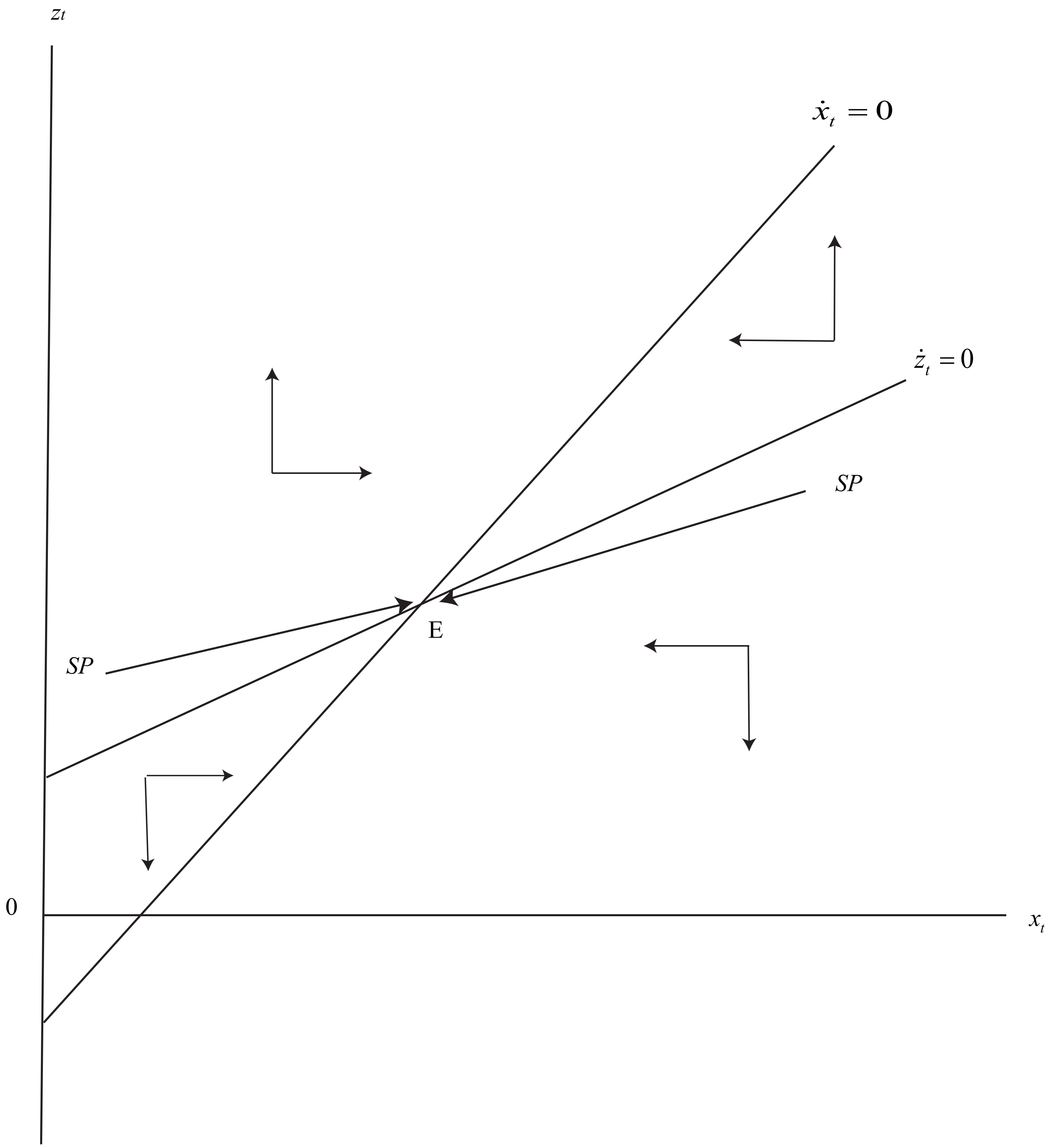


Figure 1: Phase diagram of (23) and (24)

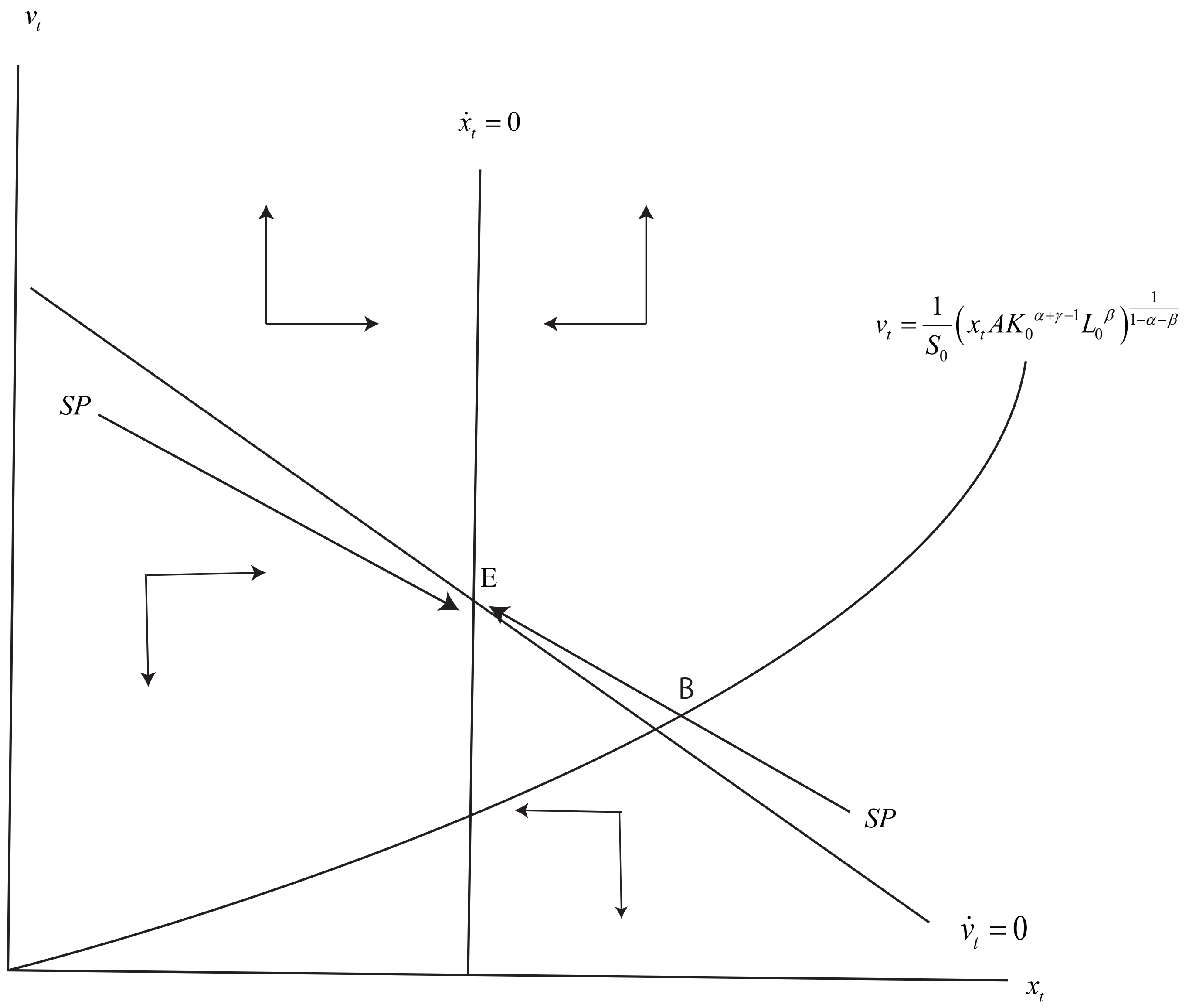


Figure 2: Phase diagram of (32) and (33)

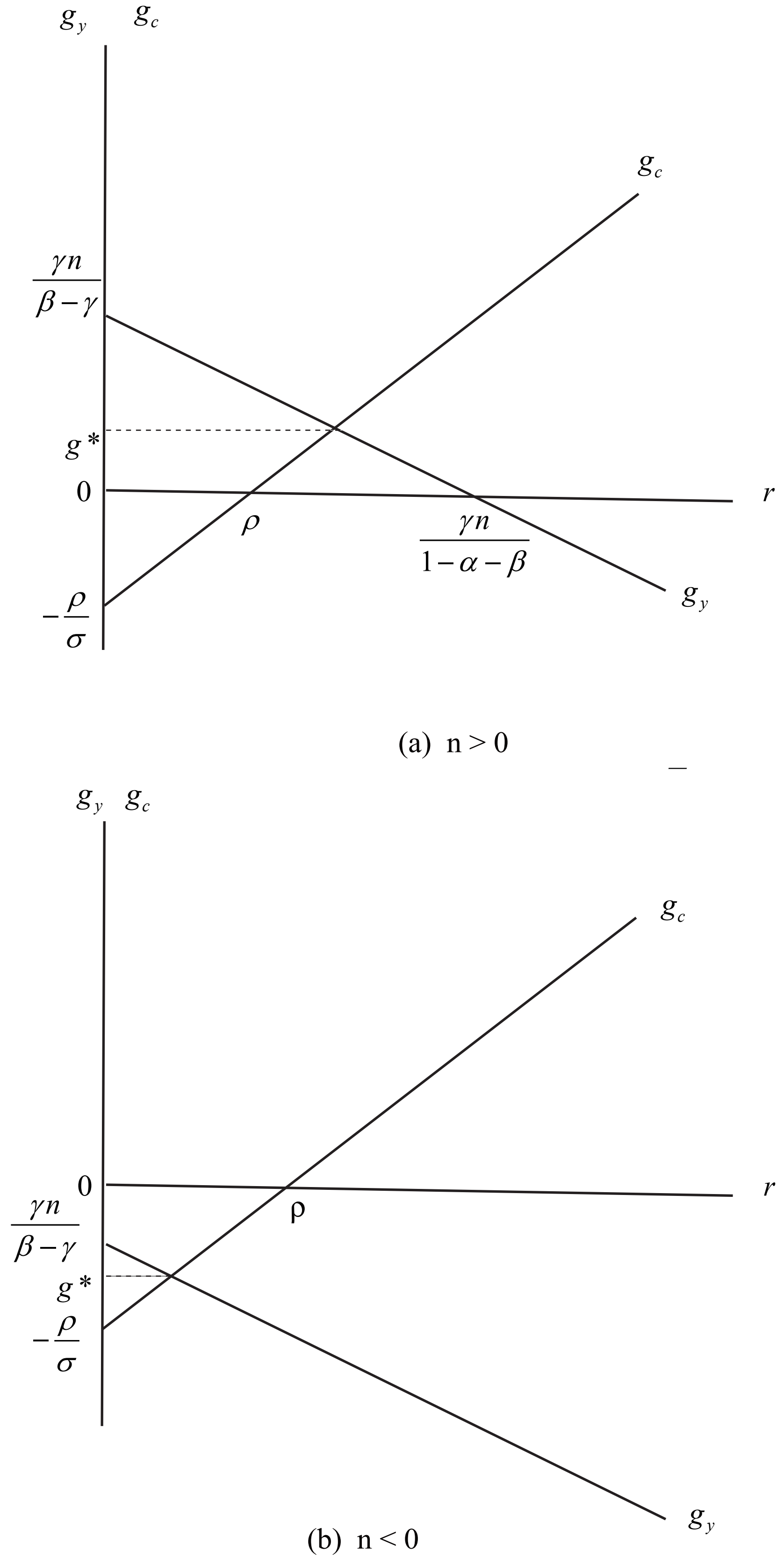


Figure 3: Determination of the balanced growth rate under $\beta > \gamma$

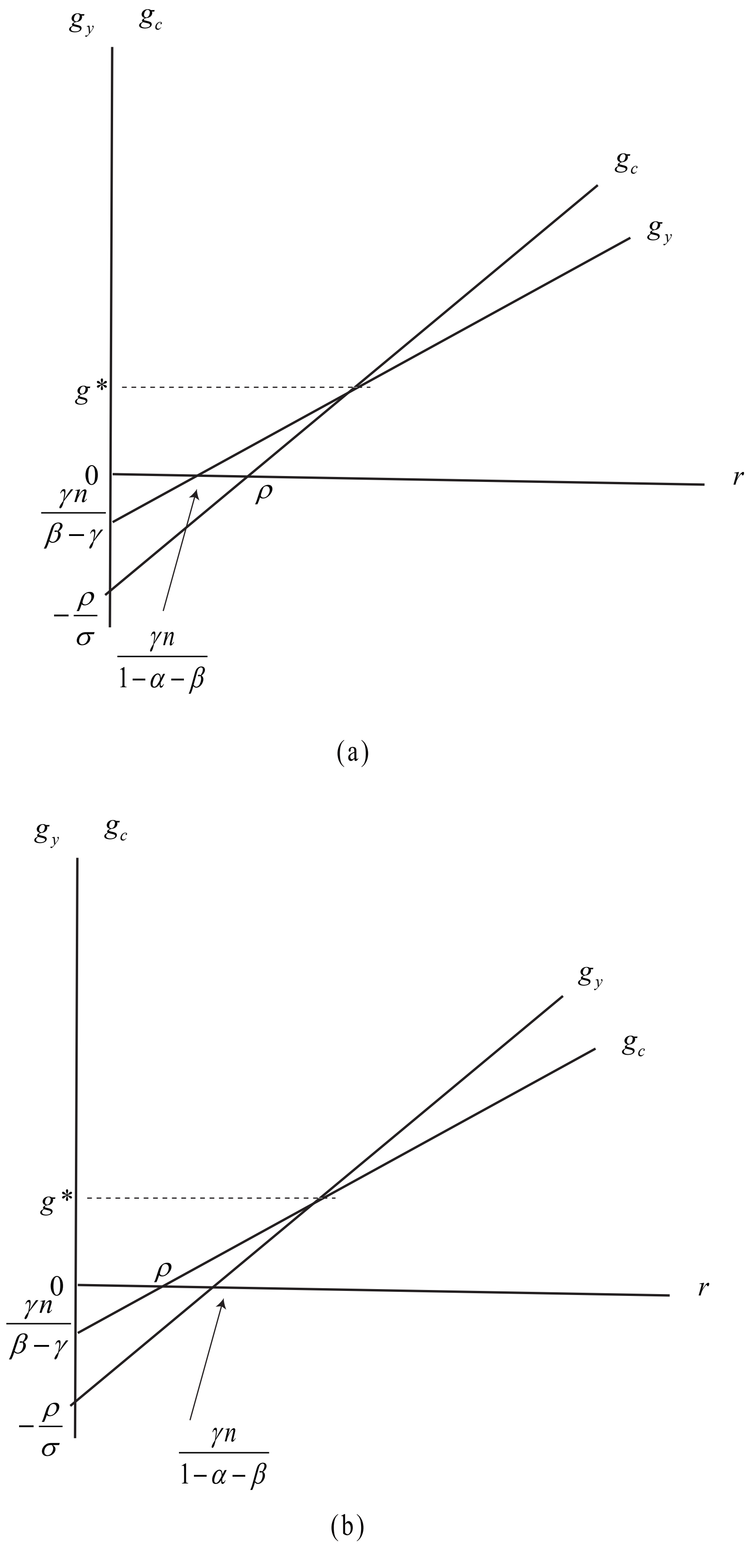
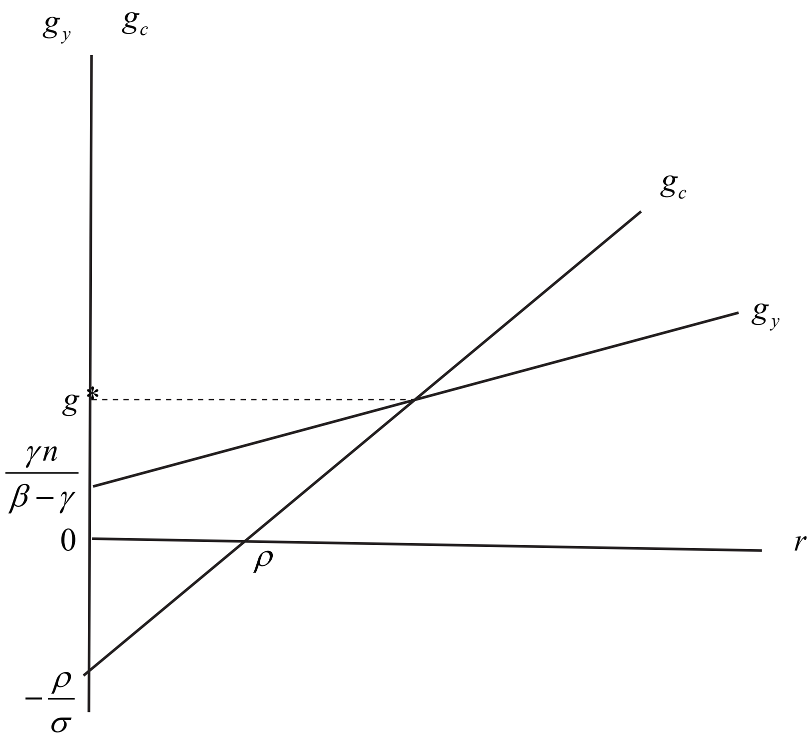
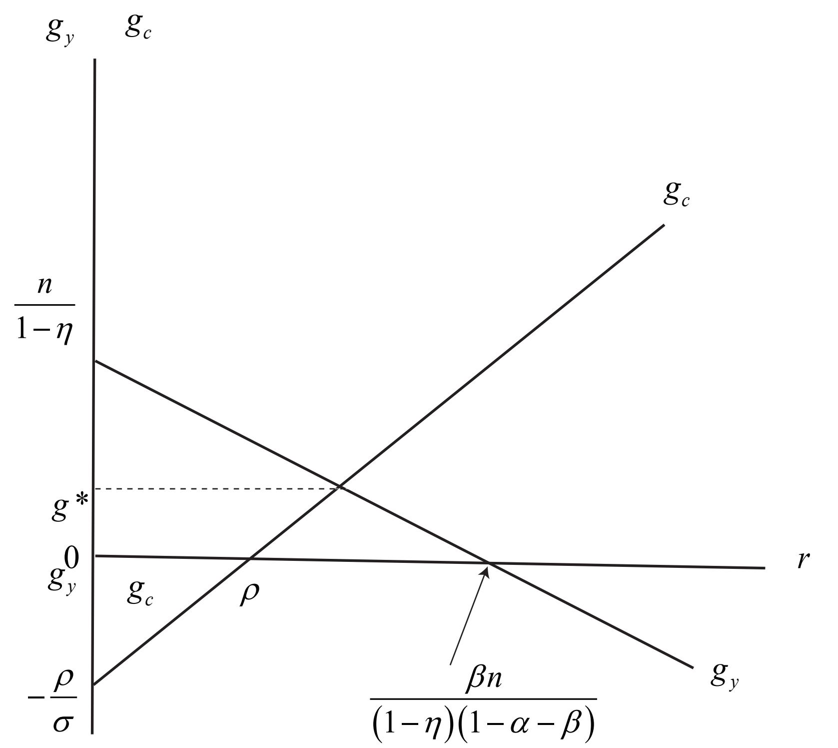


Figure 4: Determination of the balanced growth rate under $\gamma > \beta$ and $n > 0$

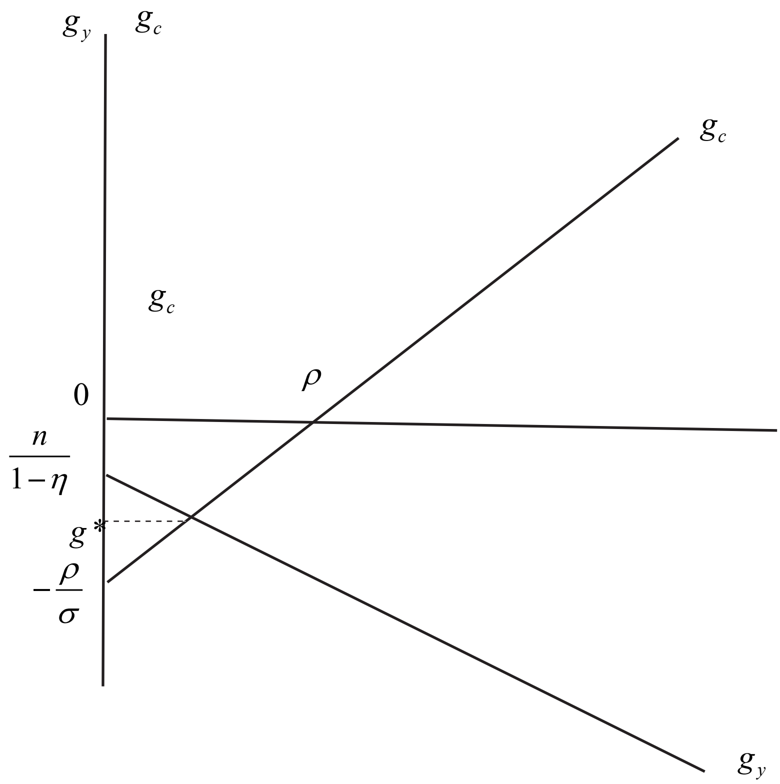


(c)

Figure 4 (continued): Determination of the balanced growth rate under $\gamma > \beta$ and $n < 0$



(a) n > 0 and 0 < η < 1; n < 0 and η > 1



(b) n < 0 and 0 < η < 1; n > 0 and η >1

Figure 5: Determination of the balanced growth rate in the model with R&D